



## PAPER

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Collective excitation of Bose–Einstein condensate of  $^{23}\text{Na}$  via high-partial wave Feshbach resonanceZhenlian Shi<sup>1,2</sup>, Ziliang Li<sup>1,2</sup>, Pengjun Wang<sup>1,2,\*</sup> , Wei Han<sup>1,2</sup>, Lianghui Huang<sup>1,2</sup>, Zengming Meng<sup>1,2</sup>, Liangchao Chen<sup>1,2</sup> and Jing Zhang<sup>1,2,\*</sup><sup>1</sup> State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-electronics, Shanxi University, Taiyuan, Shanxi 030006, People's Republic of China<sup>2</sup> Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan, Shanxi 030006, People's Republic of China

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E-mail: [pengjun\\_wang@sxu.edu.cn](mailto:pengjun_wang@sxu.edu.cn), [jzhang74@yahoo.com](mailto:jzhang74@yahoo.com) and [jzhang74@sxu.edu.cn](mailto:jzhang74@sxu.edu.cn)**Keywords:** collective excitation, Bose–Einstein condensate, Feshbach resonance

## Abstract

We experimentally observe the collective excitation (called surface-mode excitation) of Bose–Einstein condensate of  $^{23}\text{Na}$  by ramping the external magnetic field across the high-partial wave magnetic Feshbach resonance corresponding to vary the atomic interaction. We check the collective surface mode excitation of  $|1, 1\rangle$  state for the three d-wave and three g-wave Feshbach resonances below 600 G and find that only two d-wave resonances present the strong excitation, another d-wave resonance only creates a weak excitation, and all g-wave resonances do not, which reflects the strength of these magnetic Feshbach resonances. For the collective excitation, the excitation of surface modes along the axial weak-confinement and radial strong-confinement of optical dipole trap shows different characteristics. We also study the lifetime of the collective oscillation by measuring the damping rate of the oscillation amplitude, which is caused by the mechanisms of dephasing effect and collisional relaxation. This excitation method gives us a new tool for investigating the properties of ultracold quantum gases without changing the trap frequencies.

## 1. Introduction

Ultracold atomic gases are a powerful platform for providing a unique opportunity to investigate many interesting quantum phenomena in many-body physics [1], and especially study quantum matter in the presence of a variety of gauge fields [2, 3], which give deep insights into physics that are difficult to realize in solid state systems [4–9]. Collective low energy excitation, which is a small deviation from static equilibrium, has been an essential probe tool for investigating properties of an atomic gas, for example collisional and dynamical properties [10–21], equation of state [22–29], and dynamical many-body physics in the mixture of bosonic and fermionic atoms [30–34].

There are two kinds of collective excitation in ultracold atomic gases. One is the compression-mode excitation, which is created by quenching or modulating the trap frequency of the magnetic or optical trap [11, 20, 24–26, 28, 32, 35–42]. The other is the surface-mode excitation where the volume of quantum gas is not changed. This excitation can be created by perturbing the magnetic trap with far detuning light [12, 43], suddenly rotating the trapping potential [44–46], modulating the s-wave scattering length with oscillation of the magnetic field [47, 48], or quenching a series of atomic interspecies interaction with  $\pi$  radio frequency pulses [30]. Recently a surface-mode excitation method is developed by ramping the external magnetic field across a d-wave shape resonance of  $^{41}\text{K}$  (the shape resonance originates from the coupling of the bound state and the scattering atomic state in a same channel, which is different from the case of Feshbach resonance, that arises from coupling between the bound state in a closed channel and free atomic state in the open channel) due to the atom-molecule conversion. The observed oscillation provides indirect evidence for the molecular states in the same entrance channel of scattering atoms [49].

In this paper, we experimentally study the surface mode excitation of Bose–Einstein condensate (BEC) of  $^{23}\text{Na}$  via the high-partial wave magnetic Feshbach resonance. There are three d-wave and three g-wave Feshbach resonances below 600 G for  $|1, 1\rangle$  state. We check the collective surface mode excitation by ramping the external magnetic field across the magnetic Feshbach resonance and find that two d-wave resonances at 493 G and 536.7 G can strongly create the excitation, another d-wave resonance at 586 G creates a weak excitation, and all g-wave resonances do not. Therefore, the strength of these magnetic Feshbach resonances can be assigned from the excitation amplitude of the collective surface mode. According to the measurement of the loss rate of these Feshbach resonances, we may give the clue which kind of magnetic Feshbach resonance can create the surface mode excitation. The excitation of surface modes along the axial weak-confinement and radial strong-confinement of optical dipole trap is studied, which present the different excitation. The dependence of oscillation frequency and decay rate on the trap frequency is extracted. We also observe the damping induced by the mechanism of the dephasing effect and collisional relaxation, and study the dependence on the atom numbers. Furthermore, we employ the modulation of the external magnetic field to create the excitation, and observe similar collective behavior.

## 2. Experimental results

### 2.1. Loss spectroscopy of Feshbach resonances

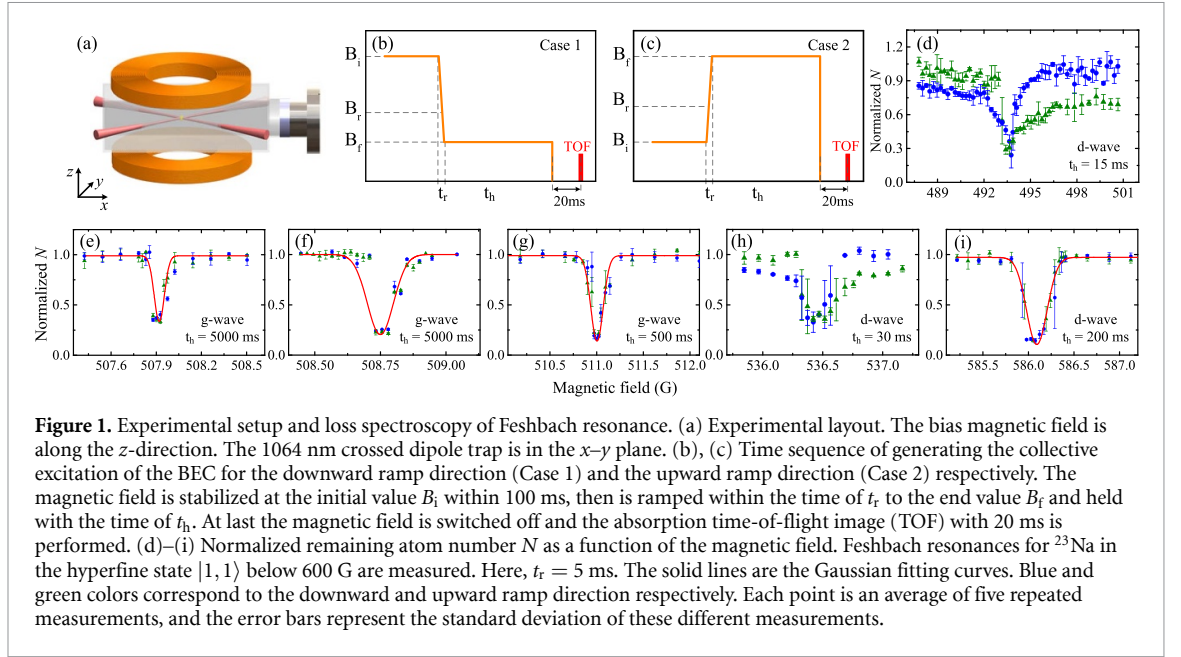
The collective excitation of BEC in experiment is shown schematically in figure 1. We prepare a weakly interacting  $a_{bg} = 54.54 a_0$  ( $a_0$  is the Bohr radius), almost pure BEC of  $^{23}\text{Na}$  atoms in the lowest internal state  $|F = 1, m_F = 1\rangle$ ,  $F$  is the total atomic angular momentum and  $m_F$  is the projection along the quantization axis  $z$ . The experimental apparatus and preparing sequence for creating BEC have been described in our previous works [50, 51]. We start our experiments with an atomic ensemble of  $N \sim 2.5 \times 10^6$   $^{23}\text{Na}$  in a crossed optical trap with trapping frequencies  $(\omega_x, \omega_y, \omega_z) = 2\pi \times (72, 114, 134)$  Hz. We apply a magnetic field gradient for 10 ms to excite the dipole oscillation of the condensate to measure the center of mass for determining the trapping frequency in three directions. The homogeneous external magnetic bias field  $B_{\text{exp}}$  is applied along the  $z$  axis (gravity direction) by a pair of Helmholtz coils, as shown in figure 1(a). We ramp the external magnetic field to an initial field  $B_i$  during 30 ms and wait the additional 100 ms for the magnetic field to stabilize. Then, by ramping the external magnetic field across the Feshbach resonance point to a target value  $B_f$ , we observe the collective oscillation of BEC.

Here, we consider the three d-wave and three g-wave Feshbach resonances below 600 G for  $|1, 1\rangle$  state, which have been predicted and measured in the previous work [52]. First, we determine these Feshbach resonances from the loss spectroscopy as a function of the magnetic field. We ramp the magnetic field quickly  $t_r = 5$  ms in two different directions from a far detuning field  $B_i$  to a  $B_f$  and wait for a holding time  $t_h$  (as shown in figures 1(b) and (c)), after which we switch off and release the atoms from the optical trap to measure the residual atom number from the absorption images after a time of flight (TOF) expansion of 20 ms. The measured loss spectroscopy as shown in figures 1(d)–(i) is fitted by a Gaussian curve to extract the Feshbach resonance location  $B_r$ . The results are listed in table 1, which are in agreement with the previously measured values [52].

It is clearly shown from the loss spectroscopy that the atom loss rate is very lower for three g-wave resonances. Therefore, it needs a long holding time  $t_h = 500$  ms and 5 s at the final magnetic field  $B_f$  for measuring the loss spectroscopy. One of the d-wave resonances at 586.09 G also has a low atom loss rate with the holding time  $t_h = 200$  ms. However, two d-wave resonances at 493.65 G and 536.43 G have the larger atom loss rate with the holding time  $t_h < 30$  ms. Due to the larger atom loss rate, we observe the asymmetry of the lineshape in the loss spectroscopy by means of the different ramping directions for the two d-wave resonances at 493.65 G and 536.43 G. The fast control of the magnetic field is needed when the Feshbach resonance with a large atom loss rate is measured. The dynamics of the magnetic field due to the fast control may induce the asymmetry in the lineshape.

### 2.2. The excited surface mode of BEC

Now we check the collective surface mode excitation by ramping the external magnetic field across the magnetic Feshbach resonance for the six Feshbach resonances below 600 G. When  $B_f$  is not ramped across the Feshbach resonance position  $B_r$ , there is not any size and atom number changes. Once the  $B_f$  is ramped across the Feshbach resonance, the collective oscillation is excited, while the atom number is decreased due to the three-body loss. We find two d-wave resonances at 493.65 G and 536.43 G can create the large excitation as shown in figures 2(a) and (b), the other d-wave resonance at 586.09 G may create a weak excitation as shown in figure 2(c), and three g-wave resonances cannot. For the collective surface mode excitation, we observe the sudden change of the atoms cloud width when ramping the magnetic field across the Feshbach resonance. At the same time, the sudden atoms loss is also observed, as shown in figure 2. For each point in



**Table 1.** Observed Feshbach resonances  $B_r$  and measured  $B_{\text{expt}}$  in [52].  $B_{\text{cc}}$  and  $\Delta_{\text{cc}}$  is the theoretical Feshbach resonance and width from coupled-channels (CC) calculation [52]. The symbol for the  $\circ$  indicates the strong collective excitation,  $\otimes$  the weak excitation and  $\times$  does not.

	Expt		CC		Exc.
	$B_r$ (G)	$B_{\text{expt}}$ (G) [52]	$B_{\text{cc}}$ (G)	$\Delta_{\text{cc}}$ (mG)	
d-wave	493.65	493.6	493.44	1.7	$\circ$
	536.43	536.6	536.5	0.3	$\circ$
	586.09	586.3	586.15	0.05	$\otimes$
g-wave	507.9	508	507.75	<0.005	$\times$
	508.74	508.8	508.59	<0.005	$\times$
	511	510.9	510.65	<0.005	$\times$

figure 2, the cloud profiles are fitted using a Thomas–Fermi profile and the cloud diameters  $s_x$  along the  $\hat{x}$  direction are extracted. It is clearly shown that the sudden change of  $s_x$  happens while the  $B_f$  is across the Feshbach resonance position  $B_r$ , marked by a vertical dashed line in figure 2.

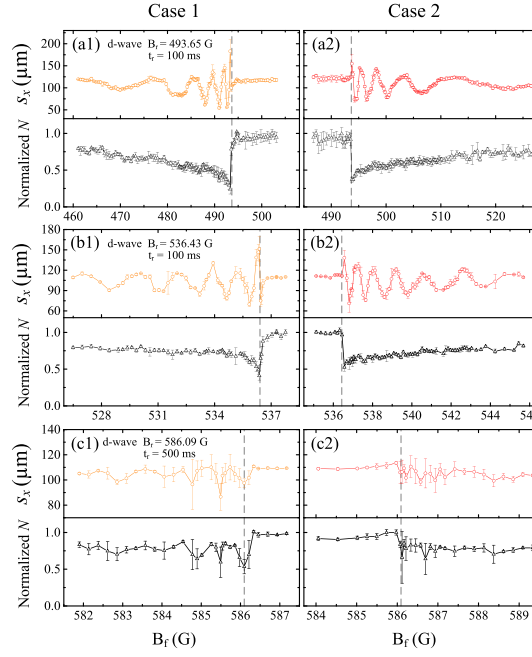
To capture the oscillation frequency and the damping rate of the atomic cloud size, a model function is applied to describe the dependence of the size oscillation on the holding time  $t_h$ ,

$$s(t) = s_0 + lt_h + A \sin(\omega t_h + \phi) \exp(-kt_h) \quad (1)$$

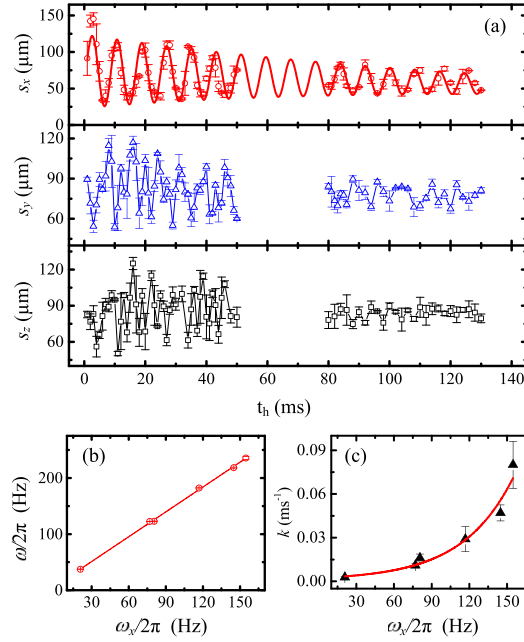
where  $s_0$  is the offset,  $l$  is a linear slope induced by the long time shift,  $A$  is the initial oscillation amplitude of the surface mode,  $\omega$  is the oscillation frequency,  $\phi$  is the initial phase, and  $k$  is the damping rate constant in the exponential decaying of the oscillation amplitude. The collective oscillation frequency and the damping rate can be well determined by equation (1) from the fitting to the experimental data.

At first, we study the time evolution of the cloud size with TOF of 20 ms after the collective excitation in three directions  $x$ ,  $y$ , and  $z$ . A typical experimental result is shown in figure 3(a). It is found that the oscillation of the cloud size along the weakest-trapping  $x$  direction shows a single-frequency with about  $1.48\omega_x$  and could be well fitted by the model function of equation (1). In contrast the time evolution of the surface mode in other strong-trapping directions ( $y$ ,  $z$ ) presents the multi-frequency oscillations.

In the following, we focus on the behavior in the weakest trapping  $x$  direction of the optical trap potential. The dependence of collective mode frequencies and the damping rates on the trapping frequency in  $x$  direction are extracted, as shown in figures 3(b) and (c). We can lineally fit the relation between the oscillation frequency and the trap frequency, and determine the ratio is about 1.48, close to the theoretical value in the limitation that the interaction energy is more predominate than the kinetic energy in the range of the aspect ratio of the trap frequency  $\omega_{y,z}/\omega_x \in (1.5, 10)$  [53–57].



**Figure 2.** The size in  $x$  direction after TOF and remaining atom number of BEC as the function of the final magnetic field. Case 1 and Case 2 correspond to the downward and upward ramping direction respectively. The d-wave Feshbach resonances at 493.65 G (a), 536.43 G (b), and 586.09 G (c) are measured. The Feshbach resonance positions  $B_f$  are indicated by vertical dashed lines. Here,  $t_h = 0$  ms.

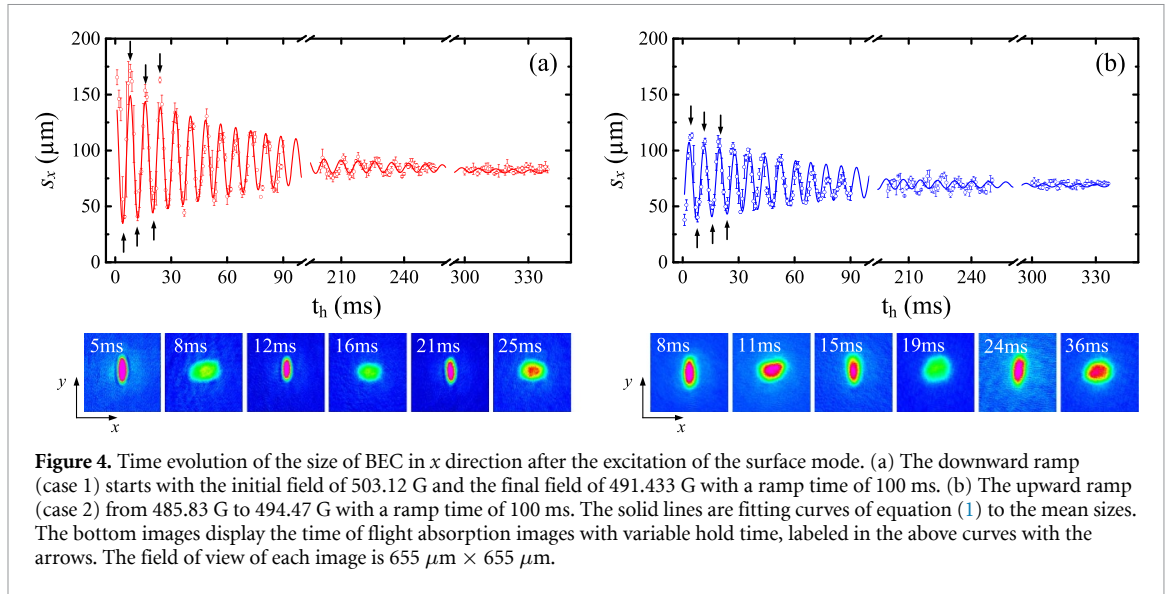


**Figure 3.** (a) Time evolution of the size of BEC after the excitation of the surface mode in an optical trap with trapping frequencies  $(\omega_x, \omega_y, \omega_z) = 2\pi \times (80, 136, 136)$  Hz. Solid lines are fitting curves based on damped sinusoidal equation (1). The extracted oscillation frequencies  $\omega$  (b) and damping rates  $k$  (c) as a function of the trapping frequency  $\omega_x$ .

The theoretical analytical expression for the excited oscillation frequency of the lowest-lying quadrupole mode [53]

$$\omega_Q = \omega_r \sqrt{2} \left[ \left( 1 + \lambda^2 - P_{2,3} \right) - \sqrt{\left( 1 - \lambda^2 + P_{2,3} \right)^2 + 8P_{3,2}^2} \right]^{1/2} \quad (2)$$

where  $P_{i,j} = P/(4u_{0r}^i u_{0x}^j)$  and  $u_{0r}$  and  $u_{0x}$  are the equilibrium half-widths in the radial and axial directions, the aspect ratio  $\lambda = \omega_x/\omega_r$  and  $\omega_x$  and  $\omega_r$  are the axial and radial trapping frequencies, the interaction



**Figure 4.** Time evolution of the size of BEC in  $x$  direction after the excitation of the surface mode. (a) The downward ramp (case 1) starts with the initial field of 503.12 G and the final field of 491.433 G with a ramp time of 100 ms. (b) The upward ramp (case 2) from 485.83 G to 494.47 G with a ramp time of 100 ms. The solid lines are fitting curves of equation (1) to the mean sizes. The bottom images display the time of flight absorption images with variable hold time, labeled in the above curves with the arrows. The field of view of each image is  $655 \mu\text{m} \times 655 \mu\text{m}$ .

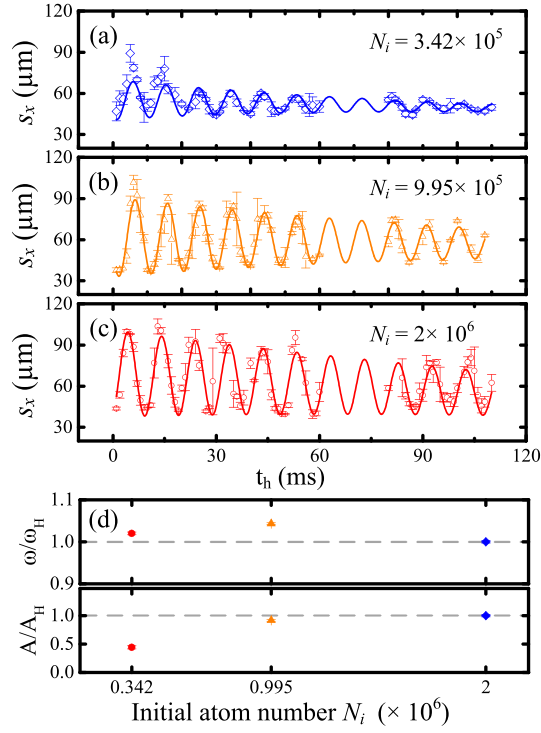
parameter  $P = \sqrt{2/\pi} Na_{bg}/l_r$  with  $l_r = \sqrt{\hbar/m\omega_r}$  being the radial harmonic oscillator size. We can see that the surface mode in the axial weak confinement presents an almost pure single-frequency oscillation with the frequency  $1.6\omega_x$ . This theoretical value is obtained at zero temperature. The finite temperature will induce the discrepancy between the theoretical prediction and measured value [10, 24, 58]. In our work, the ramping across the Feshbach resonance heats the sample, and the result deviates from the theoretical value. We also find that the damping rate of the surface mode is exponentially increased as the trap frequency increases as shown in figure 3(c), which is induced by the increased interaction between atoms.

Subsequently, we study this collective excitation in two cases with different ramping directions. We observe the evolution of the ultracold atom cloud by holding the excited gas with a variable time  $t_h$  after the ramping process across the Feshbach resonance location at 493.65 G, where the ramping speed is about  $0.12 \text{ G ms}^{-1}$  with a fixed ramping time  $t_r = 100 \text{ ms}$ . Figure 4 shows the extracted size as a function of the holding time  $t_h$  for two cases. **Case 1** corresponds to ramp the magnetic field from an initial field of  $B_i = 503.12 \text{ G}$  above the Feshbach resonance to a final field of  $B_f = 491.43 \text{ G}$ , and **Case 2** from  $B_i = 485.83 \text{ G}$  below the Feshbach resonance to  $B_f = 494.47 \text{ G}$ . After the ramping process, about 45% atoms are remained in the condensate. We observe the oscillation frequency  $\omega = 2\pi \times 123.9 \text{ Hz}$  for both cases. The lifetime of the surface modes observed for two cases is about 100 ms, corresponding to the decay rate  $k \sim 0.011 \pm 0.001 \text{ ms}^{-1}$ , which is the essential property of the quadrupole mode and induced by the coupling from collisions in different direction. Moreover, the anisotropy of the optical trap generates the coupling between the quadrupole and other excited modes, which induces the damping of quadrupole modes in the hydrodynamic regime [55].

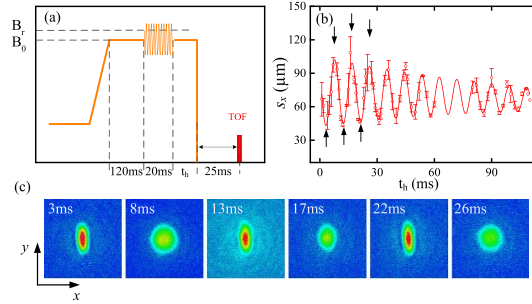
We further study the dependence of collective excitation on the atom number. We prepare the sample of BEC with different initial atom number  $N_i$  and follow the above mentioned experimental procedure of case 2 to excite the collective surface mode. We measure the number and size of remaining atoms after a variable holding time as shown in figures 5(a)–(c). Furthermore, the normalized oscillation frequency  $\omega$  and the initial oscillation amplitude  $A$  as the function of the atom number from figures 5(a)–(c) are given in figure 5(d). It is found that the oscillation frequency  $\omega$  is almost the same for the different initial atom number, but the measured initial oscillation amplitude  $A$  dramatically decreases for smaller initial atom number.

Finally, we use another excitation scheme of fast modulation as shown in figure 6. After preparing a BEC of  $^{23}\text{Na}$  atoms in state  $|1, 1\rangle$  at the lower magnetic field  $B_0 = 494 \text{ G}$ , smaller than the resonance point  $B_r$ , an additional oscillating magnetic field of 10 kHz (much larger than the trapping frequency) is then used at 20 ms in the vicinity of the d-wave Feshbach resonance, where the oscillating amplitude should be enough large to cross the Feshbach resonance field point. In this scheme, the frequency and amplitude of the oscillating field could be controlled precisely from a signal generator. After the oscillating time and a variable holding time  $t_h$ , the atoms are imaged after TOF of 25 ms. The surface mode is also observed in momentum space at a frequency of  $\omega = 2\pi \times 123.6 \text{ Hz}$ , extracted from the fitting using equation (1), while about 30% atoms are remained.

Here this modulation scheme is different from that of modulating the atomic s-wave scattering length in reference [47], in which the scattering length is at  $3a_0$  with the modulation depth of  $\delta a \sim 2a_0$  and the



**Figure 5.** Time evolution of the size of BEC after the excitation of the surface mode for the different initial atom number with (a)  $3.42 \times 10^5$  (blue diamond), (b)  $9.95 \times 10^5$  (yellow triangle), (c)  $2 \times 10^6$  (red circle), respectively. The solid lines show fitting curves from equation (1). (d) Normalized oscillation frequency  $\omega/\omega_H$  and amplitudes  $A/A_H$  as a function of the initial atom number  $N_i$ .



**Figure 6.** The excitation of surface mode through a high-frequency modulation of external magnetic field. (a) The external magnetic field is ramped to a value  $B_0$  below the Feshbach resonance  $B_r$  and the modulation of the magnetic field with frequency 10 kHz is used for a duration 20 ms to excite the surface mode. (b) Collective oscillation of the size as a function of hold time  $t_h$ . Solid curves corresponding to fit. (c) The images with 25 ms TOF display the expanded atom cloud with the different hold time labeled in curve (b) with the arrows. The field of view is  $771 \mu\text{m} \times 771 \mu\text{m}$  for all images.

modulation frequency near  $\sqrt{2}\omega_z$ ,  $\omega_z$  is the axial trapping frequency of an elongated optical trap. Here we modulate the magnetic field across the resonance point  $B_r$  with the modulation frequency much larger than the optical trapping frequency.

### 3. Conclusion

In summary, we have experimentally observed a novel collective surface mode of  $^{23}\text{Na}$  BEC, which is excited by sweeping the external magnetic field across d-wave Feshbach resonance, corresponding to vary the atomic interaction. We check this collective excitation near the six magnetic Feshbach resonances below 600 G, and find only two d-wave resonances could be used to excite and the others cannot. Therefore, the excitation amplitude of the collective mode reflects the strength of these magnetic Feshbach resonances. The excited oscillation in the momentum space by means of TOF absorption imaging is studied in detail. An almost pure single-frequency oscillation is excited in the axial weak confinement of the optical dipole trap. In contrast, multi-frequency oscillations are excited in the radial strong-confinement. We observed the damping of the



oscillation amplitude induced by the mechanisms of dephasing effect and the collisional relaxation. We also investigated the dependence of the surface mode on the atom number of BEC. A different scheme of the fast oscillating external magnetic field was used to excite this collective mode, and a similar surface mode was also observed. These excitation methods may give us more choices without changing the trap frequencies for investigating the properties of ultracold quantum gases. Especially these methods can be used to excite only one component in a mixture of multi-component ultracold atomic gases, and keep the other components unexcited.

## Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

## Acknowledgments

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