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GUP-reinforced Hawking radiation in rotating linear dilaton black hole spacetime

E Sucu and İ Sakallı*

Department of Physics, Eastern Mediterranean University, Famagusta, 99628 North Cyprus via Mersin 10, Turkey

* Author to whom any correspondence should be addressed.

E-mail: erdemsc07@gmail.com and izzet.sakalli@emu.edu.tr

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Abstract

This article investigates the influence of the Generalized Uncertainty Principle (GUP) on the emission of Hawking quanta in a rotating linear dilaton black hole spacetime. The study proposes a GUP-reinforced black hole thermal emission model that takes into account the quantum tunneling process with GUP effects. The result obtained for the corrected temperature suggests that temperature of the GUP-reinforced Hawking radiation decreases with the increasing GUP parameter and gets higher values with the increasing mass of the black hole. The study also discusses the implications of these findings on the corrected entropy and hence the information loss paradox, and the potential for experimental verification of GUP effects in astrophysical observations. Overall, this work highlights the significant role of GUP in the thermal emission of non-asymptotically flat stationary black holes and can shed light on the intricate interplay between quantum gravity and astrophysics.

1. Introduction

Black holes [1] are intriguing objects that are of great interest to physicists due to their unique properties and their potential for shedding light on fundamental questions in physics, such as the nature of spacetime and the behavior of matter under extreme conditions. The studies on black hole thermodynamics [2] have long been a subject of fascination and debate among physicists. One of the key phenomena in this field is Hawking radiation, which describes the emission of particles by black holes due to quantum effects. Despite its groundbreaking implications for the understanding of the nature of spacetime, Hawking radiation [3] remains one of the most challenging and mysterious aspects of black hole physics. Because the precise mechanism underlying Hawking radiation is still a subject of active research and many open questions are on the agenda of the current literature.

GUP [4–7] is a modification of the Heisenberg uncertainty principle that arises from the interplay between quantum mechanics and gravity. GUP predicts that there is a fundamental limit to the precision with which certain pairs of observables, such as position and momentum, can be measured. In recent years, there has been growing interest in the role of GUP in the dynamics of black holes (see for example [8–13]). The implications of GUP effects on the dynamics of black holes modify the emission of Hawking radiation (for a topical review, the reader is referred to [14] and references therein). GUP has been shown to modify the entropy of black holes and to play a role in resolving the information loss paradox [15–17], which is one of the most challenging problems in theoretical physics. The information loss paradox suggests that the quantum mechanical information contained in a matter that falls into a black hole is lost forever, leading to a violation of unitarity and a breakdown of the laws of quantum mechanics [18]. However, GUP effects can potentially resolve this paradox by leading to corrections to the black hole entropy that depends on the Planck length, which is the fundamental length scale of quantum gravity [19].

Among the notable studies about the GUP, [20] discusses an investigation into the quantum tunneling of massless particles through the quantum horizon of a Schwarzschild black hole. This study takes into account quantum gravity effects with the incorporation of natural cutoffs, including a minimal length, minimal momentum, and maximal momentum through a GUP. The research focuses on potential correlations between emitted particles to address the information loss problem. The role of these natural cutoffs in influencing the tunneling rate through the quantum horizon is also explored. Moreover, [21] explores the implications of the GUP on Hawking radiation and the final stages of black hole evaporation. By integrating the GUP into the quantum tunneling process based on the null-geodesic method, correlations emerge between the tunneling probabilities of different modes in the black hole radiation spectrum. This leads to the encoding of quantum information in Hawking radiation, allowing for potential non-thermal GUP correlations to recover the information. And [22] compares the tunneling rates from the two horizons and highlights the significant impact of the quintessence field on the thermodynamics and behavior of the black hole. Quantum corrections due to the quintessence field influence the horizon locations and prevent the singularity's approach, ultimately leaving behind a Planck scale remnant with quintessence content.

Rotating linear dilaton black holes (RLDBHs) [23–25] are a type of black hole solution in theoretical physics that have garnered attention in recent years due to their potential to shed light on some of the most fundamental questions in astrophysics and cosmology [26, 27]. In this study, we first re-explore the physical properties of RLDBHs, which are spinning on their axis. In addition to the mass and spin, the RLDBHs are also characterized by the *background* charge. One of the intriguing properties of the RLDBHs is their potential connection to dark matter [28–30]. Meanwhile, the dark matter [31] is a hypothetical form of matter that is thought to make up a significant portion of the total matter in the Universe, but which does not interact with light and thus cannot be directly observed. On the other hand, dilaton is a scalar field that arises in string theory is thought to permeate all of space and play a role in the formation of black holes [32]. Moreover, recent studies have suggested that the dilaton field may be related to dark matter and for this reason, as stated above, the RLDBHs could be a potential candidate for explaining some of the properties of dark matter. It is also worth noting that the dilaton field could interact with dark matter particles in such a way as to give rise to a force that affects the motion of stars and galaxies, leading to the observed effects of dark matter like the rotational speeds of galaxies, which are much higher than what can be accounted for by visible matter alone [33, 34].

The structure of this work is as follows. In the current section (1), we review some basic information about the Hawking radiation, GUP analysis, RLDBH spacetime, and its fundamental properties. In section (2), we introduce the metric of non-asymptotically flat (NAF) RLDBH spacetime and analyse its some physical features. In section (3), we employ the Parikh-Wilczek's quantum tunneling method [35] for the RLDBH metric and show how the Hawking temperature of the RLDBH is computed. In the sequel, we study the GUP corrected temperature and entropy of the RLDBH in section (4). Finally, we draw our conclusions in section (5).

Throughout the paper, unless stated otherwise, we use the geometrized (natural) units $G = c = \hbar = k_B = 1$ and $(+, -, -, -)$ metric signature.

2. Physical properties of RLDBH

This section provides a concise overview of the spacetime of RLDBH spacetime, which was first described by Cl é ment *et al* [23]. The theory of EMDA (Einstein-Maxwell-Dilaton-Axion) gravity can be considered as a truncated form of the bosonic portion of $D = 4, N = 4$ supergravity [36]. The EMDA gravity theory's action can be expressed as follows [23]:

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[-\mathcal{R} + 2\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}e^{4\phi}\partial_\mu\mathbb{A}\partial^\mu\mathbb{A} - e^{-2\phi}F_{\mu\nu}F^{\mu\nu} - \mathbb{A}F_{\mu\nu}\tilde{F}^{\mu\nu}\right], \tag{1}$$

where ϕ and $\mathbb{A}$ represent the dilaton and the axion (pseudoscalar) fields, respectively. $\mathcal{R}$ stands for the Ricci scalar, F and $\tilde{F}$ denote the electromagnetic field strengths of the abelian vector field A and its dual, respectively. In addition to the static black hole solution of the EMDA theory (1), [23] also provides an explicit metric of the RLDBH spacetime without the need for the NUT charge [37]:

$$ds^2 = \frac{\Delta}{r_0 r}dt^2 - r_0 r\left[\frac{dr^2}{\Delta} + d\theta^2 + \sin^2\theta\left(d\varphi - \frac{a}{r_0 r}dt\right)^2\right], \tag{2}$$

where $\Delta = r^2 - 2Mr + a^2$ and the other background fields are given by

$$F = \frac{1}{\sqrt{2}}\left[\frac{r^2 - a^2\cos^2\theta}{r_0 r^2}dr\wedge dt + a\sin 2\theta d\theta\wedge\left(d\varphi - \frac{a}{r_0 r}dt\right)\right], \tag{3}$$

$$e^{-2\phi} = r_0 r\chi, \tag{4}$$

$$\mathbb{A} = -r_0 a\cos\theta\chi. \tag{5}$$

in which $\chi = (r^2 + a^2\cos^2\theta)^{-1}$. Meanwhile, the physical parameters a and r_0 denote the rotation and background charge of the spacetime, respectively. In fact, metric (2) was derived from the Kerr metric by using a particular solution-generating technique [23]. However, the metric represented by equation (2) differs in two ways: its behavior as r approaches infinity is not flat, but the spacetime metric is nothing but the static linear dilaton metric. Its behavior near $r = 0$ is also distinct from that of the Kerr metric. In the case of the Kerr metric, a disk exists at $r = 0$ through which the metric can be extended to negative r. Conversely, in equation (2), $r = 0$ is a timelike line singularity. As a result, the Penrose diagrams of equation (2) are identical for all three cases ($a^2 < M^2$, $a^2 = M^2$, and $a^2 > M^2$), but are distinct from those of the Kerr spacetime. Instead, they resemble the Penrose diagrams of the Reissner-Nordström spacetime, where the charge is replaced by the angular momentum (or the rotation) parameter a.

Now, we want to summarize the thermodynamics features of the RLDBH. First of all, it should be noted that M appeared in the solution is no longer the ADM mass. To obtain the first law of black hole mechanics, the relevant thermodynamics quantities can be calculated by using the mass computation of Brown and York [38] who formulated the mass for NAF spacetimes. The quasilocal mass $\widetilde{M}$ [39] of the RLDBH is associated with the mass parameter M as follows

$$\widetilde{M} = \frac{M}{2}, \tag{6}$$

and the angular momentum J is given by

$$J = \frac{ar_0}{2}. \tag{7}$$

The statistical Hawking temperature T_{RLDBH}, the Bekenstein-Hawking entropy S_{RLDBH}, and the angular velocity Ω_{RLDBH} of the RLDBH are given by [40]

$$T_{RLDBH} = \frac{\kappa}{2\pi} = \frac{r_+ - r_-}{4\pi r_0 r_+}, \tag{8}$$

$$\Omega_{RLDBH} = \frac{a}{r_0 r_+}, \tag{9}$$

$$S_{RLDBH} = \pi r_0 r_+, \tag{10}$$

where $r_\pm = M \pm \sqrt{M^2 - a^2}$ are the locations of the outer (event) horizon and the inner horizon, respectively. With the thermodynamical quantities given above, one can check that the first law of black hole mechanics holds for the RLDBH:

$$d\widetilde{M} = T_{RLDBH}\, dS_{RLDBH} + \Omega_{RLDBH}\, dJ. \tag{11}$$

It should be noted that the electric charge:

$$Q = \frac{r_0}{\sqrt{2}}, \tag{12}$$

does not appear in the above thermodynamics relation. The differentiations seen in equation (11) are performed by keeping Q as a fixed value, which is a characteristic feature of linear dilaton backgrounds. Namely, the electric charge Q is nothing but the background charge (a reader can refer to [23] for more information). Moreover, the extremality condition is given by $M = a$ and the entropy [41] at extremality reads

$$S_{RLDBH}(T = 0) = \pi r_0 a. \tag{13}$$

As can be seen from above, for extremal black holes, the event horizon area is at its minimum, implying that the entropy associated with these black holes is also minimized. Thus, the entropy of an extremal black hole is typically much smaller than that of a non-extremal black hole (10).

3. Hawking radiation of RLDBH via quantum tunneling process

In the context of black hole physics, the event horizon is the boundary beyond which nothing, not even light, can escape. According to quantum mechanics, particles can exhibit wave-like behavior and have a certain probability of crossing energy barriers that would be classically forbidden. This phenomenon is known as quantum tunneling. On the other hand, the WKB (Wentzel-Kramers-Brillouin) [42] approximation is a semiclassical method that allows us to calculate the tunneling probability of a particle through a barrier. It is based on the assumption that the particle's wave function can be approximated as a rapidly oscillating wave in some regions and a slowly varying wave in others.

The tunneling rate of an s-wave from inside to outside the black hole horizon in the framework of the WKB approximation[1] is given by [43]

$$\Gamma = \Gamma_0 \exp(-2\,\mathrm{Im}\,\mathcal{I}), \tag{14}$$

[1] Details about the methodology used in this section can be found in [58].

in which $\mathcal{I}$ is the action of the tunneling particle and Γ_0 denotes the normalization factor. Since a black hole's radiation conforms to the law of Boltzmann distribution in a classical sense, i.e. as a blackbody radiation, it is possible to describe the rate at which the energetic particles are emitted from the horizon of a black hole as follows:

$$\Gamma = \Gamma_0 \exp(-\beta\mathcal{E}), \tag{15}$$

where $\beta = \frac{1}{T}$ is the inverse temperature (T) and is known as the Boltzmann constant [3]. After that, the imaginary part of a tunneling particle's action in terms of an s-wave can be estimated as [10, 11]:

$$\mathrm{Im}\,\mathcal{I} = \mathrm{Im}\int_{r_h(M)}^{r_h(M-\mathcal{E})} p_r\, dr = \mathrm{Im}\int_{r_h(M)}^{r_h(M-\mathcal{E})}\int_0^{p_r} dp_r'\, dr, \tag{16}$$

where r_h is the distance from the black hole's center ($r_h(M) = r_+$ and $r_h(M - \mathcal{E}) < r_+$), M stands for the original overall mass of the black hole, while $M - \mathcal{E}$ denotes the resultant mass of the black hole subsequent to the emission of radiation caused by the particle with energy $\mathcal{E}$ being tunneled away. By employing Hamilton's equations of motion, we arrive at the following outcome:

$$\dot{r} = \frac{dH}{dp_r} = \frac{d(M-\omega)}{dp_r}, \tag{17}$$

which can be rewritten as

$$dp_r = \frac{d(M-\omega)}{\dot{r}}. \tag{18}$$

If we substitute equation (18) in integral (16), one gets

$$\mathrm{Im}\,\mathcal{I} = \mathrm{Im}\int_0^{\mathcal{E}}\int_{r_h(M-\mathcal{E})}^{r_h(M)} \frac{dr}{\dot{r}} d\omega. \tag{19}$$

It is worth noting that the majority of the radiation spectrum is often held by zero-mass particles since black holes typically have very low Hawking temperatures [43]. A tunneling particle with negligible mass travels along a radial path characterized by a null geodesic in the context of an s-wave.

The generic metric expression of the RLDBH spacetime (2) can be redefined as follows

$$ds^2 = g_{tt}dt^2 - g_{rr}dr^2 - g_{\theta\theta}d\theta^2 - g_{\psi\psi}d\psi^2 + 2g_{t\psi}dtd\psi, \tag{20}$$

in which

$$g_{tt} = \frac{r^2 - 2Mr + a^2\cos^2\theta}{r_0 r}, \tag{21}$$

$$g_{rr} = \frac{r_0 r}{r^2 - 2Mr + a^2}, \tag{22}$$

$$g_{\theta\theta} = r_0 r, \tag{23}$$

$$g_{\psi\psi} = r_0 r\sin^2\theta, \tag{24}$$

$$g_{t\psi} = 2a\sin^2\theta. \tag{25}$$

It is reasonable to assume that the radiation emanating from a spinning black hole maintains spherical symmetry for an observer at spatial infinity. This enables us to continue describing the process of tunneling for rotating black holes using the s-wave approximation. However, as a particle tunnels through the event horizon of a rotating black hole, it will come into interaction with the black hole's spin and become influenced by it. In such a scenario, a tunneled particle will consequently display motion in the ψ direction with a non-zero rate of change, denoted as $d\psi \neq 0$. To counteract this motion, we can adopt a reference frame that rotates along with the black hole's horizon over time. For this purpose, we employ a rotational coordinate transformation:

$$\psi = \psi' + \Omega_h t, \tag{26}$$

where Ω_h represents the rotational speed of the event horizon of a spinning black hole:

$$\Omega_h = \left.\frac{g_{t\psi}}{g_{\psi\psi}}\right|_{r=r_h}. \tag{27}$$

From here on, we set $r_h = r_+$ to represent the event horizon of the RLDBH. Observers at the horizon in a co-rotating reference system will find that the black hole's angular velocity, denoted as Ω_h, is zero: $\Omega_h(r_h) = 0$. This is because they are unable to detect the black hole's revolution due to their proximity to the event horizon. In such a co-rotating reference system, a particle will not experience a pulling effect caused by the spinning of the black hole. This is because the tunneling of a particle occurs at the horizon spontaneously. As a result, the particle undergoing tunneling does not display any motion in the degrees of freedom represented by ψ'. Therefore, it is reasonable to assume $d\psi' = 0$, indicating that there is no change in the ψ' coordinate as the particle tunnels through the horizon.

On the other hand, to examine Hawking radiation in a quantum tunneling framework, let us reconsider the RLDBH metric given in equation (20) in the co-rotating reference system and set $\theta = 0$. Whence, metric (20) becomes

$$ds^2|_{\theta=0} = \frac{r^2 - 2Mr + a^2}{r_0 r}dt^2 - \frac{r_0 r}{r^2 - 2Mr + a^2}dr^2. \tag{28}$$

On the horizon, g_{rr} is singular, and we have to remove that coordinate singularity. For this purpose, we pass to the Painlevè coordinate system [44]:

$$dt = dT - \sqrt{\frac{g_{rr}(r) - 1}{\mathcal{G}_{tt}(r)}}dr, \tag{29}$$

which transforms the metric into

$$ds^2 = \mathcal{G}_{tt}(r)dT^2 - \sqrt{4\mathcal{G}_{tt}(r)(g_{rr}(r) - 1)}\,dTdr - dr^2, \tag{30}$$

in which

$$\mathcal{G}_{tt}(r) = \frac{(r - r_+)(r - r_-)}{r_0 r}. \tag{31}$$

As is well-known, in the case of null geodesics, i.e. $ds^2 = 0$, we obtain

$$\frac{\dot{r}}{Y} = \sqrt{\mathcal{G}_{tt}(r)g_{rr}(r)}, \tag{32}$$

where

$$\sqrt{1 - g^{rr}(r)} = 1 - Y. \tag{33}$$

After substituting equations (32) in (19), the imaginary part of the tunneling particle's action becomes

$$\text{Im}\,\mathcal{I} = Im\int_0^{\mathcal{E}}\int_{r_h(M-\mathcal{E})}^{r_h(M)} \frac{dr}{\sqrt{\mathcal{G}_{tt}(r)g_{rr}(r)}\,Y}d\omega. \tag{34}$$

By simultaneously multiplying the numerator and denominator of the integrand with $2 - \chi$, one gets

$$\text{Im}\,\mathcal{I} = \text{Im}\int_0^{\mathcal{E}}\int_{r_h(M-\mathcal{E})}^{r_h(M)} \frac{2 - Y}{g^{rr}(r)}\sqrt{\frac{g^{rr}(r)}{\mathcal{G}_{tt}(r)}}drd\omega. \tag{35}$$

For the metric of a four-dimensional rotating black hole, because g_{rr} is singular on the horizon, generally, we can write g_{rr} in the following form

$$g_{rr}(r) = \frac{z(r)}{r - r_h}, \tag{36}$$

where

$$z(r) = \frac{rr_0}{r - r_-}, \tag{37}$$

which is a function regular on the horizon. By substituting equations (36) into (35), we find out

$$\text{Im}\,\mathcal{I} = \text{Im}\int_0^{\mathcal{E}}\int_{r_h(M-\mathcal{E})}^{r_h(M)} \frac{z(r)\left(1 + \sqrt{1 - \frac{r - r_h}{z(r)}}\right)}{r - r_h}\sqrt{\frac{g^{rr}(r)}{\mathcal{G}_{tt}(r)}}drd\omega. \tag{38}$$

In equation (38), the parameter r_h currently represents a pole of the expression being integrated. By introducing a slight imaginary component to the variable r and allowing the integration path to encircle the pole along a semicircular route, one can compute the integral of dr. This process leads to the following outcome:

$$\frac{\text{Im}\,\mathcal{I}}{2\pi} = \int_0^{\mathcal{E}} z(r_h)\sqrt{\frac{g^{rr}(r_h)}{\mathcal{G}_{tt}(r_h)}}d\omega. \tag{39}$$

One can reasonably infer that the energy $\mathcal{E}$ of the tunneling particle is significantly smaller than the total mass M of the black hole, specifically $\mathcal{E} \ll M$. As a result, in equation (39), the expression within the integral can be approximated to a constant. Consequently, we arrive at the following outcome:

$$\frac{\mathrm{Im}\,\mathcal{I}}{2\pi\mathcal{E}} = z(r_h)\sqrt{\frac{g^{rr}(r_h)}{\mathcal{G}_{tt}(r_h)}}. \tag{40}$$

Since $(\mathcal{G}_{tt}(r),\, g^{rr}(r)) \to 0$ around the event horizon (r_h), we can expand them as follows

$$\mathcal{G}_{tt}(r) \approx \mathcal{G}'_{tt}(r_h)(r - r_h)+\ldots, \tag{41}$$

$$g^{rr}(r) \approx g^{rr'}(r_h)(r - r_h)+\ldots, \tag{42}$$

where '…' seen in equations (41) and (42) present the high order terms of $(r - r_h)$ and '′' symbol denotes the derivative with respect to r. From equation (42), we get

$$g^{rr'}(r_h) = \frac{1}{z(r_h)}. \tag{43}$$

Inserting equations (41) and (42) into (40) and making some arrangements, one can compute the near-horizon form of the action

$$\mathrm{Im}\,\mathcal{I} \approx \frac{2\pi\mathcal{E}}{\sqrt{\mathcal{G}^{\dagger}_{tt}(r_h)z^{-1}(r_h)}}. \tag{44}$$

Using equation (44) in the tunneling rate expression (14), which can be cast in the form of equation (15) that includes the Boltzmann constant, we get

$$\mathrm{Im}\,\mathcal{I} = \frac{\pi\mathcal{E}}{\kappa(r_h)}. \tag{45}$$

After matching equations (45) and (44), one can derive the surface gravity:

$$\kappa(r_h) = \frac{\sqrt{\mathcal{G}'_{tt}(r_h)z^{-1}(r_h)}}{2}, \tag{46}$$

which yields the surface temperature of the RLDBH:

$$T_H = \frac{\sqrt{\mathcal{G}'_{tt}(r_h)z^{-1}(r_h)}}{4\pi}. \tag{47}$$

Equation (47) has been computed from the quantum tunneling approach. However, by using the timelike Killing vectors [43], the surface gravity of the RLDBH spacetime can be computed as follows

$$\kappa(r_h) = \lim_{r\to r_h}\frac{\partial_r\sqrt{\mathcal{G}_{tt}}}{\sqrt{g_{rr}}} = \lim_{r\to r_h}\frac{\partial_r\mathcal{G}_{tt}}{\sqrt{4\mathcal{G}_{tt}g_{rr}}}. \tag{48}$$

Referring to laws of black hole thermodynamics [45], it is well-known that at the event horizon, $\kappa(r_h)$ remains constant. Thus, one can evaluate it at an arbitrary angle θ_0. By substituting equations (41) and (42) into (48), we obtain

$$\kappa(r_h) = \frac{\sqrt{\mathcal{G}'_{tt}(r_h)z^{-1}(r_h)}}{2}. \tag{49}$$

After making a quick comparison between equations (46) with (49), it can be easily seen that the surface gravities obtained are equal to each other. On the other hand, as is well-known, $\kappa(r_h)$ is a constant on the horizon. Therefore, the explicit result for the surface gravity of the RLDBH obtained from equation (49) should not depend on the parameter θ. Namely, the surface gravity and the Hawking temperature expressed in equations (46) and (47), respectively, will not be affected by the parameter θ.

4. GUP-corrected temperature and entropy of RLDBH

GUP is an extension of Heisenberg's uncertainty principle, which takes into account quantum gravity effects [13, 46]. It suggests the existence of a minimum observable length, leading to modifications in the uncertainty relations between position and momentum. In recent years, researchers have been exploring the implications of the GUP on various physical phenomena, including black hole thermodynamics. Specifically, they have investigated how the GUP corrections affect the entropy of charged and/or rotating black holes. Based on this point, in this section of our paper, we will examine the effects of the GUP on the entropy of RLDBH. To this end, let us consider the Lense-Thirring effect, which is a discernible gyroscopic precession [47] and can be obtained by the dragging coordinate transformation (26) that yields the following metric:

$$ds^2 = \frac{r^2 - 2Mr + a^2\cos^2\theta}{r_0 r}dt^2 - \frac{r^2 - 2Mr + a^2}{r_0 r}dr^2 - r_0 r\sin^2\theta d\psi'^2 - r_0 r d\theta^2. \tag{50}$$

The Klein–Gordon equation (KGE) with GUP for a scalar field Ψ takes the following form (see [14, 48, 49] and references therein for the details):

$$-(i\hbar)^2\partial^t\partial_t\Psi = [(i\hbar)^2\partial^i\partial_i + m_p^2] \times [1 - 2\alpha_{GUP}(i\hbar)^2\partial^i\partial_i + m_p^2]\Psi, \tag{51}$$

where m_p and α_{GUP} denote the mass of the scalar particle and the GUP parameter, respectively. The semi-classical WKB approximation method can be applied to solve the generalized KGE (51) [50]. To this end, one can use the following ansatz:

$$\Psi(t, r, \theta, \psi) = \exp\left(\frac{i}{\hbar}S(t, r, \theta, \psi)\right), \tag{52}$$

where $S(t, r, \theta, \psi)$ denotes the outlawed action of tunnelling. To account for the symmetries of metric (50), we can make use of the following Hamilton-Jacobi ansatz [51] for the action:

$$S(t, r, \theta, \psi) = -Et + W(r) + K(\theta) + j\psi + C, \tag{53}$$

where C is a complex constant, E stands for the energy, and j denotes the angular momentum of the particle [52]. By substituting action equations (53) into (52), one obtains (in the leading order of $\hbar$):

$$\frac{r_0 r}{r^2 - 2Mr + a^2}E^2 = \frac{r^2 - 2Mr + a^2}{r_0 r}W'(r)^2 + \frac{j^2}{r_0 r\sin^2\theta} + \frac{K'(\theta)^2}{r_0 r}$$
$$+m_p^2\left[1 - 2\alpha_{GUP}\left(\frac{r^2 - 2Mr + a^2}{r_0 r}\right)W'(r)^2 - \frac{2\alpha_{GUP}j^2}{r_0 r\sin^2\theta} - \frac{2\alpha_{GUP}K'(\theta)^2}{r_0 r} - 2\alpha_{GUP}m_p^2\right]. \tag{54}$$

Thus, the radial part (by disregarding the higher order parts of α_{GUP}) yields the following integral solution:

$$W(r) = \pm\int\frac{Edr}{\sqrt{\Delta(1 - 2\alpha_{GUP}m_p^2)}}, \tag{55}$$

in which, recall that, $\Delta = r^2 - 2Mr + a^2$. Afterward, the contour can be deformed to calculate the integral encircling the singularity at r_h. This provides the opportunity to attain the following expression:

$$W(r_+) = \frac{i\pi E}{2\sqrt{1 - 2\alpha_{GUP}m_p^2}}\frac{r_0 r_+}{\sqrt{M^2 - a^2}}. \tag{56}$$

Consequently, the Hawking temperature of the RLDBH can be determined with the assistance of the GUP as follows

$$T_{RLDBH}^{GUP} = \frac{\sqrt{M^2 - a^2}}{2\pi r_0 r_+}\sqrt{1 - 2m_p^2\alpha_{GUP}}. \tag{57}$$

Regarding to the above expression, when α_{GUP} approaches zero, the GUP-modified Hawking temperature reverts the temperature back to the original Hawking temperature (8). As can be seen from figure 1, as the mass of black holes increases, the GUP effect (α_{GUP}) becomes more pronounced and exhibits performance in the direction of reducing the Hawking temperature.

According to quantum physics textbooks, the conventional Heisenberg uncertainty principle ($\Delta x\Delta p \geqslant 1$) and its saturated counterpart, as mentioned in [53, 54], are derived when the GUP effect is not existed ($\alpha_{GUP} = 0$):

$$\zeta\Delta x \geqslant 1, \tag{58}$$

where ζ represents the energy of quantum-scale particle. On the other hand, by taking account of the GUP, the quantum gravity corrected (QGC) energy is given by [55]

$$\zeta_{QGC} \geqslant \zeta\left[1 - \frac{\alpha_{GUP}}{2\Delta x} + \frac{\alpha_{GUP}^2}{(\Delta x)^2} + \ldots\right]. \tag{59}$$

By refering the works of Anacleto *et al* [53, 54], the quantum tunnelling rate for a quantum particle with ζ_{QGC} is given by

$$\Gamma \simeq \exp[-\mathrm{Im}\,S] = \exp\left[-\frac{\zeta_{QGC}}{T_{QGC}}\right], \tag{60}$$

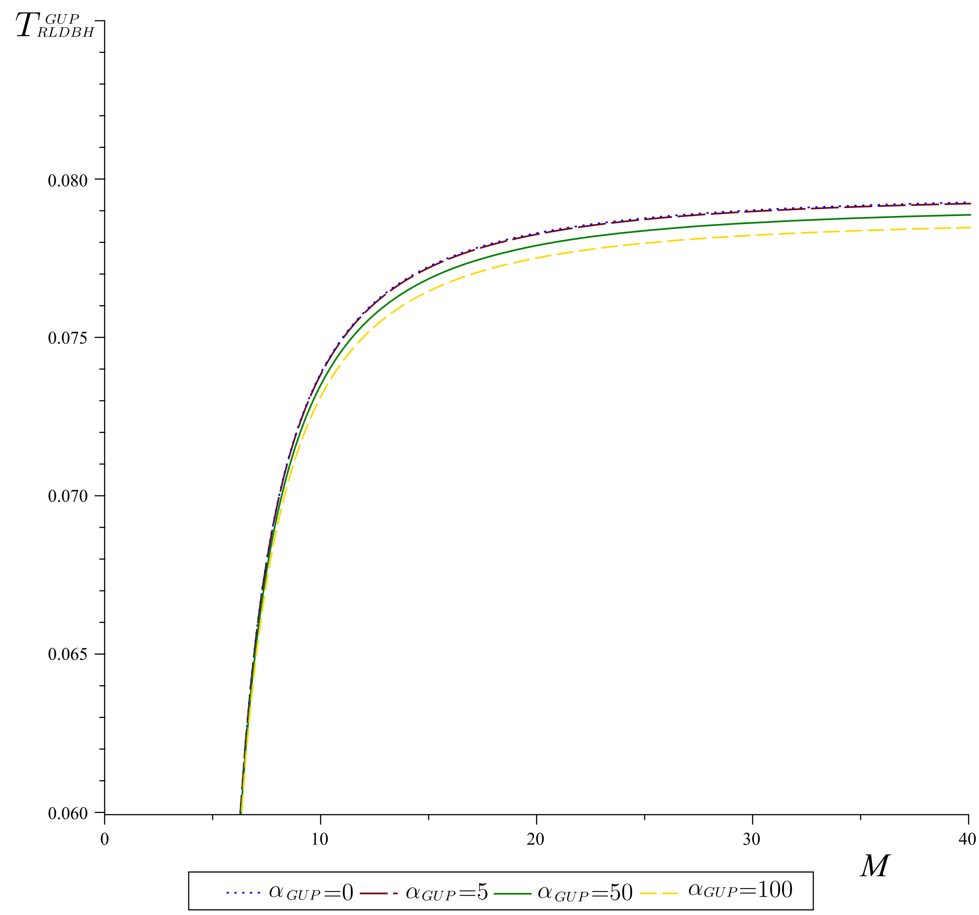

Figure 1. T^{GUP}_{RLDBH} versus M plots for various α_{GUP} values. The plots are governed by equation (57). The physical parameters are chosen as $a = 5$, $m_{GUP} = 0.01$, and $r_0 = 1$.

where T_{QGC} represents the QGC temperature and it reads

$$T_{QGC} = T_H\left[1 - \frac{\alpha_{GUP}}{2\Delta x} + \frac{\alpha^2_{GUP}}{(\Delta x)^2} + ...\right]^{-1}. \tag{61}$$

In the light of the current investigations [53, 54], we can attribute the change in x, denoted as Δx, to $\frac{A_h}{\pi}$ in which A_h represents the area of the event horizon. Therefore, by applying the first law of black hole thermodynamics, the GUP corrected entropy can be calculated as follows:

$$\begin{aligned} S^{GUP}_{RLDBH} &= \int \frac{\kappa dA_h}{8\pi T_{QGC}} = \int \frac{T_H dA_h}{4T_{QGC}} \\ &= \int \frac{dA_h}{4}[1 - \frac{\pi\alpha_{GUP}}{2A_h} + \frac{\pi^2\alpha^2_{GUP}}{A^2_h} + ...] \\ &= S_{RLDBH} - \frac{\pi\alpha_{GUP}}{8}\ln(4\pi r_0 r_+) - \frac{\pi^2\alpha^2_{GUP}}{32\pi r_0 r_+} + ..., \end{aligned} \tag{62}$$

by which S^{GUP}_{RLDBH} was given in equation (10). At this stage, we would like to provide information about the significance of the modified entropy. As is well-known, the entropy of a black hole is directly related to the number of microstates that correspond to a given macroscopic configuration. The information loss paradox comes into play because Hawking radiation appears to be thermal and lacks specific correlations to the information that fell into the black hole [15, 56]. This raises questions about whether the information about the initial state of the matter that formed the black hole is truly lost or whether it can somehow be encoded in the radiation. This contradicts the principles of quantum mechanics, which dictate that information must always be conserved. By incorporating the GUP into the calculation of black hole entropy, the corrected entropy provides a modified description of the microstates available to a black hole. This modification helps address the information paradox by suggesting that black holes may retain some remnants or traces of the information they have absorbed (see for example [15, 57]). It is clearly evident in figure 2 that entropy decreases with the increasing α_{GUP} parameter in comparison to a same massive RLDBH. On the other hand, the decrease in entropy increases

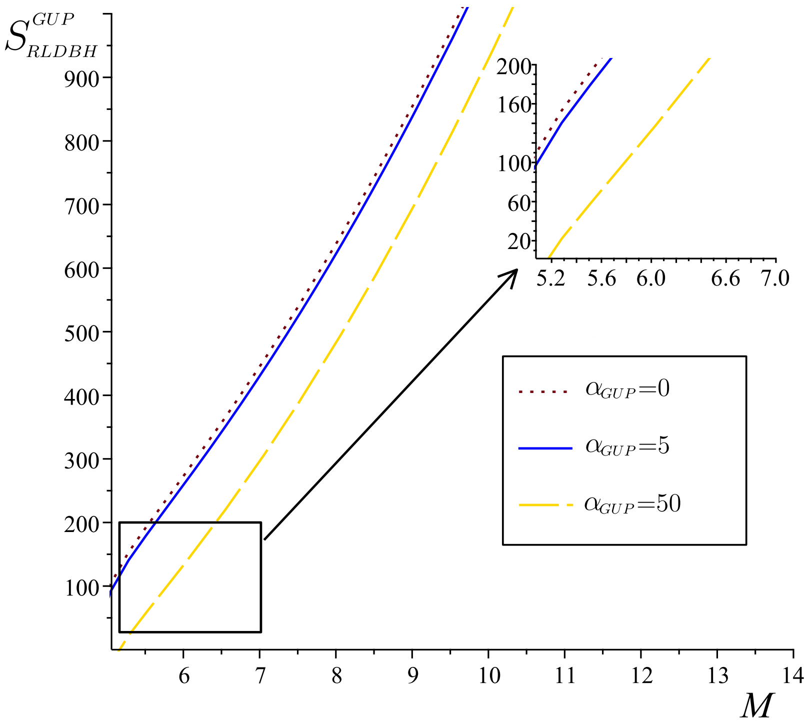

Figure 2. S^{GUP}_{RLDBH} versus M plots for various α_{GUP} values. The plots are governed by equation (62). The physical parameters are chosen as $a = 5$ and $r_0 = 1$.

the likelihood of information leakage from the black hole. This finding for the RLDBH further supports our previous study [57], which is about the static linear dilaton black hole (SLDBH) [57] had also shown us that the quantum gravity effects play a crucial role in the presence of a nonzero statistical correlation and in resolving the information paradox for the SLDBH. In short, the physical importance of the GUP-corrected entropy lies in its potential role in reconciling the behavior of black holes with the principles of quantum mechanics. It presents a potential solution to the information paradox and adds to our comprehension of the underlying characteristics of spacetime and gravity on a quantum scale.

5. Conclusion

This paper has examined the quantum thermodynamics of the RLDBH. The study has focused on two main objectives: (1) determining the Hawking temperature of the RLDBH by employing the null-geodesic tunneling technique proposed by Parikh and Wilczek [35], and (2) calculating the modified temperature and entropy of the RLDBH associated with the GUP. To accomplish these objectives, we implemented the utilization of dragging coordinate systems. Within this particular type of coordinate framework, the spacetime of a rotating black hole in four dimensions has been compressed into a three-dimensional cross-section [58]. In order to maintain the inherent topology of spacetime, a coordinate system that moves in tandem with the event horizon is utilized. This approach effectively eradicates the impact of the angular parameter (ψ) of a tunneling particle. The results obtained have shown us that the temperature of RLDBH's emitted thermal radiation, as determined through the quantum tunneling approach, is in agreement with the statistical Hawking temperature (8). Thus, we have effectively demonstrated the complete separation of the KGE coupled with the GUP in the context of a massive scalar field propagation within the geometry of the RLDBH. This separation has been realized through the utilization of the Hamilton-Jacobi method. Our attention then pivots to the framework of quantum tunneling, where we have accurately computed the Hawking temperature, as modified by the GUP, for the RLDBH, as denoted by equation (57). Moreover, leveraging the entropy derived from the GUP, as presented in section 4, we have also obtained the Hawking temperature for the QGC scenario, which has been expressed by equation (61). Notably, both temperatures converge to the standard Hawking temperature (8) when the GUP effect is no longer applicable (i.e. $\alpha_{GUP} = 0$). Similarly, as shown in figure 2, it was indicated that the S^{GUP}_{RLDBH} decreases with the increasing α_{GUP} parameter compared to an equally massive RLDBH, which implies an increased likelihood of obtaining more information from the respective black hole, referring to our previous study [57] on the SLDBH. Overall, we have shown that GUP has a measurable influence on Hawking radiation, which could potentially leave subtle signatures in the energy spectrum or other characteristics of the radiation.

Future space-based observatories, such as the planned Laser Interferometer Space Antenna (LISA) [59, 60] mission, may provide opportunities to study black holes and their radiation in more detail, potentially enabling the detection of GUP effects.

In our future research, we intend to broaden the investigation of the GUP-modified thermal radiations from particles with various spin-$s\{=0, \frac{1}{2}, 1, \frac{3}{2}, 2\}$ to encompass some particular black holes, utilizing the frameworks of both the rainbow [61, 62] and bumblebee [63] gravity theories. These theories arise from the realm of quantum gravity and introduce deformations in spacetime as well as violations of the Lorentz symmetry. By delving into these effects, our goal is to uncover new insights that enhance our understanding of quantum gravity theory and shed light on the implications they have for the phenomenon of Lorentz symmetry breaking. We shall also plan to focus on a comprehensive exploration of the potential correlations between two particles emitted from the RLDBH. In particular, by inspiring from the work of [64], which investigated the information loss problem of static linear dilaton BH, we aim to investigate the intricate concepts of mutual information processes and the remnant of RLDBH as potential sources for shedding light on the perplexing issue of information loss. This line of inquiry will be a central component of our research agenda in the near future, as we strive to contribute meaningful insights to the broader scientific community's understanding of this complex phenomenon.

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Data availability statement

The data cannot be made publicly available upon publication because no suitable repository exists for hosting data in this field of study. The data that support the findings of this study are available upon reasonable request from the authors.

ORCID iDs

E Sucu https://orcid.org/0009-0000-3619-1492
İ Sakallı https://orcid.org/0000-0001-7827-9476

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