

Effective action methods on manifolds with branes and boundaries

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Abstract. We review the developing effective action framework for field-theoretical models in the presence of branes: describe general scheme based on Dirichlet-to-Neumann reduction and present some applications and results.

1. Introduction. Braneworld setup

The field theoretical setup for models with branes motivated from string theory can be realized by the fundamental action which is the sum of bulk and brane actions

$$\mathbf{S}[\Psi, \varphi] = \mathbf{S}_{Bulk}[\Psi] + \mathbf{S}_{brane}[\varphi] \quad (1)$$

Bulk action is the integral over bulk manifold of the local invariants of fields, propagating in the bulk. Set of fields which are dynamical in the bulk we shall denote as Ψ . Brane action is the integral over brane d -dimensional manifold of invariants built of boundary values of fundamental bulk fields $\Psi|_{brane} = \varphi$. The latter are called *induced* or *brane* fields.

$$\begin{aligned} \Psi : & \quad G_{AB}, \dots & - & \quad \text{Bulk fields} \\ \varphi \equiv \Psi|_{brane} : & \quad g_{\alpha\beta}, \dots & - & \quad \text{induced (brane) fields} \end{aligned}$$

In braneworld models various additional matter and gauge fields which are binded to branes can be present. But they do not play crucial role in what follows, so we omit them and concentrate on fields, which propagate not only on branes, but in the bulk as well. The main goal is to discuss the effective dynamics of such fields as it seen by the observers living on branes.

Branes can be considered as boundaries of the bulk, no matter they really bound the bulk manifold or they are inside the bulk manifold (so-called fixed points). The only thing one has to do is to select appropriate boundary or junction conditions [1].

In what follows we restrict ourselves to the case when branes are d -dimensional manifolds embedded in the bulk of dimension $(d+1)$ – case of codimension 1 branes, which is motivated by interest in applications to braneworld scenarios such as Randall-Sundrum (RS) or Dvali-Gabadadze-Porratti (DGP) with one large extra dimension [2],[3]. Also for the sake of simplicity in what follows we skip analysis of the gauge aspects of the formalism [4],[5].

The brane effective action approach allows us to introduce the brane functional of only induced fields φ , which incorporate the dynamics of correspondent fields Ψ in the bulk [1],[6]

$$\mathbf{S}[\Psi, \varphi] \rightarrow S^{eff}[\varphi] \quad (2)$$

The main advantage of this approach, when one is allowed to deal with functional of only brane fields, compared for example with Kaluza-Klein approach, is that these induced fields can be “measured” directly by observers on the brane due to local interaction of brane gauge and matter fields with these induced fields [1],[7].

The price one pays for this is that effective action for induced fields appears to be nonlocal.

$$e^{-S^{eff}[\varphi]} = e^{-S_{brane}[\varphi]} \int_{\Psi|_{brane}=\varphi} d\Psi e^{-\mathbf{S}_{Bulk}[\Psi]} \quad (3)$$

This means that from the viewpoint of brane observer the dynamics of induced fields along branes is essentially nonlocal. On the other hand such properties as local causality, unitarity to a large extent are preserved by construction that justifies and suggests new ideas for search of consistent modifications of Einstein gravity since one accepts the possibility that our 4-dimensional universe can be the brane embedded in multidimensional bulk.

Passing on to working with brane effective action one does not completely lose the information about bulk dynamics of fundamental fields – the geometry, topology of the bulk and information of brane embedding is incorporated in nonlocality of induced fields dynamics, particularly in operator of small fluctuations. Thus brane observers are able to obtain some information about the bulk by measuring deviations from local d -dimensional behavior of fundamental fields, including gravity.

For example one can study the deviations from the Newton law on very large and very small distances. Alternative way is to observe the deformations of gravitational waves and signals from distant astrophysical objects. For the class of Randall-Sundrum type models that was analyzed in [8].

Another argument in favor of efficiency of the effective action approach is that nonlocalities governing the propagation of induced field perturbations are calculable, at least in models possessing symmetric bulk backgrounds. So one has a handy tool for explicit analysis of induced fields behavior on arbitrary energy scales. It is very helpful that by construction all objects appear to be d -dimensionally covariant.

In order to take into account quantum properties in full (i.e. to quantize brane configurations as well) one needs *braneworld quantum effective action* $\Gamma[\bar{\varphi}]$. It is by definition the functional of the mean brane field emergent as the response to arbitrary sources concentrated on branes.

$$e^{-\Gamma[\bar{\varphi}]} \equiv \int d\Psi e^{-\mathbf{S}[\Psi, \varphi] - \int_{brane} (\varphi - \bar{\varphi}) j} \Big|_{j=j[\bar{\varphi}]} = \int d\varphi e^{-S^{eff}[\varphi] - \int_{brane} (\varphi - \bar{\varphi}) j} \Big|_{j=j[\bar{\varphi}]} \quad (4)$$

Its construction differs from construction of quantum effective action of the whole theory (1) in the bulk by the fact that one introduces sources (j) only on branes. Owing to that it becomes the functional only of mean values of induced fields $\bar{\varphi} \equiv \langle \varphi \rangle$. Relevance of that object is again justified by the braneworld philosophy that observers living on branes are able to investigate the world only by means of local interactions on branes.

2. One-loop brane effective action and duality relations

For realistic models which are nonlinear in fields, especially for models such as gravity which is inevitably present in braneworld scenarios, one is unable to calculate explicitly functionals S^{eff} or $\Gamma[\bar{\varphi}]$. So, one has to use some perturbation technique. When weak field approximation is relevant it is convenient to perform loop expansion

$$\Gamma[\bar{\varphi}] = S^{tree}[\bar{\varphi}] + \Gamma^{1-loop}[\bar{\varphi}] + \dots \quad (5)$$

Interesting and nontrivial observations can be found already at one-loop order, when in path integral formalism one deals with calculable gaussian functional integrals. In one-loop approximation the effective action is given by the following path integral:

$$e^{-\Gamma^{1-loop}[\bar{\varphi}]} \equiv \int d\Psi e^{-\frac{1}{2} \int_{Bulk} \Psi \overleftrightarrow{\mathbf{F}} \Psi - \frac{1}{2} \int_{brane} \phi f \phi} \quad (6)$$

where we perform integration over fluctuations Ψ : $\Psi|_{brane} = \phi$, over background configuration $\Psi_0 = \Psi[\bar{\varphi}]$, which is the solution of:

$$\begin{cases} \frac{\delta \mathbf{S}_{Bulk}[\Psi]}{\delta \Psi} = 0, \\ \Psi|_{brane} = \bar{\varphi}. \end{cases} \quad (7)$$

i.e. the on-shell continuation of brane background to the bulk.

In r.h.s of (6) background $\bar{\varphi}$ shows itself through the operators of small fluctuations in the bulk \mathbf{F} and on branes f :

$$\mathbf{F} \delta^{d+1}(X, X') \equiv \frac{\delta^2 \mathbf{S}_{Bulk}[\Psi]}{\delta \Psi(X) \delta \Psi(X')} \Big|_{\Psi = \Psi[\bar{\varphi}]}, \quad f \delta^d(x, x') \equiv \frac{\delta^2 \mathbf{S}_{brane}[\varphi]}{\delta \varphi(x) \delta \varphi(x')} \Big|_{\varphi = \bar{\varphi}}. \quad (8)$$

In braneworld scenarios under consideration \mathbf{F} is a second order local differential nondegenerate bulk operator $\mathbf{F} = \square^{d+1} + \dots$ and f – local brane operator.

It can be proved [6] that r.h.s of (6) equals square root of functional determinant of Neumann Green function for bulk operator \mathbf{F} of fluctuations Ψ over background $\Psi_0 \equiv \Psi[\bar{\varphi}]$. It is the same as determinant of the operator \mathbf{F} to the power minus one half on space of functions with Neumann boundary conditions $(-\nabla_n + f)\Psi|_{brane} = 0$ (∇_n – covariant derivative normal to brane):

$$\int d\Psi e^{-\frac{1}{2} \int_{Bulk} \Psi \overleftrightarrow{\mathbf{F}} \Psi - \frac{1}{2} \int_{brane} \phi f \phi + \int_{brane} \phi j} = \mathbf{Det}^{1/2} \mathbf{G}_N \cdot e^{\frac{1}{2} \int_{brane} j |\mathbf{G}_N| j}. \quad (9)$$

We introduced auxiliary sources j in path integral to pick up another useful (*tree-level*) contribution [1] leading to so called Dirichlet-to-Neumann map. Notation $|\mathbf{G}_N|$ stands for boundary operator whose kernel is the restriction of bulk Neumann Green function $\mathbf{G}_N(X, X')$

$$\begin{cases} \mathbf{F} \mathbf{G}_N(X, X') = \delta^{d+1}(X, X'), \\ (-\nabla_n + f) \mathbf{G}_N(X, X')|_{X \in brane} = 0, \end{cases} \quad (10)$$

to boundary (brane):

$$|\mathbf{G}_N|(x, x') \equiv \mathbf{G}_N(X, X') \Big|_{X=x, X'=x'}.$$

Note, that there were made *no* special assumptions about boundary operator f but it selfadjointness (and preserving strong ellipticity). So our approach covers most of possible *generalized* Neumann boundary conditions such as Robin, oblique or even more involved ones, arising for example in Dvali-Gabadadze-Porratti model (where f is brane d -dimensional D'Alembertian). More detailed discussion of this subject can be found in the next section.

The second way to calculate path integral above is to split integration

$$\int d\Psi \dots \rightarrow \int d\phi \int_{\Psi|_{brane}=\phi} d\Psi.$$

Performing first integration over bulk fields with fixed boundary values generate determinant of Dirichlet operator in the bulk. Finally this way leads to [6]

$$\int d\Psi e^{-\frac{1}{2} \int_{Bulk} \Psi \overleftrightarrow{\mathbf{F}} \Psi - \frac{1}{2} \int_{brane} \phi \mathfrak{f} \phi + \int_{brane} \phi j} = \mathbf{Det}^{1/2} \mathbf{G}_D \cdot \mathbf{Det}^{-1/2} (F^{ind} + \mathfrak{f}) \cdot e^{\frac{1}{2} \int_{brane} j |F^{ind} + \mathfrak{f}|^{-1} j}. \quad (11)$$

where \mathbf{G}_D is the Dirichlet Green function:

$$\begin{cases} \mathbf{F} \mathbf{G}_D(X, X') = \delta^{d+1}(X, X'), \\ \mathbf{G}_D(X, X')|_{X \in brane} = 0, \end{cases} \quad (12)$$

and F^{ind} is the (nonlocal) effective inverse propagator for field fluctuations on the boundary generated by bulk dynamics of Ψ , which kernel is defined in terms of Dirichlet Green function as follows:

$$F^{ind} \delta^d(x, x') \equiv \left(- \overleftrightarrow{\nabla}_n \mathbf{G}_D(X, X') \overleftarrow{\nabla}_{n'} \right) \Big|_{X=x, X'=x'}. \quad (13)$$

Comparing (9) and (11) one can notice that this gives justification for the following duality relations between Green functions for different boundary value problems which is known as Dirichlet-to-Neumann map and between functional determinants of operators with different boundary conditions on manifolds with boundaries [1],[6]:

$$- \left| \overleftrightarrow{\nabla}_n \mathbf{G}_D \overleftarrow{\nabla}_n \right| + \mathfrak{f} = \left| \mathbf{G}_N \right|, \quad (14)$$

$$\mathbf{Det} \mathbf{G}_D \cdot \mathbf{Det}^{-1} (F^{ind} + \mathfrak{f}) = \mathbf{Det} \mathbf{G}_N. \quad (15)$$

Surely such formal path integral manipulations can not serve as rigorous prove for relations above. In fact more pedantic considerations show that (14,15) are correct for rather general setup [1],[6].

3. Braneworld applications

What can this approach suggest for braneworld physics?

First of all duality relations suggest handy scheme for exact calculation of one-loop effective action depending on background parameters (such as internal mass scales, interbrane distances and brane embeddings, background curvature scales, etc. which in what follows we denote as m_i) and covariant low-energy effective action expressions. The key point is that due to (15) calculation of one-loop contribution which for dynamical sector of the model is nothing but logarithm of the Neumann determinant decouples into calculation of the Dirichlet bulk determinant and brane determinant of some brane nonlocal operator.

First part (Dirichlet determinant) is well-studied problem. New and intriguing is the problem of brane determinant calculation. The approach to this problem depends on what one aims at¹. In braneworld physics one-loop effective action is interesting in the context of moduli stabilization problem, for cosmological applications it can give the preferred vacuum. In both cases one is

¹ Generic properties of brane determinant based on Schwinger-DeWitt expansion are discussed in the next section.

interested in dependence on background moduli m_i and hence is forced to calculate quantum determinants over parameterized families of highly-symmetric backgrounds.

When bulk background metric appears to be of the form $ds^2 = dy^2 + a^2(y)g_{\mu\nu}(x)dx^\mu dx^\nu$ with brane(s) located at some $y = const$ then induced boundary operator (13) can be *explicitly* expressed as some function of brane d'Alembertian (Laplacian in euclidean signature calculations) $F^{ind} = F^{ind}(\square)$ parametrically dependent on background moduli (interbrane distances, brane embeddings, bulk curvature scale, etc.).

For example in DGP-like background [3] ($(d+1)$ -dimensional flat space, flat d -dimensional brane)

$$F^{ind} = m_{DGP} \sqrt{M^2 - \square}; \quad \mathfrak{f} = -\square. \quad (16)$$

In Randall-Sundrum background [2] ($(d+1)$ -dimensional AdS in the bulk and flat d -dimensional brane) F^{ind} is some combination of Bessel functions of $\sqrt{-\square}$, $\mathfrak{f} = 0$ [1],[7].

When boundary operators are expressed as functions of brane d'Alembertian (Laplacian) \square then expressions for effective propagator and brane contribution to one-loop effective action can be obtained using Fourier transform or proper time technique (Laplace transform). Using latter representation needed quantities can be expressed in terms of weighted proper time integrals

$$\frac{1}{F^{ind} + \mathfrak{f}} = \int_0^\infty ds e^{s \square} u(s, m_i), \quad (17)$$

$$\ln \text{Det} (F^{ind} + \mathfrak{f}) = -\text{Tr} \int_0^\infty \frac{ds}{s} e^{s \square} v(s, m_i), \quad (18)$$

The advantage of the latter representation is that it is applicable also in the case of the curved brane, so that generally covariant expansion of (17)-(18) in curvatures can be directly obtained by using a well-known Schwinger-DeWitt expansion for $e^{s \square}$. Thus the lowest order approximation for the exact brane-to-brane operator in models with a curved bulk and curved branes can be considered by means of manifestly covariant technique which can be systematically extended to higher orders. Combined with the method of fixing the background covariant gauge for diffeomorphism invariance in brane models, developed in [4, 5] this will ultimately give the universal background field method of the Schwinger-DeWitt type in gravitational brane systems.

For instance in DGP-type model these weights are given in terms of the error function [9]:

$$u(s, m_{DGP}) = e^{s m_{DGP}^2} \cdot \text{erfc} (m_{DGP} \sqrt{s}); \quad v(s, m_{DGP}) = \frac{1}{2} \left(1 + u(s, m_{DGP}) \right) \quad (19)$$

Another test-application of this scheme is renormalized (using dimensional regularization) effective potential of the usual Coleman-Weinberg structure for DGP case [9]:

$$V_{\text{eff}}(m_{DGP}) = \frac{1}{2} \frac{(-1)^d}{\Gamma(\frac{d}{2} + 1)} \left(\frac{m_{DGP}^2}{4\pi} \right)^{d/2} \ln \frac{m_{DGP}}{\mu}. \quad (20)$$

Here μ is the parameter reflecting the renormalization ambiguity, and the role of the field (argument of the potential) is played by the DGP-scale m_{DGP} of the brane term in the classical action. This result confirms the boundary effective action calculation of [10].

4. New heat kernel technique

One of interesting byproducts of the brane effective action formalism briefly discussed above is a new technique for calculating the heat kernel expansion on manifolds with boundaries.

In quantum field theory heat kernel $K(s|X, Y) \equiv e^{-s\mathbf{F}}\delta(X, Y)$ for differential operator \mathbf{F} of quantum perturbations is a well-known and convenient tool for studying one-loop divergences, counterterms, quantum anomalies, Casimir effect and various asymptotics of effective actions and propagators [11]. Convenience and efficiency of heat kernel in many respects is based on the possibility of expanding it and its trace in asymptotic series in (integer) powers of proper time s in the (ultraviolet) limit $s \rightarrow 0$:

$$e^{-s\mathbf{F}}\delta(x, y) = \frac{e^{-\frac{\sigma(x,y)}{2s}}}{(4\pi s)^{d/2}} \sum_{n=0}^{\infty} s^n a_n(x, y) \quad (21)$$

where $\sigma(x, y)$ is half of the geodesic distance squared. Coefficients $a_n(x, y)$ referred as Schwinger-DeWitt coefficients are universal and can be explicitly found in the coincidence limit ($x \rightarrow y$) as local invariants of space-time curvature, fibre-bundle connection and background external fields [11] by solving recurrent equations. In particular this expansion gives rise to local low-energy expansion of the effective action in inverse powers of (auxiliary) large mass parameter M^2 when the inverse propagator is supplied with mass term $\mathbf{F} \rightarrow \mathbf{F} + M^2$.

In spacetimes with boundaries the heat kernel theory becomes complicated. The apparent modification consists in that while bulk coefficients of expansion remain unchangeable, the additional series of half-integer powers of proper time (and possible logarithmic terms) with coefficients built up of boundary invariants arise

$$\mathbf{Tr} e^{-s\mathbf{F}-sM^2} = \frac{1}{(4\pi s)^{D/2}} e^{-sM^2} \sum_{n=0}^{\infty} \left[s^n \mathbf{A}_n + s^{n/2} B_{n/2} + \ln(s\mu^2) s^{n/2} C_{n/2} \right] \quad (22)$$

where

$$\mathbf{A}_n = \int_{Bulk} a_n(X, X); \quad B_n, C_n = \int_{brane} b_n, c_n(x, x). \quad (23)$$

The crucial difficulty consists in the absence of recurrent local scheme for boundary coefficients B_n (and C_n). In contrast to the bulk coefficients A_n which are universal and independent on boundary conditions, the surface (boundary) coefficients essentially depend on the latter. Surface coefficients can be somehow regularly found only for Dirichlet and homogeneous Neumann boundary conditions [11, 12]. For more complicated boundary conditions such as Robin and oblique cases the problem of calculating the surface contributions in (22) becomes highly involved or needs the use of methods (e.g. based on conformal properties of geometric invariants) which are not universally applicable [11].

One-loop duality (15) suggests new elegant, covariant and universal way for explicit calculation of Schwinger-DeWitt boundary coefficient for generalized Neumann boundary conditions. The latter are parameterized by the boundary operator $W = -\nabla_n + \mathfrak{f}$, which determine selfadjoint extension of bulk symmetric operator \mathbf{F} of the Neumann type. In our approach, beside pure Neumann case $\mathfrak{f} = 0$, which arise for example in Randall-Sundrum type models, one can deal with the following generalized Neumann boundary conditions:

$$\begin{aligned} \mathfrak{f} = v(x) & \quad - \text{inhomogeneous Neumann (Robin) case;} \\ \mathfrak{f} = \frac{1}{2}(\Gamma^\mu \nabla_\mu + \nabla_\mu \Gamma^\mu) & \quad - \text{oblique boundary conditions (open string, Chern-Simons);} \\ \mathfrak{f} = -\frac{1}{m}\square \equiv -\frac{1}{m}g^{\alpha\beta}\nabla_\alpha\nabla_\beta & \quad - \text{Dvali-Gabadadze-Porratti (DGP) model.} \end{aligned} \quad (24)$$

The key point of new technique is that one-loop duality relation (15) being rewritten in the following form

$$\mathbf{Tr} \ln(\mathbf{F}_N + M^2) - \mathbf{Tr} \ln(\mathbf{F}_D + M^2) = \mathbf{Tr} \ln(F_M^{ind} + \mathfrak{f}) \quad (25)$$

and being expanded in inverse powers of M gives the difference between Schwinger-DeWitt coefficients of heat kernel trace of generalized Neumann operator \mathbf{F}_N in the bulk and trace of Dirichlet operator \mathbf{F}_D in terms of expansion coefficients of *boundary* operator $(F_M^{ind} + \mathfrak{f})$.

$$\begin{aligned} & \text{Tr} \ln(\mathbf{F}_{N,D} + M^2) \\ & \sim \int_0^\infty \frac{ds}{s} s^{-\frac{d+1}{2}} e^{-sM^2} \sum_{n=0}^\infty \left[s^n \mathbf{A}_n + s^{n/2} B_{n/2}^{N,D} + \ln(s\mu^2) s^{n/2} C_{n/2}^N \right] \\ & \sim M^{d+1} \sum_{n=0}^\infty \left[M^{-2n} \mathbf{A}_n + M^{-n} (B_{n/2}^{N,D} + C_{n/2}^N) - \ln(M^2/\mu^2) M^{-n} C_{n/2}^N \right] \end{aligned} \quad (26)$$

Note that in l.h.s. of (25) bulk contributions (\mathbf{A}_n) from above expansion of generalized Neumann and Dirichlet operators are identical and therefore cancel out.

Finally one comes to the following identification

$$\text{Tr} \ln (F_M^{ind} + \mathfrak{f}) \sim M^{d+1} \sum_{n=0}^\infty M^{-n} \int_{brane} \left[(b_{n/2}^N + c_{n/2}^N) - \ln(M^2/\mu^2) c_{n/2}^N - b_{n/2}^D \right] \quad (27)$$

Boundary coefficients $b_{n/2}^D$ for Dirichlet case can be considered as reference ones since there exist explicit calculational schemes and they were calculated to rather high orders n (see e.g. [11, 12]). The Dirichlet case is a special one since it is unique for a given manifold – it does not depend of boundary operator \mathfrak{f} . Mainly for the same reason Dirichlet case is the simplest one.

The only thing one needs in our scheme to obtain generalized Neumann Schwinger-DeWitt coefficients is the handy tool to calculate the large M expansion for trace of logarithm of boundary operator $(F_M^{ind} + \mathfrak{f})$ in l.h.s of (27). Despite the nonlocality of boundary operator F^{ind} great simplification comes from the fact that boundary (brane) is itself the d -dimensional manifold without boundary.

As was mentioned in previous section in braneworld scenarios where branes are symmetric manifolds F^{ind} can be expressed explicitly as the function of intrinsic and embedding geometric brane quantities and tangential boundary covariant derivatives ∇_α . This allows one to use straightforwardly the theory of asymptotic expansion of integrals with weak peculiarity to obtain the expansion in inverse powers of M . Finally, identifying terms at the same powers of M in both sides of (27) one attains the aim.

For general case of bounded manifold a recurrent scheme for calculation of l.h.s. in (27) also can be suggested. One possible way is as follows (for simplicity consider scalar case with operator $\mathbf{F}_M = -\Delta^{d+1} + M^2 = -\partial_y^2 - k(x, y)\partial_y - \square_x(y) + M^2$ in gauss normal coordinates in the bulk). With the help of proper time representation for one-dimensional formal operator $-\partial_y^2 + \hat{V}(y)$ and subsequent restriction to $y = 0$ (where the brane is placed) and integration over proper time one can expand operator (13) $F_M^{ind} = \left| \overrightarrow{\partial}_y \frac{-1}{-\partial_y^2 - k(x, y)\partial_y - \square_x(y) + M^2} \overleftarrow{\partial}_y \right|_{y=0}$ either in inverse powers of M or in inverse (odd) powers of $\sqrt{M^2 - \square(0)}$. Coefficients of this expansion are d -dimensional tangential to brane differential operators expressed in terms of extrinsic curvature, brane Riemann curvature, their normal and tangential derivatives and tangential derivatives themselves [13]. Then scheme depends on what type of operator \mathfrak{f} . When $\mathfrak{f} = -\square$ as in DGP case or when it is a potential term (case of Robin boundary value problem) then it is possible to express coefficients of inverse M expansion of l.h.s. recurrently using known Schwinger-DeWitt coefficients of \square on brane. The case when \mathfrak{f} is first order in tangential derivatives (oblique boundary conditions case) with dimensionless coefficients should be treated in slightly different way [13].

Examples of implementing this scheme for some nontrivial but particular cases already exist. For instance special (toy) DGP case is considered in [9] where for flat background inverse M expansion coefficients are found and a nonperturbative hypergeometric representation of l.h.s. (27) dependence on DGP scale is obtained. Some insight on how to treat the oblique boundary value problem case can be found in [6].

5. Conclusions

To summarize, we conclude that the technique for quantum effects in brane models is more complicated than in systems without boundaries. Moreover, it does not reduce to a simple bookkeeping of surface terms in the heat kernel expansion of [11, 12], and so on, because of the complicated square-root structure of the brane propagator F^{ind} mediating the effect of the generalized Neumann boundary conditions. The proper time method that was fundamental and efficient in models without boundaries in our approach appear as a derivative one in an alternative calculational scheme. Namely, the surface terms in the heat kernel expansion can be recovered from the $1/M$ -expansion of the action obtained by a different method.

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