DYNAMICAL MODELS INCORPORATING HIGHER SYMMETRIES

R. E. Cutkosky (*), J. Kalckar and P. Tarjanne

Nordisk Institut for Teoretisk Atomfysik, København

(presented by R. E. Cutkosky)

The resonances and bound states which have been observed suggest a multiplet structure derived from a symmetry group which is larger than the isospin group SU_2. However, many multiplet assignments can be juggled into a superficial resemblance to the observed states, since any such higher symmetry would be very approximate, and it must be expected that many characteristics of the symmetry would be masked. In order to eliminate these uncertainties in the multiplet assignment, we use dispersion theory to determine the multiplets in which the forces are strongly attractive.

We describe here some properties of the models in which there are n equivalent fundamental baryon fields, so that the symmetry group is SU_n. We have more detailed results for SU_3 [1] and have also obtained results for the groups C_2[2], $SU_2 \otimes SU_3$, G_2[2] and the octet model based on SU_3[1] in which there are, respectively, 5, 6, 7 and 8 fundamental baryon fields which are not fully equivalent.[3]

The representations of SU_n are designated by $n-1$ ordered non-negative integers $\{f_1 \dots f_{n-1}\}$ (the lengths of the rows of the Young diagram). The fundamental (n-dimensional) representation is $f\{1,0...0\}$, the adjoint representation is $R = \{2, 1 \dots 1\}$, and the one-dimensional identity representation is $I = \{0, 0 \dots 0\}$. We shall show that it is self-consistent to assume that the baryons (B) belong to f, and that the pseudoscalar mesons (Π) and vector mesons (V) belong to R. There are also bound or resonant states which belong to other multiplets.

We write the contribution of one-particle exchange graphs to the scattering amplitude as $F = ab$, where b gives the dependence on the generalized isobaric spin, and a the dependence on other factors. Attraction is implied by $F>0$.

Let us denote by g the average B^2V coupling constant, and by G the average V^3 coupling constant. If a V_- is exchanged between a B and $\bar{B}$ (note that $f\otimes f = R \oplus I$), we have $b(R) = +1$, and $b(I) = -(n^2-1)$ [4].

On the assumption that the anomalous moment terms lead to contributions of the same sign as the ordinary term, $a \sim (+g^2)$. When a V is exchanged between two V's, we consider the decomposition

$$R\otimes R=\{4,2\dots2\}\oplus\{3,1\dots1,0\}\oplus\{3,3,2\dots2\}\oplus\{2,2,1\dots1,0\} \quad (n \geqq 4)\oplus R_+ \oplus R_- \oplus I \,.$$

The factors b are (in order): $+2n, 0, 0, -2n, -n^2, -n^2, -2n^2$,[4] and $a \sim (-G^2)$.

The same factors b occur in the force between two Π's, and the contributions of the other one-particle exchange graphs are qualitatively similar. If we have a " large " symmetry, and $g \sim G$, we may ignore the $B+\bar{B}$ component of the V and Π states.

The vector mesons belong to the anti symmetric R_- combination of $V+V$ (or $\Pi+\Pi$). The vector state has, in the multiplet I, a long range attraction which is exactly twice as strong; the I multiplet is symmetric so this can not correspond to a physical particle, but may be interpreted as the origin of the Regge trajectory which governs the diffractive high energy scattering [5]. We have not yet analyzed the spin-dependence of the forces in sufficient detail to estimate the ratio of the Π mass to the V mass, or to rule out low lying scalar or axial vector states.

(*) N.S.F. Postdoctoral fellow, on leave from Carnegie Institute of Technology, Pittsburgh, Pa.

For the $B+V$ and $B+\Pi$ states, we have:

$$f \otimes R = \{3,1\ldots1\} \oplus \{2,2,1\ldots1\} \oplus f .$$

The factors b for the baryon exchange graphs are $b = n,\ -n,\ -1$; for the "diagonal" V exchange graphs, $b = n,\ -n,\ -n^2$; for the V meson exchange graph leading to $B+V \leftrightarrow B+\Pi$, $b = n-2,\ -(n+2),\ n^2-4$ [4]. In the multiplet f, we expect that the V exchange forces predominate, which implies near coincidence of the Regge trajectories with opposite signatures. These forces are attractive in f if the coupling constants have the same signs.

Simple non-relativistic arguments illustrate an important feature of the spin dependent forces arising from V exchange. The spin-orbit and central forces are related by

$$\langle V_{LS} \rangle / \langle V_{\text{centrifugal}} \rangle \sim \langle V_{\text{central}} \rangle / M,$$

where M is an appropriate average mass. If the potential is strong enough to produce a binding energy comparable to the masses of the particles, the spin-orbit force will be comparable to, or even override, the centrifugal barrier. This shows how it is possible that the $j = {}^1/_2(+)$, ${}^3/_2(-)$, and ${}^5/_2(+)$ states can be interpreted as being predominatly P, D, and F states of $B+(V, \Pi)$, and the S states disregarded [(*)].

The V exchange contribution to the BV and VV forces gives an especially strong singular attraction at small distances, which must be cut-off. It is possible to adjust the effective cut-off so that self consistent bound B and V states are obtained. We assume that by including more terms in the potential, some properties of the cut-off could ultimately be calculated. If we assume the cut-offs are similar in the B and V problems, we must have $g \sim G$.

The baryon exchange graph gives a strong ΠB attraction in the state $J = {}^3/_2(+)$, $\{3,1\ldots1\}$ (to which the (3,3) resonance belongs), and we also find a strong attraction in $J = {}^1/_2(+)$, $\{2,2,1\ldots1\}$, to which the Σ is assigned, in some models. These states must eventually be included in our self-consistent model. Note that if n were very large, these states would be expected to disappear.

We have also applied the self-consistent picture of the baryons to the octet version of SU_3, which was suggested by Gell-Mann and Ne'eman. We cannot get self-consistency if we assume R invariance, but if we assume the baryons to be a linear combination

$$\psi_B = \cos\theta\psi_+ + \sin\theta\psi_-$$

of the symmetric and the antisymmetric meson-baryon states, we can find a self-consistent value of θ. A very rough calculation gives $\theta = 45^\circ$ for the B-π coupling, which is in reasonable agreement with observed partial widths of the excited baryon states, and requires the $T = {}^3/_2 \pi$-N resonance to belong to a ten-fold super-multiplet. The details will be published later.

LIST OF REFERENCES

1. R. E. Cutkosky, J. Kalckar, and P. Tarjanne, Physics Letters *1*, 93 (1962).
2. P. Tarjanne (to be published in Ann. Acad. Scient. Fennicae, VI Physica).
3. For a discussion of these models, see R. E. Behrends, J. Dreitlein, C. Frønsdal, and W. Lee, Rev. Mod. Phys. *34*, 1 (1962); D. Speiser and J. Tarski, I.A.S. preprint, Princeton (1961); R. Sawyer, I.A.S. preprint, Princeton (1962); and other references cited in (1) and (2).
4. For the calculation of these numbers we use properties of the infinitesimal generators of SU_n and of the Casimir operator, proceeding as in reference 1.
5. G. F. Chew and S. C. Frautschi, Phys. Rev. Letters *7*, 394 (1961).

DISCUSSION

SAKURAI: First, doesn't the representation 1 for baryons lie lower than the representation 8 for baryons in this kind of approach? Secondly, is your mixture parameter θ related in any obvious way to the parameter α that characterises a linear combination of F-type couplings and D-type couplings in Gell-Mann's notation?

(*) This point was overlooked in reference [1].

CUTKOSKY: The answer to the first question is, I believe, no. As to the second question,

$$\alpha = (1+\tfrac{1}{3}\sqrt{5}\tan\theta)^{-1}$$

BREITENLOHNER: I would like to make a remark on R-in variance mentioned by Cutkosky. In my opinion the main reason to introduce this invariance is to distinguish the two equivalent octets in the (baryon octet)·(meson octet) direct product, but this gives some trouble as $\mu_n = 0$. Another possibility is to assume an interaction symmetric in the two octets and this is plausible since the interaction is mediated by vector bosons which couple in the same way in both octets. This gives no such trouble.

CUTKOSKY: The trouble with R invariance is that is gives an attraction in both of the 8-dimensional multiplets, and also in the one-dimensional one. So that you start out assuming that you have eight baryons and you end up finding more than eight. Of course, this is based on a picture in which we do not assume any gluons or things like that. I mean, it is only particles which come out that are put in at the beginning.

NE'EMAN: In our calculations of some branching ratios we have generally been able to get the right branching ratios with mixtures of the two couplings, F and D, something like the θ Dr. Cutkosky was describing. It did not come out as one to one, if the $\theta = 45°$ represents something like that, but a mixture for the case of a coupling of the vector meson with the baryons was generally necessary. When you work with a vector meson coupling with mesons you generally take only F, because then R is the same thing as charge conjugation, and you have to keep within this coupling.

CUTKOSKY: Well, the angle which we think works best for the pseudoscalar meson-baryon coupling is something like 30° actually.

NE'EMAN: Just one other point about the comparison with the Sakata model for annihilations into two mesons. The Rehovoth group has proved an identity for the Sakata model showing that the annihilation of $p\bar{p}$ into $\overline{K^0K^0}$ cannot produce an odd charge conjugation pair. As the experiments show that what is produced is a $K_1{}^0K_2{}^0$ system, the Sakata model seems to fail in this respect. I would just like to ask the experimentalists what is the present situation for the spin of the cascade, because that would probably be crucial for the knowledge about the two models.

YAMAGUCHI: Yesterday I asked specifically this question and it seems it is not settled.

CUTKOSKY: If you believe the Sakata model, then you get the Σ belonging to a sextet which lies very close to the triplet, and in the strangeness zero channel there are two doublets separated by a mass difference comparable to the $\Lambda\Sigma$ mass difference. And then you will have to assume that you have some unsymmetrical forces comparable to the Λ-nucleon mass difference. So these two doublets will certainly be strongly mixed, so I do not think that one could realistically expect, if the Sakata model were right fundamentally, that this selection rule would work.

RENORMALIZABLE COMPOSITE MODEL OF ELEMENTARY PARTICLES

Y. Miyamoto

University of Tokyo, Tokyo

(presented by Y. Miyamoto)

Recent discovery of many resonance levels of baryons and bosons stimulates an interest in the structure of elementary particles. There is the possibility that baryon isobar levels may be explained in terms of Chew-Low type theory (namely p wave $\pi(K)$ meson-hyperon resonances) and the Takeda-Ball-Frazer theory. On the other hand (ω, ρ, K^*) multiplet with spin 1 may be interpreted to be the p wave resonances among (π, K, π_0') multiplet with spin 0. But, as for this model, the coincidence of energy levels of (ω, ρ, K^*) seems to pre-suppose a rather complicated mechanism (c.f. ω and ρ are K-$\bar{K}$ bound states and K^* is K-π p wave resonance). Then the most simple model is the Fermi-Yang-Sakata model. In this model π, ω etc. mesons are compound states of P, N, $\Lambda^{(*)}$ and their antiparticles.

(*) We abbreviate (P, N, Λ) by **B**.