AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

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A B S T R A C T

Both mesons and baryons are constructed from a set of three fundamental particles called aces. The aces break up into an isospin doublet and singlet. Each ace carries baryon number $\frac{1}{3}$ and is consequently fractionally charged. SU_3 (but not the Eightfold Way) is adopted as a higher symmetry for the strong interactions. The breaking of this symmetry is assumed to be universal, being due to mass differences among the aces. Extensive space-time and group theoretic structure is then predicted for both mesons and baryons, in agreement with existing experimental information. An experimental search for the aces is suggested.

*) This work was supported by the Air Force Office of Scientific Research and the National Academy of Sciences - National Research Council, U.S.A.

1. INTRODUCTION

We wish to consider a higher symmetry scheme for the strongly interacting particles based on the group SU_3. The way in which this symmetry is broken will also concern us. Motivation, other than aesthetic, comes from an attempt to understand certain regularities, described below, in the spectra of particles and resonances. Since we deal with the same underlying group as that of the Eightfold Way [1], particle classification will be similar in the two models. However, we will find restrictions on the representations that may be used to classify particles, restrictions that are not contained in the Eightfold Way. The $(N, \Lambda, \Sigma, \Xi)$ and the pseudoscalar mesons will fall into octets; the vector mesons will be grouped into an octet and singlet, where the two representations will mix by a predictable amount when unitary symmetry is broken; while the $(N^*_{\frac{3}{2}}(1238), Y^*_1(1385), \Xi^*_{\frac{1}{2}}(1530), Z^-_0(1675?))$ will form a decuplet in the usual manner. The restriction of representations will allow us to understand certain features concerning the organization of these particles. We will also be able to obtain a deeper understanding of both the meson and baryon mass spectrum by relating one to the other.

The two symmetry schemes differ in the way particles or resonances are constructed. In the Eightfold Way, the 8 pseudoscalar mesons may be thought of as bound states of a fundamental triplet (p, n, Λ). For example, the π^+ would be represented by $\bar{n}p$, the K^- by $\bar{p}\Lambda$, etc. In the language of group theory, the 8-dimensional representation of SU_3 containing the mesons is included in the 9-dimensional baryon $\otimes$ antibaryon cross product space, i.e., $\bar{3} \otimes 3 = 8 \oplus 1$. However, if as in the Sakata model [2] we attempt to construct the baryons out of this triplet (for example $n \sim \bar{p}pn$, $\Xi^- \sim \bar{p}\Lambda\Lambda$, etc.) we are no longer able to classify them into the familiar group of 8 particles. The difficulty stems from the fact that the eight-dimensional representation describing the baryons is not contained in the 27-dimensional antibaryon $\otimes$ baryon $\otimes$ baryon cross product space, $\bar{3} \otimes 3 \otimes 3$. In the decomposition

$\overline{3} \otimes 3 \otimes 3 = 3 \oplus 3 \oplus \overline{6} \oplus 15$, only the 15-dimensional representation can accommodate all 8 baryons. Unfortunately this representation contains other particles whose masses may be predicted by the Gell-Mann - Okubo mass formula [3]

$$m = m_0 \left\{ 1 + aY + b\left[I(I+1) - 1/4\, Y^2\right] \right\} \tag{1.1}$$

Since these particles or resonances do not seem to be present in nature, we must abandon the Sakata model and work with the 8 baryons themselves as "fundamental" units.

There is, however, another possibility based on a genuine desire to keep certain elements of the Sakata model. If we build the baryons from a triplet of particles (p_0, n_0, Λ_0), (p_0, n_0) being a strangeness zero isospin doublet and Λ_0 a strangeness -1 singlet, using $3 \otimes 3 \otimes 3$ instead of $\overline{3} \otimes 3 \otimes 3$ we find that classification of baryons into a set of 8 is possible since $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$. We note that the 10-dimensional representation is present so that the $N^*_{\frac{3}{2}}$ decuplet may also be constructed from our three fundamental units. The 27-dimensional representation which occurs naturally in the Eightfold Way and which does not seem to be used by nature is suggestively absent. The only difficulty is that now the baryons seem to have baryon number 3. This we get around by assigning baryon number 1/3 to each member of the basic triplet, which leads via the Gell-Mann - Nishijima charge formula, $Q = e\left[I_z + \frac{1}{2}(B+S)\right]$, to non-integral charges for (p_0, n_0, Λ_0) [4]. The isospin doublet (p_0, n_0) contains charges $(\frac{2}{3}, -\frac{1}{3})$ while the isospin singlet Λ_0 has charge $-\frac{1}{3}$. We shall call p_0, n_0, or Λ_0 an "ace". Note that the charges of the aces are just those of (p, n, Λ), but shifted by a unit of $-\frac{1}{3}$. The isospin and strangeness content, along with space-time properties, remain the same. We will work with these aces as fundamental units from which all mesons and baryons are to be constructed.
It is quite possible that aces are completely fictitious, merely providing a convenient way of expressing a symmetry not present in the Eightfold Way. On the other hand, as we shall see, an experimental search for aces would definitely seem worthwhile.

2. THE BARYON OCTET

For convenience, let us designate the aces (p_o, n_o, Λ_o) by (A_1, A_2, A_3). In order to construct the states representing the eight baryons we consider the reduction of the 27-dimensional cross product space of "treys"[6] $A_\alpha A_\beta A_\gamma$[5] $(\alpha, \beta, \gamma = 1,2,3)$ into irreducible representations[6].

$$A_\alpha \otimes A_\beta \otimes A_\gamma \sim T_{\alpha\beta\gamma,} \oplus T_{\alpha\beta,\gamma} \oplus T_{\alpha\gamma,\beta} \oplus T_{\alpha,\beta,\gamma}$$

$$3 \otimes 3 \otimes 3 \sim 10 \oplus 8 \oplus 8 \oplus 1 \qquad (2.1)$$

Here $T_{\alpha\beta\gamma,}$ is totally symmetric in its indices and will represent members of the $N^*_{\frac{3}{2}}(1238)$ decuplet, while $T_{\alpha\beta,\gamma}$ is symmetric in α, β; being explicitly given by

$$T_{\alpha\beta,\gamma} = \frac{1}{2\sqrt{2}} \left\{ T_{\alpha\beta\gamma} - T_{\alpha\gamma\beta} + T_{\beta\alpha\gamma} - T_{\beta\gamma\alpha} \right\}$$

and will be taken to represent the nucleon octet ($T_{\alpha\gamma,\beta}$ could of course be used just as well). $T_{\alpha,\beta,\gamma}$ is totally antisymmetric in α, β, γ and allows for the existence of an $I = 0$, $S = -1$ singlet to be identified with the $Y^*_0(1405)$. The fact that the $N^*_{\frac{3}{2}}$ does not seem to belong to the 27-dimensional representation[7] of SU_3 may be taken as a prediction of this model.

We now list the baryon states :

$$n = -T_{22,1} \qquad p = T_{11,2}$$

$$\Lambda = \sqrt{2/3} \left\{ T_{23,1} - T_{13,2} \right\}$$

$$\Sigma^- = T_{22,3} \qquad \Sigma^0 = \sqrt{2}\, T_{12,3} \qquad \Sigma^+ = -T_{11,3} \qquad (2.2)$$

$$\Xi^- = -T_{33,2} \qquad \Xi^0 = T_{33,1}$$

For example, by inspection of the subscripts, $T_{22,3}$ has

$I_z = (+\frac{1}{2}) + (-\frac{1}{2}) + 0$ and strangeness $S = 0+0+(-1)$. In the limit of unitary symmetry the 3 aces are indistinguishable and all baryon states have the same structure and mass. This is represented in Fig. 1a. The mass of a baryon $T_{\alpha\beta,\gamma}$ may be thought of as $m_\alpha + m_\beta + m_\gamma - E_{\alpha\beta} - E_{\alpha,\gamma} - E_{\beta,\gamma}$ where, for example, m_α represents the mass of the ace α and the E's are binding energies. ($E_{\alpha\beta}$ is the binding energy between the two aces α and β when they are connected by a solid line. Binding energies for dashed line connections are given by $E_{\alpha,\beta}$.) In the unitary symmetric limit we have $m_\alpha = m_\beta = m_\gamma$, $E_{\mu\nu} = E_{\sigma\delta}$, $E_{\mu,\nu} = E_{\sigma,\delta}$ so that the masses of all the baryons are identical. We now assume that unitary symmetry is broken due to the fact that the singlet A_3 is heavier than the doublet (A_1, A_2) [8], in analogy to the Sakata model where the Λ was assumed heavier than the (p,n). The baryons now break up into distinguishable groups, so that instead of Fig. 1a, we have Fig. 2a. As a first approximation, neglecting differences in binding energies, we immediately find [9]:

$$\underset{(1115)}{m(\Lambda)} = \underset{(1193)}{m(\Sigma)}, \quad \underset{(1154)}{[m(\Sigma) + m(\Lambda)]/2} = \underset{(1130)}{[m(\Xi) + m(N)]/2} \tag{2.3}$$

The Σ and Λ masses are expected to differ, however, because the ace A_3 is bound differently in the two cases. To obtain more accurate results one would have to say something about the binding energy between aces.

It is interesting to note that if one assumes that the breaking of unitary symmetry by electromagnetism takes place by virtue of the fact that the $A_2 A_1$ mass difference is not zero, then independent of the values of the binding energies we have the mass difference equation [10]:

$$\underset{(5.6 \pm 1.4)}{m(\Xi^-) - m(\Xi^0)} = \underset{(7.0 \pm 0.5)}{m(\Sigma^-) - m(\Sigma^+) - [m(n) - m(p)]} \tag{2.4}$$

Assuming that A_2 (the more negative member of the doublet) is heavier than A_1 and neglecting shifts in binding energies due to the electromagnetic breaking of the symmetry we find the qualitatively correct result that within any charge multiplet, the more negative the mass, the heavier the particle.

3. THE BARYON DECUPLET

Figure 1b represents the decuplet $T_{\alpha\beta\gamma}$, in the limit of unitary symmetry. A_1, A_2, and A_3 are indistinguishable and all binding energies are equal. The 10 members are completely degenerate. As for the baryon octet, we assume that unitary symmetry is mainly broken by virtue of the fact that A_3 is heavier than A_1 and A_2. The objects of the decuplet will no longer be identical but appear as in Fig. 2b. Neglecting shifts in binding energies due to the breaking of unitary symmetry it is clear that the decuplet resonances increase their masses linearly with strangeness, i.e.,

$$\underset{(147)}{m(Y_1^*) - m(N^*_{3/2})} = \underset{(145)}{m(\Xi^*_{1/2}) - m(Y_1^*)} = \underset{(?)}{m(Z_0^-) - m(\Xi^*_{1/2})} \tag{3.1}$$

Since the decuplet and octet are constructed from the same set of particles we may try to obtain a formula relating the masses of the two different representations. The $\Xi^*_{\frac{1}{2}}\,Y^*_1$ mass difference is given by $m_3 - m_2 - E_{33} + E_{ab}$ (a,b = 1,2 depending on the charges we take) [11]. $m_\Lambda - m_N$, $m_\Sigma - m_N$, $m_\Xi - m_\Lambda$, $m_\Xi - m_\Sigma$ all contain the difference $m_3 - m_2$ and are of roughly the right order of magnitude. If we pick $\Xi - \Sigma$, the only mass difference whose binding energy term is $-E_{33} + E_{ab}$ we find :

$$\underset{(145)}{m(\Xi^*_{1/2}) - m(Y_1^*)} = \underset{(130)}{m(\Xi) - m(\Sigma)} \tag{3.2}$$

Note that we do not expect this equation to hold exactly, even in the limit of unitary symmetry, because the spins, and hence the ace dynamics or binding energies, differ for the two representations.

The Baryon Singlet

The $Y_0^*(1405)$, in the limit of unitary symmetry, is shown in Fig. 1c. Figure 2c indicates the Y_0^* when the symmetry is broken by increasing the A_3 mass. Since the Y_0^* is a unitary singlet nothing quantitative can be said about its mass.

4. THE VECTOR MESON OCTET AND SINGLET

Meson states are built from the same units (A_1, A_2, A_3) as the baryons. They are contained in the anti-ace $\otimes$ ace cross product space :

$$\begin{array}{ll} A^\alpha \otimes A_\beta \sim \left(D^\alpha_\beta - 1/3\,\delta^\alpha_\beta\, D^\gamma_\gamma \right) \oplus \delta^\alpha_\beta\, D^\gamma_\gamma \\ \bar{3} \otimes 3 \sim \quad 8 \quad \oplus \quad 1 \end{array} \tag{4.1}$$

where A^α stands for the anti-ace of A_α. Because of the nature of the decomposition of $\bar{3} \otimes 3$, mesons can only fall into groups of 8 or 1. The Eightfold Way would allow, in addition, groups of 10 and 27, possibilities which nature does not seem to take advantage of. We have pictorially represented in Fig. 1d, 1e the two possible meson representations in the limit of unitary symmetry.

The vector meson states are given by :

$$\rho^- = D^1_2 \qquad \rho^0 = 1/\sqrt{2}\,(D^1_1 - D^2_2) \qquad \rho^+ = D^2_1$$

$$K^{*0} = D^3_2 \qquad K^{*+} = D^3_1$$

$$K^{*-} = D^1_3 \qquad \bar{K}^{*0} = D^2_3$$

$$\omega_8 = 1/\sqrt{6}\,(D^1_1 + D^2_2 - 2D^3_3) \tag{4.2}$$

for the octet, and

$$\omega_0 = 1/\sqrt{3}\; D^\gamma_\gamma = 1/\sqrt{3}\,(D^1_1 + D^2_2 + D^3_3) \tag{4.2a}$$

for the unitary singlet. In the limit of unitary symmetry the masses of the singlet and octet must be the same because the binding is identical in both representations and all aces are degenerate. In fact, if the forces are such as to bind the aces into an octet, they must also bind the aces into a singlet. It is important to note that this is not the case for baryons where the singlet, octet, and decuplet bindings all differ, even in the unitary symmetric limit.

Unitary symmetry must be broken for the mesons in exactly the same way as it was broken for the baryons, that is, the isospin singlet A_3 (or its anti-ace A^3) must become heavier than the isospin doublet (A_1, A_2). Breaking the symmetry by giving A_3 a larger mass not only splits the masses of the eight vector mesons, but it also mixes the singlet ω_0 with the $I = 0$ member, ω_8, of the octet. As a result of mixing the physically observable particles ω and φ are formed. Since A_3 becomes distinguishable from A_1 and A_2, ω_0 and ω_8 must mix in such a way as to separate (A_1, A_2) from A_3. This immediately leads to

$$\varphi = D_3^3 \qquad (4.3)$$

$$\omega = 1/\sqrt{2}\,(D_1^1 + D_2^2)$$

The plus and not the minus sign that appears in the "deuce" expression for ω distinguishes the ω from the ρ^0. Figure 2d shows the vector meson states after unitary symmetry has been broken. Using the empirical fact that when dealing with mesons one must always work with squares of masses, and neglecting changes in the bindings due to the breaking of unitary symmetry we immediately have [12]

$$\begin{array}{ccc} m^2(\omega) & = & m^2(\rho) \\ (784)^2 & & (750)^2 \end{array} \qquad (4.4)$$

$$\begin{array}{ccc} m^2(\varphi) & = & 2m^2(K^*) - m^2(\rho) \\ (1018)^2 & & (1007)^2 \end{array} \qquad (4.5)$$

Mixing has made the φ as heavy as possible. The mixing angle Θ defined by

$$\varphi = \omega_0 \sin\Theta - \omega_8 \cos\Theta$$

$$\omega = \omega_0 \cos\Theta + \omega_8 \sin\Theta \tag{4.6}$$

comes out to be

$$\sin\Theta = \sqrt{1/3}\;,\quad \cos\Theta = \sqrt{2/3}\;,\quad \text{or}\quad \Theta = 35.3^\circ \tag{4.7}$$

as compared with the empirical value of $\Theta \approx 38^\circ$ [13].

Only now has the real power of dealing with three basic objects become apparent. When working with the baryons, one could easily say, for example, that the more strangeness a particle carries, the heavier it is. But by using the basic triplet of aces we are able to say, after inspecting the baryons, that for an octet and singlet of mesons it is a <u>non-strange</u> particle that is heaviest of all; for it contains more A_3 than the strangeness carrying meson does.

Interestingly enough, we are able to improve equations (4.4) and (4.5). If we define the traceless matrix V of the vector meson octet in the conventional way :

$$V = \begin{pmatrix} \omega_8/\sqrt{6} + \rho^0/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & \omega_8/\sqrt{6} - \rho^0/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -2\omega_8/\sqrt{6} \end{pmatrix}$$

and let the matrix G be given by

$$G_{\alpha\beta} = V_{\alpha\beta} + \delta_{\alpha\beta}\,\omega_o/\sqrt{3} \tag{4.8}$$

then the mass formulae (4.4), (4.5) may be alternatively derived by assuming that

$$H^2 \approx H_1^2 = m_1^2\,\mathrm{Tr}\,\bar{G}G - m_2^2\,\mathrm{Tr}\,\bar{G}\left[G\lambda_8 + \lambda_8 G\right] \tag{4.9}$$

for the mass terms in the square of the Hamiltonian H [14]. Here Tr stands for trace while $m_1^2 = (2m^2(K^*)+m^2(\rho))/3$, $m_2^2 = (m^2(K^*)-m^2(\rho))/\sqrt{3}$, and

$$\lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Note that we have suppressed all terms involving $\mathrm{Tr}\,G = \sqrt{3}\,\omega_o$.

More generally, however, we may write for the mass terms in the square of the Hamiltonian,

$$\begin{aligned} H^2 = H_1^2 + m_3^2\,\mathrm{Tr}\,\bar{G}\,\mathrm{Tr}\,G + m_4^2\left[\mathrm{Tr}\,\bar{G}\,\mathrm{Tr}\,G\lambda_8 + \mathrm{Tr}\,G\,\mathrm{Tr}\,\bar{G}\lambda_8\right] + \\ m_5^2\,\mathrm{Tr}\,\bar{G}\lambda_8\,\mathrm{Tr}\,G\lambda_8 + m_6^2\,\mathrm{Tr}\,\bar{G}\lambda_8 G\lambda_8\,, \end{aligned} \tag{4.10}$$

where we treat the terms in m_3^2 to m_6^2 as perturbations to H_1^2. Since the term $m_3^2\,\mathrm{Tr}\,\bar{G}\,\mathrm{Tr}\,G$ is invariant under SU_3 while the terms multiplying m_4^2, m_5^2, and m_6^2 are not, we might expect that to a good approximation we only need keep the perturbation $\mathrm{Tr}\,\bar{G}\,\mathrm{Tr}\,G$, i.e.,

$$H^2 = H_1^2 + m_3^2\,\mathrm{Tr}\,\bar{G}\,\mathrm{Tr}\,G \tag{4.11}$$

Doing this we immediately arrive at

$$\left[m^2(\omega) - m^2(\varphi)\right]/2 = m^2(\varphi) + m^2(\rho) - 2m^2(K^*) \tag{4.12}$$

5. THE PSEUDOSCALAR MESONS

In the limit of unitary symmetry we have nine pseudoscalar mesons of equal mass, just like the vector meson case. The members of the octet we call (π, K, η_8) while the singlet is denoted by η_0. Breaking the symmetry by increasing the A_3 mass yields relations analogous to (4.4) and (4.5), i.e.,

$$m^2(\pi_0^0) = m^2(\pi) \tag{5.1}$$

$$m^2(\eta) = 2m^2(K) - m^2(\pi) \tag{5.2}$$

$$(690)^2$$

where η and π_0^0 are the physically observable particles that result from mixing η_8 and η_0, just as φ and ω are mixtures of ω_8 and ω_0. Furthermore, by using arguments identical to those given in the vector meson case we obtain the analogue of the mass relation (4.12)

$$\left[m^2(\pi_0^0) - m^2(\pi)\right]/2 = m^2(\eta) + m^2(\pi) - 2m^2(K) \tag{5.3}$$

Substituting the physical masses for π, K, and η we see that $m^2(\pi_0^0)$ comes out negative !

Fortunately we have an argument that alleviates the difficulties. After increasing the A_3 mass we found $m^2(\pi_0^0) = m^2(\pi)$. Therefore in this approximation, and this is the crucial point, $m^2(\pi_0^0)$ is very small compared to the mass square differences that exist among the pseudoscalar mesons. A small perturbation (one which changes mass squares by an amount small compared to changes initiated by the A_3 mass increase) may be enough to shift the mass square of the π_0^0 down to zero or even negative values. We might say that the π_0^0 is formed from two very massive objects that are extremely tightly bound. Energy conservation leaves the π_0^0 with a small positive energy or mass. If we introduce a perturbation that decreases the mass of the fundamental objects or increases the binding strength then the π_0^0 may no longer possess a net positive energy and cannot correspond to a physical particle. This, or something like it is evidently the situation in the pseudoscalar meson case.

It is interesting to note that we would not expect the removal of the ω in analogy to the elimination of the π_0^0. The perturbation given by (4.11) is expected to shift $m^2(\omega)$ by an amount small compared to the mass square splittings induced by the increase of the A_3 mass. Since $m^2(\omega)$ is larger than the vector meson mass square splittings there is no danger of the ω's disappearing through the introduction of a perturbation.

With the removal of the π_0^0 we expect that the pseudoscalar mesons behave as an isolated octet. This is indicated in Fig. 3. Neglecting changes in the binding energies due to the breaking of unitary symmetry we immediately obtain, by counting squares, the celebrated Gell-Mann - Okubo formula :

$$m^2(K) = 3/4\, m^2(\eta) + 1/4\, m^2(\pi) \tag{5.4}$$

Neglecting differences of binding energies within octets it is clear that we have the relation

$$\underset{(.22\,\mathrm{GeV}^2)}{m^2(K^*) - m^2(\rho)} = \underset{(.22\,\mathrm{GeV}^2)}{m^2(K) - m^2(\pi)} \tag{5.5}$$

6. COMMENTS

The degree to which unitary symmetry is violated seems precarious: it appears to change from one representation to another. For the pseudoscalar mesons, for example, the violation seems enormous. Unitary symmetry gives $m^2(\pi) = m^2(K) = m^2(\eta)$, yet, for physical particles $m^2(\pi) \ll m^2(K) \approx \approx m^2(\eta)$. For the baryons, on the other hand, unitary symmetry works reasonably well, predicting $m(N) = m(\Lambda) = m(\Sigma) = m(\Xi)$. In spite of these differences, our model suggests that the strength of unitary symmetry violation is the same in both cases: for the breaking of unitary symmetry is measured by ace mass splittings, i.e.,

$$[m(A_3) - m(A_1)]/m(A_1)$$

and not by $(m^2(K)-m^2(\pi))/m^2(\pi)$ or $(m(\Lambda)-m(N))/m(N)$. The amount of unitary symmetry breaking is universal, it is the same for mesons as baryons, it is identical for octets and decuplets. This accounts for roughly the same mass differences within the meson octets, the baryon octet, and the baryon decuplet, irrespective of the masses of the members of these representations.

Although our aces p_0, n_0, Λ_0 have "peculiar" baryon number and charge, their space-time properties should be identical to p, n, Λ (in this respect we may think of them as p, n, Λ with charge translated by a unit of $-\frac{1}{3}$). This places a restriction on the quantum numbers that a meson may possess. For example, for spin 0 or spin 1 non-strange mesons, the following J^{PG} are excluded :

1) 0^{+-}, 0^{--}, 1^{-+} for isospin 0 states;
2) 0^{++}, 0^{-+}, 1^{--} for isospin 1 states.

Up to now no resonances have been found with these quantum numbers.

It is natural to associate the baryons with the lowest energy state of the trey system that represents them. This presumably means that the 3 aces are all in orbital angular momentum S states with the spin of one pair summing to 0. Similarly, the pseudoscalar mesons would correspond to an ace and anti-ace whose orbital angular momentum and total spin are both 0 (i.e., $^1S_0$ state). Since the parity of a nucleon (ace) and antinucleon (anti-ace) state are opposite, we see that the intrinsic parity of the pion should be odd while that of the nucleon should be even.

We have obtained the result that Λ_o is heavier than (p_o, n_o) by an amount characteristic of the gross mass splittings within an octet, i.e., ~ 200 MeV. We therefore expect that Λ_o, if it exists, would undergo the β decays

$$\begin{aligned} \Lambda_o^{-1/3} &\rightarrow p_o^{+2/3} + e^- + \nu \\ &\rightarrow p_o^{+2/3} + \mu^- + \nu \end{aligned}$$

just as

$$\begin{aligned} \Lambda &\rightarrow p + e^- + \nu \\ &\rightarrow p + \mu^- + \nu \end{aligned}$$

On the basis of the electromagnetic mass splittings within a given isotopic spin multiplet we are also tempted to conjecture that n_o is heavier than p_o, making p_o completely stable (like p) but allowing the decay

$$n_o^{-1/3} \rightarrow p_o^{+2/3} + e^- + \nu$$

just as

$$n \rightarrow p + e^- + \nu$$

An experimental search for the p_o, n_o, Λ_o might prove interesting.

7. CONCLUSIONS

The scheme we have outlined has given, in addition to what we already know from the Eightfold Way, a rather loose but unified structure to the mesons and baryons. In view of the extremely crude manner in which we have approached the problem, the results we have obtained seem somewhat miraculous.

A universality principle for the breaking of unitary symmetry has been suggested. From this followed a qualitative understanding of the meson mass splittings in terms of the baryon mass spectrum, e.g., $m(\Lambda) > m(N)$ implies that $m(\varphi) > m(K^*) > m(\omega) \approx m(\rho)$. The proportionately larger mass splittings within the pseudoscalar meson octet have been explained. Mass formulae relating members of different representations have been suggested, e.g., $\left[m^2(\omega)-m^2(\rho)\right]/2 = m^2(\varphi)+m^2(\rho)-2m^2(K^*)$, $m^2(K^*)-m^2(\rho) = m^2(K)-m^2(\pi)$.

Nature's seeming choice of 1, 8, and 10-dimensional representations for baryons along with 1 and 8-dimensional representations for the mesons has been accounted for without dynamical or "bootstrap" considerations. The existence of a unitary singlet ω_0 which mixes with the octet of vector mesons has been predicted (along with the amount of mixing), while the absence of a unitary singlet for pseudoscalar mesons has been made plausible. For the baryons the model predicted that there were no analogues of $\omega - \varphi$ mixing for either the octet or decuplet, even though there might be singlet baryon states.

The quantum numbers available to a meson have been restricted to those which may be formed from the p, n, Λ and their antiparticles. Finally, the odd intrinsic parity of the pion and opposite nucleon parity fit naturally into the model.

There are, however, a number of unanswered questions. Do aces bind to form only deuces and treys ? What is the particle (or particles) that is responsible for binding the aces ? Why must one work with masses for the baryons and mass squares for the mesons ? And more generally, why does so simple a model yield such a good approximation to nature ?

Our results may be viewed in several different ways. We might say :

1) The relationships we have established are accidents and our model is completely wrong. The formula $m(\Xi) = [3m(\Sigma)-m(N)]/2$ is correct to electromagnetic mass splittings and yet seems entirely "accidental". It certainly would be no great surprise if our mass formulae were accidents too.

2) There is a certain simplicity present, additional to that supplied by the Eightfold Way, but this simplicity has nothing to do with our model [15]. For example, the Gell-Mann - Okubo mass formula may be written for any SU_3 representation as :

$$m^2 = m_0^2 \left\{ 1 + b'(m_0^2)\left[I(I+1) - 1/4\, Y^2\right] \right\}$$

for mesons,

$$m = m_0 \left\{ 1 + a(m_0)\, Y + b(m_0)\left[I(I+1) - 1/4\, Y^2\right] \right\}$$

for baryons, where m_0, b', a, b vary from one representation to another. The quantities b', a, and b may be considered functions of m_0 or m_0^2. Equation (5.5) may be "explained" by postulating that $b'(m_0^2)$ goes like : $b'(m_0^2) \sim 1/m_0^2$. Equation (3.2) would follow if $a(m_0)$ and $b(m_0)$ were any slowly varying function of m_0, going for instance like $1/m_0$. Relations of this type could undoubtedly result from many different theories.

3) Perhaps the model is valid inasmuch as it supplies a crude qualitative understanding of certain features pertaining to mesons and baryons. In a sense, it could be a rather elaborate mnemonic device.

4) There is also the outside chance that the model is a closer approximation to nature than we may think, and that fractionally charged aces abound within us.

ACKNOWLEDGEMENTS

We would like to thank Dr. J. Prentki and Dr. L. Van Hove. Their comments and criticisms have been very useful.

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4) Dr. Gell-Mann in a recent preprint, Physics Letters, to be published, has independently speculated about the possible existence of these particles. His primary motivation for introducing them differs from ours in many respects.

5) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".

6) R.E. Behrends, J. Dreitlein, C. Fronsdal and W. Lee, Rev.Mod.Phys., 34, 1 (1962).

7) S.L. Glashow and A.H. Rosenfeld, Phys.Rev. Letters 10, 192 (1963).

8) Since A_1 and A_2 form an isospin doublet their mass difference must be electromagnetic in origin and hence negligible in first approximation.

9) These formulae are obtained by counting the number of shaded squares (the number of A_3's) that are present in each baryon. We have averaged the masses of the Λ and Σ somewhat arbitrarily in Eq. (2.3).

10) This formula was first derived with other techniques by : S. Coleman and S.L. Glashow, Phys.Rev. Letters 6, 423 (1961).

11) Charge independence would require $E_{11} = E_{22} = E_{12}$ and $E_{1,1} = E_{2,2} = E_{1,2}$, but not $E_{11} = E_{1,1}$, etc.

12) For example, we would write $m^2(\rho^+) = m_1^2+m_2^2-(E_1^2)^2$ where E_1^2 is the binding between ace 1 and anti-ace 2. The author has no explanation for why squares of masses or binding energies should appear when working with mesons. This is especially mysterious in any model, like ours, where particles are treated as composite.

13) J.J. Sakurai, Phys.Rev. 132, 434 (1963).

14) For a discussion of the G matrix see : S. Okubo, Physics Letters 5, 165 (1963).

15) In a recent preprint S. Coleman and S.L. Glashow have considered another mass formulae producing model.

FIGURE CAPTIONS

Figure 1 These deuces and treys correspond to all known particle representations in the limit of unitary symmetry.

a) This trey stands for a member of a baryon octet. The shaded circles at the vertices are aces, while the solid and dashed lines denote two different types of binding. The trey is symbolically given by $T_{\alpha\beta,\gamma}$ while the binding energies are $E_{\alpha\beta}$ (solid line); $E_{\alpha,\gamma}$, and $E_{\beta,\gamma}$ (dashed lines). In the unitary symmetry limit the three aces are indistinguishable.

b) This trey represents a member of a baryon decuplet and is written as $T_{\alpha\beta\gamma,}$.

c) The trey stands for $T_{\alpha,\beta,\gamma}$, a unitary singlet.

d)–e) The deuces shown correspond to members of meson octets $D^{\alpha}_{\beta} - (\delta^{\alpha}_{\beta}/3)D^{\gamma}_{\gamma}$; and singlets, $(1/\sqrt{3})D^{\gamma}_{\gamma}$. The open circles are anti-aces.

Figure 2 We view the particle representations with unitary symmetry broken. One of the three aces has now become distinguishable from the other two. It is pictured as a shaded square. The open squares are anti-aces. The mass splittings within representations are induced by making the squares heavier than the circles. Since the same set of aces are used to construct all particles, mass relations connecting mesons and baryons may be obtained.

Figure 3 The isolated octet of pseudoscalar mesons is represented after unitary symmetry has been broken and the π^{0}_{0} has been removed.

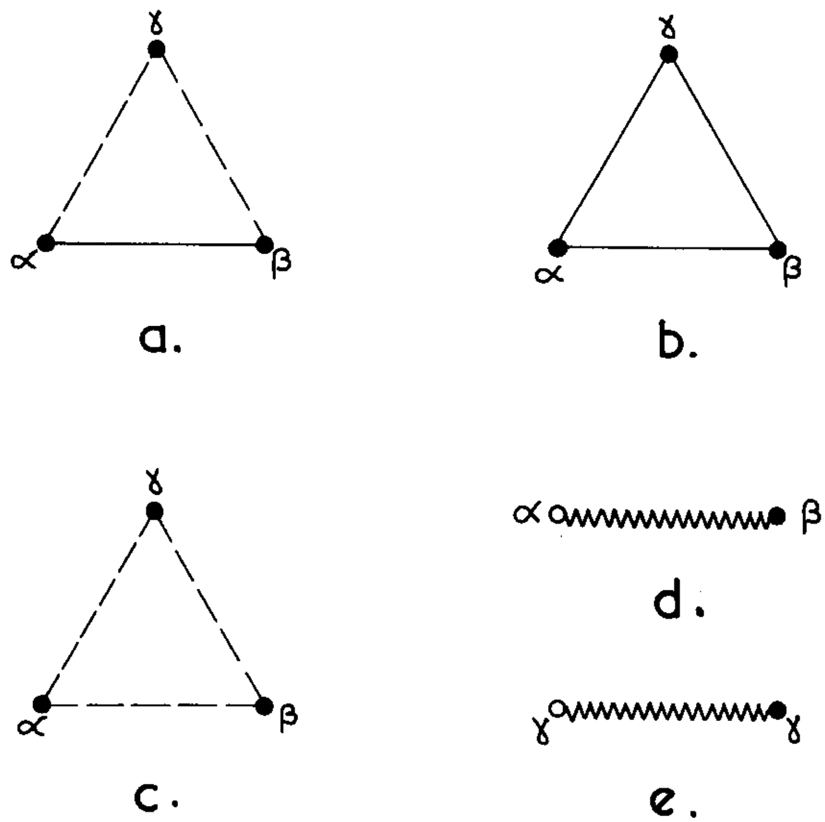

FIG. 1

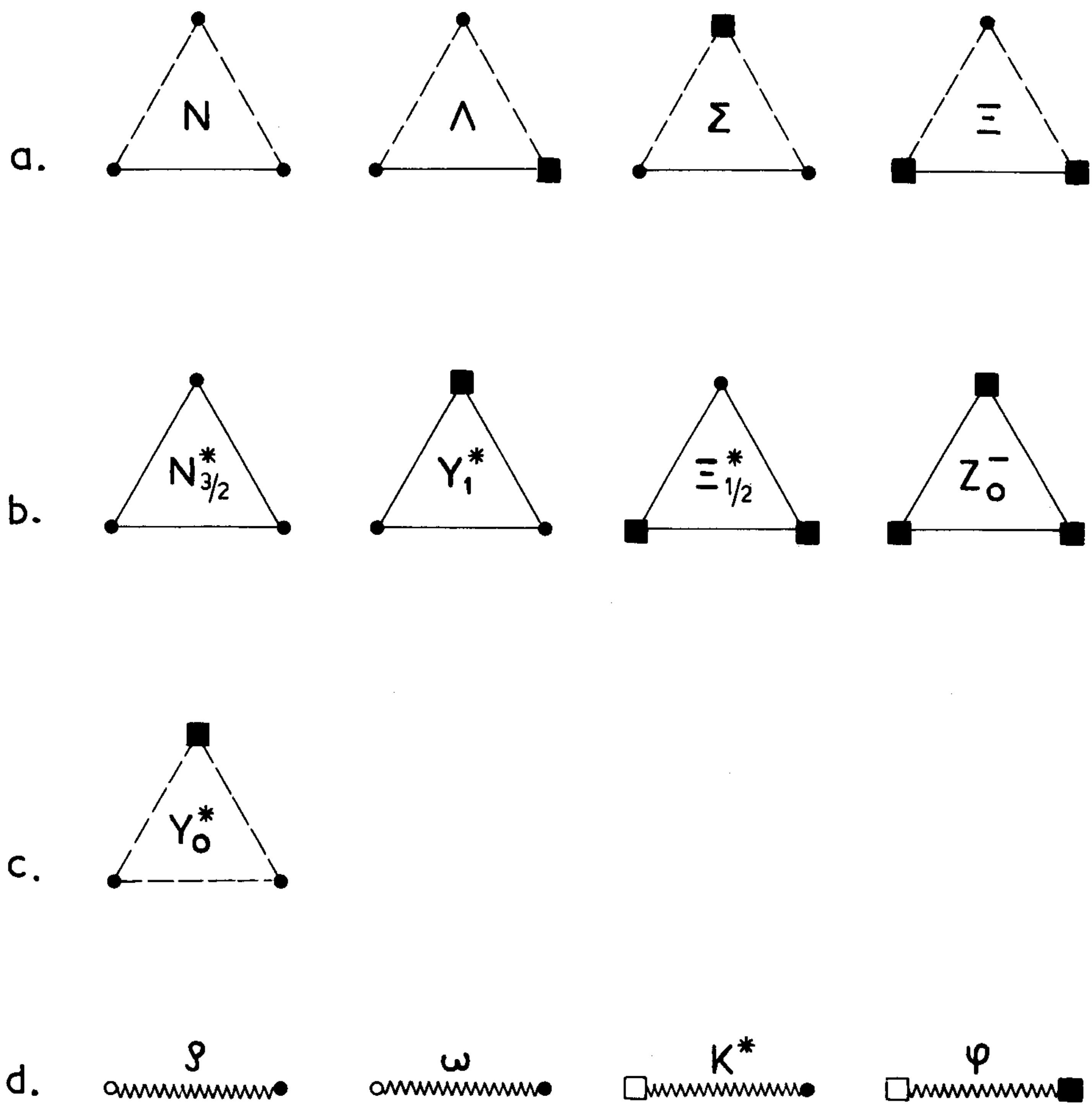

FIG. 2

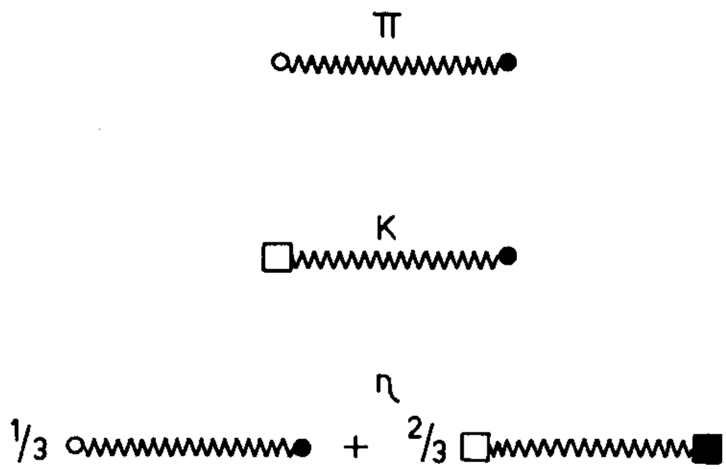

FIG. 3