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To cite this article: Y Rybakov 2016 *J. Phys.: Conf. Ser.* **731** 012012

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Solitons in Skyrme – Faddeev spinor model and quantum mechanics

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Abstract. We discuss the possibility of unification of Skyrme and Faddeev approaches for the description of baryons and leptons respectively as topological solitons within the scope of 16-spinor model. The motivation for such a unification is based on a special 8-semispinor identity invented by the Italian geometrician F. Brioschi. This remarkable identity permits one to realize baryon or lepton states through the effect of spontaneous symmetry breaking emerging due to special structure of the Higgs potential in the model. At large distances from the particle – soliton small excitation of the vacuum satisfies Klein – Gordon equation with some mass that permits one to establish the correspondence with quantum mechanics in special stochastic representation of the wave function for extended particles – solitons. Finally, we illustrate the peculiar properties of stochastic representation by the famous T. Young's experiment with n slits in soliton realization.

1. Introduction. Brioschi identity and 16-spinor field model

The Skyrme's fruitful idea [1] to describe baryons as topological solitons was based on the identification of the baryon number B with the topological charge of the degree type $B = \text{deg}(S^3 \rightarrow S^3)$, which serves as the generator of the homotopy group $\pi_3(S^3) = \mathbb{Z}$. The similar idea to describe leptons as topological solitons was announced by Faddeev [2], who identified the lepton number L with the Hopf invariant Q_H , which serves as the generator of the homotopy group $\pi_3(S^2) = \mathbb{Z}$. The unification of these two approaches was suggested in [3], baryons and leptons being considered as two possible phases of the effective 8-spinor field model, for which the special 8-spinor Brioschi identity [4] holds:

$$j_\mu j^\mu - \tilde{j}_\mu \tilde{j}^\mu = s^2 + p^2 + \vec{v}^2 + \vec{a}^2, \quad (1)$$

where the quadratic spinor quantities are introduced:

$$s = \bar{\Psi} \Psi, \quad p = i \bar{\Psi} \gamma_5 \Psi, \quad \vec{v} = \bar{\Psi} \vec{\lambda} \Psi, \quad \vec{a} = i \bar{\Psi} \gamma_5 \vec{\lambda} \Psi, \quad j_\mu = \bar{\Psi} \gamma_\mu \Psi, \quad \tilde{j}_\mu = \bar{\Psi} \gamma_\mu \gamma_5 \Psi,$$

with $\bar{\Psi} = \Psi^+ \gamma_0$ and $\vec{\lambda}$ standing for Pauli matrices in the isotopic spinor space. Here γ_μ , $\mu = 0, 1, 2, 3$, designate the unitary Dirac matrices acting on Minkowski spinor indices. One can verify the time-like character of the Dirac 4-current j_μ and, in view of the identity (1), use the special structure of the Higgs potential V implying the spontaneous symmetry breaking in our model:

$$V = \frac{\sigma^2}{8} (j^\mu j_\mu - \mathfrak{a}_0^2)^2, \quad (2)$$

where σ and \mathfrak{a}_0 stand for some constant parameters. If one searches for localized soliton-like configurations, one should use the natural boundary condition at space infinity:

$$\lim_{|\vec{x}| \rightarrow \infty} j_\mu j^\mu = \mathfrak{a}_0^2, \quad (3)$$



which determines the fixed (vacuum) point in the phase space of our model. In particular, if for the vacuum state Ψ_0 one gets $s(\Psi_0) \neq 0$, then the chiral invariant $s^2 + \vec{a}^2$ determines sphere S^3 as the field manifold, that corresponds to the Skyrme model and the baryon phase. If $v_3(\Psi_0) \neq 0$, then the $SO(3)$ invariant \vec{v}^2 determines the sphere S^2 as the field manifold, that corresponds to the Faddeev model and the lepton phase.

However, the vacuum state Ψ_0 should be universal, that is common both for baryon and lepton phases. It means that conditions $v_3(\Psi_0) \neq 0$ and $s(\Psi_0) \neq 0$ should be compatible. Taking into account that the Brioschi identity (1) holds only for 8-spinor fields, one concludes that the realistic model should include the two 8-spinor fields Ψ_1 and Ψ_2 , which can be naturally unified forming 16-spinor field $\Psi = \Psi_1 \oplus \Psi_2$. Finally, each topological sector should be described by the effective 8-spinor field.

In view of these arguments, using the analogy with Skyrme – Faddeev model, we consider the following Lagrangian density for the 16-spinor field model resembling the one used in [3]:

$$L_{spin} = \frac{1}{2\lambda^2} \overline{\partial_\mu \Psi} \gamma^\nu j_\nu \partial^\mu \Psi + \frac{\varepsilon^2}{4} f_{\mu\nu} f^{\mu\nu} - V, \quad (4)$$

where $f_{\mu\nu}$ stands for the anti-symmetric tensor of Skyrme – Faddeev type:

$$f_{\mu\nu} = (\overline{\Psi} \gamma^\alpha \partial_{[\mu} \Psi) (\partial_{\nu]} \overline{\Psi} \gamma_\alpha \Psi), \quad (5)$$

with λ and ε being constant parameters of the model. It should be underlined that the first term in (4) is similar to the sigma-model one and includes the projector $P = \gamma^0 \gamma^\nu j_\nu$ on the positive energy states. We also remark that the Dirac 16-spinor current conserves its time-like character due to additive property $j_\mu = j_\mu^{(1)} + j_\mu^{(2)}$.

In 16-spinor space we introduce the two kinds of internal Pauli matrices:

$$\vec{\lambda} = I_4 \otimes \vec{\sigma} \otimes I_2, \quad \vec{\Lambda} = I_8 \otimes \vec{\sigma}, \quad (6)$$

where I_n designates the n -dimensional unity and $\vec{\sigma}$ - usual Pauli matrices. We suppose also that the vacuum state has the following structure:

$$\Psi_0 = \Psi_{10} \oplus 0, \quad (7)$$

with the projective property:

$$\Lambda \Psi_0 = 0, \quad \Lambda = \frac{1}{2}(1 - \Lambda_3). \quad (8)$$

Now the main question arises: how to choose the lepton sector or the baryon one? To find the answer, we consider the topological structure of our model implying the distinction between these sectors. First, we pay attention to the fact that in the lepton sector $\vec{a} = i \overline{\Psi} \gamma_5 \vec{\lambda} \Psi = 0$ that implies $B = 0$. This condition can be satisfied if we suppose the mirror symmetry of the lepton sector:

$$\Psi \rightarrow \gamma_0 \Psi. \quad (9)$$

Therefore, if we introduce the following structure of the 16-spinor:

$$\Psi = \bigoplus_{j=1}^2 (\varphi_j \oplus \chi_j \oplus \xi_j \oplus \zeta_j), \quad (10)$$

where $\varphi_j, \chi_j, \xi_j, \zeta_j$ stand for 2-spinors, then (9) implies the invariance condition $\Psi = \gamma_0 \Psi$, that is $\varphi_j = \chi_j, \xi_j = \zeta_j$ for the Weyl representation of γ -matrices. Thus, we conclude that the lepton sector is described by the effective 8-spinor:

$$\Psi_L = \bigoplus_{j=1}^2 (\varphi_j \oplus \varphi_j \oplus \xi_j \oplus \xi_j), \quad (11)$$

which is evidently consistent with the Brioschi identity, since the new vector $\vec{V} = \bar{\Psi} \vec{\Lambda} \Psi \neq 0$.

In particular, one gets

$$j_0 = 2 \sum_{j=1}^2 (|\varphi_j|^2 + |\xi_j|^2); \quad \vec{j} = 0; \quad V_1 = 4 \operatorname{Re} (\varphi_1^+ \varphi_2 + \xi_1^+ \xi_2);$$

$$V_2 = 4 \operatorname{Im} (\varphi_1^+ \varphi_2 + \xi_1^+ \xi_2); \quad V_3 = 2(|\varphi_1|^2 + |\xi_1|^2 - |\varphi_2|^2 - |\xi_2|^2),$$

with the evident property:

$$\frac{1}{4} (j_0^2 - \vec{V}^2) = 4(|\varphi_2|^2 + |\xi_2|^2) (|\varphi_1|^2 + |\xi_1|^2) - 4|\varphi_1^+ \varphi_2 + \xi_1^+ \xi_2|^2 \geq 0$$

due to the Schwartz inequality. In view of the boundary condition (3) we conclude that the correspondent localized configuration can be endowed with the nontrivial Hopf index $Q_H = L$, the S^2 -manifold being determined by the invariant structure \vec{V}^2 .

Let us now consider the baryon sector. In this case we take into account the charge independence of strong interactions, which is equivalent to the mirror symmetry in the isotopic space:

$$\Psi \rightarrow \gamma_0 \gamma_5 \gamma_2 \lambda_2 \Psi^*. \quad (12)$$

Inserting (10) into the invariance condition for the group (12), one finds the following structure for the baryon sector 16-spinor field:

$$\Psi_B = \bigoplus_{j=1}^2 (\varphi_j \oplus \chi_j \oplus i\sigma_2 \varphi_j^* \oplus i\sigma_2 \chi_j^*), \quad (13)$$

which is again equivalent to the effective 8-spinor. In this case we find the following bilinear spinor quantities determining the topology of the baryon sector:

$$j_0 = 2 \sum_{j=1}^2 (|\varphi_j|^2 + |\chi_j|^2); \quad \vec{j} = 0; \quad s = 4 \sum_{j=1}^2 \operatorname{Re} (\varphi_j^+ \chi_j);$$

$$a_1 = -4 \sum_{j=1}^2 \operatorname{Re} (\varphi_j^T \sigma_2 \chi_j); \quad a_2 = -4 \sum_{j=1}^2 \operatorname{Im} (\varphi_j^T \sigma_2 \chi_j); \quad a_3 = 4 \sum_{j=1}^2 \operatorname{Im} (\varphi_j^+ \chi_j).$$

Therefore, we can calculate the chiral invariant structure determining S^3 -manifold:

$$s^2 + \vec{a}^2 = 16 (|\varphi_1^+ \chi_1 + \varphi_2^+ \chi_2|^2 + |\varphi_1^T \sigma_2 \chi_1 + \varphi_2^T \sigma_2 \chi_2|^2).$$

Using the Schwartz inequality, one can prove that $j_0^2 \geq s^2 + \vec{a}^2$. This structure is again consistent with the Higgs potential (2).

2. Interaction with physical vector fields

The Lagrangian (4) admits very large group of transformations of the 16-spinor field Ψ , including phase transformations with some charge generator Γ_e and left, right isotopic rotations with generators $P_{L,R} \lambda^a \Lambda / 2$; $a = 1, 2, 3$, where $P_{L,R} = (1 \pm \gamma_5) / 2$ stand for the corresponding projectors. These symmetries give rise to the interactions with the electromagnetic field A_μ

and left, right Yang – Mills fields $A_\mu^{aL,R}$ respectively, the latter ones being responsible for the strong interactions. Due to the general gauge invariance principle these interactions can be included in the Lagrangian through the extension of the derivative $\partial_\mu \rightarrow D_\mu$, where

$$D_\mu \Psi = \partial_\mu \Psi - i e_0 \Gamma_e A_\mu \Psi + (A_\mu^L + A_\mu^R) \Psi. \quad (14)$$

We adopt the following structure of Γ_e and $A_\mu^{aL,R}$:

$$\Gamma_e = P_3 \Lambda, \quad P_3 = (1 - \lambda_3)/2, \quad A_\mu^{L,R} = P_{L,R} \frac{e_{1L,R}}{2i} A_\mu^{aL,R} \lambda^a \Lambda, \quad (15)$$

which is in accordance with the breaking of isotopic symmetry by the electromagnetic interaction, e_0, e_{1L}, e_{1R} being the corresponding coupling constants. However, the fields $A_\mu^{aL,R}$ seem to be not independent due to the natural condition that Yang – Mills fields should not give any contribution to the lepton sector. This condition can be satisfied, in view of (9) and (14), only if the sum $A_\mu^L + A_\mu^R$ is proportional to γ_5 . Taking into account (15), one concludes that this is possible, only if the following constraint holds:

$$e_{1L} A_\mu^{aL} + e_{1R} A_\mu^{aR} = 0. \quad (16)$$

In view of (16) one can easily find that

$$A_\mu^L + A_\mu^R = \frac{e_{1L}}{2i} A_\mu^{aL} \gamma_5 \lambda^a \Lambda \equiv g_0 \gamma_5 A_\mu, \quad (17)$$

where the denotation is used:

$$e_{1L} \equiv g_0; \quad A_\mu \equiv \frac{\lambda^a}{2i} \Lambda A_\mu^a; \quad A_\mu^{aL} \equiv A_\mu^a.$$

If one introduces the intensity of the Yang – Mills fields:

$$F_{\mu\nu}^{L,R} = \partial_\mu A_\nu^{L,R} - \partial_\nu A_\mu^{L,R} + [A_\mu^{L,R}, A_\nu^{L,R}]$$

and use the natural supposition that $e_{1L} = e_{1R}$, then the standard Yang – Mills Lagrangian can be rewritten, in view of the constraint (16), as follows:

$$L_{YM} = \frac{1}{32\pi g_0^2} Sp(F_{\mu\nu}^L F^{\mu\nu L} + F_{\mu\nu}^R F^{\mu\nu R}) = \frac{1}{32\pi} Sp\{(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + g_0^2 [A_\mu, A_\nu][A^\mu, A^\nu]\},$$

where the symbol Sp signifies the corresponding matrix trace.

Now the extended derivative (14) takes the form:

$$D_\mu \Psi = \partial_\mu \Psi - i e_0 \Gamma_e A_\mu \Psi + g_0 \gamma_5 A_\mu \Psi, \quad (18)$$

the effective Yang – Mills interaction being pseudo-vector one.

3. Solitons and stochastic representation of quantum mechanics

Let us consider small excitations of our soliton near the vacuum: $\Psi = \Psi_0 + \xi$, $\xi \rightarrow 0$ as $|\vec{x}| \rightarrow \infty$. Then the linearized equation for ξ after the substitution $\xi = k \Psi_0$ takes the form:

$$\partial_\mu \partial^\mu k + \frac{1}{2} M_0^2 (k * + k) = 0, \quad M_0 \equiv 2\sigma\lambda \alpha_0. \quad (19)$$

Thus, from (19) one derives the following equations for real and imaginary parts of $k = k_1 + i k_2$:

$$\partial_\mu \partial^\mu k_1 + M_0^2 k_1 = 0, \quad \partial_\mu \partial^\mu k_2 = 0. \quad (20)$$

According to (20), our model admits two types of vacuum excitations: massive and massless ones. For massive soliton the first equation in (20) coinciding with the well-known Klein – Gordon one, the spinor excitation ξ could be interpreted as the wave function of the point-like particle representing the motion of the center of the soliton, if the mass parameter M_0 corresponded to the real mass M of the particle - soliton (or inverse Compton length in natural units $\hbar = c = 1$). To satisfy this condition, we first introduce the interaction with the gravitational field via the new extended derivative generalizing (14):

$$\nabla_\mu \Psi = (D_\mu - \Gamma_\mu) \Psi, \quad (21)$$

where Γ_μ stands for spinor connection with gravity. Then we intend to generalize the Higgs potential (2), in which the constant multiplier σ^2 will be replaced with the special invariant:

$$\sigma^2 = -\frac{16D^3}{\lambda^2 G^2 K^2} \mathfrak{a}_0^{-2}, \quad (22)$$

where G stands for the Newton gravitational constant, D is the sigma-model part of the Lagrangian L_{spin} :

$$D = \overline{\nabla_\mu \Psi} \gamma^\alpha j_\alpha \nabla^\mu \Psi \quad (23)$$

and K is the so-called Kretschmann invariant constructed with the help of the Riemannian curvature tensor:

$$K = \frac{1}{48} R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda}. \quad (24)$$

The validity of the choice (22) can be verified by calculating invariants (23) and (24) for Schwarzschild metric at large distances $r = |\vec{x}| \rightarrow \infty$:

$$D = -\frac{r_g^2}{4r^4} \mathfrak{a}_0^2; \quad K = \frac{r_g^2}{r^6}; \quad r_g = GM. \quad (25)$$

The resulting Lagrangian of our 16-spinor model reads:

$$L = L_{spin} + L_{em} + L_{YM} + L_g, \quad (26)$$

where L_{em} stands for the Maxwellian electromagnetic part and the gravitational Lagrangian L_g coincides with the Einstein one: $L_g = R / (16\pi G)$, R being the scalar curvature.

It is worth-while to stress that the gravitational field plays an important role in our model, since the wave-particle duality principle of quantum mechanics has the gravitational origin. As for the vacuum excitation ξ , one can prove [5, 6] that it plays the role of the wave function in the special stochastic representation of quantum mechanics. To this end, we discuss the fundamental problem of statistical description of solitons stimulated by experimental observations confirming wave - particle duality relations and other quantum properties. The usefulness of the stochastic representation of the wave function will be illustrated by the famous T. Young experiment in soliton realization.

Let us consider an impenetrable screen with n slits, for which the width $d \gg l$, where $l^3 = V_0$ denotes the proper volume of particles - solitons pushed to the screen one by one with some initial velocity. Our aim is to find the probability $dW(x) = w(x) dx$ for observing a center of a particle in the vicinity dx of some point x behind the screen at large distance from it. Let

us also introduce the probability $dW_j(x) = w_j(x) dx$ for the soliton to pass through the j -th slit, and therefore $dW = \sum_{j=1}^n dW_j$. However, the wave - particle duality means that there should be the influence of the k -th slit on the j -th one, i. e. on the passing through the j -th slit. That is why the following relation holds:

$$w = \sum_{j=1}^n w_j = \sum_{j,k=1}^n w_{jk}. \quad (27)$$

As well known, for classical particles $w_{jk} = 0$, if $j \neq k$, contrary to quantum particles.

How to construct the wave function for 1-particle states in stochastic representation? First we introduce the Lagrangian density $L(\phi, \partial_\mu \phi)$ for an abstract real field $\phi(t, \vec{x})$ with an arbitrary number of components. Suppose that the equations of motion admit 1-particle soliton-like solution $\phi(t, \vec{x})$, define its canonical momentum $\pi(t, \vec{x}) = \partial L / \partial \dot{\phi}$ and also the auxiliary complex function

$$\varphi(t, \vec{x}) = \frac{1}{\sqrt{2}} (\nu \phi + i \pi / \nu), \quad (28)$$

where the number ν can be found from the following normalization condition:

$$\hbar = \int d^3x |\varphi|^2, \quad (29)$$

with \hbar being the Planck constant. Assuming the experiment with scattered soliton to be repeated $N \gg 1$ times, we define the wave function $\psi(t, \vec{x})$ as follows:

$$\psi(t, \vec{x}) = \frac{1}{\sqrt{\hbar N}} \sum_{i=1}^N \varphi_i(t, \vec{x}), \quad (30)$$

where $\varphi_i(t, \vec{x})$ denotes the function (28) for the i -th trial. To show that $\psi(t, \vec{x})$ plays the role of probability density $\rho(t, \vec{x})$, let us consider a small volume ΔV with the center \vec{x} and calculate the following quantity:

$$\Delta V \rho = \int_{\Delta V} d^3x |\psi|^2 = \frac{1}{\hbar N} \left(\sum_{i=1}^N a_{ii} + S \right),$$

where

$$S = \sum_{i \neq k=1}^N a_{ik}, \quad a_{ik} = \frac{1}{2} \int_{\Delta V} d^3x (\varphi_i^* \varphi_k + \varphi_k^* \varphi_i).$$

If one takes into account the normalization condition (29), one gets

$$\sum_{i=1}^N a_{ii} = \hbar \Delta N, \quad \Delta V \rho = \frac{1}{\hbar N} (\hbar \Delta N + S), \quad (31)$$

where ΔN is the number of trials, for which the center of our soliton can be observed in the domain ΔV . Taking into account the arbitrariness of initial conditions for solitons φ_i in the i -th trial, one concludes that the quantities a_{ik} for $i \neq k$ can be considered as independent random variables with zero mean values. Therefore, in view of Chebyshev's inequality [7], one can estimate the probability for $|S|$ to surpass $\hbar \Delta N$ as follows:

$$P(|S| > \hbar \Delta N) \leq \frac{1}{(\hbar \Delta N)^2} \langle S^2 \rangle = \frac{1}{(\hbar \Delta N)^2} \sum_{i \neq k=1}^N \langle a_{ik}^2 \rangle. \quad (32)$$

However, the functions φ_i and φ_k can overlap only in the domain of the volume V_0 , that permits one to deduce from (29) and (32) the estimate:

$$\langle S^2 \rangle \leq \alpha \hbar^2 \frac{\Delta N}{\Delta V} V_0 \Delta N,$$

where $\alpha \sim 1$ stands for the "packing" coefficient. Thus, (32) implies the estimate

$$P(|S| > \hbar \Delta N) < \alpha \frac{V_0}{\Delta V} \ll 1, \quad (33)$$

whence with the probability close to unity the quantity $\rho = |\psi|^2$ can be interpreted as the probability density w in accordance with the Born's hypothesis.

Applying now the definition (30) to the Young's experiment, one gets

$$\psi = \sum_{j=1}^n \psi_j, \quad \psi_j = \frac{1}{\sqrt{\hbar N}} \sum_{i(j)} \varphi_i, \quad (34)$$

where indices $i(j)$ correspond to the solitons passing through the j -th slit. The formula (34) expresses the well-known Dirac's superposition principle that permits one to obtain the explicit structure of the "influence" function w_{jk} . In fact, inserting (34) into (27), one gets

$$\sum_{j,k=1}^n w_{jk} = \left| \sum_{j=1}^n \psi_j \right|^2 = \frac{1}{2} \sum_{j,k=1}^n (\psi_j^* \psi_k + \psi_k^* \psi_j) \equiv \sum_{j,k} s_{jk}. \quad (35)$$

Substituting $w_{jk} = s_{jk} + s_j \delta_{jk}$ into (35), one infers that $\sum_{j=1}^n s_j = 0$. Assuming that

$$s_j = s - \sum_{k=1}^n s_{jk},$$

one derives the relation

$$s n = \sum_{j,k=1}^n s_{jk} = |\psi|^2$$

implying the structure of w_{ik} :

$$w_{ik} = s_{ik} + \delta_{ik} \left(\frac{1}{n} |\psi|^2 - \sum_{j=1}^n s_{ij} \right). \quad (36)$$

Let us now discuss the problem of measurement of some physical observable A , which can be represented as a bilinear form, in view of E. Noether's theorem, if the generator \hat{M}_A of the corresponding symmetry group is given:

$$A = \int d^3x \pi i \hat{M}_A \phi = \int d^3x \phi^* \hat{M}_A \phi. \quad (37)$$

Applying (37) to the j -th trial, one can write the mean value over N trials as follows:

$$\langle A \rangle = \frac{1}{N} \sum_{j=1}^N A_j = \frac{1}{N} \int d^3x \sum_{j=1}^N (\varphi_j^* \hat{M}_A \varphi_j). \quad (38)$$

Using the definition (30) of the wave function and the estimate (33), one can derive from (38) that with the probability close to unity

$$\langle A \rangle = \int d^3x \psi^* \hat{A} \psi, \quad \hat{A} \equiv \hbar \hat{M}_A. \quad (39)$$

As can be seen, the formula (39) expresses the quantum mechanical rule for calculating the mean value of some observable A generated by the Hermitian operator $\hat{A} = \hbar \hat{M}_A$.

What will change if one considers the case of K particles - solitons? In this case a new definition of the wave function in the configuration space of $3K$ dimensions should be given as generalization of (30):

$$\psi(t, \vec{x}_1, \dots, \vec{x}_K) = (\hbar^K N)^{-1/2} \sum_{j=1}^N \prod_{k=1}^K \varphi_j^{(k)}(t, \vec{x}_k), \quad (40)$$

where $\varphi_j^{(k)}$ denotes the soliton configuration of the form (28) for the k -th particle - soliton in j -th trial. In this case the mean value of some observable A will be given by similar formula:

$$\langle A \rangle = \int d^{3K} x \psi^* \hat{A} \psi, \quad \hat{A} \equiv \sum_{k=1}^K \hbar \hat{M}_A^{(k)}.$$

It is worth-while now to explain, on the basis of the definition (40), the spin - statistics correlation. Suppose that all K particles are identical, that is $\varphi_j^{(k)} = \varphi_j(\vec{x}_k - \vec{r}_k)$, where \vec{r}_k denotes the center of the k -th particle - soliton and functions φ_j belong to the irreducible representation $D^{(J)}$ with the weight J of the rotation group $SO(3)$. One can also identify the weight J and the spin of our particle - soliton. Let us now choose two arbitrary particles with the centers \vec{r}_1, \vec{r}_2 :

$$\varphi_j^{(1)} = \varphi_j(\vec{x}_1 - \vec{r}_1), \quad \varphi_j^{(2)} = \varphi_j(\vec{x}_2 - \vec{r}_2),$$

and perform their transposition. First of all we rotate our two-particle system over angle π around the mediatrice of the central line $\vec{r}_1 - \vec{r}_2$. However, in view of extended character of our particles - solitons, to restore the previous state of the system it would be necessary to perform two additional rotations over angle π around particles proper axes parallel to the mediatrice. The latter operation being equivalent to the relative rotation of particles over angle 2π , it implies the multiplication of ψ by $(-1)^{2J}$. Therefore, the wave function should be symmetrical under transpositions $1 \Leftrightarrow 2$ for integer J and anti-symmetrical - for semi-integer J . As can be seen, this fact corresponds to the well-known spin - statistics correlation in quantum mechanics.

4. Conclusions

We show the effectiveness of the 16-spinor realization of the popular in particle physics Skyrme and Faddeev models for the description of baryons and leptons as topological solitons. It is worth-while to underline that the gravitational field plays an important role in our model, since the wave - particle duality principle of quantum mechanics has the gravitational origin and the vacuum excitation ξ can be considered as a tool for constructing wave functions in the special stochastic representation [5, 6] in accordance with the definitions (30) and (40). On the other hand, at small distances, due to the inclusion of the Kretschmann invariant containing higher derivatives of the metric tensor, the gravity becomes strong and provides the existence of topologically nontrivial field configurations with closed string structure [2].

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