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Review

Dynamical Casimir Effect: 55 Years Later

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Abstract: The paper represents a brief review of the publications in 2020 to 2024 related to the phenomena combined under the name of dynamical Casimir effect.

Keywords: moving boundaries; photon generation from vacuum; parametric amplification; atomic excitations; time-dependent qubits; cavity and circuit QED; hybrid cavities; optomechanical interactions; effective Hamiltonians; decoherence; entanglement; nonstationary media; quantum friction; Casimir–Polder effects

1. Introduction

In quite a broad field of matter–radiation interactions, the group of phenomena nowadays combined under the name of dynamical Casimir effect (DCE) occupies quite an important place. The history of these phenomena began fifty five years ago, when Gerald Moore published his paper [1], where he showed that motions of ideal boundaries of a one-dimensional cavity can result in the generation of quanta of the electromagnetic field from the initial vacuum quantum state. Since that time, these remarkable quantum phenomena (frequently referred to as the DCE afterwards [2,3]) have attracted high attention.

Nowadays, the term "dynamical Casimir effect" is applied to various phenomena, whose common is the amplification of vacuum quantum fluctuations of different fields (mainly electromagnetic) due to temporal variations in some parameters characterizing macroscopic systems. Professor Francesco Persico made significant contributions to these fields, especially such areas as atom–radiation interaction, quantum vacuum fluctuations, and Casimir and Casimir–Polder interactions; see Refs. [4–13].

References of several hundred publications related to the DCE in a broad sense can be found in the reviews [14–18]. One of the goals of the present paper is to provide an outline of the main results achieved in the five years after the most recent review [18]. Hence, this paper can be considered an extension of Ref. [18].

2. Single-Mirror DCE

The DCE with single moving mirrors became an attractive topic after the pubications [19–21]. Fifty years later, this area continues to attract high attention.

2.1. One-Dimensional Models

The simplest models correspond to $(1+1)$-dimensional space-time. The creation of a thermal radiation by a moving mirror has been considered in Refs. [22–25]. The vacuum radiation of a massive scalar field in the case of a single flat mirror moving in $(1+1)$ and $(3+1)$ dimensions has been calculated in Ref. [26].

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Different models of partially reflecting moving mirrors [27] or static mirrors with time-dependent reflection coefficients [28–32] have been considered. Often, the influence of mirrors is described using Klein–Gordon equations of the form

$$\left[\partial_t^2 - \partial_x^2 + U(x,t)\right]\phi(x,t) = 0, \tag{1}$$

where $\partial_a \equiv \partial/\partial a$, and $\phi(x,t)$ represents the scalar field as a function of the space (x) and temporal (t) coordinates. In the case of thin mirrors, the effective potential $U(x,t)$ is taken as a combination of the Dirac delta function and its space-derivative:

$$U(x,t) = \lambda_1(t)\delta[x - x_0(t)] + \lambda_2(t)\delta'[x - x_0(t)], \tag{2}$$

where $\lambda_1(t)$ and $\lambda_2(t)$ can be arbitrary functions of time. Such models have been studied in Refs. [28–32]. A microscopic model of a moving mirror in a single space dimension has been studied in Ref. [33]. The mirror is modeled as a quantum oscillator with ponderomotive coupling to the field. Thus, the back-reaction from a quantum field to the dynamics of a moving mirror is taken into account. Another model taking into account the back-reaction has been considered in Ref. [34].

2.2. Moving Mirrors in Several Dimensions

The DCE in the presence of a single thin and inhomogeneous flat mirror, modeled with a delta potential in four-dimensional (4D) space-time, has been studied in Refs. [35,36]. Space-time quantum metasurfaces and microscopic models of moving dielectric mirrors as a collection of dipoles in which the center-of-mass coordinates are modulated in space and in time have been studied in Refs. [37,38]. The sudden creation of a fast-moving mirror has been considered in Ref. [39]. A detailed study of pair production by semitransparent moving mirrors with Neumann boundary conditions has been performed in Ref. [40]. Time-dependent Dirichlet surfaces in $(d+1)$ dimensions have been considered in Ref. [41]. The DCE, due to the motion of a single mirror with a finite transverse size, has been studied in Refs. [42,43]. It is shown that models using infinite metallic surfaces drastically overestimate DCE radiation.

3. Cavity DCE

The "most conventional " directions of studies on the DCE are related to the evolution of the quantum electromagnetic field inside cavities with moving boundaries.

3.1. Single-Dimensional (1+1) Models

One-dimensional models of the DCE inside cavities have been extensively studied since Moore's paper [1]. The following is a summary of the publications since 2020.

The discrete spectrum of photons created due to the DCE when one of two mirrors oscillates with relativistic maximal velocities has been calculated in Ref. [44]. A case of two resonantly coupled modes has been considered in Ref. [45]. Effective Hamiltonians have been used in Refs. [46–48]. Time-dependent boundary conditions have been considered in Ref. [49]. A massive scalar field inside a 1D cavity with ideal oscillating boundaries has been the subject of study [50]. The field amplification and generation of ultrashort narrow pulses between oscillating mirrors have been considered in Refs. [51,52]. A case where the boundaries of a one-dimensional cavity move randomly has been studied in Ref. [53]. The DCE for fermionic fields has been considered in Refs. [54–57]. The possibility of converting vibrational excitations of the wall into fermion pairs has been investigated in Ref. [58]. Numerical methods have been used in Ref. [59]. A short review is given in Ref. [60]. The behavior of a confined quantum field with moving boundaries in space-time with a screened scalar field has been studied in Ref. [61], where the related Bogoliubov coefficients are calculated. The classical wave equation in a 1D cavity has been considered in Ref. [62] in a case when a single wall moves under radiation pressure force (and apparently other external forces). Specifically, the phenomenology of this system is outlined when the moving boundary achieves large displacements and velocities (comparable with the wave velocity).

3.2. Hybrid Cavities

The term "hybrid" or "optomechanical" cavities is used for models taking into account the quantization of the cavity wall's motion and/or the back-reaction of the electromagnetic field on the walls [63]. Such models have been considered, e.g., in Refs. [64–66]. Typical single-mode hybrid Hamiltonians have the following structure:

$$H = \omega_c a^\dagger a + \omega_b b^\dagger b - g a^\dagger a \left(b^\dagger + b \right) - (g/2) \left(a^\dagger a^\dagger + aa \right) \left(b^\dagger + b \right), \tag{3}$$

where the first term represents the free Hamiltonian of photons (cavity mode), the second term gives the free Hamiltonian of phonons (vibration of cavity wall), the third term represents the optomechanical interaction term, and the fourth term describes the DCE interaction; $a(a^\dagger)$ and $b(b^\dagger)$ represent the annihilation (creation) operators of photons and phonons, respectively; g is the coupling constant; ω_c and ω_b denote the frequencies of photons and phonons, respectively. More complete multi-mode Hamiltonians have been used in Refs. [67,68]. The statistical properties of the electromagnetic field created in the cavity due to the DCE (such as squeezing effects) have been studied in Ref. [69], where the models of the classical moving wall and its quantized counterpart are compared.

The models considered in Refs. [70,71] include a two-level atom (with levels $|g\rangle$ and $|e\rangle$) inside the optomechanical cavity, adding the term

$$H_{\text{at}} = \omega_e |e\rangle\langle e| + \lambda \left(a^2 |e\rangle\langle g| + a^{\dagger 2} |g\rangle\langle e| \right) \tag{4}$$

to the Hamiltonian (3). Here, ω_e denotes the transition frequency between the atomic states $|g\rangle$ and $|e\rangle$, and λ represents the field-atom coupling constant.

Equation (4) means that the atomic transition $|g\rangle \leftrightarrow |e\rangle$ couples the atom to the cavity mode through a two-photon process. The effect of two-photon hopping in the system of two cavities separated by a vibrating two-sided perfect mirror has been considered in Ref. [72]. The correlations between the fields in two cavities separated by a moving mirror (whose motion is quantized) have been studied in Refs. [73,74]. A one-dimensional "membrane-in-the-middle" optomechanical model has been considered in Ref. [75]. The optical nonreciprocity phenomenon due to the DCE in a three-mode and two-port optomechanical cavity has been studied in Ref. [76]. Ref. [77] has proposed a protocol for inducing and observing real mechanical excitations of a mirror enabled by the virtual photons in the ground state of a tripartite system, where a resonant optical cavity is ultrastrongly coupled to a two-level system (qubit) and, at the same time, optomechanically coupled to a mechanical resonator. The back-reaction problem in the cavity DCE has been considered in Ref [78]. Re. [79] has proposed a protocol to study the DCE in a cavity optomechanical system under the condition that the resonator frequency is much larger than the mechanical frequency. Another proposal in the same direction has been given in Ref. [80], where it is claimed that "the mechanical frequency can be about two orders of magnitude smaller than the output photons".

4. Circuit and Waveguide DCE

The DCE in quantum circuits has continuously attracted high attention. As a continuation of the breakthrough experiment [81], Ref. [82] has investigated the entanglement of a stream of photon pairs, generated in a semi-infinite broadband transmission line, terminated by a superconducting quantum interference device (SQUID). Controllable multipartite Einstein–Podolsky–Rosen steering via the dynamical Casimir effect in the frame of superconducting quantum networks has been studied in Refs. [83–85]. Bell's inequality violation through dynamical Casimir radiation in a circuit quantum electrodynamical setup, where a relativistically moving mirror is simulated by variable external magnetic flux in a superconducting quantum interference device terminating a superconducting microwave waveguide, has been the subject of Ref. [86]. A theory of parametric photon generation in the waveguides coupled to arrays of quantum emitters with temporally modulated resonance frequencies, interpreted as a dynamical Casimir effect, has been developed in Ref. [87]. In Ref. [88], it has been shown that a transmission line coupled to an externally driven superconducting quantum interference device (SQUID), exhibiting the dynamical Casimir effect, can function as an autonomous cooler where the SQUID can be used as a work source to cool down the cavity modes. Strong amplification of particle creation in left-handed metamaterial transmission lines has been predicted in Ref. [89]. Ref. [90] has proposed the use of a superconducting quantum circuit to study photon generation via the dynamical Casimir effect in an effective (simulated) one-dimensional double cavity divided by a dielectric membrane.

5. Interaction with Atoms and Detectors

When a two-level atom interacts resonantly with a coherent electromagnetic field, the polarizability of the atom can be modulated due to the Rabi oscillations between atomic levels. In the quantum vacuum, such time-dependent atomic polarizability can lead to the resonant generation of photon pairs, a phenomenon reminiscent of photon emission in the dynamical Casimir effect. This possibility has been discussed in Ref [91]. The average number of photons generated by the DCE in the dissipative cavity containing a two-level atom has been calculated in Ref. [92].

A proposal to put a "cloud" of photodetectors (achieved using nitrogen vacancy color center defects) on a diamond membrane undergoing gigahertz flexural motion within a superconducting microwave cavity has been considered in Ref. [93]. When the number of detectors exceeds some critical value, the system undergoes a transition to an inverted lasing phase, signaled by a "burst" peak in the average cavity photon number, yielding significantly enhanced, collective photon production from a vacuum.

A microscopic model of the single-mirror DCE, where a collection of atoms moving in phase mimics the oscillation of a material planar surface, has ben studied in Ref. [94]. A similar model, where real boundaries (mirrors) are replaced with "metasurfaces" constructed from 2D atom arrays [95], has been developed in Ref. [96]. Detailed reviews of recent advances in the amplification of quantum light–matter interaction and simulation of ultrastrong light–matter interaction, particularly in cavity and circuit quantum electrodynamics and in cavity optomechanics, can be found in Refs. [97,98].

5.1. Dynamical Casimir–Polder Effects

The emission of photons by moving atoms in the presence of mirrors has been studied in Refs. [99–102]. The dynamical atom–wall Casimir–Polder effect after a sudden change in the atomic position has been considered in Ref. [103]. The dynamical Casimir–Polder force between a two-level atom and the walls of different 1D cavities has been calculated in Refs. [104,105]. The influence of the initial states of the atom on the force is analyzed.

The case of a semi-infinite waveguide has been considered in Ref. [106]. The friction force on an atom moving near a conducting plate has been calculated in Ref. [107].

Effective Hamiltonians describing the interaction of the quantum electromagnetic field with atoms or molecules have been elaborated on in Ref. [108]. These Hamiltonians (whose form depends on the field states considered) allow for considerable simplification of the calculation of the Casimir–Polder interactions, including in the presence of boundary conditions. Nonlocal dynamical vacuum field correlations in the Casimir–Polder interactions have been studied in Ref. [109].

A promising platform to observe the dynamical Casimir effect using a graphene nanosheet has been suggested in Ref. [110]. The Casimir friction force on an atom which moves close to a graphene plate has been calculated in Ref. [111].

5.2. DCE and Qubits

A fast scheme for anti-DCE (a process in which photons or other system excitations are annihilated from the thermal state of the field due to variation in the system parameters), based on coupling a single-mode cavity to a qubit with time-dependent coupling strength, has been suggested in Ref. [112]. The DCE in cavities with some modes resonantly coupled through a qubit (a two-level system) has been studied in Ref. [113]. In this case, a limited number of photons can be created due to the modulation of the qubit's transition frequency. This protocol admits several modulation frequencies. As a result, complex tripartite entangled states can be formed. A case where the modulation is applied to an artificial two-level atom that can be located even outside the cavity has been considered in Ref. [114].

A general approach to the efficient description of resonant transitions in periodically modulated quantum systems has been developed in Ref. [115]. A microscopic model for the dynamical Casimir effect without time-dependent boundary conditions has been considered in Ref. [116]. The photons are produced due to the motion of a qubit inside a cavity. It is shown that under certain conditions regarding the qubit's movement that do not depend on its physical properties, a large number of photons may be generated without changing the qubit's state, as should be expected for a microscopic model of the mirror. The experimental requirements are discussed in detail.

The synchronization of qubits via the dynamical Casimir effect in a shared coplanar waveguide resonator terminated at one end by a superconducting quantum interference device has been considered in Ref. [117]. A quantum Rabi model for a two-level system coupled to a quantized cavity mode under periodic modulation of the cavity–dipole coupling in an ultrastrong coupling regime has been considered in Ref. [118]. It is shown how purely mechanical driving can produce real photons, depending on the strength and frequency of the periodic coupling rate.

6. DCE in Time-Dependent Media

The wave propagation and photonics in time-varying media has been the subject of a number studies considered in the reviews [119–122]. Here, I only address the recent publications somehow related to the DCE or similar effects. Luminal space-time crystals, where the structure moves at or close to the velocity of light, have been considered in Ref. [123]. The emission of light from a radiation source placed inside a photonic time crystal has been studied in Ref. [124]. It is found that radiation corresponding to the momentum bandgap is exponentially amplified, whether initiated by a macroscopic source, an atom, or vacuum fluctuations, drawing the amplification energy from the modulation.

A theory of weakly modulated "dynamical vacuum effects" in arbitrary nanostructured, dispersive, and dissipative systems has been presented in Refs. [125,126]. Vacuum amplification effects at anisotropic temporal boundaries have been investigated in Ref. [127].

Dynamical quantum vacuum amplification effects in temporal metamaterials have been considered in Ref. [128]. Various aspects of the dynamical Casimir effect in moving media have been considered in Ref. [129].

The possibility of ultra-strong light–matter coupling in the THz range has been demonstrated in Ref. [130]. The results of Ref. [130] may be of importance for studies of quantum vacuum radiation because such resonators can be optically modulated at ultrafast rates, apparently leading to the generation of non-classical light via the dynamic Casimir effect. The "photonic conductivity" effect and "photonic Hall effect" have been investigated in Ref. [131]. Ref. [132] has used shortcuts to the adiabatic method and proposes implementing fast multi-photon down-conversion, which can rapidly create $2N$ photons from the quantum vacuum based on the counter-rotating effect of ultrastrong light–matter coupling. The energy for the produced photons is given by a high-frequency pump field. Such accelerated evolution can restrain the influence of decoherence during the evolution so as to generate Fock states from the vacuum with high fidelities.

Quantum vacuum amplification effects in time-varying media with an arbitrary time modulation profile have been considered in Ref. [133]. It has been shown in Ref. [134] that considering material nonlocality is necessary for obtaining an accurate, physically satisfactory, and self-consistent description of dynamical Casimir effects in time-varying dispersive systems. The relations between the DCE and "time refraction" effects have been discussed in Ref. [135]. The "time reflection" phenomenon and its connection with the DCE and Cherenkov radiation have been considered in Ref. [136]. The parametric amplification of the zero-point fluctuations in the spin modes of a two-component Bose–Einstein condensate, triggered by the dynamical evolution of the condensate density, has been investigated in Ref. [137]. Analogs of the DCE in Bose–Einstein condensates has been considered in Ref. [138].

7. Various Applications and Connections with the DCE

The main features of the DCE as a part of the wide area of "Casimir physics" have been briefly described in the recent reviews [139–142]. Some connections with the DCE (mainly in the context of cosmological/gravitational problems and their laboratory simulations) can be found in Refs. [143–163]. For other remote connections, one is addressed to Refs. [164–171]. A combination of two vibrating plates and a strong perpendicular electric field between them have been considered in Ref. [172]. The authors predict a dramatic mutual enhancement of the DCE and the Schwinger effect (the creation of charged particles from a vacuum in strong electric fields [173]). Particle-pair creation by laser fields has been considered in Ref. [174]. The quantum vacuum Sagnac effect has been the subject of Ref. [175]. The ways to eliminate the negative influence of the DCE on the efficiency of quantum heat engines have been considered in Ref. [176]. A quantum field heat engine powered by phonon–photon interactions has been considered in Ref. [177].

Solutions to Moore equations [1] simulating adiabatically moving boundaries (when no DCE happens) have been found in Refs. [178–180]. The relations between the DCE, shortcuts to adiabaticity, and quantum heat engines have been studied in Ref. [181]. Some solutions to the Moore equation have been found in Ref. [182]. Solutions to wave equations in domains with a moving boundary have been obtained in Ref. [183]. The quantum dynamics of a Dirac particle in a 1D-box with a moving wall has been studied in Ref. [184]. The dynamical confinement in a quantum box with a moving wall, described in terms of the Schrödinger equation with time-varying boundary conditions, has been considered in Refs. [185,186].

Nonlinear effects in the DCE have been studied in Refs. [187–191]. The normal (perpendicular to the planes) Casimir force between two conductive planes with isotropic conductivity that move laterally and with a constant relative velocity has been calculated in Refs. [192,193]. The effects of quantum friction between moving bodies have been considered in Refs. [194–203]. The relations between the quantum friction and geometric phase have been studied in Ref. [204].

A graph model of the DCE has been considered in Ref. [205]. In Ref. [206], it is shown that the average number of created photons may be substantially increased if time-dependent *non-Hermitian* Hamiltonians could be engineered. Non-Hermitian Hamiltonians have been also used in Ref. [207] to describe a quantum system of two super-conducting qubits that move at relativistic speeds and where each qubit is coupled to a resonator mode. In this model, the relativistic motion of the qubits resulted from the modulation of the time-varying coupling between the qubits and the resonator. Analogies with the DCE in connection with enhanced superconducting correlations after photoexcitation have been discussed in Ref. [208]. Long-lived oscillations arise from the parametric generation of plasmon pairs due to pump-induced perturbation of the superconducting order parameter. A scheme of generating extremely weak squeezed light from the DCE process is proposed in Ref. [209].

Entanglement and Decoherence

The decay and decoherence of the field after switching off the effective boundary modulation that generates the DCE is the subject of the study [210]. The enhancement of the decoherence of a two-level neutral particle sliding on a metallic surface in a vacuum has been discovered in Ref. [211]. The dissipative evolution of quantum correlations due to the DCE in a superconducting waveguide has been considered in Ref. [212]. The entanglement degradation between field modes in two cavities due to the dynamical Casimir effect, when one of the cavities is harmonically shaken, has been investigated in Ref. [213]. It is shown that as the moving cavity is three dimensional, only two modes inside become coupled, and the entanglement either degrades asymptotically with time or oscillates, depending on the driving. On the other hand, as the cavity has an equidistant spectrum, the entanglement either vanishes asymptotically if it is driven with its fundamental frequency or gets a sudden death if it is driven with an uneven harmonic frequency. The dynamical Casimir effect in a double superconducting cavity in a circuit quantum electrodynamics architecture has been considered in Ref. [214]. The parameters in the quantum circuit are chosen in such a way that the superconducting cavity mimics a double cavity, formed of two perfectly conducting outer walls and a dielectric wall, with arbitrary permittivity separating both halves.

Tripartite entanglement generated by the dynamical Casimir array composed of three superconducting waveguides has been considered in Ref. [215]. The squeezing and entanglement due to the DCE in a cavity optomechanical system with optical bistability has been studied in Ref. [216]. The entanglement dynamics in an optomechanical cavity has been studied in Ref. [217]. Ref. [218] has considered a cavity resonator containing a two-sided perfect mirror. Although the mirror separates the cavity modes into two independent confined electromagnetic fields, the radiation pressure interaction gives rise to high-order effective interactions across all subsystems. Depending on the chosen resonant conditions, which are also related to the position of the mirror, $2n$-photon entanglement generation and bilateral photon pair emission have been studied.

8. Conclusions

This brief review gives the current panorama of studies related to the DCE. One can find that the interest in various manifestations of the DCE continues growing. Although the majority of publications discussed are purely theoretical, several suggestions on possible experiments have been put forward [77,79,80,93,96,110,116], though they have not yet been realized. A dynamic version of the magnonic Casimir effect [219] may also be apparently tested. One could also consider possible nonstationary versions of the "emergent experimental approaches" described in Ref. [220] having in mind other areas of Casimir physics.

It might be appropriate to mention two possible ideas (although extremely speculative at first glance). One of the main difficulties in observing the DCE is the extremely small ratio v/c between the achievable velocity of the boundary v and the light propagation velocity c. The *phase velocity* can be diminished in quantum circuits and waveguides. This factor has already been used in experiments [81]. On the other hand, it is known that the *group velocity* can be nevertheless diminished almost to zero (down to a few m/s) in certain experiments [221]. It could be said that the group velocity has no relevance to the DCE, since it is related to the propagation of wave packets. However, it is known that the electromagnetic field arising inside the cavity due to the DCE, under certain conditions (at least in single-dimensional models), concentrates in narrow packets moving between the reflecting boundaries [51,52,222]. It is of interest considering such an implausible possibility in more detail. (For other proposals of using "slow light" to simulate relativistic effects, see, e.g., [223].)

Another possibility is to use thin slabs made of vanadium oxides and illuminate them periodically with powerful short laser pulses. Then, the initial dielectric slab becomes highly conducting, diminishing the effective optical length of the cavity periodically. Experimentally it was shown that this transition may be achieved in a sub-picosecond regime [224,225]. Hence, the idea deserves to be addressed in more detail. On the other hand, the possibility that just achieving high conductivity in thin layers will be not sufficient to transform the dream of the DCE in cavities into reality is not excluded [226].

An impressive number of more than 200 publications related to the DCE in the past five years shows that significant progress in this area may be expected in quite near future. Surely, experimental progress will be of the most importance. As it looks, certain breakthroughs to come from the area of hybrid cavities. From this perspective, the proposals of Refs. [77,79,80,93,96,110,116] should be addressed in detail. Those proposals can presumably be implemented in laboratories. What concerns the theory, one of the challenging problems remains the completing the theory of field quantization in real dissipative cavities with time-dependent (e.g., moving) boundaries. A more moderate goal could be generalization of the effective approximate analytic solutions for a 1D ideal cavity with oscillating boundaries and Dirichlet boundary conditions [227,228] to a more realistic case, taking into account the effects of dissipation and more general (e.g., Robin) boundary conditions.

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