

## Shape evolution in odd-A Ge isotopes: the effect of $g_{9/2}$ orbital

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### Introduction

Recently, the experimental results show interesting shape properties for the Zn ( $Z = 30$ ) and Ge ( $Z = 32$ ) nuclei near  $N = 40$  [1-4]. The shape evolution in the odd-A isotopes in these nuclei is governed by the neutron  $g_{9/2}$  orbital. The systematic of the  $g_{9/2}$  orbitals in Ge isotopes, shown in Fig.1, indicates that  $9/2^+$  is the ground state for the  $N = 41$  isotope  $^{73}\text{Ge}$  and its excitation energy increases on either side of it. So,  $N = 40$  behaves like a transitional point in Ge isotopes.

The shape evolution of the Ge isotopes as a function of the  $g_{9/2}$  occupation of the odd neutron has been investigated in this work. For this, the total Routhian surfaces (TRS) were calculated for the positive parity neutron configuration in the odd-A Ge isotopes spanning the entire  $fpg$  shell of the neutrons between the shell closures at  $N = 28$  and 50. Calculations are also performed for  $^{59}\text{Ge}$  ( $N = 27$ ), for which the neutron Fermi level lies below the  $N = 28$  shell closure, for comparison. The method described in Ref. [5,6] and references there in, has been adopted in this work.

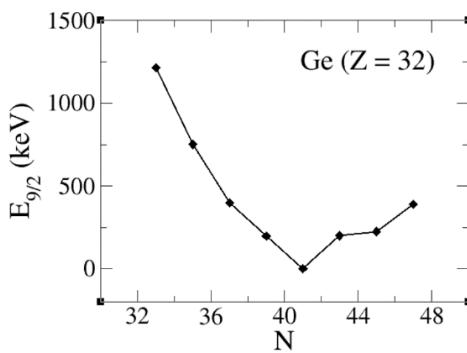


Fig.1: Systematics of the measured excitation energy ( $E_{9/2}$ ) of the  $9/2^+$  state in Ge isotopes as a function of neutron number  $N$ .

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### Results and Discussion

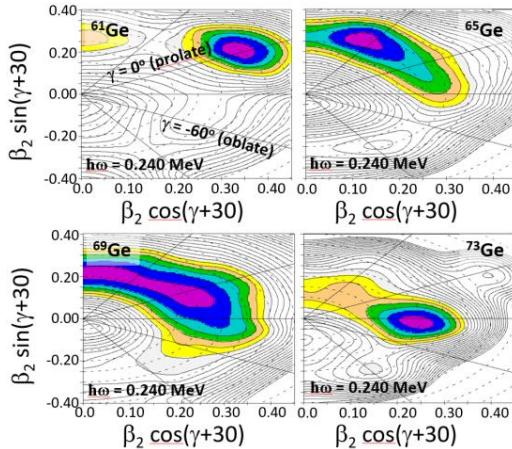
The results are summarized in Table 1. It contains the deformation parameters  $\beta_2$  and  $\gamma$  for different nuclei, corresponding to the minimum in the TRS. These parameters determine the shapes of these nuclei for the one quasi-particle  $vg_{9/2}$  configuration. Here, the deformation parameter  $\beta_2$  gives the amount of deformation and  $\gamma$  gives the nature of deformation. In the Lund convention, used here,  $\gamma = 0^\circ$  ( $-60^\circ$ ) corresponds to collective prolate (oblate) shapes and  $\gamma$  value in between these limits signifies triaxial shape with  $\gamma = \pm 30^\circ$  is considered as maximum triaxiality.

**Table 1:** Deformation parameters of the Ge isotopes obtained in this work at  $\hbar\omega = 0.240$  MeV.

N	Nucleus	$\beta_2$	$\gamma$
27	$^{59}\text{Ge}$	0.348	-58.1
29	$^{61}\text{Ge}$	0.394	3.1
31	$^{63}\text{Ge}$	0.344	4.5
33	$^{65}\text{Ge}$	0.284	36.5
35	$^{67}\text{Ge}$	0.274	86.0
37	$^{69}\text{Ge}$	0.256	-5.2
39	$^{71}\text{Ge}$	0.261	-32.5
41	$^{73}\text{Ge}$	0.241	-33.5
43	$^{75}\text{Ge}$	0.232	-37.5
45	$^{77}\text{Ge}$	0.124	-19.4
47	$^{79}\text{Ge}$	0.130	-28.0
49	$^{81}\text{Ge}$	0.114	-116.0

It is interesting to note from Table 1 that  $^{59,61}\text{Ge}$  has oblate and prolate shapes, respectively, with large deformation parameter  $\beta_2$  compared to the other isotopes. This may be understood from the fact that certain Nilsson components of the  $g_{9/2}$  orbital can be accessible for the odd neutron in these isotopes, only for large  $\beta_2$ . It can also be observed that the deformation parameter  $\beta_2$

decreases with the increase in neutron number as the  $g_{9/2}$  orbital becoming more accessible for near spherical shape.

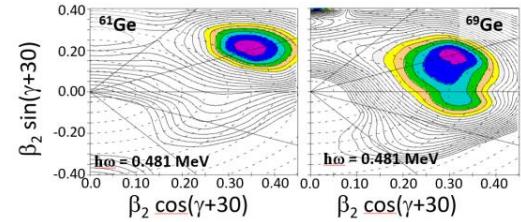


**Fig. 2:** TRS plots for  $^{61,65,69,73}\text{Ge}$  calculated at the rotational frequency  $\hbar\omega = 0.24$  MeV. The contours are 250 keV apart.

The TRS plots for some of the Ge isotopes, calculated in this work for the 1 quasi-particle positive parity, positive signature configuration, are presented in Fig.2. The large prolate shapes for  $^{61-65}\text{Ge}$  are clearly evident in this plot (TRS plot for  $^{63}\text{Ge}$  is similar to that of  $^{65}\text{Ge}$  but is not shown in Fig.2). As the neutron number increases, the minimum in the potential energy shifts towards large triaxial shape for neutron number around  $N \sim 40$ , as shown in the plot for  $^{73}\text{Ge}$ , through gamma-soft shape, as shown for  $^{69}\text{Ge}$ . The shape of  $^{71}\text{Ge}$  (not shown in Fig.2) and  $^{73}\text{Ge}$  with  $N = 39$  and 41, respectively, becomes triaxial with  $\gamma$  values close to maximum triaxiality (see Table 1). Therefore, the lowering of the  $9/2^+$  energy of the Ge nuclei around  $N = 40$  (as shown in Fig. 1) perhaps, due to their triaxial shapes. It would be interesting to investigate the nature of the  $g_{9/2}$  orbital as a function of the deformation parameter  $\gamma$ . With further increase of neutrons towards  $N = 50$ , both triaxiality and deformation decrease towards near spherical shape.

Structural evolution of the Ge isotopes as a function of rotational frequency has also been studied in this work and the TRS plots for two isotopes,  $^{61}\text{Ge}$  and  $^{69}\text{Ge}$ , calculated at  $\hbar\omega = 0.48$  MeV are shown in Fig.3. It can be seen that the prolate shape in  $^{61}\text{Ge}$  at lower  $\hbar\omega$  continues to

retain its shape even at higher rotational frequency, though the  $\beta_2$  value reduces slightly. On the other hand, a significant change in shape is observed for  $^{69}\text{Ge}$  and  $^{67}\text{Ge}$  isotopes. The gamma-soft shapes in these nuclei at lower frequency change to a near prolate shape with larger  $\beta_2$  at higher frequency. This indicates that the gamma-soft shape stabilizes at an axially symmetric prolate shape by the effect of the coriolis force in these isotopes with  $N = 35, 37$ .



**Fig. 3:** Same as Fig.2 but for  $^{61,69}\text{Ge}$  at  $\hbar\omega = 0.48$  MeV.

## Conclusion

The shape evolution of the positive parity configuration in the odd- $A$  Ge isotopes has been studied by the TRS calculations. The calculations are done for neutron number ranging between two shell closures at  $N = 28$  and 50. A change in shape from large prolate for the neutron deficient isotopes near  $N = 28$  to a near spherical one close to  $N = 50$  has been obtained. The isotopes with neutron number around  $N = 40$  are predicted to be triaxial. The gamma-soft shapes for  $^{65,69}\text{Ge}$  at lower rotational frequency get stabilized to a near prolate shape at higher frequency. The experimental evidence of triaxial shapes and shape evolution will be interesting.

## References

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