

Graviton-to-photon conversion effect in magnetized relativistic plasma

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Abstract. The graviton-to-photon conversion effect in a magnetized relativistic lepton plasma is considered. This effect can be important for the possible generation of electromagnetic radiation accompanying coalescence of relativistic compact neutron star – black hole binaries. The relativistic electron-positron plasma can be generated near the surface of a rotating magnetized neutron star (a radio pulsar). The formation of a relativistic compact binary containing a pulsar and a black hole is predicted by the evolution of massive binary stars. Prior to the coalescence of such a binary due to gravitational wave (GW) emission, a fraction of the GW power can be converted in the plasma outflow into a low-frequency electromagnetic (EM) waves, which can lead to additional radio power prior to the coalescence. Using the graviton-to-photon conversion mechanism in an external magnetic field, we calculate the fraction of GW power converted into the EM radiation in the relativistic plasma. The result is found to depend on the neutron star spin period P , plasma Lorentz factor γ and the cascade multiplicity λ , but independent of the neutron star magnetic field: $K \simeq 10^{-35} (P/1 \text{ s})^2 (\gamma/10^5)^2 (\lambda/10^5)^{-2}$. The possibility of the detection of the non-thermal EM counterparts from neutron star – black hole coalescences in the forthcoming GW observations by aLIGO/Virgo detectors is briefly discussed.

1. Introduction

One of the mechanisms for the emergence of electromagnetic radiation during the coalescence of compact binary neutron stars or the absorption of a neutron star by a black hole can be the transformation of gravitational waves (GWs) into electromagnetic ones in an external magnetic field by converting gravitons into photons [1, 2, 3].

The plasma frequency of a thermal cosmic plasma is of the order of $\Omega_e = 60 \sqrt{n_e}$ kHz, where n_e is the electron number density. According to the present observational data of the international scientific collaboration LIGO, the frequency of the detected GWs is $\omega/2\pi \sim 100 - 200$ Hz [4], and the generated electromagnetic wave with the GW frequency cannot propagate in the plasma once the GW frequency ω is less than the plasma frequency Ω_e . Instead, as was shown in [1], gravitons will produce new photons over and over again, which will transfer energy to the plasma, which may eventually lead to a flash of electromagnetic radiation.

The conversion effect is important to estimate the possible electromagnetic counterparts of coalescing compact binaries detected by their GW emission. Electromagnetic counterparts enables more accurate localize GW source because the angular resolution of GW detectors is



in the best case of the order of several square degrees [5]. Multiwavelength follow-up of GW sources is a key goal of the modern multimessenger astronomy.

2. Graviton-to-photon conversion effect

Here we briefly remind the essence of the graviton-to-photon conversion effect in the external electromagnetic (EM) field [3]. Let a GW

$$h_{ij} = \sum_{\lambda=x,+} h_{\lambda}(x) e_{ij}^{\lambda} e^{-i(\omega t - \mathbf{k}x)} \quad (1)$$

propagates in a uniform external magnetic field. Here λ is the polarization index of the gravitational wave, and e_{ij} is the polarization tensor of the GW.

Varying the total action of the gravitational and electromagnetic fields [3] yields the following system of equations:

$$\begin{aligned} (\omega^2 - k^2)h_{\lambda}(\mathbf{k}) &= \kappa k A_{\lambda}(\mathbf{k}) B_T, \\ (\omega^2 - k^2 - m^2)A_{\lambda}(\mathbf{k}) &= \kappa k h_{\lambda}(\mathbf{k}) B_T. \end{aligned} \quad (2)$$

where $A_{\lambda}(\mathbf{k})$ is the EM vector potential, B_T is the component of the external magnetic field perpendicular to the propagation of gravitons, $h_{\lambda}(k)$ are canonically normalized field of GW perturbations of space-time such that the kinetic term in Lagrangian can be written as $(\partial_{\mu} h_j)^2$,

$$g_{\mu\nu} = \eta_{\mu\nu} + \tilde{h}_{\mu\nu}, \quad h_j = \tilde{h}_j / \kappa, \quad (3)$$

with $\kappa^2 = 16\pi/m_{Pl}^2$, $m_{Pl} \approx 2 \times 10^{19}$ GeV is the Planck mass and m is the effective mass of a photon in the medium. Here and below we will use the natural units $\hbar = G = c = k_B = 1$. The electron charge is $e^2 = 4\pi\alpha$, $\alpha \approx 1/137$ is the fine structure constant.

These equations were derived by assuming the characteristic scale of the external magnetic field to be much greater than the photon wavelength $\lambda \gg \lambda_p$, $(-i\partial_x) = k$ and the dielectric constant of the external medium to be of the order of one, $\omega + k \approx 2\omega$.

3. Non-relativistic conversion effect

Consider a neutron star in a coalescing binary system. We start with the case where the neutron star is not a pulsar and the surrounding plasma is non-relativistic. First we find the eigenvalues of the wavevector k of the system of equations (2):

$$\begin{aligned} k_1 &= \pm\omega\sqrt{1 + (\kappa B)^2/m^2}, \\ k_2 &= \pm im\sqrt{(1 - \frac{(\kappa B)^2}{m^2})(1 - \omega^2/m^2)}. \end{aligned} \quad (4)$$

The effective photon mass m in the medium includes the plasma frequency Ω_e and the Heisenberg-Euler correction, but under the conditions of the problem the latter addition is small due to the smallness of the GW frequency ω compared to the plasma frequency Ω_e :

$$m^2 = \Omega_e^2 - \frac{2\alpha C\omega^2}{45\pi} \left(\frac{B}{B_{cr}}\right)^2 \approx \Omega_e^2 \quad (5)$$

Here $B_{cr} = \frac{m_e^2}{e} \approx 4.4 \times 10^{13}$ G is the Schwinger critical magnetic field. The non-relativistic plasma frequency reads

$$\Omega_e^2 = \frac{n_e e^2}{m_e}. \quad (6)$$

The first eigenvalue k_1 in equation (4) is real and describes an electromagnetic wave created by constant conversion of gravitons into photons in an external magnetic field, and the second

eigenvalue k_2 in equation (4) is purely imaginary if the EM frequency is less than the plasma frequency and describes damped EM oscillations.

The amplitude of EM wave A_j created by gravitons h_j in the external magnetic field corresponds to the eigenvalue k_1 (see [1]) and can be written in the form:

$$A_j \approx \frac{\kappa B \omega}{\omega^2 (1 - \epsilon) - \Omega_e^2} h_j \quad (7)$$

where ϵ is the dielectric permittivity, $k^2 = \epsilon \omega^2$.

Consider the plasma dielectric constant. We are interested in the imaginary part of the dielectric constant $\text{Im} \epsilon$ that is responsible for the EM wave damping in plasma.

Under the condition $\omega \ll kv_{Ti} \ll kv_{Te}$ which is relevant for low-frequency EM wave, the transverse component of the dielectric constant of a collisionless Maxwell electron-ion plasma reads:

$$\epsilon_T - 1 = -\frac{1}{(ka_e)^2} - \frac{1}{(ka_i)^2} + i \sqrt{\frac{\pi}{2}} \frac{\Omega_e^2}{\omega ka_e} \quad (8)$$

where a_e and a_i is the Debye radius of electrons and ions, respectively; for electrons $a_e = \sqrt{\frac{T_e}{e^2 n_e}}$.

Substituting the imaginary part

$$\text{Im} \epsilon_T = \sqrt{\frac{\pi}{2}} \frac{\Omega_e^2}{\omega ka_e} \quad (9)$$

into equation (7) yields the EM field amplitude created by graviton-to-photon conversion which damps in the plasma:

$$A_j \approx \frac{\kappa B \omega a_e}{\Omega_e} h_j \quad (10)$$

Therefore, the part of the GW energy flux damping in the magnetized plasma reads:

$$K \equiv \frac{\rho_\gamma}{\rho_{GW}} = \left(\frac{\kappa B \omega a_e}{\Omega_e} \right)^2 \approx 10^{-46} \left(\frac{\omega}{\Omega_e} \right)^2 \left(\frac{a_e}{1 \text{ cm}} \right)^2 \left(\frac{B}{1 \text{ G}} \right)^2 \quad (11)$$

4. Relativistic conversion effect

Consider now the case where the neutron star in a coalescing compact binary is a pulsar with strong magnetic field. To estimate the graviton-to-photon conversion effect in this case, however, equation (11) should be modified because ultra-relativistic electron-positron plasma is generated near the pulsar surface (see [11] and references therein).

To calculate the GW to EM conversion rate in the relativistic lepton plasma equation (7) can be used if the transverse component of the dielectric permeability of relativistic pair plasma is known.

In the case of a collisionless relativistic plasma with temperature $T_e \gg mc^2$ [8]:

$$\epsilon_T^R - 1 = \frac{\pi e^2 n_e}{\omega k T_e} \left[\left(1 - \frac{\omega^2}{k^2} \right) \ln \left(\frac{\omega - k}{\omega + k} \right) - \frac{2\omega}{k} \right] \quad (12)$$

Clearly, there is no imaginary part, and in the collisionless plasma no attenuation of electromagnetic waves occurs. There will also be no Landau damping because the dispersion law of relativistic plasma implies $\omega > ck$ for any k [7].

Consider now a plasma with collision frequency ν between electrons, positrons, and photons [9]. Let ν vary from 0 to 1 in units of γm_e , where γ is the plasma Lorentz factor. In this case

$$\epsilon_T^R - 1 = \frac{2\pi e^2 n_e}{\omega(\omega + i\nu)} \left[1 + \left(\frac{(\omega + i\nu)^2}{k^2} - 1 \right) \left(1 - \frac{\omega + i\nu}{2k} \ln \frac{\omega + i\nu + k}{\omega + i\nu - k} \right) \right] \quad (13)$$

In the limit $\nu = 0$ we get equation (12) for the dielectric permittivity of collisionless plasma.

Now consider the limiting case of $\nu = 1$ in units of γm_e and take the imaginary part of the dielectric permeability of relativistic plasma:

$$Im \epsilon_T = \frac{3}{4} \frac{\Omega_{rel}^3}{\omega^4} \left[\Omega_{rel} Arg \left(1 - \frac{2i\omega}{\Omega_{rel}} \right) + 2\omega \left(1 - \frac{1}{2} \ln \frac{\Omega_{rel}^2 + 4\omega^2}{\Omega_{rel}^2} \right) \right] \quad (14)$$

where $\Omega_{rel} = \frac{4\pi e^2 n_e}{3T_e}$ is relativistic plasma frequency and $Arg \left(1 - \frac{2i\omega}{\Omega_{rel}} \right) = \arctan \left(-\frac{2\omega}{\Omega_{rel}} \right)$.

Considering that $\frac{2\omega}{\Omega_{rel}} \ll 1$, we expand the arctangent in power series in small parameter $\frac{2\omega}{\Omega_{rel}}$ and the logarithm in series in parameter $\left(\frac{\omega}{\Omega_{rel}} \right)^2$ to obtain

$$Im \epsilon_T \approx \frac{\Omega_{rel}}{\omega}. \quad (15)$$

Ultimately, by substituting equation (15) into equation (7) we find

$$A_j \approx \frac{\kappa B \omega}{\omega^2 \left(\frac{\Omega_{rel}}{\omega} \right) - \Omega_{rel}^2} h_j = \frac{\kappa B \omega}{\Omega_{rel}^2 \left(\frac{\omega}{\Omega_{rel}} - 1 \right)} h_j \approx \frac{\kappa B \omega}{\Omega_{rel}^2} h_j \quad (16)$$

Note that formula (16) can be readily derived from equation (10) by substituting the expression for the Debye radius of the electron-positron plasma [10]:

$$a_{e,rel}^2 = \frac{\gamma}{\Omega_{rel}^2} \quad (17)$$

It is easy to check that in the non-relativistic limit $\gamma \rightarrow 1$ equation (16) reduces to equation (10):

$$\frac{\kappa B \omega}{\Omega_{rel}^2} = \frac{\kappa B \omega}{\Omega_{rel} \Omega_{rel}} = \kappa B \omega \sqrt{\frac{T_e}{n_e e^2}} \sqrt{\frac{T_e}{n_e e^2}} = \kappa B \omega \sqrt{\frac{\gamma m_e}{n_e e^2}} \sqrt{\frac{T_e}{n_e e^2}} = \frac{\kappa B \omega a_e}{\Omega_e}. \quad (18)$$

Next, we estimate the magnetic field. Inside the light cylinder the magnetic field behaves as dipole field, at a distance of R from the pulsar we get

$$B = \frac{B_0 R_0^3}{R^3}, \quad (19)$$

where B_0 is the field near the pulsar surface and R_0 is the pulsar radius (assumed to be 10 km). For estimates, consider the magnetic field at the wave zone boundary (the light cylinder): $R_l = \frac{c}{\omega_{NS}}$ where ω_{NS} is the pulsar spin frequency (the pulsar spin period is $P = \frac{2\pi}{\omega_{NS}}$):

$$B = \frac{B_0 R_0^3}{R_l^3} \approx 1 \text{ G} \left(\frac{P}{1 \text{ s}} \right)^{-3} \left(\frac{B_0}{10^{12} \text{ G}} \right). \quad (20)$$

The electron number density created near the pulsar surface can be estimated from the Goldreich-Julian model (see [11])

$$n_{e,rel} = \lambda n_{GJ} \quad (21)$$

where λ is multiplicity of electron-positron pairs (usually taken to be $\sim 10^4 - 10^5$), and n_{GJ} is the Goldreich-Julian electron number density:

$$n_{GJ} = \frac{\omega_{NS} B_0}{2\pi e}. \quad (22)$$

The plasma generated near the pulsar surface propagates in the dipole field magnetic tube with the velocity of light, and therefore from the mass flux conservation the ratio n_e/B remains constant. Thus from equation (19) and equation (20) we get for the electron number density near the light cylinder

$$n_{e, rel} = 10^{16} \text{ cm}^{-3} \left(\frac{P}{1 \text{ s}} \right)^{-4} \left(\frac{B_0}{10^{12} \text{ G}} \right). \quad (23)$$

As $\Omega_{rel}^2 \sim n_e$, we find from equation (15) that the magnetic field of the pulsar cancels out, and the graviton-to-photon conversion efficiency in ultrarelativistic pair plasma is

$$K_{rel} = \left(\frac{\kappa B \omega}{\Omega_{rel}^2} \right)^2 \approx 10^{-35} \left(\frac{\omega}{100 \text{ rad s}^{-1}} \right)^2 \left(\frac{P}{1 \text{ s}} \right)^2 \left(\frac{\lambda}{10^5} \right)^{-2} \left(\frac{\gamma}{10^5} \right)^2. \quad (24)$$

5. Conclusion

We have considered the graviton-to-photon conversion effect in a magnetized relativistic plasma. As an astrophysical application, we estimated the efficiency of such a conversion in the vicinity of the light cylinder of an active pulsar located in a coalescing compact binary (e.g. BH+PSR) system. The effect was found to be independent on the magnetic field of the neutron star but strongly dependent on the plasma multiplicity factor near the pulsar.

While the relativistic plasma the GW-to-EM conversion efficiency in the GW wave zone is found to be 10 orders of magnitude higher than in the non-relativistic one, it is still too low for the present-day EM detector to observe it from ~ 100 Mpc distances the advanced LIGO/Virgo detectors will be sensitive to. However, the effect can be potentially interesting in future GW-EM observations with more sensitive EM detectors. It is not excluded that the effect can be amplified in the strong gravity zone with low plasma density in the case of early jet launching before the NS is fully disrupted and an accretion disc around black hole is formed [12].

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