Measurement of the ratio of branching fractions $\mathcal{B}(B^0 \to D^{*-}\tau^+\nu_\tau)/\mathcal{B}(B^0 \to D^{*-}\mu^+\nu_\mu)$ with τ three-prong decays

Guy Wormser, on behalf of the LHCb Collaboration
Laboratoire de l'Accélérateur Linéaire, IN2P3/CNRS, Université Paris-Sud, Université Paris-Saclay,91898 Orsay, France

The prospects of a measurement of the branching fraction ratio $\mathcal{R}(D^*) \equiv \mathcal{B}(B^0 \to D^{*-}\tau^+\nu_\tau)/\mathcal{B}(B^0 \to D^{*-}\mu^+\nu_\mu)$ is performed by using a data sample, corresponding to 3 fb^{-1} integrated luminosity, collected by LHCb at center-of-mass energies of 7 and 8 TeV during the LHC Run 1. The τ lepton is reconstructed in the hadronic decay with three charged pions. Backgrounds from prompt $B^0 \to D^*3\pi(X)$ decays are rejected by requiring that the τ decay vertex lies downstream of the B^0 decay vertex. Other physical backgrounds due to B decays to double charmed hadrons are suppressed by partial reconstruction techniques, isolation criteria, and the dynamics of the 3π system. LHCb result based on the Run1 statistics will be published soon with a statistical precision of 6.7%, the best achieved for a single measurement so far. The systematic error will be likely larger than this but can be reduced by joint efforts of the LHCb, BELLE and BES collaborations. This analysis will also enable the search for SM deviations in the event distributions, in addition to the event yield, thanks to its unique capability to select high statistics (a few thousands events) highly enriched (50%) in semi-tauonic decays.LHCb will use the exact same method to perform the measurement of all other B hadrons semitauonic decays, including those coming from Λ_b^0 and B_c^+ hadrons.

In the Standard Model (SM), the electroweak couplings are independent of the lepton family. This lepton universality can be violated in many models that extend the SM by adding interactions with stronger couplings to the third generation. Being mediated by a single W boson, semileptonic decays of b-hadrons are a sensitive probe of SM extensions with mass-dependent couplings, *e.g.* models with an enlarged Higgs sector or leptoquarks. In particular, the ratios of branching fractions of semi-tauonic decays of B mesons relative to decays involving lighter lepton families, $\mathcal{R}(D^{(*)}) \equiv \mathcal{B}(B \to D^{(*)}\tau^+\nu_\tau)/\mathcal{B}(B \to D^{(*)}\mu^+\nu_\mu)$, are computed in the SM with a precision at the percent level [1,2], due to the cancellation of the dominant uncertainties from hadronic effects. The experimental determination of such ratios is also clean, due to the cancellation of many systematic uncertainties.

Measurements of $\mathcal{R}(\mathcal{D})$ and $\mathcal{R}(D^*)$ have been reported by the Babar [3], Belle [4,5] and LHCb experiments [6], that are consistently higher and average at about 4σ above the SM predictions[7]. In these measurements, the τ lepton was always reconstructed in its leptonic decay to electron or muon. A first simultaneous measurement of $\mathcal{R}(D^*)$ and the τ polarization using the hadronic

1-prong decay was recently released by Belle[8].

This proceeding reports the sensistivity of the first measurement of $\mathcal{R}(D^{*+})$ where the τ lepton decays into three charged particles (3-prong) in the final state, by using a data sample of proton-proton collisions, corresponding to $3.0 fb^{-1}$ integrated luminosity, collected by the LHCb detector at center-of-mass energies $\sqrt{s} = 7$ and 8 TeV during the LHC Run 1 in 2011-2012. The D^{*+} meson is reconstructed through the $D^{*-} \to \overline{D}^0(\to K^+\pi^-)\pi^-$ decay chain. The final state consists of a D^* meson and 3 pions (plus X). In order to minimize the experimental systematic uncertainties, the $B^0 \to D^{*-}\pi^+\pi^-\pi^+$ decay is chosen as normalization channel, thus leading to a measurement of $\mathcal{R}_{had}(D^*) \equiv \mathcal{B}(B^0 \to D^{*-}\tau^+\nu_\tau)\mathcal{B}(\tau^+ \to 3\pi\nu_\tau)/\mathcal{B}(B^0 \to D^{*-}3\pi)$ that gives a measurement of $\mathcal{R}(D^*)$ by taking the branching fraction of the normalization channel and the well-measured semileptonic decay of the B^0 meson in lighter leptons as external inputs. The major background due to $B^0 \to D^*3\pi(X)$ decays (100 times larger than the signal) can be rejected by three orders of magnitude by requiring a decay topology where the τ decay vertex lies downstream of the B^0 decay vertex. This technique is not effective on physics backgrounds due to B decays into double charm events, the largest one being due to $B \to D^*D_s^+(X)$, since the 3π vertex is transported away from the B vertex in a similar way as for the signal. Observables based on the kinematics, dynamics and topological structure of these backgrounds are used to suppress them. A multidimensional fit, where template distributions determined from simulation, corrected by using data control samples when needed, provides the statistical separation of signal from the residual background.

The LHCb detector[9,10] is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, designed for the study of particles containing b or c quarks.

The online event selection is performed by a trigger[11], which consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction. At the hardware trigger stage, events are selected because either particles other than those in the D^* 3π system pass any trigger requirements, or the D^* decay products satisfy the hadron trigger requirement. The software trigger is topologically based and requires a two-, three- or four-track secondary vertex with a significant displacement from the primary pp interaction vertices (PV). A multivariate algorithm[12] is used for the identification of secondary vertices consistent with the decay of a b hadron. In addition, at least one of the following conditions must be met: the D^* 3π system must pass the topological trigger defined above, or the D^0 and its decay products must satisfy selection criteria based on particle momenta and transverse momenta, D^0 momentum pointing to a PV, and D^0 reconstructed mass.

In the simulation, pp collisions are generated using PYTHIA[13] with a specific LHCb configuration[15]. Decays of hadronic particles are described by EVTGEN[16], in which final-state radiation is generated using PHOTOS[17]. The interaction of the generated particles with the detector, and its response, are implemented using the GEANT4 toolkit[18] as described in Ref.[20]. In the generation of signal decays, form factors are used that are derived from Heavy Quark Effective Theory[21]. The experimental values of the corresponding parameters are taken from[7], except for an unmeasured helicity-suppressed component, that is taken from[?].

In the offline selection, D^0 candidates are built from well identified kaons and pions, with good track quality. The invariant mass of the D^0 candidate must be compatible within 20 MeV with its known value. The D^* candidates are obtained from D^0 candidates by adding a low-energy pion. The difference between the invariant mass of the D^* and D^0 candidates must lie within 2.5 MeV of the known value. Three well identified and well reconstructed pions are used to build τ candidates. They also must satisfy IP-$\chi^2 > 15$. Both D^0 and τ candidates must have vertices with a good χ^2, their momentum vectors must approximately point to the location of one PV. The impact parameter of the D^0 candidate with respect to any PV must be greater than 10σ. The flight distance of the τ candidate with respect to any PV must be greater than 10σ in the beam direction, and between 0.2 and 0.5 mm in the transverse plane. The B^0

candidate is built by performing a least-square fit of its decay chain [22]. A requirement for the τ vertex to be downstream the B^0 vertex by at least 4σ significance rejects the background due to $B \to D^* 3\pi X$ decays by three orders of magnitude. A requirement for the D^0 vertex to be downstream the τ vertex with at least 4σ significance is used to select the normalization sample.

Physics backgrounds due to partially reconstructed B decays, where at least one additional particle originates from either the 3π, the B^0 vertex, or both, are suppressed by requiring a single B^0 candidate per event, and by applying an isolation algorithm as follows. If any other charged track in the event, with transverse momentum larger than $250\,\mathrm{MeV}/c$ and impact parameter IP with respect to all PVs larger than 2σ, have an IP with respect to either the B^0 or τ vertex smaller than 5σ, the event is rejected. This criterium rejects backgrounds due to B decays with a $D^* D^0$ in the final state by 95% and is still 80% efficient on signal.

Another isolation algorithm, used in the following, computes the multiplicity of reconstructed tracks, neutral objects and the sum of neutral energy contained in cones of different sizes around the direction of the τ candidates.

The reconstruction of the kinematics of the signal decay is crucial for signal and background discrimination. Even in the presence of an unreconstructed neutrino, the measurable flight length of the τ allows to determine its momentum with a 2-fold ambiguity

$$|\vec{p}_\tau| = \frac{(m_{3\pi}^2 + m_\tau^2)|\vec{p}_{3\pi}|\cos\theta \pm E_{3\pi}\sqrt{(m_\tau^2 - m_{3\pi}^2)^2 - 4m_\tau^2|\vec{p}_{3\pi}|^2\sin^2\theta}}{2(E_{3\pi}^2 - |\vec{p}_{3\pi}|^2\cos^2\theta)}, \tag{1}$$

where θ is the angle between the 3 charged pions and the τ direction; $m_{3\pi}$, $|\vec{p}_{3\pi}|$ and $E_{3\pi}$ are the invariant mass, 3-momentum and energy of the 3π system, respectively; and m_τ is the τ mass. To avoid this ambiguity, the kinematic point where the argument of the root square vanishes can be used. This corresponds to the angle:

$$\theta_{max} = \arcsin\left(\frac{m_\tau^2 - m_{3\pi}^2}{2m_\tau|\vec{p}_{3\pi}|}\right). \tag{2}$$

This angle is used to estimate the τ momentum $|\vec{p}_\tau| = |\vec{p}_\tau(\theta_{max})|$. A similar argument is used to infer the B^0 momentum, thereby allowing to reconstruct rest frame variables, such as the τ decay time and the squared four-momentum transfer of the B to the lepton system $q^2 = (p_B - p_{D^*})^2$ with p_B and p_{D^*} being the four momenta of the B and D^* mesons, with no biases and resolutions sufficient enough to preserve a good discrimination between signal and backgrounds. A partial reconstruction is performed also in the background hypothesis where $B \to D^* D_s^+(\to 3\pi N)$, N being a massive neutral system, by solving the equation for momentum conservation in two possible ways by applying vectorial algebra.

The Dalitz structure of the 3π system is a powerful discriminant against backgrounds due to B decays in a D^* and another charm hadron in the final state, especially the D_s^+ meson. The three-prong decays of the τ lepton are dominated by the a_1 resonance, therefore the Dalitz plane will exhibit two ρ bands. The D_s decays with 3π in the final state, instead, are dominated by the η and η' resonances to a large extent, leading to an enhancement of the Dalitz structure at lower masses.

The suppression of double-charm backgrounds is achieved by combining observables related to the cone-based isolation algorithm (5 variables), to the partial reconstruction in the signal (2 variables) and background (6 variables) hypotheses, to the Dalitz structure of the 3π system (2 variables), and to the B^0 decay kinematics (3 variables), in a multivariate analysis (MVA) method using a boosted decision tree (BDT) [23,24]. The BDT is trained using signal and background simulations. The background rejection is validated by using three data control samples: a $B \to D^* D_s^+ X$ sample obtained by using the partial reconstruction technique in the background hypothesis; a $B \to D^* D^0 X$ sample obtained by removing the charged particle isolation criterium and requiring a charged kaon around the 3π vertex with a mass of the K3π system compatible with a D^0 meson; a $B \rightarrow D^* D^+$ sample obtaining requiring positive kaon identification for the pion candidate with charge opposite to that of the other two and a mass of the K$\pi\pi$ system compatible with a D^+ meson. A good agreement in the distribution of the input variables to the BDT is observed in data and simulation for all three control samples.

The yield for the normalization mode is determined by fitting the invariant mass distribution of the D^* 3π system around the B^0 peak in the normalization sample. Fig.1 exhibits this mass peak, as well as the 3 others which are used to control the backgrounds induced by the D_s^+,D^0 and D^+ mesons. The fitting function is the sum of a Crystal Ball and Gaussian functions for signal, an exponential function for the combinatorial background. A total $17657 \pm 164_{stat} \pm 64_{syst} \pm 22_{D_s sub}$ candidates are found, where the first uncertainty is statistical, the second systematic and the third refers to the subtraction of a small contribution of 151 ± 22 candidates due to $B \rightarrow D^* D_s^+ (\rightarrow 3\pi)$ decays.

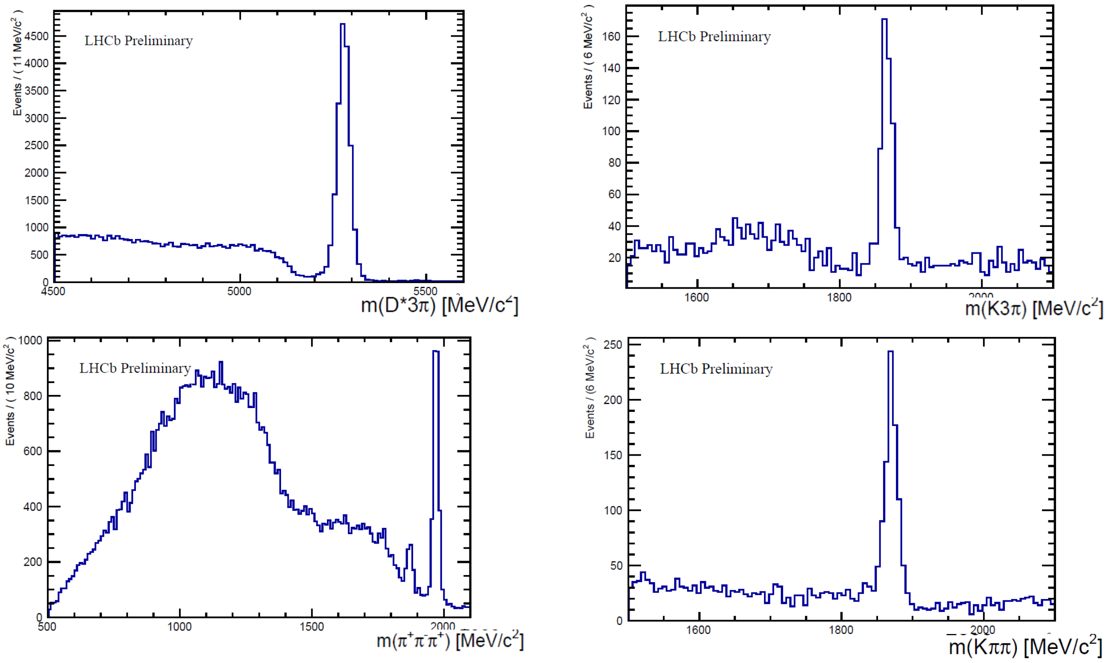

Figure 1 – Invariant mass distributions : (Top left) D^* 3π in normal topology (Bottom right) 3pion in detached topology, (Top right) K3π in detached topology when an extra charged kaon has been identified as compatible with the 3π vertex (Bottom Right) $K^-\pi^+\pi^-$ in detached topology where the particle with charge opposite to the two others' has been identified as a good Kaon candidate

At low BDT values, the data sample consists about 85% of $D^* D_s^+$ events and is used to refine the D_s^+ decay model used in the MC simulation. A simultaneous fit of 4 mass spectra (min mass($\pi^+\pi^-$), max mass ($\pi^+\pi^-$), mass($\pi^+\pi^+$) , mass (3π) allows to fix the relatives proportions of the four main categories of D_s^+ decays into 3pions : $\eta\pi$ +X, $\eta'\pi$ +X, ϕ, ω+X, R$\pi^+\pi^-\pi^+$. The distribution (min mass($\pi^+\pi^-$) is playing a specially useful role because it constrains the η' component through the low mass enhancement specific to the $\eta' \rightarrow \eta\pi^+\pi^-$, and therefore constrains the rate of only signal-like decay channel of the D_s^+ meson, $D_s^+ \rightarrow \eta'\pi$ +X with $\eta' \rightarrow \rho\gamma$. Fig.2 shows the minimum mass($\pi^+\pi^-$) with the four D_s^+ components mentionned above and the very good agreement obtained in that fit. The relative contribution of each mode in the high BDT output region used for the signal fit are inferred using simulation.

The signal yield is obtained by a three-dimensional fit to the data, in a region above the threshold previously mentioned in the BDT output, by using templates obtained on simulation. The fit variables are: the τ decay time, the squared invariant mass of the lepton pair q^2, and the output of the BDT. The control samples defined above are also used to check that the simulation well reproduces the expected distributions on data, and to correct the simulation otherwise.

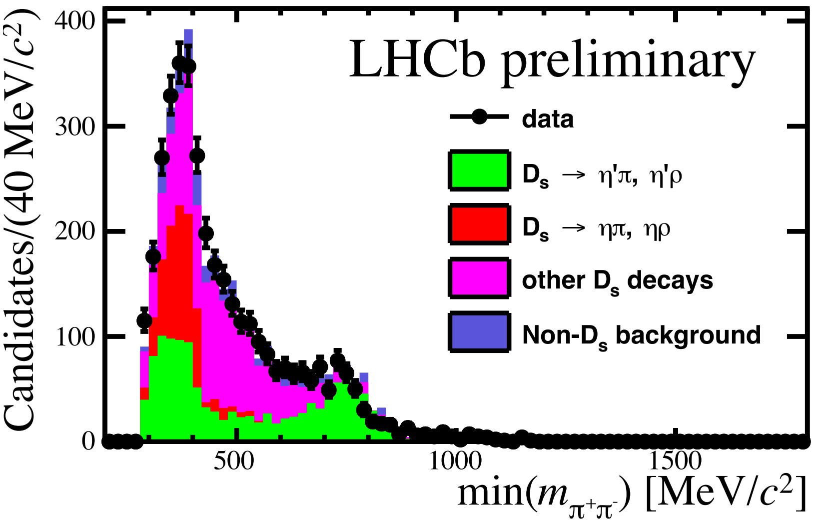

Figure 2 – Fit to the $min(m_{\pi^+\pi^-})$ distribution in a sample enriched in D^*D_s decays, obtained by requiring the BDT output below a threshold. The different fit components correspond to D_s decays with η (red), η' (green), all the rest (magenta) in the final state, and non-D_s decays (grey). The total fit model is the solid histogram, points correspond to data.

The signal template is the sum of two terms, due to $\tau \to 3\pi$ and $\tau \to 3\pi\pi^0$ decay, where the yield for the latter scaled with the former through a proportionality factor determined by their relative branching fractions and selection efficiencies. A contribution due to $B \to D^{**}\tau\nu_\tau$ decays, with the D^* being produced in the D^{**} decay chain, is also linked to the signal yield through a proportionality factor determined from simulation, and validated on a sample where the narrow $D_1^0(2420)$ and $D_2^{*0}(2460)$ resonances are reconstructed in their $D^*\pi$ decay.

The background due to $H_b \to D^*D_s^+X$ decays, H_b being a generic b hadron, is divided into contributions from $B^0 \to D^{*-}D_s^+$, $B^0 \to D^{*-}D_s^{*-}$, $B^0 \to D^{*-}D_{s0}^{*+}$, $B^0 \to D^{*-}D_{s1}^{\prime +}$, $B^{0,+} \to D^{**}D_s^+X$, $B_s^0 \to D^{*-}D_s^+X$. Their relative contributions are constrained by using the results of a fit, to the D^* 3π invariant mass on a data control sample consisting of these decays, where the D_s^+ meson is reconstructed through its exclusive decay in 3π. The relative amount of each contribution obtained from this fit is used to rescale the simulation when preparing the templates to be used in the final fit.

The number of floating parameters in the fit is 11. The $B \to D^*D^0X$ background is subdivided in two contributions, according to whether the 3π originate from the same D^0 vertex, or where at least one pion originate from the D^0 vertex and the other two from elsewhere. The contribution of the former is constrained, based on the yield obtained on the $D^0 \to K3\pi$ control sample where a kaon compatible with the 3π vertex is found. The template shape is also taken from the same control sample. The yield of the latter is a free parameter in the fit. The yield of the $B \to D^*D^+X$ background is a free parameter. The template shape is taken from the corresponding control sample. A residual contribution of $B \to D^*3\pi X$ decays is included, its contribution constrained by means of the $B^0 \to D^*3\pi$ exclusive peak.

The combinatorial background is divided in two contributions, depending on whether the D^* meson is real or fake. In the first case, the D^* and the 3π originate from different B decays. The shape is taken from simulation. A sample of candidates where the D^* and the 3π system have the same charge is used to normalize data and simulation above the B meson mass. The fake D^* background is parameterized and constrained by using the events in the D^0 sidebands.

Uncertainties in the template shape, that originate from the finite size of the simulation sample, are taken into account in the fit likelihood by using the Beeston-Barlow procedure [?].

By taking into account the ratio of efficiencies between signal and normalization $\varepsilon_{sig}/\varepsilon_{norm}$, correction factors due to PID and trigger mismodelling between data and MC, the sum of the branching fractions for the $\tau \to 3\pi$ and $\tau \to 3\pi\pi^0$ decays, properly reweighted to take into account efficiency differences, the value for the $B^0 \to D^*3\pi$ and $B^0 \to D^{*-}\mu\nu_\mu$, R(D*) can be determined. We obtain a statistical error of 6.7% for the Run1 data sample. This is the smallest statistical reported reporte so far for a single measurement of R(D*).

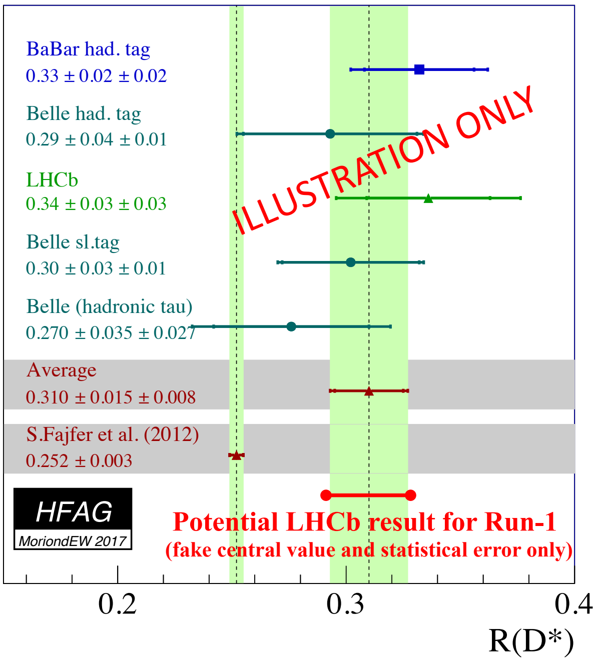

Figure 3 – Potential sensitivity of this analysis (statistical error only) reported at an arbitrary value , together with all R(D*) measurements reported so far with the HFAG determination of the World average (WA), and compared to the SM prediction

This can be appreciated on Fig. 3 where are reported all previous results on R(D*), the world average determination by the HFAG group[25] and the LHCb potential result described in this paper, when its yet unknown mean value is for this illustration purpose chosen to coincide with the present WA result. The systematic uncertainties attached to this measurements consist of an external part of 4.8% due to the knowledge of the 2 external B^0 branching fractions into the D^* 3π and D^* lν needed to extract R(D*) for our measurement, and of internal part which will be dominated by the uncertainties coming the MonteCarlo statistics and those coming from the imperfecct knowledge for the D_s^+, D^0 and D^+ background models. The total systematic error is likely to exceed the statistical error. The impact of this measurement should be competitive with the other single measurements and, depending on its central value, can noticeably influence the present significance of the deviation of the WA with respect to the SM prediction. In addition, the Run2 data sample contains about 3 times more D^* events per fb^{-1} because of the higher $b\bar{b}$ cross section at 13 TeV and more efficient trigger conditions. This means that we can quadruple our data sample using data already on tape today, and therefore reach a statistical precision significantly better than the present world average. More over, LHCb can and will study using the τ muonic and hadronic channels all the other semitauonic decays, namely B_s^0 to $D_s^+\tau\nu$, Λ_b^0 to $\Lambda_c^+\tau\nu B_c^+$ to $J/\psi\tau\nu$, B^0 and B^+ to $D^0\tau\nu$ and $D^+\tau\nu$. The relative precision of the R measurement in all these channels will depend of the specific combination of production yield, reconstructible final states' branching fractions, trigger conditions and background levels, but one could estimate that they can end up to be rather similar at the 10% level. The diversity of all these channels, as well as the different values of the spin of spectator particle (0 for D_s^+,D^0 and D^+, 1 for D^* and J/ψ and 1/2 for Λ_c^+) will offer an uncomparable probe of deviations from SM due to Lepton Universality Violation. It has also be stressed that the 3π hadronic τ analysis channel will offer, in all these cases, the possibility to study potential deviations from SM prediction beyond the mere event yield, by a study of relevant angular and kinematic distributions thanks to the large purity (typically 50%) and high statistics (typically 1000 events) that can be uniquely reached with this new method.

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