Clustering in nuclei

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Abstract. I stress a role of clustering in describing the structure of such state that is difficult to understand from the mean field. The examples include the neutron-halo of ^{11}Li, ^{3}H+p and ^{3}He+n clustering in ^{4}He, and a unified description of the ground and first excited 0^+ states of ^{16}O.

1. Introduction

Cluster model has a long history. Especially a microscopic version of the cluster model is known for a long time as a unified theory of both structure and reactions for light nuclei [1]. Quite often a semimicroscopic cluster model taking into account the essence of the Pauli principle has been employed to reproduce those states which are difficult to understand from the shell model. One well-known example is the first excited 0^+ state of ^{16}O. The state together with other low-lying states of ^{16}O are reproduced well by an $\alpha+^{12}$C cluster model [2] in which the rotational motion of ^{12}C is taken into account. The same model also successfully describes low-energy $\alpha+^{12}$C scattering [3].

Since it stresses a particular type of nuclear excitations, the cluster model misses many other modes of excitations, e.g. monopole and quadrupole excitations that are well described in a symplectic model [4]. The coupling between the cluster and symplectic models was studied more than 20 years ago [5] using the SU(3) algebra [6], and it was applied to the ^{16}O and ^{20}Ne region [7]. This problem appears to attract some attention even today [8] as experimental observation of the monopole strength reaches higher in the excitation energy.

Some evidence indicating the importance of a cluster concept appears in heavy nuclei as well. The recent observation of the unnatural parity states in ^{212}Po [9] that have enhanced electric dipole transition strength is interpreted in an $\alpha+^{208}$Pb cluster model [10] in which ^{208}Pb can be excited to the 3^- vibrational state. Note, however, that a microscopic calculation with the cluster model becomes more difficult as the mass number of the nuclear system increases. See, for example, a calculation for the α-decay of the ^{212}Po ground state [11].

I focus on three topics performed from the cluster viewpoint: the neutron halo structure of ^{11}Li [12], ^{3}H(t)+p and ^{3}He(h)+n clustering in the excited states of ^{4}He [13], and the shell-cluster competition in ^{16}O [14]. In each of these I stress different roles of the cluster model. In the first case we want to quantify how much the distortion of ^{9}Li improves the halo nature of ^{11}Li in a multi-cluster approximation. In the second topic I point out the emergence of a cluster structure in ^{4}He based on the results of a four-body calculation for ^{4}He with realistic potentials. In the last case I attempt at describing both the ground and first excited states of ^{16}O in a

^{12}C$+p+p+n+n$ five-body calculation. The basis functions we use are explicitly correlated Gaussians [15] that were proposed more than a half century ago, and further augmented greatly by developing a global vector representation of the angular part responsible for a rotational motion [16, 17, 18, 19]. An optimization of the basis functions is performed by a stochastic variational method [16, 17].

2. Neutron halo structure of ^{11}Li

A basic feature of ^{11}Li can be understood from its Borromean structure of ^{9}Li$+n+n$. It is interesting to know the extent to which the excitation of the ^{9}Li core plays a role in binding the halo neutrons. To examine this problem, we need to break the core. For the moment no *ab initio* calculation is available for ^{11}Li, obviously because ^{11}Li is so fragile that an accurate calculation is beyond a present theory. We used a microscopic multi-cluster approach in which ^{9,10,11}Li are described assuming α, t, and n clusters. That is, ^{9}Li and ^{11}Li are treated as four- and six-body systems of $\alpha+t+n+n$ and $\alpha+t+n+n+n+n$, respectively. The inter-cluster motion is solved as accurately as possible while the intrinsic wave functions of α and t are kept fixed as $0s$ wave functions. In this way we can quantify the distortion of the ^{9}Li core. The nucleon-nucleon interaction employed is the Minnesota potential, and the total wave function is properly antisymmetrized. This model is consistent with an earlier description of ^{7,8,9}Li [20].

The wave function of ^{9}Li is constructed in the $\alpha+t+n+n$ model by carefully optimizing the parameters of the correlated Gaussians. The properties obtained for ^{9}Li are listed in Table 1. They are in excellent agreement with experiment. The basis functions for ^{11}Li are constructed by adding two neutrons to ^{9}Li. They are characterized by Y- and T-type arrangements as well as the orbital and spin angular momenta of the neutrons. Two types of calculations are compared in Table 1. One is a frozen (fr.) model in which the ^{9}Li core is fixed to its ground state configuration. The other one called a full model takes into account the excited configurations, which allows for the distortion of the core. It is seen that a significant gain of the energy is obtained by accounting for the core distortion. The inclusion of the distortion makes the proton radius (r_p) considerably larger (by about 0.3 fm) and the neutron radius slightly smaller. This seems to be understood from a mechanism similar to what we found in ^{6}He [21]: The halo neutron receives more binding by gaining the attraction with the protons inside the ^{9}Li core as the np interaction is more attractive than the nn interaction. For this to happen, the core has to be distorted so as to have more chance that the protons can approach the halo neutrons more frequently. On the other hand the neutron size would shrink because of stronger binding. Though no accurate values on the proton radii were available when we published the results in 2002, our full model result agrees very well with the recently measured radius of ^{11}Li. The binding of ^{11}Li receives approximately equal contributions from p- and s-wave single-particle

Table 1. Properties of ^{9}Li and ^{11}Li. The energies ϵ are given from the two-neutron thresholds. See text for the frozen (fr.) and full models.

| Nucleus		ϵ (MeV)	r_p (fm)	r_n (fm)	r_m (fm)	Q ($e$fm^2)	μ (μ_N)	$\sigma_{\rm R}$ (mb)
^{9}Li		-5.91	2.12	2.54	2.41	-3.37	3.40	799$\pm$3
	exp.	-6.09	2.11 [22]			-3.06	3.44	796$\pm$6
^{11}Li	fr.	-0.12	2.15	3.21	3.15	-3.52	3.21	1054$\pm$2
	full	-0.34	2.43	3.09	3.03	-3.71	3.23	1023$\pm$2
	exp.	-0.37	2.38 [22]			$|Q|$=3.33 [23]	3.67	1056$\pm$14

states. The deviation of the calculated magnetic moment from experiment is probably due to that of t. See Ref. [12] for detail.

3. Clustering due to parity inversion in ^{4}He

^{4}He is a doubly magic nucleus. This system is small enough to make it possible for us to perform an *ab initio* calculation using realistic nucleon-nucleon interactions. Its first excited state with 0^+ is located between the $t+p$ and $h+n$ thresholds. This state is found to have large spectroscopic amplitudes to those decay channels, consistent with the $3N + N$ cluster structure [24]. We find that in this state the spins of the $3N$ and N clusters are coupled to $I = 0$, and the orbital angular momentum of the relative motion between them is $\ell = 0$ [13].

There are several negative-parity states above the excited 0^+ state. Though they are conventionally considered on the shell-model basis, an inversion doublet picture may be useful to interpret some of their structure. That is, a negative-parity state that has basically the same $3N + N$ configuration may appear near the threshold [25]. Unlike in the case of $\alpha+^{16}$O or $\alpha+^{12}$C, the clusters are, however, not spinless in the present case. Actually when the $3N + N$ clusters are coupled to $I = 1$, the odd ℓ value is favored for the relative motion, especially $\ell = 1$ is most interesting. The coupling of I and ℓ suggests that some $3N + N$ cluster states with $J^\pi = 0^-, 1^-, 2^-$ may show up. The calculation using a realistic nucleon-nucleon potential confirms that the lowest $0^-, T = 0$ and 2^- states can in fact be well understood from the cluster model rather than the shell model [13]. Figure 1 displays the spectroscopic amplitudes of the three states. As expected, each of three curves shows behavior indicating the $3N + N$ cluster structure. The peaks are centered around 2 fm near the $3N$ surface. The $3N + N$ spectroscopic factors are large, 0.58, 0.52, and 0.53 for the 0^-0, 2^-0, and 2^-1 states.

The negative-parity states of ^{4}He actually have broad widths. Though a bound-state method used in Ref. [13] may be useful to obtain the level spectrum qualitatively, one should in principle treat the continuum more properly. For example, strength functions for electric dipole [26] and spin-dipole operators give us information on the relevant resonance states.
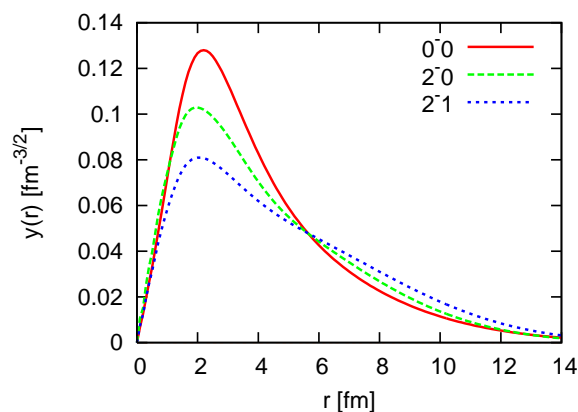
Figure 1. Spectroscopic amplitudes of the three lowest-lying negative-parity states of ^{4}He for the p-wave $h + n$ decay with $I = 1$.

4. The 0^+ states of ^{16}O

A description of states that show prominent clustering on the basis of single-particle orbits is extremely hard because it requires in general enormous major-shell excitations. See, e.g.,

Refs. [27, 28]. It is thus obviously very interesting to attempt at describing such states without assuming the clusters *a priori*. The case of ^{4}He discussed in the previous section is one such attempt. Here we focus on the two 0^+ states of ^{16}O. The ground state is again doubly magic, and a typical shell-model state. The first excited state defies a shell-model description. We ask a question of if both states can be described well with a model built on a ^{12}C core plus four valence nucleons $(p + p + n + n)$.

^{12}C is treated by a rotational model. The valence nucleons are constrained not to include those orbits that are occupied by the core. A brief formulation is given in Ref. [14]. The Hamiltonian reads

$$H = T + H_{core} + \sum_i U_i + \sum_{i<j} V_{ij}, \tag{1}$$

where the potential U_i acts between the core and valence nucleon and V_{ij} is the potential acting among the valence nucleons. Letting ψ_F denote the orbits occupied by the core, the total wave function has to satisfy the condition

$$\langle \psi_F | \Psi \rangle = 0 \qquad \text{for all valence nucleons.} \tag{2}$$

As a first attempt we ignore the excitation of ^{12}C, and thus H_{core} is a constant. The potential U_i is determined following the rotational model and its strength is set to reproduce the single-particle energies of ^{13}C. The orbits ψ_F are taken as $0s_{1/2}$ and $0p_{3/2}$. The total wave function is assumed to be given as a combination of the correlated Gaussians with the global vector representation for the angular part. Figure 2 displays the energies of the two 0^+ states. The energy convergence is rather slow. This is probably because fulfilling the condition (2) is hard at least in a small basis dimension.

The calculation further continues up to the basis dimension 8500. The ground state is almost converged well, and its energy turns out to be close to experiment. After choosing the basis functions so as to optimize the excited state, the first excited state reaches only slightly above the threshold. The state still contains some forbidden-state components, which pushes up the energy. Estimating this energy loss, we think that the first excited state is obtained just below the α threshold. It appears that we thus obtain the two 0^+ states at the expected positions. An analysis of their structure is in progress.
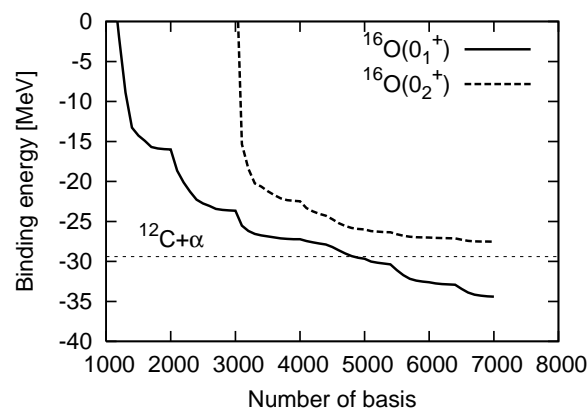
Figure 2. The ground and first excited 0^+ states of ^{16}O as a function of the basis dimension. The dotted line is the calculated α threshold. The Minnesota potential is used. The experimental energies of the two states are 7.16 and 1.11 MeV below the threshold.

5. Summary

In this talk I showed three examples. In a microscopic multi-cluster description of ^{11}Li the distortion of ^{9}Li is vital to gain the binding of the halo neutrons. An *ab initio* calculation using realistic forces confirms that some positive- and negative-parity states of ^{4}He show $3N + N$ cluster structure. Though a unified approach of ^{12}C$+p + p + n + n$ for ^{16}O is quite expensive, we have an indication that the ground and first excited states emerge at the expected energies.

Clustering is an important and useful concept in both structure and reactions. It certainly deviates from a mean field picture. Thus a fully microscopic description of clustering is quite expensive. It is a challenge of next generation to found clustering on innovative microscopic theories. With a progress of research for exotic nuclei, I expect physics of clustering will play a vital role furthermore.

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