Semi-Leptonic Form Factors of $B_s \to D_s$ Transitions in the Light Front Quark Model

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Introduction

Exclusive semileptonic decays of bottom mesons provide a clean probe of the weak interaction while being sensitive to strong dynamics through hadronic form factors. These processes offer a valuable testing ground for nonperturbative QCD and for extracting fundamental parameters such as CKM matrix elements with increasing precision from experiments like LHCb and Belle II [1]. Alongside these achievements, theoretical efforts remain essential to provide reliable predictions on form factors across the full kinematic range, which are also crucial for testing the consistency of experimental findings and uncovering possible hints of physics beyond the Standard Model. In semileptonic decays, the vector form factor $f_+(q^2)$ can be reliably extracted from the "good" $J^+ = J^0 + J^3$ current, since it avoids complications from vacuum fluctuations and zero modes. In contrast, the scalar form factor $f_-(q^2)$ (and subsequently $f_0(q^2)$) involves the "bad" currents $J^- = J^0 - J^3$ and $J^\perp = (J^x, J^y)$, which are sensitive to instantaneous fermion contributions and require careful treatment. In this work, we focus on the transverse current component $J^\perp = (J^x, J^y)$ and employ the Bakamjian-Thomas (BT) construction within the standard light front quark model, treating mesons as bound states of on-shell quark-antiquark pairs and encoding all interaction effects in the invariant mass M_0, allowing consistent computation of the scalar form factor $f_0(q^2)$.

Theoretical Formalism

In semileptonic transitions of pseudoscalar mesons, the weak hadronic current can be expressed in terms of two independent form factors, $f_+(q^2)$ and $f_-(q^2)$ as

$$\mathcal{J}^\mu_{\text{Weak}} = f_+(q^2)(P_1 + P_2)^\mu + f_-(q^2)q^\mu$$

where P_1 and P_2 are the four-momenta of the initial and final mesons and $q^\mu = P_1^\mu - P_2^\mu$ is the four-momentum transferred to the lepton pair ($\ell\nu_\ell$). The allowed kinematic range is $m_\ell^2 \le q^2 \le (M_1 - M_2)^2$ with m_ℓ being the charged lepton mass. M_1 and M_2 is the mass of initial and final meson. The scalar form factor $f_0(q^2)$ defined by

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_1^2 - M_2^2} f_-(q^2)$$

ensures the smooth behavior at $q^2 = 0$ and satisfies $f_0(0) = f_+(0)$ consistent with vector current conservation. We adopt the Drell Yan West frame ($q^+ = 0$ and $\mathbf{P}_\perp = \mathbf{0}$) so that the momentum transfer is purely transverse, $q^2 = -\mathbf{q}_\perp^2 \equiv -Q^2$. The four-momenta of initial and final mesons are then expressed as $P = \left(P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp\right)$ and $P' = \left(P^+, \frac{M'^2 + \mathbf{q}_\perp^2}{P^+}, -\mathbf{q}_\perp\right)$ giving $q = P - P' = \left(0, \frac{M^2 - M'^2 - \mathbf{q}_\perp^2}{P^+}, \mathbf{q}_\perp\right)$. For the transition $P(q_1\bar{q}) \to P'(q_2\bar{q})$, the light front momenta of the active quark and spectator antiquark are assigned as $p_1^+ = p_2^+ = xP^+$, $p_{\bar{q}}^+ = p'^+_{\bar{q}} = (1-x)P^+$, $\mathbf{p}_{1\perp} = x\mathbf{P}_\perp + \mathbf{k}_\perp$, $\mathbf{p}_{2\perp} = x\mathbf{P}'_\perp + \mathbf{k}'_\perp$, $\mathbf{p}_{\bar{q}\perp} = (1-x)\mathbf{P}_\perp - \mathbf{k}_\perp$, $\mathbf{p}'_{\bar{q}\perp} = (1-x)\mathbf{P}'_\perp - \mathbf{k}'_\perp$. Imposing spectator momentum conservation, $p_{\bar{q}}^+ = p'^+_{\bar{q}}$ and $\mathbf{p}_{\bar{q}\perp} = \mathbf{p}'_{\bar{q}\perp}$, yields the intrinsic transverse momentum relation $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$. Within the LFQM, employing noninteracting $q\bar{q}$ Fock-state representation compatible with the BT framework, the weak current matrix element at one loop order takes the form of a convolution of the light-front wave functions of the initial and final mesons with the corresponding spinor transition amplitude as

$$\mathcal{J}^\mu_{\text{Weak}} = \int_0^1 dp_1^+ \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\, \phi'(x, \mathbf{k}'_\perp)\, \phi(x, \mathbf{k}_\perp) \mathcal{T}^\mu$$

Here, the integration measure satisfies $dp_1^+ = P^+ dx$. The $\mathcal{T}^\mu$ denotes the helicity trace structure which can be further decomposed into helicity conserving (hc) and helicity violating (hv) contributions. Accordingly, we derive analytic expressions for the hc and hc contributions to the current components following the Ref. [2]. For the

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plus component J^+, the hv term vanishes, leaving only the hc contribution. In contrast, the transverse components $J^\perp$ receive contributions from both hc and hv terms, reflecting the more intricate spinor structures in the perpendicular direction. The radial wave function $\phi(x, \mathbf{k}_\perp)$ is taken in the Gaussian form as

$$\phi(x, \mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}} \sqrt{\frac{\partial k_z}{\partial x}} \exp\left(-\frac{k^2}{2\beta^2}\right)$$

where β is the variational parameter to be determined from meson spectroscopy studies. The three momentum $\mathbf{k} = (k_z, \mathbf{k}_\perp)$ can be written in terms of $(x, \mathbf{k}_\perp)$ as $k_z = \left(x - \frac{1}{2}\right) M_0 + \frac{m_q^2 - m_1^2}{2M_0}$. The change of variables $\{k_z, \mathbf{k}_\perp\} \rightarrow \{x, \mathbf{k}_\perp\}$ introduces the Jacobian factor $\frac{\partial k_z}{\partial x} = \frac{M_0}{4x(1-x)} \left[1 - \left(\frac{m_1^2 - m_q^2}{M_0^2}\right)^2\right]$. The radial wave function also satisfy the normalization condition $\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} |\phi(x, \mathbf{k}_\perp)|^2 = 1$. The explicit expressions for the form factors $f_+(q^2)$ and $f_-(q^2)$ are derived within the standard LFQM incorporating the BT construction as

$$f_+(q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi_1(x, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2 + \mathbf{k}_\perp^2}} \frac{\phi_2(x, \mathbf{k}'_\perp)}{\sqrt{\mathcal{A}_2^2 + \mathbf{k}'^2_\perp}} \times \left(\mathcal{A}_1\mathcal{A}_2 + \mathbf{k}_\perp \cdot \mathbf{k}'_\perp\right)$$

$$f_-^{(\perp)}(q^2) = \int_0^1 \bar{x}\, dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi_1(x, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2 + \mathbf{k}_\perp^2}} \frac{\phi_2(x, \mathbf{k}'_\perp)}{\sqrt{\mathcal{A}_2^2 + \mathbf{k}'^2_\perp}} \times \Bigg[-\bar{x}M_0^2 + (m_2 - m_q)\mathcal{A}_1 - m_q(m_1 - m_q) - \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{q^2}\left(M_0^2 + M_0'^2 - 2(m_1 - m_q)(m_2 - m_q)\right)\Bigg]$$

We note that $\bar{x} = 1 - x$ and $\mathcal{A}_i = (1-x)m_i + xm_q (i = 1, 2)$. The boost invariant mass squared of the quark–antiquark system within this framework is given by $M_0^2 = \frac{\mathbf{k}_\perp^2 + m_i^2}{x} + \frac{\mathbf{k}_\perp^2 + m_q^2}{1-x}$. The parameters m_b = 5.2 GeV, m_c = 1.8 GeV, m_s = 0.45 GeV, $B_s(1^1S_0)$ = 5.366 GeV, $D_s(1^1S_0)$ = 1.968 GeV, β_{bs} = 0.571 GeV and β_{ds} = 0.502 GeV are taken from previous study based on harmonic oscillator potential [5].

Results and Discussion

In the present study of semileptonic decays $B_s \rightarrow D_s l \nu_l$, we compute the form factors using the standard light front quark model accompanying the BT construction. The evaluated form factors at zero and maximum momentum transfer are summarized in Table I. We found that at $q^2 = 0$, our predictions are higher than those of RQM [3] and CLFQM [4]. As illustrated in Figure 1, the contributions from hc and hv terms exhibit distinct behavior. While the hc component remains positive, the hv contribution, arising from relativistic spin rotations and intrinsic transverse motion of quarks, dominates in magnitude, resulting in a negative total contribution. This underscores the importance of relativistic effects in shaping the form factors and highlights the significant role of quark orbital motion in semileptonic transitions. The work of HMC is supported by the NRF (2023R1A2C1004098).

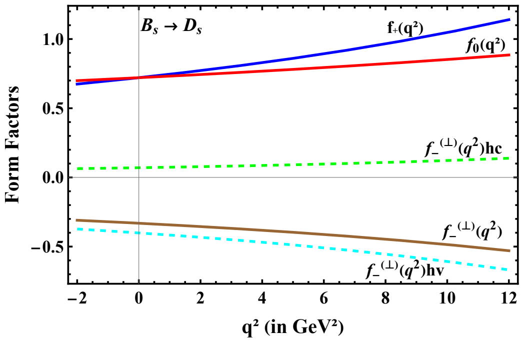

FIG. 1: The form factors for the semileptonic decays of $B_s \rightarrow D_s \ell \nu_\ell$ over the full kinematic range. The analytic continuation is performed via $\mathbf{q}_\perp^2 \rightarrow -q^2$.

TABLE I: Form factors evaluated at $q^2 = 0$ and $q^2 = q^2_{\rm max}$ for $B_s \rightarrow D_s \ell \nu_\ell$ transition.

	This Work	RQM [3]	CLFQM [4]
$f_+(0)$	0.68	0.66	0.67
$f_+(q^2_{\rm max})$	1.12	1.23	1.20
$f_-^{(\perp)}(0)$	-0.32	···	···
$f_-^{(\perp)}(q^2_{\rm max})$	-0.54	···	···
$f_0(0)$	0.68	0.66	0.67
$f_0(q^2_{\rm max})$	0.87	0.87	0.92

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