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Cite this article: Kuzenko SM. 2023 Local supersymmetry: variations on a theme by Volkov and Soroka. *Proc. R. Soc. A* **479**: 20230022.
<https://doi.org/10.1098/rspa.2023.0022>

Received: 10 January 2023

Accepted: 23 February 2023

Subject Areas:

gauge theory, high-energy physics,
mathematical physics

Keywords:

local supersymmetry, supergravity, Goldstino

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Local supersymmetry: variations on a theme by Volkov and Soroka

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We revisit the work by Volkov and Soroka on spontaneously broken local supersymmetry. It is demonstrated for the first time that, for specially chosen parameters of the theory, the Volkov–Soroka action is invariant under two different local supersymmetries. One of them is present for arbitrary values of the parameters and acts on the Goldstino, while the other supersymmetry emerges only in a special case and leaves the Goldstino invariant. The former can be used to gauge away the Goldstino, and then the resulting action coincides with that proposed by Deser and Zumino for consistent supergravity in the first-order formalism. In this sense, pure $\mathcal{N} = 1$ supergravity is a special case of the Volkov–Soroka theory, although it was not discovered by these authors. We also explain how the Volkov–Soroka approach allows one to naturally arrive at the 1.5 formalism. Our analysis provides a nonlinear realization approach to construct unbroken $\mathcal{N} = 1$ Poincaré supergravity.

1. Introduction

Supergravity was discovered in 1976 [1,2]. In 1994, less than 2 years before his death, Volkov posted two preprints to the hep-th archive [3,4], both of which contained ‘supergravity before 1976’ in the title. He argued that the crucial ingredients of $\mathcal{N} = 1$ supergravity in four dimensions had appeared in his earlier work with Soroka [5,6].¹ Since the concept of local supersymmetry

¹Similar thoughts were also expressed by Soroka a few years later [7,8], see also [9,10].

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has played a fundamental role in modern theoretical physics, it is suitable to have a fresh critical look at the Volkov–Soroka construction.

In this paper, we revisit the Volkov–Soroka approach to spontaneously broken local supersymmetry [5,6]. We demonstrate for the first time that, for specially chosen parameters of the theory, the Volkov–Soroka action is invariant under two different local supersymmetries. One of them can be used to gauge away the Goldstino, and then the resulting action coincides with that proposed by Deser and Zumino to describe consistent supergravity in the first-order formalism. In this sense, pure $\mathcal{N} = 1$ supergravity is a special case of the Volkov–Soroka theory. We also explain how the Volkov–Soroka approach allows one to naturally arrive at the 1.5 formalism [11,12].²

Before we turn to the technical part of this paper, it is appropriate to make a few historical comments about Dmitry V. Volkov, one of the co-discoverers of supersymmetry. His most prominent results on rigid and local supersymmetry are the Goldstino model (jointly with Akulov) [13–15] and the super-Higgs mechanism (jointly with Soroka) [5,6]. His approach to nonlinear realizations of internal and space–time symmetries [16], which paved the way to [5,6,13–15], has also been highly influential. However, it is less known that Akulov & Volkov [15] also pioneered the following fundamental concepts of modern theoretical physics: (i) the \mathcal{N} -extended super-Poincaré group and, hence, the \mathcal{N} -extended super-Poincaré algebra³ for $\mathcal{N} > 1$; and (ii) \mathcal{N} -extended Minkowski superspace. It is quite remarkable that all these results had appeared before the first paper by Wess and Zumino on supersymmetry [18] was published on 18 February 1974.⁴

2. A review of the Volkov–Soroka construction

This section is devoted to a pedagogical review of the Volkov–Soroka construction [5,6].

Let $\mathfrak{P}(4|\mathcal{N})$ denote the four-dimensional \mathcal{N} -extended super-Poincaré group introduced in [15]. Any element $g \in \mathfrak{P}(4|\mathcal{N})$ is a $(4 + \mathcal{N}) \times (4 + \mathcal{N})$ supermatrix of the form⁵

$$g = S(b, \varepsilon)h(M, U) \equiv Sh, \quad (2.1a)$$

$$S(b, \varepsilon) := \left(\begin{array}{c|c|c} \mathbb{1}_2 & 0 & 0 \\ \hline -i\tilde{b}_{(+)} & \mathbb{1}_2 & 2\epsilon^\dagger \\ \hline 2\epsilon & 0 & \mathbb{1}_{\mathcal{N}} \end{array} \right) = \left(\begin{array}{c|c|c} \delta_\alpha^\beta & 0 & 0 \\ \hline -i\tilde{b}_{(+)}^{\alpha\beta} & \delta^{\dot{\alpha}}_{\dot{\beta}} & 2\bar{\epsilon}^{\dot{\alpha}j} \\ \hline 2\epsilon_i^\beta & 0 & \delta_i^j \end{array} \right) \quad (2.1b)$$

and

$$h(M, U) := \left(\begin{array}{c|c|c} M & 0 & 0 \\ \hline 0 & (M^{-1})^\dagger & 0 \\ \hline 0 & 0 & U \end{array} \right) = \left(\begin{array}{c|c|c} M_\alpha^\beta & 0 & 0 \\ \hline 0 & (\bar{M}^{-1})_{\dot{\beta}}^{\dot{\alpha}} & 0 \\ \hline 0 & 0 & U_i^j \end{array} \right), \quad (2.1c)$$

where $M = (M_\alpha^\beta) \in \text{sl}(2, \mathbb{C})$, $U = (U_i^j) \in \mathfrak{u}(\mathcal{N})$, and

$$\tilde{b}_{(\pm)} = \tilde{b} \pm 2i\epsilon^\dagger \epsilon. \quad (2.2)$$

The tilde notation in (2.1b) and (2.2) reflects the fact that there are two types of relativistic Pauli matrices, σ_a and $\bar{\sigma}_a$, see the appendix. The group element $S(b, \varepsilon)$ is labelled by 4 commuting real parameters b^a and $4\mathcal{N}$ anti-commuting complex parameters $\varepsilon = (\epsilon, \epsilon^\dagger)$, where $\epsilon = (\epsilon_i^\alpha)$ and $\epsilon^\dagger =$

²The terminology ‘1.5 formalism’ originated from [11].

³The $\mathcal{N} = 1$ super-Poincaré algebra was discovered in 1971 by Golfand and Likhtman [17].

⁴The Akulov–Volkov paper [15] was submitted to the journal *Theoretical and Mathematical Physics* on 8 January 1973 and published in January 1974, before the publication of the first paper on supersymmetry by Wess & Zumino [18] and long before the work by Salam & Strathdee [19] devoted to the $\mathcal{N} = 1$ superspace approach. It remains largely unknown, perhaps because it was published in a Russian journal.

⁵Our parametrization of the elements of $\mathfrak{P}(4|\mathcal{N})$ follows [20].

$(\bar{\epsilon}^{\dot{\alpha}i}, \bar{\epsilon}^{\dot{\alpha}i} := \overline{\epsilon_i^\alpha}$. In the vector notation, equation (2.2) reads

$$b_{(\pm)}^a := b^a \pm i\epsilon_i\sigma^a\bar{\epsilon}^i = b^a \pm i\epsilon_i^\alpha(\sigma^a)_{\alpha\dot{\alpha}}\bar{\epsilon}^{\dot{\alpha}i}. \quad (2.3)$$

Introduce Goldstone fields $Z^A(x) = (X^a(x), \Theta_i^\alpha(x), \bar{\Theta}^{\dot{\alpha}i}(x))$ for space–time translations (X^a) and supersymmetry transformations ($\Theta_i^\alpha, \bar{\Theta}^{\dot{\alpha}i}$). They parametrize the homogeneous space (\mathcal{N} -extended Minkowski superspace)

$$\mathbb{M}^{4|4\mathcal{N}} = \frac{\mathfrak{P}(4|\mathcal{N})}{\text{Sl}(2, \mathbb{C}) \times \text{U}(\mathcal{N})} \quad (2.4)$$

according to the rule:

$$\mathfrak{S}(Z) = \left(\begin{array}{c|c|c} \mathbb{1}_2 & 0 & 0 \\ \hline -i\tilde{X}_{(+)} & \mathbb{1}_2 & 2\Theta^\dagger \\ \hline 2\Theta & 0 & \mathbb{1}_{\mathcal{N}} \end{array} \right) \implies \mathfrak{S}^{-1}(Z) = \left(\begin{array}{c|c|c} \mathbb{1}_2 & 0 & 0 \\ \hline i\tilde{X}_{(-)} & \mathbb{1}_2 & -2\Theta^\dagger \\ \hline -2\Theta & 0 & \mathbb{1}_{\mathcal{N}} \end{array} \right), \quad (2.5)$$

where

$$\tilde{X}_{(\pm)} = \tilde{X} \pm 2i\Theta^\dagger\Theta. \quad (2.6)$$

We define gauge super-Poincaré transformations by

$$g(x): Z(x) \rightarrow Z'(x), \quad g\mathfrak{S}(Z) = \mathfrak{S}(Z')h, \quad (2.7)$$

with $g = Sh$. This is equivalent to the following transformations of the Goldstone fields:

$$S(b, \epsilon): \quad \tilde{X}' = \tilde{X} + \tilde{b} + 2i(\epsilon^\dagger\Theta - \Theta^\dagger\epsilon), \quad (2.8a)$$

$$\Theta' = \Theta + \epsilon \quad (2.8b)$$

and

$$h(M, U): \quad \tilde{X}' = (M^\dagger)^{-1}\tilde{X}M^{-1}, \quad (2.9a)$$

$$\Theta' = U\Theta M^{-1}. \quad (2.9b)$$

Introduce a connection $\mathfrak{A} = dx^m\mathfrak{A}_m$ taking its values in the super-Poincaré algebra,

$$\mathfrak{A} := \left(\begin{array}{c|c|c} \Omega & 0 & 0 \\ \hline -i\tilde{e} & -\Omega^\dagger & 2\psi^\dagger \\ \hline 2\psi & 0 & iV \end{array} \right) = \left(\begin{array}{c|c|c} \Omega_\alpha^\beta & 0 & 0 \\ \hline -ie^{\dot{\alpha}\beta} & -\bar{\Omega}^{\dot{\alpha}\dot{\beta}} & 2\bar{\psi}^{\dot{\alpha}j} \\ \hline 2\psi_i^\beta & 0 & iV_i^j \end{array} \right), \quad (2.10)$$

and possessing the gauge transformation law

$$\mathfrak{A}' = g\mathfrak{A}g^{-1} + g\text{d}g^{-1}. \quad (2.11)$$

Here the one-forms Ω_α^β and $\bar{\Omega}^{\dot{\alpha}\dot{\beta}}$ are the spinor counterparts of the Lorentz connection $\Omega^{ab} = dx^m\Omega_m^{ab} = -\Omega^{ba}$ such that

$$\Omega_\alpha^\beta = \frac{1}{2}(\sigma^{ab})_\alpha^\beta\Omega_{ab} \quad \text{and} \quad \bar{\Omega}^{\dot{\alpha}\dot{\beta}} = -\frac{1}{2}(\bar{\sigma}^{ab})^{\dot{\alpha}\dot{\beta}}\Omega_{ab}. \quad (2.12)$$

The Lorentz connection is an independent field. It is expressed in terms of the other fields on the mass shell. The one-form $e^{\dot{\alpha}\beta}$ is the spinor counterpart of the vierbein $e^a = dx^m e_m^a$. The fermionic one-forms ψ_i^β and $\bar{\psi}^{\dot{\alpha}j}$ describe \mathcal{N} gravitini. Finally, the anti-Hermitian one-form iV is the $\text{U}(\mathcal{N})$ gauge field.

Associated with \mathfrak{G} and \mathfrak{A} is another connection

$$\mathbb{A} := \mathfrak{G}^{-1}\mathfrak{A}\mathfrak{G} + \mathfrak{G}^{-1}d\mathfrak{G}, \quad (2.13)$$

which is characterized by the gauge transformation law

$$\mathbb{A}' = h\mathbb{A}h^{-1} + h dh^{-1}, \quad (2.14)$$

for an arbitrary gauge parameter $g = Sh$. This transformation law tells us that \mathbb{A} is invariant under all gauge transformations of the form $g = S(b, \varepsilon)$ which describe local space–time translations and supersymmetry transformations. The connection \mathbb{A} is the main object of the Volkov–Soroka construction. It has the form

$$\mathbb{A} := \left(\begin{array}{c|c|c} \Omega & 0 & 0 \\ \hline -i\tilde{E} & -\Omega^\dagger & 2\Psi^\dagger \\ \hline 2\Psi & 0 & iV \end{array} \right), \quad (2.15)$$

where we have defined

$$\Psi := \psi + D\theta, \quad (2.16a)$$

$$\Psi^\dagger := \psi^\dagger + D\theta^\dagger, \quad (2.16b)$$

and
$$\tilde{E} := \tilde{e} + D\tilde{X} + 4i\left(\psi^\dagger - \frac{1}{2}D\theta^\dagger\right)\theta - 4i\theta^\dagger\left(\psi - \frac{1}{2}D\theta\right), \quad (2.16c)$$

and D denotes the covariant derivative,

$$D\theta = d\theta - \theta\Omega + iV\theta, \quad (2.17a)$$

$$D\theta^\dagger = d\theta^\dagger - \Omega^\dagger\theta^\dagger - i\theta^\dagger V \quad (2.17b)$$

and
$$D\tilde{X} = d\tilde{X} - \Omega^\dagger\tilde{X} - \tilde{X}\Omega. \quad (2.17c)$$

Equation (2.14) is equivalent to the following gauge transformation laws:

$$\Omega' = M\Omega M^{-1} + M dM^{-1}, \quad (2.18a)$$

$$iV' = U(iV)U^{-1} + U dU^{-1} \quad (2.18b)$$

and

$$\tilde{E}' = (M^\dagger)^{-1}\tilde{E}M^{-1}, \quad (2.19a)$$

$$\Psi' = U\Psi M^{-1}. \quad (2.19b)$$

We see that the supersymmetric one-forms E^a and Ψ_i^β transform as tensors with respect to the Lorentz and $U(\mathcal{N})$ gauge groups.

Making use of (2.8), we deduce the local supersymmetry transformation of the gravitini and the vielbein

$$\psi' = \psi - D\epsilon \quad (2.20a)$$

and

$$\tilde{e}' = \tilde{e} + 4i(\epsilon^\dagger\psi - \psi^\dagger\epsilon) + 2i(D\epsilon^\dagger\epsilon - \epsilon^\dagger D\epsilon). \quad (2.20b)$$

In the infinitesimal case, this transformation can be rewritten in the form

$$\delta_\epsilon\psi = -D\epsilon \quad \text{and} \quad \delta_\epsilon e^a = 2i \operatorname{tr}[\sigma^a(\psi^\dagger\epsilon - \epsilon^\dagger\psi)]. \quad (2.21)$$

As pointed out by Volkov [3], the transformation laws in (2.21) coincide with those used by Deser and Zumino in their construction of $\mathcal{N} = 1$ supergravity [2]. We should remark that the supersymmetry transformations of the Goldstone fields X^a and Θ_i^β are given by the relations (2.8).

Let us consider a local Poincaré translation, $S(b, 0)$. It only acts on the Goldstone vector field X^a and the vierbein e^a ,

$$X'^a = X^a + b^a \quad \text{and} \quad e'^a = e^a - \mathcal{D}b^a. \quad (2.22)$$

We have two types of gauge transformations with vector-like parameters, the general coordinates transformations and the local Poincaré translations. The latter gauge freedom can be fixed by imposing the condition $X^a = 0$, and then we stay only with the general coordinate invariance. However, in what follows we will keep X^a intact.

The curvature tensor is given by

$$\mathbb{R} = d\mathbb{A} - \mathbb{A} \wedge \mathbb{A} \quad \text{and} \quad \mathbb{R}' = h\mathbb{R}h^{-1}. \quad (2.23)$$

Its explicit form is

$$\mathbb{R} := \left(\begin{array}{c|c|c} R & 0 & 0 \\ \hline -i\tilde{\mathbb{T}} & -R^\dagger & 2\mathcal{D}\Psi^\dagger \\ \hline 2\mathcal{D}\Psi & 0 & iF \end{array} \right), \quad (2.24)$$

where $R = (R_\alpha^\beta)$ and $R^\dagger = (\tilde{R}^{\dot{\alpha}}_{\dot{\beta}})$ form the Lorentz curvature, $F = (F_i^j)$ is the Yang–Mills field strength,

$$\mathcal{D}\Psi = d\Psi - \Psi \wedge \Omega - iV \wedge \Psi \quad (2.25a)$$

and

$$\mathcal{D}\Psi^\dagger = d\Psi^\dagger + \Omega^\dagger \wedge \Psi^\dagger - i\Psi^\dagger \wedge V \quad (2.25b)$$

are the gravitino field strengths, and

$$\tilde{\mathbb{T}} = d\tilde{E} - \tilde{E} \wedge \Omega + \Omega^\dagger \wedge \tilde{E} - 4i\Psi^\dagger \wedge \Psi = \mathcal{D}\tilde{E} - 4i\Psi^\dagger \wedge \Psi \quad (2.26)$$

is the supersymmetric torsion tensor. In the vector notation, the torsion tensor reads

$$\mathbb{T}^a = \mathcal{D}E^a + 2i\Psi \wedge \sigma^a \tilde{\Psi}. \quad (2.27)$$

It should be pointed out that the exterior derivative is defined to obey the property

$$d(\Sigma_p \wedge \Sigma_q) = \Sigma_p \wedge d\Sigma_q + (-1)^q d\Sigma_p \wedge \Sigma_q, \quad (2.28)$$

which is used for superforms [21].

The above results allow one to engineer gauge-invariant functionals that can be used to construct a locally supersymmetric action. The invariants proposed in [5,6] are the following:

— the Einstein–Hilbert action

$$S_{\text{EH}} = \frac{1}{4} \int \varepsilon_{abcd} E^a \wedge E^b \wedge R^{cd}, \quad (2.29)$$

— the Rarita–Schwinger action

$$S_{\text{RS}} = \frac{1}{2} \int \left(\Psi_i \wedge E^a \wedge \sigma_a \mathcal{D}\tilde{\Psi}^i - \mathcal{D}\Psi_i \wedge E^a \wedge \sigma_a \tilde{\Psi}^i \right); \quad (2.30)$$

— the cosmological term

$$S_{\text{cosmological}} = \frac{1}{24} \int \varepsilon_{abcd} E^a \wedge E^b \wedge E^c \wedge E^d, \quad (2.31)$$

— $0(\mathcal{N})$ -invariant mass term

$$S_{\text{mass}} = \frac{i}{4} \int E^a \wedge E^b \wedge \left(\delta^{ij} \Psi_i \wedge \sigma_{ab} \Psi_j - \delta_{ij} \tilde{\Psi}^i \wedge \tilde{\sigma}_{ab} \tilde{\Psi}^j \right). \quad (2.32)$$

The functionals S_{EH} , S_{RS} and $S_{\text{cosmological}}$ are $\mathcal{U}(\mathcal{N})$ invariant. The cosmological term, equation (2.31), also contains the kinetic term for the Goldstini [13–15]. The mass term (2.32) is invariant under local internal transformations only if the group $\mathcal{U}(\mathcal{N})$ is replaced with $0(\mathcal{N})$, and the gauge

connection iV takes its values in the Lie algebra $\mathfrak{so}(\mathcal{N})$. The Yang–Mills action associated with V is obviously supersymmetric, but it will not be used in what follows.

In the $\mathcal{N} = 1$ case, the Volkov–Soroka theory is described by the general action

$$S = S_{\text{EH}} + 4cS_{\text{RS}} + 4mS_{\text{mass}} + \lambda S_{\text{cosmological}}, \quad (2.33)$$

with c , m and λ coupling constants.

3. Second local supersymmetry

In this section, our consideration is restricted to the $\mathcal{N} = 1$ case, and the $U(1)$ gauge field is switched off, $V = 0$. We are going to demonstrate that the action

$$S_{\text{SUGRA}} = S_{\text{EH}} + 4S_{\text{RS}} \quad (3.1)$$

is invariant under a new local supersymmetry transformation described by the parameter $\epsilon = (\epsilon^\alpha, \bar{\epsilon}^{\dot{\alpha}})$. It acts on the composite fields Ψ^α and E^a , defined by equation (2.16), and the Lorentz connection as follows:

$$\delta_\epsilon \Psi^\alpha = -\mathcal{D}\epsilon^\alpha \quad \text{and} \quad \delta_\epsilon E^a = 2i(\Psi \sigma^a \bar{\epsilon} - \epsilon \sigma^a \bar{\Psi}) \quad (3.2a)$$

and

$$\frac{1}{4}\epsilon_{abcd}\delta_\epsilon \Omega^{bc} \wedge E^d = \epsilon \sigma_a \mathcal{D}\bar{\Psi} + \mathcal{D}\Psi \sigma_a \bar{\epsilon}. \quad (3.2b)$$

The Goldstone fields are inert under this transformation,

$$\delta_\epsilon X^a = 0 \quad \text{and} \quad \delta_\epsilon \Theta^\alpha = 0. \quad (3.2c)$$

The elementary field ψ^α and e^a transform as follows:

$$\delta_\epsilon \psi^\alpha = -\mathcal{D}\epsilon^\alpha + \Theta^\beta \delta_\epsilon \Omega_\beta^\alpha \quad (3.2d)$$

and

$$\begin{aligned} \delta_\epsilon e^a = & 2i(\Psi \sigma^a \bar{\epsilon} - \epsilon \sigma^a \bar{\Psi}) + 2i(\Theta \sigma^a \mathcal{D}\bar{\epsilon} - \mathcal{D}\epsilon \sigma^a \bar{\Theta}) - \delta_\epsilon \Omega^a_b X^b \\ & + \frac{1}{2}\epsilon^{abcd}\delta_\epsilon \Omega_{bc} \Theta \sigma_d \bar{\Theta}. \end{aligned} \quad (3.2e)$$

It should be pointed out that the transformation laws in (3.2a) can be viewed as a natural generalization of the Volkov–Soroka local supersymmetry (2.21).

The dependence on $\delta_\epsilon \Omega$ in (3.2d) and (3.2e) is such that the composite fields Ψ^α and E^a remain unchanged when the connection gets the displacement $\Omega \rightarrow \Omega + \delta_\epsilon \Omega$. We should point out that the action (3.1) involves only the one-forms Ψ^α , $\bar{\Psi}^{\dot{\alpha}}$, E^a and Ω^{ab} and their descendants. We also remark that the transformations (3.2a) and (3.2b) reduce to those given by Deser & Zumino [2] if the Goldstone fields X^a and Θ^α are switched off.

It should be pointed out that equation (3.2b) uniquely determines $\delta_\epsilon \Omega^{bc}$. Indeed, given a vector-valued two-form

$$\Sigma_a = \frac{1}{2}E^c \wedge E^b \Sigma_{a,bc}, \quad (3.3)$$

the following equation:

$$\frac{1}{2}\epsilon_{abcd}\omega^{bc} \wedge E^d \equiv \tilde{\omega}_{ab} \wedge E^b = \Sigma_a, \quad \tilde{\omega}_{ab} = E^c \tilde{\omega}_{c,ab} \quad (3.4)$$

on the one-form $\omega^{ab} = E^c \omega_c^{ab}$, which takes its values in the Lorentz algebra, has the unique solution

$$\tilde{\omega}_{c,ab} = \frac{1}{2}(\Sigma_{a,bc} - \Sigma_{b,ac} - \Sigma_{c,ab}). \quad (3.5)$$

Now we turn to demonstrating that the action (3.1) is invariant under the supersymmetry transformation (3.2). Let us first take into account the variations in (3.2a). Varying the Einstein–Hilbert action (2.29) for $\epsilon \neq 0$ and $\bar{\epsilon} = 0$ gives

$$\delta_{\epsilon}^{(1)} S_{\text{EH}} = -i \int \varepsilon_{abcd} R^{ab} \wedge E^c \wedge \epsilon \sigma^d \bar{\psi}. \quad (3.6)$$

Varying the Rarita–Schwinger action (2.30) gives

$$\begin{aligned} \delta_{\epsilon}^{(1)} S_{\text{RS}} = & - \int \left\{ \frac{1}{2} D E^a \wedge \epsilon \sigma_a D \bar{\psi} - i \epsilon \sigma^a \bar{\psi} \wedge \psi \wedge \sigma_a D \bar{\psi} \right\} \\ & + \frac{1}{2} \int \left\{ E^a \wedge \epsilon \sigma_a R^{\dagger} \wedge \bar{\psi} - E^a \wedge \epsilon R \wedge \sigma_a \bar{\psi} \right\}, \end{aligned} \quad (3.7)$$

where we have used the relations

$$D D \epsilon = -\epsilon R \quad \text{and} \quad D D \psi^{\dagger} = -R^{\dagger} \wedge \psi^{\dagger}. \quad (3.8)$$

Here $\delta^{(1)}$ means that we have taken into account only the variations (3.2a).

Making use of the identities (A13), the curvature-dependent contributions in (3.7) can be rearranged as

$$\frac{1}{2} \int \left\{ E^a \wedge \epsilon \sigma_a R^{\dagger} \wedge \bar{\psi} - E^a \wedge \epsilon R \wedge \sigma_a \bar{\psi} \right\} = \frac{i}{4} \int \varepsilon_{abcd} R^{ab} \wedge E^c \wedge \epsilon \sigma^d \bar{\psi}. \quad (3.9)$$

In order for the curvature contributions in (3.6) and (3.7) to cancel each other out, we must consider the action (3.1), for which we obtain

$$\delta_{\epsilon}^{(1)} (S_{\text{EH}} + 4S_{\text{RS}}) = -2 \int \left(D E^a + 2i \psi \wedge \sigma^a \bar{\psi} \right) \wedge \epsilon \sigma_a D \bar{\psi} = -2 \int \epsilon \sigma_a D \bar{\psi} \wedge \mathbb{T}^a, \quad (3.10)$$

with \mathbb{T}^a being the supersymmetric torsion tensor (2.27). Adding the complex conjugate part gives

$$\delta_{\epsilon}^{(1)} (S_{\text{EH}} + 4S_{\text{RS}}) = -2 \int \left(\epsilon \sigma_a D \bar{\psi} + D \psi \sigma_a \bar{\epsilon} \right) \wedge \mathbb{T}^a. \quad (3.11)$$

Next, let us vary the action (3.1) with respect to the Lorentz connection Ω^{ab} . We give the Lorentz connection a small disturbance, $\Omega \rightarrow \Omega + \delta_{\epsilon} \Omega$, with $\delta_{\epsilon} \Omega$ to be determined below, and assume that the fields ψ^{α} and e^a also acquire $\delta_{\epsilon} \Omega$ -dependent variations given in (3.2d) and (3.2e). We denote $\delta^{(2)}$ the corresponding variation. Direct calculations give

$$\delta_{\epsilon}^{(2)} (S_{\text{EH}} + 4S_{\text{RS}}) = \frac{1}{2} \int \varepsilon_{abcd} \delta_{\epsilon} \Omega^{bc} \wedge E^d \wedge \mathbb{T}^a. \quad (3.12)$$

Combining the results (3.11) and (3.12), we end up with

$$\delta_{\epsilon} (S_{\text{EH}} + 4S_{\text{RS}}) = -2 \int \left(\epsilon \sigma_a D \bar{\psi} + D \psi \sigma_a \bar{\epsilon} - \frac{1}{4} \varepsilon_{abcd} \delta_{\epsilon} \Omega^{bc} \wedge E^d \right) \wedge \mathbb{T}^a. \quad (3.13)$$

This variation vanishes if $\delta_{\epsilon} \Omega$ is given by equation (3.2b).

We have demonstrated that the theory (3.1) has two types of local supersymmetry. The original Volkov–Soroka supersymmetry is described by the relations

$$\delta_{\epsilon} \psi^{\alpha} = -D \epsilon^{\alpha} \quad \text{and} \quad \delta_{\epsilon} e^a = 2i(\psi \sigma^a \bar{\epsilon} - \epsilon \sigma^a \bar{\psi}) \quad (3.14a)$$

and

$$\delta_{\epsilon} \Theta^{\alpha} = \epsilon^{\alpha} \quad \text{and} \quad \delta_{\epsilon} X^a = i(\Theta \sigma^a \bar{\epsilon} - \epsilon \sigma^a \bar{\Theta}). \quad (3.14b)$$

The second supersymmetry, introduced in this work, is given by equation (3.2). The gauge transformations (2.22) and (3.14) allow us to gauge away the Goldstone fields $Z^A(x) = (X^a(x), \Theta^{\alpha}(x), \bar{\Theta}^{\dot{\alpha}}(x))$ by imposing the conditions

$$X^a = 0 \quad \text{and} \quad \Theta^{\alpha} = 0. \quad (3.15)$$

As a result, the action (3.1) turns into the supergravity action proposed by Deser & Zumino [2], and the local supersymmetry transformations (3.2a) turn into those given in [2].

4. Conclusion

A few years ago, it was shown⁶ [23] that the Volkov–Soroka theory (2.33), with non-vanishing parameters c , m and λ , is equivalent to spontaneously broken $\mathcal{N} = 1$ supergravity [24,25] which was called de Sitter supergravity in [26].

In this paper, we have demonstrated that the action (3.1) is a Stückelberg-type extension of the $\mathcal{N} = 1$ supergravity theory in the first-order formalism proposed by Deser and Zumino. Equivalently, the latter theory is a gauged-fixed version of (3.1). Therefore, pure $\mathcal{N} = 1$ supergravity is a special case of the Volkov–Soroka theory.

The new local supersymmetry (3.2) of the action (3.1) is the main original result of this paper. The Goldstino is just a compensator for the first local supersymmetry in this theory. In the gauge (3.15), the action (3.1) turns into the supergravity action in the first-order formalism.

In the above analysis, the Lorentz connection Ω was an independent field. The alternative approach is to work with a composite connection obtained by imposing the constraint

$$\mathbb{T}^a = DE^a + 2i\psi \wedge \sigma^a \bar{\psi} = 0, \quad (4.1)$$

which is an example of the so-called inverse Higgs mechanism [27]. The constraint (4.1) is invariant under the first supersymmetry transformation. In the gauge (3.15), the constraint (4.1) is uniquely solved to give the standard expression for the connection, $\Omega = \Omega(e, \psi)$, in terms of the vielbein and the gravitino, see e.g. [28] for a review.⁷ Under the second supersymmetry transformation, the connection varies in such a way that (4.1) is preserved. It follows from the relations (3.11) and (3.12) that the action (3.1) is invariant under the second supersymmetry transformation. We observe that the Volkov–Soroka approach allows one to naturally arrive at the 1.5 formalism [11,12].⁸

The novelty of our work is that we have developed a new nonlinear realization approach to constructing unbroken simple Poincaré supergravity theories.⁹ We only studied the case of $\mathcal{N} = 1$ supergravity in four dimensions, but it seems that the same approach can be used in other dimensions. Specifically, as in [5,6] one introduces Goldstone fields $Z^A(x) = (X^a(x), \Theta^{\hat{\alpha}}(x)(x))$ for space–time translations (X^a) and supersymmetry transformations ($\Theta^{\hat{\alpha}}$), with $\hat{\alpha}$ denoting a spinor index. These fields parametrize the coset space, that is Minkowski superspace. To describe unbroken supergravity, the Goldstone fields must describe compensating degrees of freedom. This means there should be two types of gauge transformations with vector parameters, and also two types of local supersymmetry transformations, in order to be able to gauge away the Goldstone fields. By construction, there are always two types of gauge transformations with vector parameters, the general coordinates transformations and the local Poincaré translations. The latter gauge freedom can be fixed by imposing the condition $X^a = 0$, and then we stay only with the general coordinate invariance. By construction, there is always one type of local supersymmetry. A second local supersymmetry emerges only for a special choice of the parameter in the action.¹⁰ The described approach can definitely be used to provide a new derivation of $\mathcal{N} = 1$ topologically massive supergravity in three dimensions originally constructed in [40].

⁶Similar conclusions had been obtained earlier in [22].

⁷For non-vanishing Goldstone fields X^a and $\Theta^{\hat{\alpha}}$, the constraint (4.1) should also allow one to uniquely determine the connection in terms of the other fields. But in this case equation (4.1) becomes highly nonlinear, and its explicit solution is hard to derive.

⁸Our consideration clearly shows that the work by Volkov & Soroka [6] contained all prerequisites that could, in principle, be used to discover supergravity before 1976. It is natural to wonder why they did not discover supergravity. Of course, they did not ask the right question in [5,6]. It seems more important, however, that their ideas were well ahead of time, and the scientific community in the Soviet Union was not ready to accept the novel concepts put forward in these publications. This is similar to the discovery of rigid supersymmetry in four dimensions by Golfand & Likhthman [17] (see [29] for a historical account) whose work was not appreciated in the Soviet Union.

⁹In the framework of the Ogievetsky–Sokatchev approach [30,31] to the old minimal formulation for $\mathcal{N} = 1$ supergravity [32–34], Ivanov & Niederle [35] described $\mathcal{N} = 1$ supergravity as a nonlinear realization. Their construction is completely different from ours.

¹⁰It is worth pointing out that similar ideas have been used to describe AdS gravity [36–38] and AdS supergravity [39].

As pointed out earlier, the model (3.1) is a Stückelberg reformulation of the unbroken $\mathcal{N} = 1$ supergravity in the first-order formalism. It is known that the Stückelberg formalism is often useful in the quantum theory. It would be interesting to revisit the quantization of $\mathcal{N} = 1$ supergravity using the novel formulation (3.1).

Data accessibility. This article has no additional data.

Authors' contributions. S.K.: conceptualization, formal analysis, investigation, methodology, project administration, writing—original draft, writing—review and editing.

Conflict of interest declaration. I declare I have no competing interests.

Funding. This work was supported in part by the Australian Government through the Australian Research Council, project No. DP200101944.

Acknowledgements. I thank Evgeny Buchbinder, Ian McArthur, Dmitri Sorokin, and especially Paul Townsend and Arkady Tseytlin for discussions, and Darren Grasso and Michael Ponds for comments on the manuscript. I also gratefully acknowledge email correspondence with Ioseph Buchbinder, Stepan Douplii, Jim Gates, Evgeny Ivanov, Ulf Lindström and Martin Roček. My special thanks go to Jake Stirling for checking the calculations and correcting typos in §3.

Appendix A. Two-component spinor formalism

In this appendix, we collect the key formulae of the two-component spinor formalism. Our notation and two-component spinor conventions correspond to those used in [21,41]. In particular, the Minkowski metric is $\eta_{ab} = \text{diag}(-1, +1, +1, +1)$, and the Levi-Civita tensor ε^{abcd} is normalized by $\varepsilon^{0123} = 1$.

Given a four-vector p^a , it can equivalently be described as an Hermitian 2×2 matrix with lower spinor indices

$$p := p^a \sigma_a = p^\dagger = (p_{\alpha\dot{\beta}}) \quad \text{and} \quad \sigma_a = (\not{x}_2, \boldsymbol{\sigma}), \quad (\text{A } 1)$$

or as an Hermitian 2×2 matrix with upper spinor indices

$$\tilde{p} := p^a \tilde{\sigma}_a = \tilde{p}^\dagger = (p^{\dot{\alpha}\beta}) \quad \text{and} \quad \tilde{\sigma}_a = (\not{x}_2, -\boldsymbol{\sigma}), \quad (\text{A } 2)$$

with $\boldsymbol{\sigma}$ being the Pauli matrices. The two sets of the relativistic Pauli matrices, σ_a and $\tilde{\sigma}_a$, are related to each other by the rule

$$(\tilde{\sigma}_a)^{\dot{\alpha}\alpha} = \varepsilon^{\dot{\alpha}\beta} \varepsilon^{\alpha\beta} (\sigma_a)_{\beta\dot{\beta}}, \quad (\text{A } 3)$$

where $\varepsilon^{\alpha\beta}$ and $\varepsilon_{\alpha\beta}$, $\varepsilon^{\dot{\alpha}\dot{\beta}}$ and $\varepsilon_{\dot{\alpha}\dot{\beta}}$ are antisymmetric spinor metrics normalized as $\varepsilon^{12} = \varepsilon_{21} = 1$ and $\varepsilon^{\dot{1}\dot{2}} = \varepsilon_{\dot{2}\dot{1}} = 1$. These are used to raise and lower the spinor indices,

$$\psi^\alpha = \varepsilon^{\alpha\beta} \psi_\beta \quad \text{and} \quad \psi_\alpha = \varepsilon_{\alpha\beta} \psi^\beta, \quad (\text{A } 4)$$

and similarly for the dotted spinors.

Let $\mathfrak{P}(4)$ be the universal covering group of the restricted Poincaré group $\text{ISO}_0(3, 1)$. It is usually realized as the group of linear inhomogeneous transformations (M, b) acting on the space of 2×2 Hermitian matrices $x = x^a \sigma_a = x^\dagger$ as follows:

$$x \rightarrow x' = x'^a \sigma_a = MxM^\dagger + b, \quad b = b^a \sigma_a = b^\dagger, \quad M = (M_{\alpha\beta}) \in \text{SL}(2, \mathbb{C}). \quad (\text{A } 5)$$

Here $M^\dagger := \bar{M}^T$ is the Hermitian conjugate of M , and $\bar{M} = (\bar{M}_{\dot{\alpha}\dot{\beta}})$ the complex conjugate of M , with $\bar{M}_{\dot{\alpha}\dot{\beta}} := \bar{M}_{\alpha\beta}$. The group $\mathfrak{P}(4)$ is equivalently realized as the group of linear inhomogeneous transformations acting on the space of 2×2 Hermitian matrices $\tilde{x} := x^a \tilde{\sigma}_a = \tilde{x}^\dagger$ as follows:

$$\tilde{x} \rightarrow \tilde{x}' = x'^a \tilde{\sigma}_a = (M^{-1})^\dagger \tilde{x} M^{-1} + \tilde{b}, \quad \tilde{b} = b^a \tilde{\sigma}_a. \quad (\text{A } 6)$$

In Minkowski space $\mathbb{M}^4 \equiv \mathbb{R}^{3,1}$, the transformation (A 5) or, equivalently, (A 6) looks like

$$x'^a = (\Lambda(M))^a_b x^b + b^a, \quad (\Lambda(M))^a_b = -\frac{1}{2} \text{tr}(\tilde{\sigma}^a M \sigma_b M^\dagger). \quad (\text{A } 7)$$

It is also possible to realize $\mathfrak{P}(4)$ as a subgroup of $SU(2, 2)$ consisting of all block triangular matrices of the form:

$$(M, b) := \left(\begin{array}{c|c} M & 0 \\ \hline -i\tilde{b}M & (M^{-1})^\dagger \end{array} \right) = (\mathbb{1}_2, b)(M, 0), \quad (\text{A } 8)$$

with M and \tilde{b} as in (A 5) and (A 6). Minkowski space is the homogeneous space

$$\mathbb{M}^4 = \frac{\mathfrak{P}(4)}{\text{sl}(2, \mathbb{C})}, \quad (\text{A } 9)$$

compare with equation (2.4) defining the \mathcal{N} -extended Minkowski superspace. Its points are naturally parametrized by the Cartesian coordinates x^a corresponding to the coset representative:

$$(\mathbb{1}_2, x) = \left(\begin{array}{c|c} \mathbb{1}_2 & 0 \\ \hline -i\tilde{x} & \mathbb{1}_2 \end{array} \right). \quad (\text{A } 10)$$

Given an antisymmetric tensor field $F_{ab} = -F_{ba}$, it can be equivalently described by a symmetric rank-two spinor $F_{\alpha\beta} = F_{\beta\alpha}$ and its conjugate $\bar{F}_{\dot{\alpha}\dot{\beta}}$. The precise correspondence $F_{ab} \longleftrightarrow (F_{\alpha\beta}, \bar{F}_{\dot{\alpha}\dot{\beta}})$ is given by

$$F_{ab} = (\sigma_{ab})^{\alpha\beta} F_{\alpha\beta} - (\tilde{\sigma}_{ab})^{\dot{\alpha}\dot{\beta}} \bar{F}_{\dot{\alpha}\dot{\beta}}, \quad F_{\alpha\beta} := \frac{1}{2} (\sigma^{ab})_{\alpha\beta} F_{ab}$$

and

$$\bar{F}_{\dot{\alpha}\dot{\beta}} := -\frac{1}{2} (\tilde{\sigma}^{ab})_{\dot{\alpha}\dot{\beta}} F_{ab}.$$

Here the matrices $\sigma_{ab} = ((\sigma_{ab})_{\alpha\beta})$ and $\tilde{\sigma}_{ab} = ((\tilde{\sigma}_{ab})^{\dot{\alpha}\dot{\beta}})$ are defined by

$$\sigma_{ab} = -\frac{1}{4} (\sigma_a \tilde{\sigma}_b - \sigma_b \tilde{\sigma}_a) \quad \text{and} \quad \tilde{\sigma}_{ab} = -\frac{1}{4} (\tilde{\sigma}_a \sigma_b - \tilde{\sigma}_b \sigma_a). \quad (\text{A } 11)$$

These matrices are (anti) self-dual,

$$\frac{1}{2} \varepsilon^{abcd} \sigma_{cd} = -i\sigma^{ab} \quad \text{and} \quad \frac{1}{2} \varepsilon^{abcd} \tilde{\sigma}_{cd} = i\tilde{\sigma}^{ab}. \quad (\text{A } 12)$$

The important identities involving σ_{ab} and $\tilde{\sigma}_{ab}$ are:

$$\sigma_{ab}\sigma_c = -\frac{1}{2} (\eta_{ac}\sigma_b - \eta_{bc}\sigma_a) - \frac{i}{2} \varepsilon_{abcd}\sigma^d \quad (\text{A } 13a)$$

and

$$\sigma_c\tilde{\sigma}_{ab} = \frac{1}{2} (\eta_{ac}\sigma_b - \eta_{bc}\sigma_a) - \frac{i}{2} \varepsilon_{abcd}\sigma^d. \quad (\text{A } 13b)$$

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