VARIANCE OF DIFFERENT MODELS IN ELECTRON-POSITRON ANNIHILATIONS

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The average of event shape variables is studied in e^+e^- annihilation within the context of next to leading order (NLO) perturbative QCD prediction. We measure the strong coupling constant $\alpha_S(M_{(Z^0)})$ by fitting the NLO model with the variance distribution. Then, the parameters of non-perturbative part of theory are measured. These parameters are extracted from the dispersive model for (α_0) and also from the shape function model for (λ). We explain all these features in this article.

Key words: Strong coupling constant, Perturbative theory, Non-perturbative calculations.

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1. INTRODUCTION

Quantum Chromo Dynamics (QCD) is generally believed to be the correct theory for describing the strong force between quarks and gluons. QCD is the quantum field theory of Natures strong force. The property of asymptotic freedom makes it possible to use perturbation theory at large momentum transfer and allows for a precise description of many scattering experiments as carried out in the big accelerator facilities [1]. This theory predicts first and second moments of an event shape variable y.

In probability theory and statistics, the variance is a measure of how far a set of numbers is spread out. It is one of several descriptors of a probability distribution, describing how far the numbers lie from the mean (expected value). In particular, the variance is one of the moments of a distribution. In that context, it forms part of a systematic approach to distinguishing between probability distributions. While other such approaches have been developed, those based on moments are advantageous in terms of mathematical and computational simplicity.

Although, as have explained in [2], the perturbative stage of a hard collision is distinct from the non-perturbative regime characterizing the hadron structure, experimental observations suggest that, in specific kinematic regimes, both the perturbative and non-perturbative stages arise almost ubiquitously, in the sense that the non-perturbative description follows the perturbative one. This third concept is known as parton- hadron duality, and we will here understand it as being yet another manifestation of the perturbative to non-perturbative transition in QCD.

The outline of the paper is as the following: In sect. 2, we study the variance parameter; also we explain the event shape variables. Section 3 describes the obtained physics results for the coupling constants in NLO theory, dispersive model and shape function model. Finally we summarize the obtained values and our conclusions in the next section.

2. THE VARIANCE PARAMETER

The variance is a parameter describing in part either the actual probability distribution of an observed population of numbers, or the theoretical probability distribution of a sample (a not-fully-observed population) of numbers. In the latter case a sample of data from such a distribution can be used to construct an estimate of its variance: in the simplest cases this estimate can be the sample variance [3]. The formula of the variance for the event shape variable y is:

$$Var(y) = \langle y^2 \rangle - \langle y \rangle^2 \tag{1}$$

The variance of event shape distributions on hadron level is calculated in the dispersive model. Power corrections suppressed less than α_S/Q^2 are found to cancel. So the variance is described in the dispersive model perturbatively without significant power corrections. This would open the possibility of an accurate $\alpha_S(M_{(Z^0)})$ determination [4]. This simple prediction for the variance of event shape variables on hadron level is deduced [4]:

$$Var(y) = \langle y^2 \rangle_{NLO} - \langle y \rangle_{NLO}^2 \tag{2}$$

We can obtain a purely perturbative expression for the variance in the dispersive model [5] and the shape function model [6], up to strongly suppressed corrections $O(\alpha_S(Q^2))$. These models are two ways that include the perturbative theory and the non-perturbative prediction. First we define the event shape variables, for calculation of variance in these models followed by computing the coupling constants. Event shape variables measure geometrical properties of hadronic final states at high energy particle collisions. They have been studied at e^+e^- collider experiments. Apart from distributions of these observables, we can also study the mean values as well as higher orders for the moments of event shape variables [7]. These variables have been explained in ref [5]. The most common observables (y) are: the heavy jet mass (ρ), the total jet broadening (B_T) and the wide jet broadening (B_W). The first and second moment predictions give the identical prediction in NLO model. However the NLO model does not include non-perturbative part. Thus in case of the heavy jet mass variable, we have:

$$Var(\rho) = \langle(\rho)^2\rangle_{NLO} - \langle\rho\rangle^2_{NLO} \quad (3)$$

Also we have similar formulas for the other event shape variables, such as:

$$Var(B_T) = \langle(B_T)^2\rangle_{NLO} - \langle B_T\rangle^2_{NLO} \quad (4)$$

We do similar analysis for the wide jet broadening (B_W) observable. Furthermore we can expand this analysis for different event shape variables in two other models. The dispersive model and the shape function model are including perturbative theory (NLO) and non-perturbative prediction. Consequently, we can extend our results to non-perturbative part in above equations. For example we obtain the variance in the different models for heavy jet mass observable:

$$Var(\rho) = \langle(\rho)^2\rangle_{total} - \langle\rho\rangle^2_{total} \quad (5)$$

Total includes the perturbative part as well as the non-perturbative part in both models.

In next section, we perform our calculations using the variance distribution; also we obtain the coupling constants.

3. PHYSICS RESULTS

In this article we measure the QCD parameters by using the variance distribution on different models. These models are, NLO, dispersive and shape function. The first model contains the perturbative part of our calculations. On the other hand for the two other models, in addition to the perturbative part of the calculations they also contain the non-perturbative part of the theory. This ansatz provides an additive term to the perturbative $O(\alpha_S^2)$ QCD prediction [4].

$$\langle y\rangle = \langle y^{pert}\rangle + \langle y^{pow}\rangle = \frac{1}{\sigma_{tot}}\int_y \frac{dy}{d\sigma}d\sigma \quad (6)$$

3.1. THE NLO THEORY

Figure 1 shows the variance distributions against the centre of mass energy using NLO theory for different event shape variables. In this analysis, we are using the AMY data (the range of energy between 52-60 GeV) obtained by a detector at the KEK storage ring, TRISTAN accelerator, Japan [8]. We are also using the simulated events (PYTHIA) which is based on the use of random numbers and probability statistics to investigate problems concerned with the hadronic interactions [9]. For an observable event (y) we are using the perturbative prediction to determine α_s from mean event shape variables. This ansatz provides the perturbative $O(\alpha_s^2)$ QCD prediction.

$$\langle y^{pert}\rangle = \overline{A}_F\left(\frac{\alpha_s(\mu)}{2\pi}\right)+\left(\overline{B}_F+\overline{A}_F\beta_0\log\left[\frac{\mu^2}{E_{cm}^2}\right]\right)\left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \quad (7)$$

where $\overline{A}_F = A_F$, $\overline{B}_F = B_F - 3/2C_F A_F$, $\beta_0 = (33-2N_F)/12\pi$, and μ being the renormalization scale. The coefficient A_F and B_F were determined from the $O(\alpha_s^2)$ perturbative calculations [10].

In all figures we observe an increase in variance of the observables with the centre of mass energy. We conclude that the results obtained from B_W, B_T and ρ are in good agreement with each other. They are also consistent with the results obtained from other experiments [4].

Table 1

The values of the coupling constant in NLO model.

Event Shape Variables	$\alpha_s(M_{Z^0})$
B_W	0.13347 ± 0.00248
B_T	0.11716 ± 0.01068
ρ	0.11945 ± 0.02492

By fitting the variance distributions based on NLO model, with above diagrams we measure the coupling constant in perturbation theory. Table 1 shows our results.

As the table indicates, the values of the coupling constant obtained by this method are within the statistical errors consistent with each other. They are also in good agreement with QCD predictions [11].

3.2. THE DISPERSIVE MODEL

In this subsection, we achieve the variance distribution by suing the dispersive model. As explained in [5], this model combines the perturbative $\langle y^{pert}\rangle$ (in eq.7) as well as the non-perturbative $\langle y^{pow}\rangle$ parts of the theory. That $\langle y^{pow}\rangle$ is defined:

$$\begin{aligned}\langle y^{pow}\rangle &= a_y.P\\ &= a_y.\frac{4c_F}{\pi^2}M\frac{\mu_I}{E_{cm}}\left[\alpha_0(\mu_I)-\alpha_s(\mu_I)-\left(log\left(\frac{\mu}{\mu_I}\right)+1+\frac{K}{4\pi\beta_0}\right)2\beta_0\alpha_0^2(\mu_I)\right]\end{aligned} \quad (8)$$

where α_0 is a non-perturbative parameter accounting for the contributions to the event shape below an infrared matching scale $\mu_I \cong 2$. In the $(\overline{MS})$ renormalization scheme

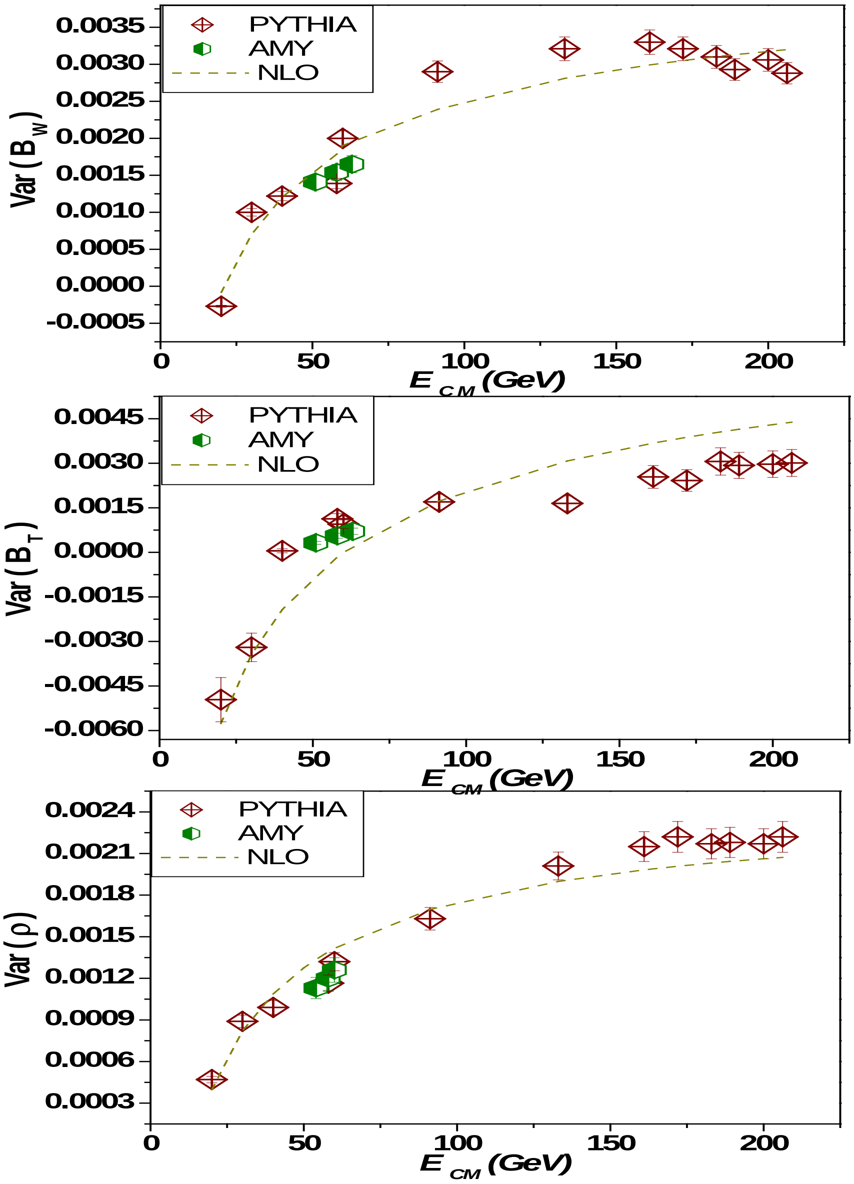

Fig. 1 – The variance of event shape variables by fitting the NLO model.

the constant k has the value $k = (\frac{67}{18} - \frac{\pi^2}{6})C_A - \frac{5}{9}N_F$, with $N_F = 5$ at the studied energies. The Milan Factor M is known in two loops $M = 1.49 \pm 20\%$ [12].

Thus we can compute the α_0 furthermore the α_s in this model. Figure 2 shows the variance distribution of the dispersive model for both the simulated MC (PYTHIA) and the AMY real data. All figures indicate that the variance of the dispersive model fits more accurately the distributions than the NLO model. The main reason behind this is that the dispersive model includes both the perturbative and the non perturbative part of our model, while the NLO model just contains the perturbative part of the model.

Tables 2 and 3 show our results for perturbative $\alpha_S(M_{Z^0})$ and non-perturbative $\alpha_0(\mu_I)$ constants, respectively.

Table 2

The strong coupling constant values in the dispersive model.

Event Shape Variables	$\alpha_s(M_{Z^0})$
B_W	0.10158 ± 0.0097
B_T	0.10323 ± 0.01054
ρ	0.10323 ± 0.01054

The non-perturbative coupling constant $\alpha_0(\mu_I)$ values in the dispersive model:

Table 3

The non-perturbative coupling constant $\alpha_0(\mu_I)$ values in the dispersive model.

Event Shape Variables	$\alpha_0(\mu_I)$
B_W	0.61832 ± 0.07653
B_T	0.61595 ± 0.10184
ρ	0.48155 ± 0.03954

Table 4

The strong coupling constant values in the shape function model.

Event Shape Variables	$\alpha_s(M_{Z^0})$
B_W	0.1063 ± 0.00062
B_T	0.11232 ± 0.05424
ρ	0.12453 ± 0.0429

Table 2 indicates that the values of the coupling constant obtained by this method are acceptable within the context of the QCD theory. Table 3 also shows that the mean value of the non-perturbative parameter obtained by this method is consistent with that obtained in our previous calculations based on dispersive model itself (ref [5] and Table 4).

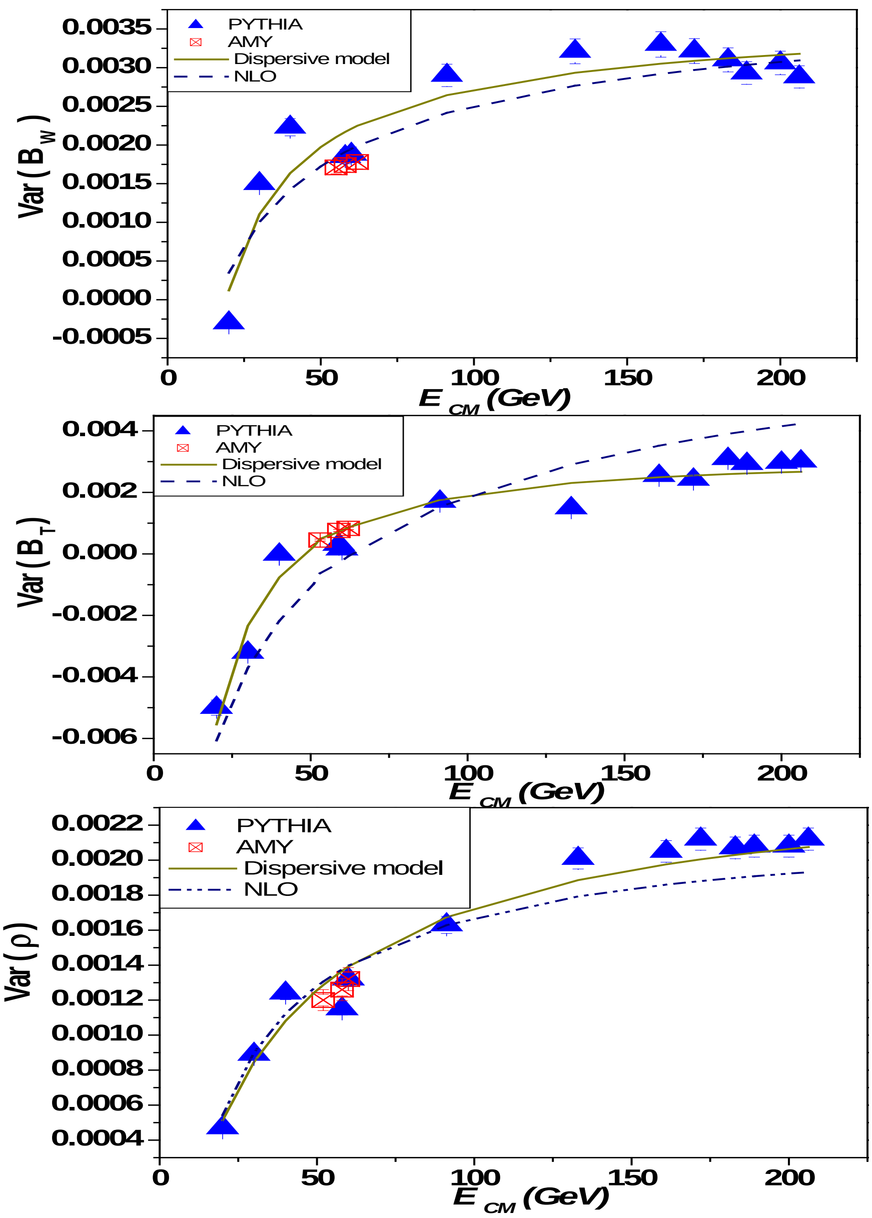

Fig. 2 – The variance distributions for three event shape variables by fitting the dispersive model.

Table 5

The non-perturbative parameter in the shape function model.

Event Shape Variables	$\lambda_1(GeV)$
B_W	1.26903 ± 0.08538
B_T	0.92708 ± 0.1187
ρ	0.56547 ± 0.043295

3.3. THE SHAPE FUNCTION MODEL

Next, we do our analysis of variance for the shape function model [4, 6]. We remind that this model also includes NLO prediction in perturbative theory as well as the power corrections in non-perturbative part of the theory.

For instance, non-perturbative scales λ_p parametrizing power corrections to $\langle y^{pow} \rangle$ in equation (6) defined by the moment $\int dy\, y f_{had}(Q, y)$ [13] is:

$$\langle y \rangle = \langle y \rangle_{PT} + \frac{\lambda_p}{Q^p} \tag{9}$$

Figure 3 shows the variance distributions for the above model. They are very similar to the distributions obtained using the variance of the dispersive model (see Figure 2).

At this stage, by fitting the variance distributions with the data, we calculate the strong coupling constant (α_s) in perturbative theory. Our results are presented in Table 4. Also we extend our analysis by measuring the free parameter in non-perturbative part of the theory, in Table 5.

As Table 4 indicates, the obtained values extracted from B_W and B_T are very similar to the values in Table 1, for NLO model. However our result for the variable B_W shows a slight decrease; but it is still in the range of the values predicted by the QCD theory.

If we compare the above values with those obtained by using the shape function model [6] we observe that within the statistical errors they are very similar to each other. We conclude that the method of the variance for measuring the QCD parameters conforms well to the different methods used elsewhere.

4. CONCLUSION

We have studied the variance distribution for different event shape variables by using the NLO model, the dispersive model and the shape function model. The strong coupling constant (α_s) in perturbative theory is calculated by fitting the variance with the above models. Also by fitting the variance distribution on the non-perturbative region, the non-perturbative parameter is measured. This parameter is (α_0) for the

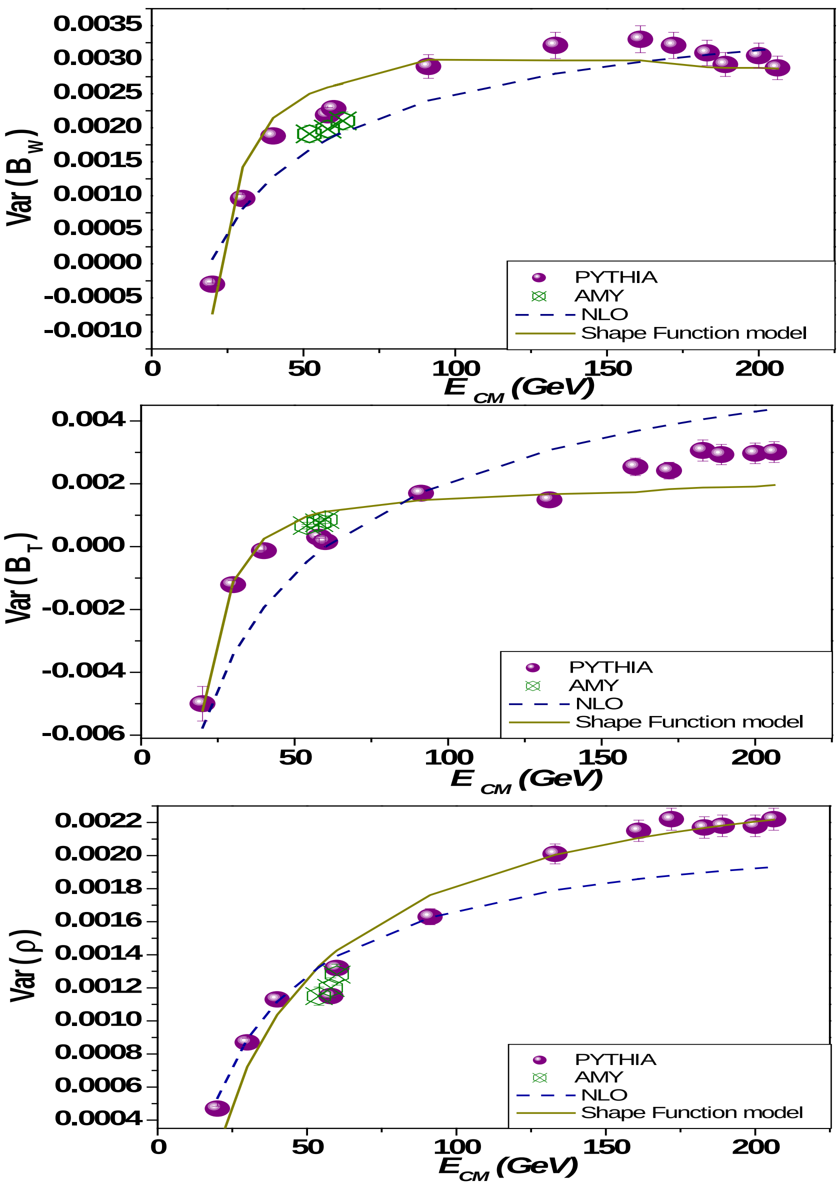

Fig. 3 – The variance distributions for four event shape variables by fitting the shape function model.

dispersive model and (λ) for the shape function model. Finally we summarized the obtained values and we see that these measurements are consistent with the previously results [5, 6] and also with the QCD prediction [10].

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