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Quantizing dynamics

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Abstract. In theoretical physics, quantization means reducing a continuum structure to discrete structure. This process was widely used in the development of quantum mechanics since Planck's convention of the discretization of the energy of electromagnetic (EM) waves $e = nhf$ as a solution to the crisis of the blackbody radiation. When combined with Einstein's relativistic particle energy equation $e = mc^2$ it became a most fundamental process of the 20th century theoretical physics. Planck was reluctant to consider his energy quanta $e = nhf$ as a physical particle. His concern was forgotten in the process of development of quantum mechanics, which was Einstein's relativity theory dynamics combined with Planck's wave-particle duality. This framework was later extended by Dirac into the quantization of the entire EM field theory of Maxwell in which the EM fields, which are mathematically a continuous structure, themselves were quantized in terms of Einstein's special theory of relativity dynamics using Fourier expansions. As it appears that the mathematical error of using wave numbers, which form a real continuum, as indexes of the Fourier expansion for the discretization of waves went unnoticed, this quantization of electrodynamics, called quantum electrodynamics (QED), became a model for further quantizing continuum based physical theories. This error was thus passed down to all of the successors of QED. The theory of quantum gravity is yet another attempt to quantize a major force field theory of gravitational forces. Here as well the issue of the difference between the continuum and the discrete was overlooked.

1. Prelude: energy

It has been said that that energy is the most fundamental concept that governs physics. That Hamilton reduced all physical phenomena to just energy is an evidence for a reliance on energy in modern physics. More recently it was Einstein who reduced the concept of relativistic energy as the most important result of relativity theory to the equation $e = mc^2$. Reportedly, Einstein said that if this equation fails, the entire relativity theory fails.

Is it really the case that energy is the most important fundamental concept in physics? In classical physics, (kinetic) energy is defined as the work needed to accelerate from $m0$ to mv. It has been believed that the work needed is $mv^2/2$. Recently, we [2, 3, 4] presented an argument that this definition is invalid as the work needed to perform this acceleration varies depending upon how we accelerate from $m0$ to mv and it is $mv^2/2$, as claimed, only under constant acceleration. Furthermore, together with the general definition that energy is a potential to do work, the energy conservation law fails.

Contrary to this reliance to energy, Landau and Lifshitz used momentum as the most fundamental concept in theoretical physics in their legendary book "Quantum Mechanics". However, they were not aware of the deficiency of the concept of energy to fail the conservation property.

2. Electromagnetic waves and photons

Electromagnetic wave equations were derived purely mathematically from Maxwell's classical (non-relativistic) axioms of electromagnetic field equations without electric current. Actual wave functions, which are solutions to these wave equations, were obtained from Maxwell's axioms with an accelerating electric current (or, equivalently, electric charges).

The electromagnetic field, which is a spatial distribution of electromagnetic force per unit charge, is not a physical reality but a counter factual modality. This makes electromagnetic waves imaginary modal waves.

The speed of such a wave is of concern. It was shown mathematically that the speed of light in vacuum without any electric current is constant c. However, the theory also showed that without accelerating current, which is a flow of charges, there can be no electromagnetic waves. This issue has been brought up by us very recently in [2]. When confronted with a question that if we consider very far away from the source of the emission of the wave, we could safely assume that the conducting current is 0 and the speed of the electromagnetic wave is c, the answer is straightforward. The theory certainly claims that the speed is c wherever light goes. The theory predicts the emission of light in this way and never says that the speed of light wave is affected by the distance from the accelerating charge.

3. Relativistic theory of photons

Following Einstein's belief, Planck's hf is considered to be a particle called *photon*. Naturally, one expects that this photon travels with speed c as the speed of the electromagnetic wave is assumed to be c. However then the photon violates the assumption of the special theory of relativity which declares that no particle travels with speed c. Moreover, this particle makes its relativistic energy

$$e = mc^2 = m_0c^2/\sqrt{1-(v/c)^2} \tag{1}$$

diverge. In an attepmt to resolve the second problem, Einstein postulated that the rest mass of the photon is 0. An explanation of this choice can be found in [1].

Furthermore, Einstein's proposal that hf is a particle called photon of frequency f that carries energy hf brings us to an unexpected mathematical difficulty. Namely, as f is a frequency of an electromagnetic wave and f varies over all positive real numbers, there must be uncountably many particles called photons and this violates the ontology that there are only countably many particles in the universe.

As we have shown above, the attempt at quantizing electromagnetic waves fails. The pilot project of quantizing continuum failed. It is a huge surprise that it took more than a century to come to the realization that photons are not legitimate on pain of contradiction.

4. The problem with the special theory of relativity

As a resistance against absolutism in the global movement of Lutheran religious revolution, the proponents of Galileo promoted the concept of relativism in which a reference frame can move inside another reference frame. In this way they tried to remove reference to the absolute frame, which was formulated by Newton.

Despite its apparent appeal, relativism had some fundamental flaws. Namely, Galileo did not seem to be aware that a reference frame F_1 as geometric space moving inside another frame F_2 means that at each moment a point in F_1 is also a point in F_2.

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The following contradiction leads to the rejection of the concept of moving reference frames, inertial or accelerating: Assume a running train. When the tip of the power pole touches the power line at point P, spark occurs at this point. An observer standing right below the point P, which is a point of the train, will observe that the light comes straight down to him/her. Moreover, as P is a stationary point on the power line the observer will observe that the light comes diagonally from the point P of the power line. This is a contradiction which we shall call *power-line power-point paradox*, or PPP in short. As Galileo's relativity theory is inconsistent, logically speaking, extending this theory by adding the axiom of constancy of the light speed yields an inconsistent theory. Any extension of an inconsistent theory is inconsistent. This phenomenon is called the *monotonicity of logic*.

Remark As Newton pointed out, the geometric space (3D space) was introduced to theoretical physics as metaphysics. It was to introduce the concept of location of a body represented as a point and the motion of such body as a function from time to geometric space. As a point in a reference frame F is not a physical body, we cannot place this point in a reference frame F' and move it. This completely makes the concept of reference frames in relative motion illegitimate. Motion is not a geometric concept. It is a physical concept.

There are two versions of the special theory of relativity (STR), namely *STR kinematics* and *STR dynamics*. As both of them rely upon the concept of relatively moving reference frames, they are both inconsistent. In STR dynamics, where Einstein allowed accelerating frames, the most fundamental assumption of the theory of relativity that all relative motions of reference frames must be inertial was violated. This restriction is essential because otherwise, the theory of relativity violates the action-reaction law. So, even if we chose to ignore the PPP paradox, STR dynamics is invalid.

It is informative to see at how Einstein's equation $E = mc^2$ was obtained. First, the relativistic mass $m = m_0/\sqrt{1-(v/c)^2}$ was calculated, so that the momentum conservation holds in the relativistic collision. With this, the relativistic second law as $\mathbf{F} = d(m\mathbf{v})/dt$ was obtained. This leads to

$$dE = \mathbf{F} \cdot d\mathbf{r} = (d(m\mathbf{v})/dt) \cdot d\mathbf{r}. \tag{2}$$

From this as $\mathbf{v}$ is constant, we obtain $E = 0$ instead of the claimed $E = mc^2$.

5. Quantum mechanics and quantum electrodynamics

Motivated by Einstein's attempt to quantize electromagnetic waves in terms of photons, de Broglie and Schrödinger launched a project of quantizing Hamiltonian mechanics. This lead to the development of what is now called quantum mechanics. Using Einstein's STR, which we now know is inconsistent, de Broglie, who wasn't aware of the inconsistency of STR, obtained a relativistic general correspondence between waves and particles, which became a guiding light house for the development of quantum mechanics.

De Broglie considered relativistic waves whose phase is invariant under the Lorentz transformation. He then obtained the following wave transformation

$$k'_x = (\omega - v\frac{\omega}{c^2})/\sqrt{1-v^2/c^2}, \quad k'_y = k_y, \quad k'_z = k_z, \quad \omega' = (\omega - vk_x)/\sqrt{1-v^2/c^2}. \tag{3}$$

In analogy to the momentum-energy transformation

$$p'_x = (\omega - v\frac{E}{c^2})/\sqrt{1-v^2/c^2}, \quad p'_y = p_y, \quad p'_z = p_z, \quad E' = (E - vp_x)/\sqrt{1-v^2/c^2}. \tag{4}$$

he then obtained the following duality between waves and particles

$$\mathbf{p} = \hbar\mathbf{k} \qquad E = \hbar\omega. \tag{5}$$

So, a wave with wave propagation vector $\mathbf{k}$ and angular frequency ω now has a particle counterpart with relativistic momentum $\mathbf{p}$ and relativistic energy E.

The particle-wave duality became common accepted in theoretical physics. Below we outline some issues with this framework.

(i) De Broglie's result is obtained from just an analogy between phase wave transformation and the momentum-energy transformation. Questioning whether the validity of such a deduction from analogy is scientifically sound is legitimate.

(ii) The entire development is based upon Einstein's STR, which we now know, is undoubtedly inconsistent.

(iii) Though this resembles Planck-Einstein's particle-wave duality

$$E = h\omega = pc \quad p = h/\lambda \tag{6}$$

where λ is the wave length, the two are fundamentally different. Unlike the photon-light duality, where the speed of the photon and that of light are equal, the phase speed of a matter wave and the speed of a particle can be different. De Broglie further assumed that, associated with a particle with speed V was a wave having phase speed $w = \omega/k$. He also assumed that the energy in the wave traveled along with a group speed $v_g = d\omega/dk$ which was identical to the particle's speed V. Here it is not quite clear what did he mean by energy in the wave. As in (i), the de Broglie relation above is a hypothesis based upon just the above mentioned analogy of the wave vector-frequency transformation and the momentum-energy transformation. Certainly, this does not yield the concept of energy in the wave.

Anyhow, it can be readily shown that $c^2(\omega/c^2 - k^2)$ is invariant under the relativistic transformation as an analogy to the relativistic invariance of $c^2(E/c^2 - p^2)$. We set

$$c^2(\omega/c^2 - k^2) = constant = C. \tag{7}$$

From this, we have

$$2\omega/c^2 d\omega/dk - 2k = 0 \quad \text{and} \quad v_g = d\omega/dt = c^2k/\omega. \tag{8}$$

As the phase speed is $w = \omega/k$, we have

$$v_g = c^2/w. \tag{9}$$

It now follows that the either phase speed w or group speed v_g could exceed c but not both. We do not know what this means for STR which asserts that nothing moves with a speed faster than c. After all, de Broglie used STR to obtain this result.

According to de Broglie, as a convention, the energy in a wave travels with a group speed. It is quite clear that we are dealing with something completely different from waves in wave mechanics where waves travel through wave medium. Clearly none of us knows what is the wave medium for the matter waves of de Broglie. Einstein's and Planck's duality between the EM wave and the photon itself is problematic enough already.

All of this mathematical complication is relative to the hypothesis that a particle with speed v has a wave dual called matter wave whose group speed is $w = \omega/k$. A particle in motion is to carry energy and so it is expected that the wave dual of this particle also carries energy of the same amount if the energy conservation law is to be respected. But for a wave to carry energy it has to have a wave medium. A concern we have is that de Broglie's wave is a mathematical wave which appears to have no wave medium. One may argue that the electromagnetic wave carries energy without having a wave medium. We have already pointed out that the electromagnetic field, which carries electromagnetic waves, is a counter-factual modality, and, hence, plays no ontological role in physics. So, what happened to the energy issue of the matter waves? This question was answered by Schrödinger in a limited context and presented in in the following section.

6. Schrödinger's wave mechanics

Schrödinger used Hamilton's energy dynamics for the particle theory and applied de Broglie's pilot wave theory to produce a wave-particle duality which looks after the energy issue of de Broglie's relation.

All waves propagated along the x-axis obey the following wave equation

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{\omega^2}\frac{\partial^2 \Psi}{\partial t^2} \tag{10}$$

where $\Psi(x,t)$ is the wave function and ω is the wave speed.

Remark Considering that a wave is a function of time and location whose value is the displacement from the x-axis at time t, this general statement is reasonable.

Here, we consider the wave function Ψ whose square yields the probability of locating a particle at any point in the space. We consider only systems whose total energy E is constant and whose particles move along the x-axis and are bound in space. Then the frequency associated, through the de Broglie relation, with the bound particle is also constant, and we can take the wave function $\Psi(x,t)$ to be

$$\Psi(x,t) = \psi(x) f(t) \tag{11}$$

as the frequency is assumed to be precisely defined, $f(t) = \cos 2\pi\nu t$. So, we have

$$\frac{\partial^2 \psi}{\partial x^2} = -\left(\frac{2\pi}{\lambda}\right)^2 \psi = -\left(\frac{p}{h}\right)^2 \psi \tag{12}$$

where the wave length is $\lambda = \omega/\nu$ and the momentum of the particle is $p = h/\lambda$.

We take the particle of mass m to be interacting with its surroundings through a potential-energy function $V(x)$. The total energy of the system is given by

$$E = E_k + V = \frac{p^2}{2m} + V \tag{13}$$

where E_k is the kinetic energy of the particle. Then we have $p^2 = 2m(E - V)$ and we have

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = E = i\hbar\frac{\partial \psi}{\partial t}. \tag{14}$$

This equation is called (non-relativistic) Schrödinger's wave equation as the energy equation involves non-relativistic mass m and it is not invariant under the Lorentz transformation. Oddly this does not mean that quantum mechanics is a non-relativistic theory. The derivation of Schrödinger's wave equation involved the de Broglie relation which is nothing but a relativistic theory.

Remark This equation of Schrödinger is not what should be called wave equation. The wave equation is of the form:

$$\frac{\partial^2\Psi}{\partial x^2} = \frac{1}{\omega^2}\frac{\partial^2\Psi}{\partial t^2} \tag{15}$$

What is more puzzling is that it is the potential-energy (which is hypothetical) function that makes an extra contribution to the wave equation. What about the speed of Schrödinger's wave? It appears that $V\psi$ is adding an extra element to the speed of Schrödinger wave. Maybe the only issue here is that the energy wave, which is not a real but a modal wave. Nevertheless, Schrödinger was aware of this problem and tried to make his wave equation relativistic, albeit without success. This problem was later tackled by Gordon, Klein and Dirac in the development of quantum electrodynamics.

The first quantum relativistic basis

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} + (\hbar c)^2\nabla^2\psi = (m_0c^2)^2\psi \tag{16}$$

was obtained by Gordon-Klein by inserting the energy operator and momentum operator into the relativistic energy-momentum relation:

$$E^2 - (pc)^2 = (m_0c^2)^2. \tag{17}$$

Unfortunately this equation is invalid as the equation $E = mc^2$ is invalid.

Moreover, here the energy operator and the momentum operator do not commute. This should mean that energy and momentum are not observables. This itself is rather expected considering that

(i) Energy is not a physical reality, it is a counter-factual modality.
(ii) The claimed energy-momentum is a false equation coming from the inconsistency of STR dynamics.

As this result appeared to resolve the difficulty, all these questions were ignored and quantum electrodynamics moved on. What is astounding here is that now the quantization is reduced to the simpler process of mechanically replacing classical variables with operators. Certainly, in the argument of Gordon-Klein there is no concern on the commutativity. Of course this is a natural consequence of using non-measurable physical quantities such as energy and, consequently, momentum.

It is plausible to assume that Gordon-Klein borrowed the idea from Heisenberg-Jordan's operator-based formalism of quantum mechanics. However, this solution did not make Schrödinger's wave equation relativistic. So, Heisenberg-Jordan's formalism and Schrödinger's formalism are not equivalent, contrary to what was claimed. Yet in effect, from here on these two different formalisms were mixed up together under the illusion that they are in essence the same thing.

Most importantly, the energy-momentum relation of Einstein is false as $e = mc^2$ is false. Moreover, categorically, energy is a modality and momentum is a physical reality. How is it possible to have an equation that relates modality and reality? In philosophy, equality holds only between compatible categories.

When putting many different equations together, one may end up with inconsistency, which will make the theory meaningless. STR is a perfect example of this. It may be reasonable to say that it means that the extraction of some of these equations from experiments went wrong, or some of the experiments (or their interpretation) went wrong or both. The connection between experimentation and conceptualization is not as simple as we would hope. A good example appears in Maxwell's EM field theory. This theory is inconsistent at the most fundamental level as it assumes the Lorentz force and the second law as its axioms. It appears that both of them were extracted from experiments. So, it is not clear at all why we ended up with this confusing situation.

Remark Interestingly, the contradiction between the second law and the Lorentz force may well be a consequence of the mixing up of physical, ontological laws and modal laws. The second law is an ontological law and the Lorentz force is a modal law. More interestingly, on the one hand, Newton's second law was obtained from induction over the experimental data accumulated by Kepler and the Lorentz force was obtained experimentally in terms of the abstract modal concept of the magnetic force field.

7. Quantum electrodynamics (QED)

As discussed in the foregoing, EM field theory, lead by Maxwell and Lorentz, considered energy to be a harmonic oscillator without providing a good reason for this. From this assumption, EM theory was developed, yet in the end it did not solve the problem of blackbody radiation. To resolve this problem, after a lengthy battle, a convention of the Planck constant was arrived at. Planck showed that under the assumption that the EM wave energy is transmitted only as nhf, where n is a natural number, f is the frequency of the EM wave and h is a special constant now called the Planck constant, the blackbody radiation problem is resolved as this fits the expectation curve for the blackbody radiation well. Planck himself accepted his own proposal only reluctantly and did not agree with Einstein who wanted to upgrade the status of hf from a convention to a particle called *photon*. Upon this convention, Einstein went on to develop relativistic dynamics which fundamentally violated the basic restriction that the speed of a reference frame must be constant, and not time-varying, in order to avoid the violation of the third law of dynamics.

7.1. Harmonic oscillators

Following Gordon-Klein's wrong response to Schrödinger's failed attempt to show that his wave equation is relativistic, Dirac quantized classical Hamiltonian H for the harmonic oscillator by replacing physical quantities in it with self-adjoint operators as

$$H_{osc} = p^2/2m + m\omega^2 q^2/2m \tag{18}$$

where $[p, q] = i\hbar$, m is the mass of oscillating object and ω is the frequency of the oscillation. Although the connection between this purely formal quantization and de Broglie's (or Schrödinger's) quantization is not understood as clearly as it should be, Dirac's quantization prevailed and became standard in contemporary quantum field theory, albeit without a convincing explanation therefor.

The first question to be asked is whether m is a relativistic mass or not. If it is relativistic, then the entire theory of photons as a quantization of harmonic oscillators is false, as the theory of relativity is false.

Remark Interestingly, de Broglie's quantization and Dirac's quantization have one thing in common. Both of them are invalid, albeit for different reasons as we discussed in the foregoing. It took a century to come to this realization. It took more than one century to realize that the theory of relativity is completely invalid. It is reasonable to ask why did it take so long.

Dirac's computation is essentially the following

$$a = (m\omega p + ip)/\sqrt{2\hbar m\omega} \qquad a^+ = (m\omega p - ip)/\sqrt{2\hbar m\omega} \qquad [a, a^+] = 1. \tag{19}$$

Now we have

$$H_{osc} = (1/2)\hbar\omega(a^+a + aa^+) = \hbar\omega(a^+a + 1/2). \tag{20}$$

Define N as $N = a^+a$. It follows:

(i) Eigenvalues of N are $n = 0, 1, 2, ...$

(ii) If $|n\rangle$ is normalized then so are $|n\pm1\rangle$ defined as $a|n\rangle = \sqrt{n}|n-1\rangle$ and $a^+|n\rangle = \sqrt{n+1}|n+1\rangle$.

(iii) If $|0\rangle$ is normalized, the normalized eigenvectors of N are $|n\rangle = ((a^+)^n/\sqrt{n!})|0\rangle$ for $n = 0, 1, 2, ...$

These are also eigenvectors of H_{osc} with eigenvalues $E_n = \hbar\omega(n + 1/2)$, $n = 0, 1, 2, ...$ The operators a and a^+ are called the *annihilation operator* and the *creation operator*, respectively, under the assumption that $|n\rangle$ represent a quantum state with n quanta.

In summary, the quantized Hamiltonian for the harmonic oscillator can be expressed as

$$H_{osc} = (1/2)\hbar\omega(a^+a + aa^+). \tag{21}$$

The intention of Dirac was to represent energy (harmonic oscillator) as a collection of countably many particles called photons or, equivalently, the associated annihilation and creation operators.

It would appear though that all of this has no clear ontological meaning. How can one say that (21) is the quantization of the Hamiltonian operator (18) without being asked what is meant by this quantization? It is not quite clear what quantization of a Harmonic operator means in this context. One of the major difficulties in following quantum mechanics is that often times no explanation is given as to what is being done when the formalism is used.

We can safely trace this problem back to where Gordon and Klein replaced the troubled Schrödinger's quantum mechanics which, despite using the theory of relativity (dynamics), failed to be relativistic, with the Klein-Gordon equation.

Gordon and Klein quantized a most fundamental relativistic equation of the energy-momentum relation $E = \sqrt{(cp)^2 + m_0^2 v^4}$, which is a consequence of $E = mc^2$, without knowing that $E = mc^2$ is false, thereby yielding a quantization of relativistic dynamics. They essentially replaced all physical variables in the energy-momentum equation with the corresponding self-adjoint operators of quantum mechanics as it was the case in the quantization of the Hamiltonian equation.

Unfortunately, Gordon-Klein's quantization of the invalid relativistic energy-momentum equation of Einstein does not offer a convincing solution to the fundamental problem of Schrödinger's wave equation not being relativistic. Replacing relativistic energy variable and relativistic momentum variable with an energy operator and a momentum operator in the faulty relativistic energy-momentum relation is not what one should call quantization.

Remark We now know that Einstein's theory of relativity is fundamentally inconsistent. This means that Schrödinger's wave equation not being relativistic is not an issue at all. The real issue for Schrödinger's work is that it came from de Broglie's relation, which is relativistic and therefore invalid.

Moreover, the Gordon-Klein equation does not conserve probability, which is one major requirement imposed by the usual interpretation of quantum mechanics. Quantum mechanics interprets the square of the modulus of a wave function's amplitudes as probabilities. For that

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reason, Schrödinger's equation was made to make sure that the coefficients of wave functions were normalized at every point in time. This unfortunately is not the case for the Gordon-Klein equation.

All of this cannot thus be seen as a valid replacement for a relativistic version of Schrödinger's equation. In order to conserve probability, a time evolution equation needs to satisfy the following condition with regards to a wave function

$$\int \left|\psi(x,t)^2\right| dx = 1. \tag{22}$$

Furthermore, as the conservation must hold at any point in time, it has to be independent of time evolution. This is to say that the Gordon-Klein equation must satisfy the following equation as well

$$\frac{\partial}{\partial t}\int \left|\psi(x,t)^2\right| dx = 0. \tag{23}$$

Now, consider the Gordon-Klein equation

$$\frac{1}{c^2}\frac{\hbar^2\partial^2}{\partial t^2}\psi(x,t) = (\hbar^2\nabla^2 - m^2c^2)\psi(x,t). \tag{24}$$

Since it involves the second derivative with respect to time, it is clear that the first derivative term in the probability conservation expression will in general not disappear. Hence, the expression will not produce the required value 0 and so, contrary to their claim, the Gordon-Klein equation does not describe the probability wave that the Schrödinger equation describes.

Notwithstanding, the most important issue is that Einstein's theory of relativity relativity is inconsistent and there is no point trying to make classical theories relativistic. Classical field theories such as the theory of electromagnetism have their own problems, e.g. with the Lorentz force versus the second law. The theory of relativity is a wrong answer to the problem of the classical electromagnetic theory.

We should be given a clear meaning of the quantization of H_{osc}. If it was meant to be the particle representation of the continuum operator H_{osc}, then we do have some serious questions to ask. Namely, how is it possible to represent an operator that acts upon a continuum with countably many particles? This is asking if photons as above are really particles at all. This is an open question on a fundamental issue of quantum mechanics. If photons are not really particles and they are infinitary continuum entities, how is it possible that they appears in particle experiments as trajectories?

In addition to this issue of continuum versus discreteness, there is one issue which was never dealt with by quantum physics. There is no clear explanation as to why quantum mechanics took harmonic oscillators as the continuum model of energy and quantized it. It is clear that this created the crisis of blackbody radiation, which was a manifestation of the mismatch between this theoretical modeling and empirical reality known as the blackbody radiation. The accepted resolution to this problem was the relativistic dynamics as proposed by Einstein. Unfortunately, as we have shown recently in [2, 3, 4], this solution is untenable as the relativistic dynamics is inconsistent.

Remark Regarding the problem of using harmonic oscillators, a recent study by Suntola [5] showed that when energy is represented with monochromatic oscillator the blackbody problem does not arise.

The issue of energy being a modality rather than a physical concept emerges again. We cannot see the modal nature of energy in the equation

$$H_{osc} = p^2/2m + m\omega^2q^2/2m \tag{25}$$

can we? Energy is a modality, as we now know. This equation came from Gordon-Klein's equation, which was obtained by replacing Einstein's false equation of the energy-momentum relation, which was obtained from the false equation $e = mc^2$. From $e = mc^2$ Einstein obtained the equation that expresses momentum in terms of energy. Momentum is a physical entity, while energy is a modality. Notwithstanding, the entire confusion is due to the lack of understanding that energy is not a physical reality. What is then the point in arguing whether it is a harmonic oscillator, a monochromatic oscillator or any other oscillator that should represent energy. They are all physical realities. One cannot express a modality as a physical reality.

The problems with Dirac's work can be summarized as follows:

(i) There is a lack of awareness that energy as potential to do work is a modal concept.

(ii) Contrary to the claim, the work needed to accelerate from $m0$ to mv is not always $mv^2/2$. It depends upon how we accelerate.

(iii) Hence, the concept of energy as the potential to do work is invalid.

(iv) As a consequence of all of this, the law of conservation of energy is false.

In the end, as we pointed out, the original concept of energy as the potential to do work is invalid. This brings an even bigger issue to resolve, namely, having no clear concept of energy, it is not enough to claim that energy is a monochromatic oscillator. At least, however, it is plausible to say that one cannot claim that energy is a system of particles called photons.

7.2. Quantization of the electromagnetic field: Dirac's aether theory

Planck quantized the energy of electromagnetic waves to deal with the problem of blackbody radiation. He then furthered this result in the context of the theory of relativity to quantize energy as harmonic oscillators. Then, he went on to quantize the electromagnetic field, which is supposed to be the medium for Maxwell's electromagnetic waves. This is called the *second quantization.*

As we have been discussing throughout this work, the electromagnetic field is not a physical concept but a modality. What is concerning here is that Dirac's second quantization, which we will discuss in what follows, produced physical particles called photons from this modal concept of EM fields. Through Fourier expansion of the electromagnetic field represented by the vector potential, Dirac induced photons as harmonic oscillators in the space together with the creation and annihilation operators.

7.2.1. Vector potential According to the classical theory of electromagnetism, the electric field **E** and the magnetic field **B** of Maxwell can be obtained as

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\phi, \quad \mathbf{B} = \nabla \times \mathbf{A}. \tag{26}$$

from a scalar potential ϕ and a vector potential $\mathbf{A}$. If there is no source of the field, we choose a gauge (Coulomb gauge) such that $\phi = 0, \nabla \cdot \mathbf{A} = 0$. From these equations one can derive the following equation for vector potential

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = 0. \tag{27}$$

This means that vector potential $\mathbf{A}$ for a charge-free space is a wave. But as both $\mathbf{E}$ and $\mathbf{B}$ are modalities, $\mathbf{A}$ also is a modality. Therefore, $\mathbf{A}$ is not a physical but a modal wave. Let us call this wave (vector) the *potential wave.*

Remark There are infinitely many vector potentials $\mathbf{A}$ that satisfy this wave equation. Considering that $\mathbf{E}$ and $\mathbf{B}$ are modal vectors, the ontological meaning of such a vector potential is up in the air.

7.2.2. Quantization of the electromagnetic field Following Dirac, let us perform a Fourier expansion of the electromagnetic field in a large cube of volume $\Omega = L^3$ and take the Fourier coefficients as the field variables. We choose the boundary conditions to be periodic on the walls of the cube. This is

$$\mathbf{A}(L,y,z,t) = \mathbf{A}(0,y,z,t), \quad \mathbf{A}(x,L,z,t) = \mathbf{A}(x,0,z,t), \quad \mathbf{A}(x,y,L,t) = \mathbf{A}(x,y,0,t). \tag{28}$$

The Fourier series of $\mathbf{A}$ is given by

$$\mathbf{A}(\mathbf{x},t) = \sum_{\substack{\mathbf{k} \\ k_z>0}} \sum_{\sigma=1,2} \sqrt{2\pi\hbar c^2/\Omega\omega_k}\,\mathbf{u}_{\mathbf{k}\sigma}(a_{\mathbf{k}\sigma}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}\sigma}(t)e^{-i\mathbf{k}\cdot\mathbf{x}}) \tag{29}$$

where $\mathbf{k}$ is a wave vector, $\omega_k = kc$ and $k = \langle \mathbf{k}\cdot\mathbf{k}\rangle$. The factor $\sqrt{2\pi\hbar c^2/\Omega\omega_k}$ is a normalization factor. $\mathbf{u}_{\mathbf{k}\sigma}$,$\sigma = 1,2$, are two orthogonal unit vectors. Due to the second condition of the Coulomb gauge, they must be orthogonal to the wave vector $\mathbf{k}$ which has the components $2\pi(n_x, n_y, n_z)/L$ where n_i are integers.

From (29) and (27) and with

$$a_{\mathbf{k}\sigma}(0) = \textit{ if } k_z > 0 \textit{ then } a^{(1)}_{\mathbf{k}\sigma}(0) \textit{ else } a^{(2)}_{-\mathbf{k}\sigma}(0), \text{ where } a_{\mathbf{k}\sigma}(t)e^{i\mathbf{k}\cdot\mathbf{x}} = a_{\mathbf{k}\sigma}(0)e^{-i\mathbf{k}\cdot\mathbf{x}} \tag{30}$$

we have

$$\mathbf{A}(\mathbf{x},t) = \sum_{\mathbf{k},\sigma} \sqrt{2\pi\hbar c^2/\Omega\omega_k}\,\mathbf{u}_{\mathbf{k}\sigma}[a_{\mathbf{k}\sigma}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + a^{(1)^*}_{\mathbf{k}\sigma}(t)e^{-i\mathbf{k}\cdot\mathbf{x}})] \tag{31}$$

This leads to $da_{\mathbf{k}\sigma}(t)/dt = -i\omega_t a_{\mathbf{k}\sigma}$. This equation for all wave vectors $\mathbf{k}$ and $\sigma = 1,2$ can be considered as the equation of motion of the electromagnetic field.

Now, the energy in the electromagnetic field (radiation Hamiltonian) is

$$\begin{aligned} H_{rad} = \frac{1}{8\pi}\int_\Omega d^3\mathbf{x}(E^2 + B^2) &= \int_\Omega d^3\mathbf{x}\left(\frac{1}{c^2}\left|\frac{\partial \mathbf{A}}{\partial t}\right|^2 + |\nabla\times\mathbf{A}|^2\right) \\ &= \frac{1}{2}\sum_{\mathbf{k},\sigma}\hbar\omega_k(a_{\mathbf{k}\sigma}a^*_{\mathbf{k}\sigma} + a^*_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}). \end{aligned} \tag{32}$$

With this, one can consider the EM field to be an infinite collection of harmonic oscillators. So, now we have

$$H_{rad} = \sum_{\mathbf{k},\sigma}\hbar\omega_k(\frac{1}{2} + a^*_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}) \tag{33}$$

and

$$\hbar\omega_k/2 \tag{34}$$

is the zero-point energy of an oscillator. Then, the zero-point energy of the radiation field $\sum_{\mathbf{k},\sigma}\frac{1}{2}\hbar\omega_k$ diverges as there are infinitely many oscillators. Dose this not suggest that something went wrong with the above mentioned theory of Dirac's? Fourier expansion expands a wave function to a sum of countably many wavefunctions. As we pointed out previously, the above wave number ranges over continuum but the electromagnetic waves in the nature seem to display a continuum of wavelengths. Therefore we cannot use Fourier expansion. This mathematically means that (29) is invalid. Separating continuumly infinite and countably infinite is essential in modern mathematics.

In what follows we will discuss some of the fundamental issues in modern mathematics that are essential for understanding the difference between countable infinity and continuum infinity.

Remark Countable infinity and uncountable infinity are entirely different mathematical structures. 1. The set E of all even numbers is countable, as the function $f : N \rightarrow E$ such that $f(n) = 2n$ is a bijection. Similarly the set of all odd numbers is countable. The set of all rational numbers is also countable. To show this we first remember that all rational numbers can be expressed as n/m where n and m are natural numbers and $m \neq 0$. Now we can list all rational numbers using the dove tailing enumeration method:

$$\begin{array}{cccccc} \frac{1}{1} & & \frac{1}{2} & \rightarrow & \frac{1}{3} & \\ \downarrow & \nearrow & & \swarrow & & \nearrow \\ \frac{2}{1} & & \frac{2}{2} & & \frac{2}{3} & \\ & \swarrow & & \nearrow & & \swarrow \\ \frac{3}{1} & & \frac{3}{2} & & \frac{3}{3} & \\ \downarrow & \nearrow & & \swarrow & & \nearrow \end{array} \tag{35}$$

2. Cantor hypothetically listed all elements of an open interval $(0, 1)$ as follows:

$$\begin{array}{l} 0.d_{11}d_{12}d_{13}, \ldots\ldots\ldots \\ 0.d_{21}d_{22}d_{23}\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array} \tag{36}$$

He created a new real number as $x = 0.d_1d_2d_3, ...$ such that $d_1 \neq d_{11}$, $d_2 \neq d_{22}$, $d_3 \neq d_{33}$, etc. Clearly x is in $(0, 1)$. But it cannot appear in the listing above on pain of contradiction. So Cantor rightly concluded that the set R of all real numbers is not countable. Indeed, as we discussed above, one can enumerate all rational numbers but one can *not* enumerate all irrational numbers. Indeed, we can show that almost all real numbers are irrational numbers using the Weierstrass function, which is defined over the interval $(0, 1)$ as

$$w(x) = \textbf{if } x \text{ is rational } \textbf{then } 0 \textbf{ else } 1 \tag{37}$$

The Lebesgue integral of this function over $(0, 1)$ is 1.

Remark There is one more issue that should be brought up to the attention of theoretical physics. We say a set X of real numbers is dense if for all $x, y \in X$, such that $x < y$ we can always find a real number z such that $x < z < y$. Clearly, the set N of all natural numbers and the set I of all integers are not dense. However the set Q of all rational numbers and the set R of all real numbers are dense. This is because for all rational (real) numbers $x < y, (x + y)/2$ is a rational (real number) such that $x < (x + y)/2 < y$. There is a lot more to mathematics than it just being a language for physics. The concept of density says that in between any two numbers of a dense set there are infinitely many numbers of that set. The density of rational numbers Q created the Zeno's paradox and the density of the much richer structure R, which is closed under bounded limit made it possible for Newton to resolve this paradox.

7.2.3. Annihilation and creation operators Following the steps of Gordon-Klein, Dirac further quantized the quantization of the classical radiative field by replacing the classical quantities $a_{\mathbf{k}\sigma}$ and $a^*_{\mathbf{k}\sigma}$ with self-adjoint operators. We may write $a_\sigma(\mathbf{k})$ and $a^*_\sigma(\mathbf{k})$ for $a_{\mathbf{k}\sigma}$ and $a^*_{\mathbf{k}\sigma}$. We just consider the quantum operators $a_\sigma(\mathbf{k})$ and $a^*_\sigma(\mathbf{k})$. We also assume that the operators referring to different oscillators commute, that is $[a_\sigma(k), a^*_\sigma(k')] = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\sigma\sigma'}$.

The operator $N_\sigma(\mathbf{k}) = a^*_\sigma(\mathbf{k}).\ \ a_\sigma(\mathbf{k})$ then has eigenvalues $n_\sigma(\mathbf{k})$, $n = 0, 1, 2, ...$ and eigenvectors defined as

$$a_\sigma(\mathbf{k})|n_\sigma(\mathbf{k})\rangle = \sqrt{n_\sigma(\mathbf{k})}|n_\sigma(\mathbf{k}) - 1\rangle, \quad a^*_\sigma(\mathbf{k})|n_\sigma(\mathbf{k})\rangle = \sqrt{n_\sigma(\mathbf{k})}|n_\sigma(\mathbf{k}) + 1\rangle. \tag{38}$$

Indeed,

$$|n_\sigma(\mathbf{k})\rangle = [[a^*_\sigma(\mathbf{k})]^{n_\sigma(\mathbf{k})}/\sqrt{n_\sigma(\mathbf{k})!}]|0\rangle. \tag{39}$$

The eigenvector of the radiation Hamiltonian given as equation (33) is a tensor product of such states, i.e.,

$$|\cdots n_\sigma(\mathbf{k})\cdots\rangle = \prod_{\mathbf{k},\sigma} |n_\sigma(\mathbf{k})\rangle \tag{40}$$

with the energy eigenvalues

$$E = \sum_{\mathbf{k},\sigma} \hbar\omega_k(n_\sigma(\mathbf{k}) + \frac{1}{2}). \tag{41}$$

The interpretation of these equations is a straight forward generalization from one harmonic oscillator to a superposition of independent oscillators, one for each radiation mode $(\mathbf{k}, \sigma)$. $a_\sigma(\mathbf{k})$ operating on the state (40) will render occupational numbers unchanged. Indeed, we have

$$|a_\sigma(\mathbf{k})|\cdots n_\sigma(\mathbf{k})\cdots\rangle = \sqrt{n_\sigma(\mathbf{k})}|\cdots n_\sigma(\mathbf{k}) - 1\cdots\rangle. \tag{42}$$

Correspondingly, the energy (41) is reduced by $\hbar\omega_k = hc|\mathbf{k}|$.

We interpret $a_\sigma(\mathbf{k})$ as an annihilation operator which annihilates one photon in the model $(\mathbf{k}, \sigma)$, i.e. with momentum $\hbar\mathbf{k}$, energy $\hbar\omega_k$ and linear polarization vector $\mathbf{u}_{\mathbf{k}\sigma}$. Similarly, $a^*_\sigma(\mathbf{k})$ is interpreted as a creation operator of such a photon. We have

$$|a_\sigma(\mathbf{k})|\cdots n_\sigma(\mathbf{k})\cdots\rangle = \sqrt{n_\sigma(\mathbf{k})}|\cdots n_\sigma(\mathbf{k}) + 1\cdots\rangle \tag{43}$$

The state of the lowest energy of the radiation field is the vacuum state $|0\rangle$ in which all occupational numbers $n_\sigma(k)$ are zero. In lieu of (41), this state has energy $\frac{1}{2}\sum_{\mathbf{k},\sigma}\hbar\omega_k$.

Quantum field theory works only for the systems for which the zero-point energy of the radiative field cancels out. For some cases, this infinite energy of vacuum cancels out when physically meaningful quantities are calculated. Therefore, we assume $H_{rad} = \sum_{\mathbf{k},\sigma}\hbar\omega_k a^*_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}$.

Remark Is this diverging zero-point energy as part of the formal mathematical representation of the classical electromagnetic field not an indication of the deficiency of Dirac's theory of quantization of the electromagnetic field?

The eigenvalues of this H_{rad} operator are $E = \sum_{\mathbf{k},\sigma}\hbar\omega_k n_\sigma(\mathbf{k})$. The momentum operator is $\mathbf{P} = \sum_{\mathbf{k},\sigma}\hbar\mathbf{k}(a^*_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}) = \sum_{\mathbf{k},\sigma}\hbar\mathbf{k}(N_\sigma(\mathbf{k}))$ whose eigenvalues are $\sum_{\mathbf{k},\sigma}\hbar\mathbf{k}(n_\sigma(\mathbf{k}))$.

In conclusion, the following picture of the electromagnetic field emerges: The field consists of photons, each of which has energy $\hbar\omega_k$ and momentum $\hbar$. $n_{\mathbf{k}\sigma}$ is the number of photons with momentum $\hbar\mathbf{k}$. The polarization is given by the vector $\mathbf{u}_{\mathbf{k}\sigma}$. The annihilation operator $a_{\mathbf{k}\sigma}$ decreases the number of photons with the momentum $\hbar\mathbf{k}$ by one and the creation operator $a^*_{\mathbf{k}\sigma}$ increases the number of photon with the momentum $\hbar\mathbf{k}$ by one.

7.3. Dirac's quantization of Schrödinger's wave equation (the second quantization)
Without being aware of the problems with his quantization of electromagnetic fields, Dirac went on to apply the same idea to the Schrödinger wave equations.

Consider Schrödinger's equation

$$-\frac{\hbar}{i}\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V(\mathbf{x})\Psi \tag{44}$$

for a particle in a potential $V(\mathbf{x})$. Let Ψ_n and E_n be the eigenvectors and eigenvalues of the operator $-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x})$. This is to say

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x})\right)\Psi_n = E_n\Psi_n. \tag{45}$$

The Fourier expansion of the wave function is $\Psi(\mathbf{x},t) = \sum_n b_n(t)\Psi_n(\mathbf{x})$. Substituting this into Schrödinger's equation yields

$$\frac{d}{dt}b_n = -\frac{1}{h}E_n b_n. \tag{46}$$

The expected value of energy is

$$H = \int d^3x\Psi^*(\mathbf{x},t)\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x})\right]\Psi(\mathbf{x},t). \tag{47}$$

Putting all of these equations together and, with the orthogonality of Ψ_n, we have

$$H = \sum_n E_n b_n^* b_n. \tag{48}$$

This is the Hamiltonian for a collection of harmonic oscillators with frequencies $E_n/\hbar$.

Remark Hamiltonians are not relativistic. They are classical energy operators. Schrödinger attempted to relativize the Hamiltonian in the limited context and failed. This lead to de Broglie's wave theory which in essence showed that a relativization of the Hamiltonian is untenable. De Broglie's relativistic theory showed that the phase speed of light can exceed c, thereby contradicting the most fundamental assumption of relativity.

If we consider b_n as an operator then b_n^* can be considered as the adjoint of b_n, in symbols b_n^+. Under the commuting relations

$$[b_n, b_{n'}] = [b_n^+, b_{n'}^+] = 0, \qquad [b_n, b_n^+] = 0 \tag{49}$$

from Heisenberg's equation

$$-\frac{\hbar}{i}\frac{d}{dt}b_n = [b_n, H] \tag{50}$$

we can derive the Fourier version of the Schrödinger equation.

In this way, Schrödinger's wave equation, which was obtained from the Hamiltonian, is represented by an infinite system of oscillating particles, as postulated by Dirac.

Here is a list of overall concerns regarding this apparently more successful quantization of wave equations. It lays out some differences between the quantization of EM waves, EM fields and Hamilton's energy equation of particle systems.

(i) Dirac is using the claimed empirical equivalence between Heisenberg-Jordan's quantum mechanics and Schrödinger's quantum mechanics. Putting aside the question of validity of this equivalence, it is in general a highly irregular mathematical and logical exercise to mix up two different theories under an empirical equivalence, because none of us really understands what empiricism is.

(ii) As discussed above, we do not fully understand the connection between Planck-Einstein's quantization of EM waves and Dirac's quantization of EM fields. To make matters even more convoluted, Suntola [5] rightly pointed out that if we assume that EM waves are monochromatic operators instead of harmonic oscilators, the blackbody problem does not arise. All we know now is that both of them (Planck-Einstein and Dirac) are equally wrong.

(iii) Dirac quantized Schrödinger's wave equation (which is about particle's mechanical energy). It apparently has little to do with the Planck-Einstein's quantization of EM wave equation due to Maxwell. There is not even an analogy between the two. Is it the case that they are experimentally equal for some selected experiments?

(iv) Quantum particles were introduced from various different continuum physics and thrown into the melting pot of the quantum world. QED is a mathematical theory of this melting pot in the obscurity of probability. It is highly questionable how much one should trust a mixing up of such magnitude.

Remark Despite Dirac's claim, it still is the case that we have not been shown that Schrödinger's formalism is relativistic despite that it came from de Broglie's relation, which is relativistic and invalid. Gordon-Klein's symbolic substitution of classical variables with quantum operators is not a solution at all. In the end, we know that as STR is inconsistent, it could be a good sign for Schrödinger that he failed to establish that his theory is relativistic. Nevertheless, Schrödinger's theory still used the de Broglie relation, which is invalid due to the invalidity of STR.

The operators $b_n^+ b_n$ have the eigenvalues $N_n = 0, 1, 2, ...$, indicating that any natural number of particles may occupy the eigenstate Ψ_n. The eigenvalue of H then is $E = \sum_n E_n N_n$. Dirac's theory obeys the Bose-Einstein statistics and these particles are called *bosons*. Dirac's theory excludes particles that obey the Fermi-Dirac statistics. These particles are called *fermions*. A minor change of the theory above will derive a theory of fermions. Namely, we keep the Hamiltonians as $H = \sum_n E_n b_n^* b_n$. We expect the Heisenberg equation of motion to yield $db_n/dt = -E_n b_n/\hbar$. The only change involves replacing the commuting relations

$$[b_n, b_{n'}] = [b_n^+, b_{n'}^+] = 0, \qquad [b_n, b_n^+] = 0 \tag{51}$$

with the new commuting relations

$$[b_n, b_{n'}]_+ = [b_n^+, b_{n'}^+]_+ = 0, \qquad [b_n, b_n^+]_+ = \delta_{n,n'} \tag{52}$$

where $[A, B]_+ = AB + BA$. Now we have

$$-\frac{\hbar}{i}\frac{d}{dt} b_n = [b_n, H] = \sum_m E_m \{b_n b_m^+ b_m - b_m^+ b_m b_n\} = \sum_m E_m \delta_{nm} b_m = E_n b_n. \tag{53}$$

So, we have obtained the Heisenberg equation of motion. Note that

$$(b_n^+ b_n) b_n^+ b_n = b_n^+ (1 - b_n^+ b_n) b_n = b_n^+ b_n - b_n^+ b_n b_n^+ b_n = b_n^+ b_n. \tag{54}$$

If λ is an eigenvalue of $b_n^+ b_n$ then

$$b_n^+ b_n|\lambda\rangle = \lambda|\lambda\rangle \qquad b_n^+ b_n b_n^+|\lambda\rangle = \lambda^2|\lambda\rangle = \lambda|\lambda\rangle. \tag{55}$$

Thus $\lambda^2 = \lambda$. This is to say $\lambda = 1$ or $\lambda = 0$. This means that at most one particle can occupy the eigenstate Ψ_n. We may denote this eigenstate with $|n\rangle$. This theory obeys the Fermi-Dirac statistics.

To express all of this on λ, we may write $b_n^+ b_n|N_n\rangle = N_n|N_n\rangle$ where $N_n = 0, 1$. Now we have

$$b_n^+ b_n b_n^+|N_n\rangle = b_n^+(1 - b_n b_n^+)|N_n\rangle = (1 - N_n)b_n^+|N_n\rangle. \tag{56}$$

This implies that $b_n^+|N_n\rangle$ is an eigenvector of $b_n^+ b_n$ with the eigenvalue $1 - N_n$. It can only differ from $|1 - N_n\rangle$ by a constant. We write $b_n^+|N_n\rangle = C_n|1 - N_n\rangle$. The constant C_n can be evaluated by taking the inner product of $b_n^+|N_n\rangle$ with itself.

$$\langle b_n^+|N_n\rangle, b_n^+|N_n\rangle\rangle = (1 - N_n) = C_n^* C_n. \tag{57}$$

Thus we have $C_n = \theta_n\sqrt{1 - N_n}$ where θ_n is the phase factor. This leads to

$$b_n^+|N_n\rangle = \theta_n\sqrt{1 - N_n}|1 - N_n\rangle \qquad b_n|N_n\rangle = \theta_n\sqrt{N_n}|1 - N_n\rangle. \tag{58}$$

In summary, we have

(i) For bosons:

$$\begin{aligned} b_n|\cdots, N_n, \cdots\rangle &= \sqrt{N_n}|\cdots, N_n - 1, \cdots\rangle \\ b_n^+|\cdots, N_n, \cdots\rangle &= \sqrt{N_n}|\cdots, N_n + 1, \cdots\rangle \end{aligned} \tag{59}$$

(ii) For fermions:

$$\begin{aligned} b_n|\cdots, N_n, \cdots\rangle &= \theta_n\sqrt{N_n}|\cdots, 1 - N_n, \cdots\rangle \\ b_n^+|\cdots, N_n, \cdots\rangle &= \theta_n\sqrt{1 - N_n}|\cdots, 1 - N_n, \cdots\rangle \end{aligned} \tag{60}$$

where $N_n = 0, 1$. In both cases, b_n is an annihilation operator and b_n^+ is a creation operator.

Some questions arise with regard to the above. Namely,

(i) Why did Dirac perform the second quantization of Schrödinger's wave equations? Why did not he start directly with Hamiltonians? Was it because Hamiltonians are just classical equations of energy? Did he think that Schrödinger's wave equations are relativistic?

(ii) Dirac likely wanted to quantize energy fields in general as he did to electromagnetic fields so that the same theoretical frame would apply to energies in general. He thought that waves are fields (mediums). There is a vicious circularity in his reasoning. Waves assume mediums but not *vice versa.*

7.4. Interactions of quantum particles

One can add the Hamiltonians for several free particle fields and introduce appropriate interaction terms to study interacting particle fields. The most common such interaction is that of photons with charged particles. We use the theory of second quantization to represent a charged particle field by the following Hamiltonian:

$$\int d^3x\Psi^+(\mathbf{x}, t)\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\Psi(\mathbf{x}, t). \tag{61}$$

The quantized electromagnetic field is represented by the following radiation (photon) Hamiltonian:

$$\int d^3x \frac{1}{8\pi}(E^2 + B^2). \tag{62}$$

The interaction of these two fields will be obtained by adding these two Hamiltonians and prescribing the following replacement:

$$\frac{\hbar}{i}\nabla \Longrightarrow \frac{\hbar}{i}\nabla - \frac{e}{c}\mathbf{A}(\mathbf{x}). \tag{63}$$

This leads to

$$\begin{aligned} H = \int d^3\mathbf{x}\Psi^+(\mathbf{x},t)\left[-\frac{\hbar^2}{2m}\left|\frac{\hbar}{i}\nabla - \frac{e}{c}\mathbf{A}(\mathbf{x})\right|^2 + V\right]\Psi(\mathbf{x},t) \\ + \int d^3x\frac{1}{8\pi}(E^2+B^2) = H_P + H_{rad} + H_I \end{aligned} \tag{64}$$

where

$$\int d^3x\Psi^+(\mathbf{x},t)\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\Psi(\mathbf{x},t) = \sum_n E_n b_n^+ b_n \tag{65}$$

is the particle Hamiltonian,

$$H_{rad} = \int d^3\mathbf{x}\frac{1}{8\pi}(E^2+B^2) = \sum_{k,\sigma}\hbar\omega_k a^+_{\mathbf{k},\sigma}a_{\mathbf{k},\sigma} \tag{66}$$

is the Hamiltonian for the radiation field, and

$$H_I = \int d^3x\Psi^+(\mathbf{x},t)\left[-\frac{\hbar^2}{imc}\mathbf{A}\cdot\nabla^2 + \frac{e^2}{2mc^2}A\right]\Psi(\mathbf{x},t) \tag{67}$$

is the interaction Hamiltonian. We can divide H_I into a part H' proportional to A and a part A' proportional to A^2 such that $H_I = H' + H''$. Expanding A and Ψ in terms of $a_{\mathbf{k},\sigma}$ and b_n gives

$$H' = \sum_{\mathbf{k},\sigma}\sum_n\sum_{n'}\left[M(\mathbf{k},\sigma,n,n')b_n^+b_{n'}a_{\mathbf{k},\sigma} + M(-\mathbf{k},\sigma,n,n')b_n^+b_{n'}a^+_{\mathbf{k},\sigma}\right] \tag{68}$$

and

$$\begin{aligned} H'' = \sum_{\mathbf{k}_1,\sigma_1}\sum_{\mathbf{k}_2,\sigma_2}\sum_n\sum_{n'} & M(\mathbf{k}_1,\sigma,\mathbf{k}_2,\sigma_2,n,n')a_{\mathbf{k}_1,\sigma_1}a_{\mathbf{k}_2,\sigma_2} \\ & + M(\mathbf{k}_1,\sigma_1,-\mathbf{k}_2,\sigma_2,n,n')a_{\mathbf{k}_1,\sigma_1}a^+_{\mathbf{k}_2,\sigma_2} \\ & + M(-\mathbf{k}_1,\sigma_1,\mathbf{k}_2,\sigma_2,n,n')a^+_{\mathbf{k}_1,\sigma_1}a_{\mathbf{k}_2,\sigma_2} \\ & + M(-\mathbf{k}_1,\sigma_1,-\mathbf{k}_2,\sigma_2,n,n')a^+_{\mathbf{k}_1,\sigma_1}a^+_{\mathbf{k}_2,\sigma_2} \end{aligned} \tag{69}$$

where

$$M(\mathbf{k},\sigma,n,n') = \sqrt{\frac{2\pi\hbar c^2}{\Omega\omega_k}\int d^3\mathbf{x}\Psi_n^*\left[-\frac{e\hbar}{imc}e^{i\mathbf{k}\cdot\mathbf{x}}\mathbf{u}_{k,\sigma}\cdot\nabla\right]}\Psi_{n'} \tag{70}$$

and

$$M(\mathbf{k}_1, \sigma, \mathbf{k}_2, \sigma_2, n, n') = \sqrt{\frac{2\pi\hbar c^2}{\Omega\omega_k}} \sqrt{\frac{1}{\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_2}}} \int d^3\mathbf{x}\Psi_n^* \left[-\frac{e\hbar}{2mc^2} e^{i\mathbf{k}\cdot\mathbf{x}} \mathbf{u}_{\mathbf{k}_1,\sigma_1} \cdot \mathbf{u}_{\mathbf{k}_2,\sigma_2} e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{x}} \right] \Psi_n. \tag{71}$$

The part of the Hamiltonian $H_p + H_{rad}$ can be considered the unperturbed part with eigenvectors and eigenvalues

$$|\cdots N_n \cdots\rangle_p \, |\cdots n_{\mathbf{k},\sigma} \cdots\rangle_{rad}\,, \quad \sum_n E_n N_n + \sum_{\mathbf{k},\sigma} \hbar\omega_{\mathbf{k}} n_{\mathbf{k}\sigma} \tag{72}$$

respectively.

The interaction Hamiltonian H_I induces transitions between these states as follows:

(i) The term $b_n^+ b_{n'} a_{\mathbf{k},\sigma}$ in $H^{'}$: (1) annihilates a photon of momentum $\hbar\mathbf{k}$ and polarization σ, (2) annihilates a particle in state $|n'\rangle$, (3) creates a particle in state $|n\rangle$.

(ii) The term $b_n^+ b_{n'} a_{\mathbf{k},\sigma}$ in $H^{'}$: (1) creates a particle in state $|n\rangle$, (2) annihilates a particle in state $|n'\rangle$, (3) annihilates a photon of momentum $\hbar\mathbf{k}$ and polarization σ.

(iii) The term $b_n^+ b_{n'} a_{\mathbf{k},\sigma}$ in $H^{''}$: (1) creates a particle in state $|n'\rangle$, (2) annihilates a particle in state $|n\rangle$, (3) annihilates a photon of momentum $\hbar\mathbf{k}_1$ and polarization σ_1, (4) annihilates a photon of momentum $\hbar\mathbf{k}_2$ and polarization σ_2.

(iv) The term $b_n^+ b_{n'} M a_{\mathbf{k}_1,\sigma_1} a^+_{\mathbf{k}_2,\sigma_2}$ in $H^{''}$: (1) creates a particle in state $|n\rangle$, (2) annihilates a particle in state $|n'\rangle$, (3) annihilates a photon of momentum $\hbar\mathbf{k}_1$ and polarization σ_1, (4) creates a photon of momentum $\hbar\mathbf{k}_2$ and polarization σ_2.

(v) The term $b_n^+ b_{n'} M a^+_{\mathbf{k}_1,\sigma_1} a_{\mathbf{k}_2,\sigma_2}$ in $H^{''}$: (1) creates a particle in state $|n\rangle$, (2) annihilates a particle in state $|n'\rangle$, (3) creates a photon of momentum $\hbar\mathbf{k}_1$ and polarization σ_1, (4) annihilates a photon of momentum $\hbar\mathbf{k}_2$ and polarization σ_2.

(vi) The term $b_n^+ b_{n'} M a^+_{\mathbf{k}_1,\sigma_1} a_{\mathbf{k}_2,\sigma_2}$ in $H^{''}$: (1) creates a particle in state $|n\rangle$,(2) annihilates a particle in state $|n'\rangle$, (3) creates a photon of momentum $\hbar\mathbf{k}_1$ and polarization σ_1, (4) creates a photon of momentum $\hbar\mathbf{k}_2$ and polarization σ_2.

Remark As discussed above, all these results suffer from serious ontological and theoretical setbacks. To begin with, none of what Dirac called particles have much to do with particles. They are just waves. Waves and particles are entirely different categories and they cannot be unified by the wrong application of Fourier transformations. Fourier transformation never reduces waves to particles. It reduces a wave into a system of waves. So, Dirac's claim of a duality between a wave and a particle is a fallacy. Secondly, even in terms of energy, there is no ground to the claim that the energy of the particle dual to a component wave of the Fourier expansion is the kinetic energy of the particle as it is not a particle. A particle's kinetic energy is the work needed to accelerate the particle from speed 0 to speed v. How can a wave accelerate?

Remark Indeed, classical physics, namely wave mechanics and classical particle dynamics, never agreed that they are equivalent or in a dual relation. Both fields are legitimate (way more so than any modern relativistic physics) fields and are both supported by experimental physics and engineering. Yet they both agree that they cannot find much of a relationship between mechanics and wave mechanics.

Remark A similar pragmatic situation exists in pure mathematics too. Pure mathematicians do not believe that the proposed axiomatic set theory ZFC can be shown to be consistent. Nearly a century long attempt to prove this theory consistent failed.

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Remark Particles have both energy and momentum. The two are of different category: energy is modality and momentum is physical reality. But waves have not momentum as no mass moves in the direction of the wave's motion. This by itself tells us that the dream of a wave-particle duality is untenable in theoretical physics. Dirac seems to have believed in the duality of waves and particles, which is based upon the energy-momentum relation of Einstein. As we discussed in the foregoing, energy is not a legitimate concept for physics. It is a modality as is defined as the *potential* to do work.

7.5. Did Dirac really established the wave-particle duality?

Upon a careful study of Fourier analysis, one will realize that it does not support Dirac's claim that there exists a duality between a particle and a wave. Each term in the Fourier expansion is not a particle. It is another wave. Fourier analysis is a theory of decomposition of a wave function into wave functions, but not into particles. Dirac picked the useful component waves and called them particles without making it clear which waves are particles. In this sense Dirac's argument is viciously circular.

Clearly, particles are local entities that do not form a dense structure such that there is another particle between any two particles. Because of this, particles do not form a continuum. A continuum is dense, and waves possess a continuum structure. This problem is closely connected with what mathematical logic calls *type lowering* which was studied by Gödel as Gödelization (Gödel numbering) in the context of recursive function theory. The idea is to represent all computable functions (partial recursive functions) of natural numbers as natural numbers and it worked well to bring out a workable theory of computation over natural numbers. This was a first mathematical model of what we now call computers.

However, the work of Gödel showed a tight limitation of this idea. Given two partial recursive (computable) functions of natural numbers, say $f1$ and $f2$, from the Gödel numbers of $f1$ and $f2$, say $n1$ and $n2$, one cannot decide if $f1$ and $f2$ compute the same function or not. This means that by representing a computable function of natural numbers as a natural number (Gödel number), we loose some vital information about the function n. Of course, this is totally expected as Cantor's set theory tells us that the structure of the space $[X \to Y]$ of functions from set X to set Y is way more complicated and way bigger than X and Y. Philosophical logicians say that the type of $[X \to Y]$ is higher than that of X or Y.

Going back to the work of Dirac, his idea was to consider the lower type objects of particles representing higher type objects of waves as waves, which appear in the Fourier expansion, of the original higher type wave. Such an attempt inevitably leads to either a vicious circle or a contradiction. Gödel's discovery told us that this is untenable because we are violating the type hierarchy.

The above establishes that, contrary to what Dirac did, one cannot derive particles as waves on pain of vicious circularity. Putting aside this general philosophical argument, let us consider the internal coherence of Dirac's theory of quantum physics. The process of quantizing the electromagnetic wave does not involve Dirac's quantization. It is just a curve fitting hypothesis of Planck upgraded to the wave-particle duality. Moreover, Dirac's quantization of the EM field and his quantization of Schrödinger's wave equation are entirely different theories. The former has nothing to do with quantization of waves. The latter is a quantization of Schrödinger's waves. Each of these three developments suffer from problems of their own and there is no unification of the theory of particles involved in this vast development.

8. Quantum field theory

The reason why we spent a considerably long time on analyzing QED is because of the current trend of applying the quantization of the electromagnetic field to the gravitational field, the so called *quantum gravity*. However, contrary to the common belief, quantum gravity is just a special example of a more general category called *quantum field theory*. This is because gravity is treated as a force field instead of an action at a distance force.

The generalization of QED into quantum field theory (QFT) started ontologically through high energy particle collision experiments using particle detectors. The theory of quantum mechanics predicts that there should be no trajectories of particles in any experiment because of the momentum-position uncertainty. This uncertainty asserts that once we localize a particle we will not be able to find the next location the particle will appear on. However, particle experiments always show the trajectories of the particles in the particle detection chamber such as the Wilson chamber. In short, all quantum field theories are experimentally refuted.

Nonetheless, one of the major generalizations of QED in QFT is the theory of non-Abelian gauge theories (Young-Mills theories). In QED charged particles interact through the exchange of photons and in no-Abelian gauge theory, particles interact via the exchange of massless gauge bosons. Although photons do not carry charges, non-Abelian gauge bosons carry charges. That gauge bosons are mass-less indicates, again, that they are not real, like photons, on pain of contradiction.

There are more irregularities associated with the development of quantum field theory. As a quantum theory that is relativistic, QFT naturally rejected the action-reaction law (the concept of reference frames violates action-reaction law). It was Yukawa who explicated the π-meson connecting a proton and a neutron using the action-reaction law.

Among the subjects of this vast area of quantum field theory, a most controversial one is the theory of quantum gravity, to which we will turn in what follows.

9. Quantum gravity: theory and history

9.1. Classical field theory

Gravitational wave theory was started as an analogy to the EM wave theory of Maxwell. Research in neither field though realized that a gravitational (or electric) force field, which is a spatial distribution of gravitational force (or electric force) per unit mass (or charge), is not a physical entity. It is a counter-factual modality. As such it is not a physical entity like e.g. water for water waves. For example in *The secret history of gravitational waves* (American Scientist March-April, 2018) Tony Rothman wrote

> $\cdots$ a [force] field is a continuously varying plane of action through which disturbances propagate, eliminating the conceptual knot of action at a distance. Today no one doubts the reality of [force] fields, anyone who has sprinkled iron filings on a piece of paper above a bar magnet has perceived a field pretty directly. Back then, the existence of [force] field was less obvious.

Modal logic is a branch of logic that studies the logic of necessity and possibility. Now, it is a most important branch of logic. Logicians understand that a force field is nothing but a special case of a counter-factual modality. Rothman's description of the force field as a physical reality is incorrect. It is impossible to physically distribute force per unit charge (or mass) all over the continuum space. One cannot even directly see the force exerted at a position in a space. How can one see the force field? As regards the example of the magnetic force line around a magnetic dipole using iron powder sprayed around the magnetic dipole, the observed pattern is not the pattern of the magnetic field as the interaction between the sprayed iron powder and the magnetic force changes the magnetic field. This is a good example of what Goethe called *impossibility of measurement.* In contemporary physics it is called the uncertainty principle. To be precise, in between the iron particles, we have space where the magnetic field still exists. This space is not the same as it was before the iron particle was sprayed. We do not see this in the experiment. Moreover, we do not see what is happening inside the metal particle. It is not a point. It is a spherical entity.

Physical continuum is different from mathematical continuum. Mathematical continuum is dense: in between two points, there are infinitely many other points. Moreover, mathematical continuum is closed under the limit. It means that for each sequence of elements, there is an element which is the limit of the sequence. This structure can never be realized by using particles! Furthermore, when we place charges on every point of the field space, this certainly changes the field itself. So, we cannot observe the field structure in the way described in the *Scientific American* article. This is the same problem as the uncertainty of measurement in quantum mechanics.

The continuum is a mathematical metaphysics that is needed to provide calculus, which is a basic metaphysics for the theory of motion. The reason why Newton separated calculus and dynamics is simple. Newton namely understood that it does not help to complicate the issue one wants to understand. The fact that real wave mechanics relies upon the wave medium is not an advantage of this discipline over particle mechanics. It is a disadvantage which brought a mathematical and conceptual nightmare to wave mechanics. Philosophically, it is much more desirable to consider just the action (change) at a distance than the global spatial change due to the local change. The simpler the better. It appears that Feynman is one of very few who took this to heart. He rejected the force field in his QED. However, he still failed to recognize that then the EM waves are not waves.

A natural consequence of not knowing what EM waves and gravitational waves are is that we might have some difficulty in explaining why EM waves get “absorbed” as they travel through matter while gravitational waves do not.

There is no physical medium for gravity, is there? According to the tradition of empiricism, there is a medium through which electromagnetic force and electromagnetic waves travel. Empiricism is not interested in why there are the parameters ε and μ for such mediums. Empiricism says that these numbers are obtained from experiments for each medium. It was Russell who warned physics that empiricism is a vicious circle: to experimentally verify a theory, we use the theory to be verified to make the apparatus for the experiment.

Empiricism appears not to be concerned with that the EM medium (except vacuum) is nothing but a massive sea of charged particles, which goes back to the Greek philosophy of atomism. This is why the EM waves are affected by the medium. These tiny particles are strong enough in charge to affect the so-called EM waves. So, to break this vicious circularity, what is missing here is a convincing scientific explanation of how the sea of particles forming the medium for EM phenomena yields ε and μ. We have not seen it.

Certainly the same thing happens to the gravitational forces (gravitons). But due to the for tiny mass of a particle consisting what we call mass, this effect of gravity upon the gravitational wave (graviton) is negligible. This is why the gravity wave does not get absorbed by the massive body.

As Newton already pointed out, if we want to maintain our sanity, we must reduce mass and charge to point objects to avoid this problem. Then there is no interference between mass (charge) and gravitational (electromagnetic) waves. By all of this we want to say that in correct dynamics, there are only point masses or point charges. According to the theory, there is no such thing as objects (bodies) through which EM waves or gravitational waves travel.

In this section, we discussed shortly the issues and the solutions to the early stage gravitational wave theory initiated by Heaviside and further developed by such researchers as Poincaré and Einstein. Though very short, we summed up the issues (and confusions) in the discussions by these early pioneers of the field. We wish theoretical physics had more concern for the difference between what constitutes physical reality and what is a modality.

Remark Notwithstanding, history of physics shows some serious negative consequences of using this misunderstood concept of the force field. It was just recently that we managed to point out that the Lorentz force used in Maxwell's EM field theory is in conflict with the second law of dynamics, which is the most fundamental axiom of dynamics. As this force appears in the context of the magnetic field, it was overlooked in physics.

9.2. Early works on gravitational waves

Using the analogy between Newton's law of gravity and Coulomb's law of electromagnetic force, Heaviside suggested the possibility of gravitational waves.

Poincaré, inspired by the Lorentz transformation from the classical theory of electromagnetism, postulated that one cannot consider a body that moves with a speed faster than c. Due to the algebraic analogy between Coulomb's law and Newton's law, Poincaré proposed that the acceleration of a mass should produce gravitational waves as that of a charge produces electromagnetic waves.

One interesting question is this: What if we closed our eyes to the invalidity of Einstein's STR dynamics, which considered acceleration in the frame of STR (while STR is limited to only kinematics for the reason of otherwise violating the principle of relativity), and studied the gravitational waves in STR dynamics? For some reason, Einstein did not seem to have worked on this problem. As we will discuss in the next section, Einstein went straight into the gravitational wave theory within his general theory of relativity. It is plausible that he did not see a need for the gravitational wave theory within the context of STR dynamics as he was developing the general theory of relativity, which is supposed to be a relativistic theory of the generalized gravitational field.

9.3. General relativistic gravitational wave theory

Einstein pointed out that the analogy between Coulomb and Newton breaks down at the dipole. Unlike the magnetic dipole, there is no gravitational dipole. Einstein explored the idea and came up with the proposal of three different kinds of gravitational waves, namely, longitudinal-longitudinal, transverse-longitudinal, and transverse-transverse.

Remark The most fundamental reason why Newtonian dynamics and Coulomb's dynamics diverge is the so-called generalized Ampère's law in the latter which connects the world of the electric and the magnetic field. A most fundamental difference between the two is that the former is logically consistent while the latter is logically inconsistent, yielding the Lorentz force which contradicts the second law of Newton. As long as this connection is not there, there isn't much difference between the electric world and the magnetic world.

Eddington showed that the first two types of the waves proposed by Einstein can propagate at any speed by choosing an appropriate coordinate system. He also showed that the third type of waves always propagates with speed c regardless of the choice of the coordinate system.

Einstein and Rosen discovered that in full generality of the general theory of relativity, solutions of gravitational wave equations have a singularity. Robertson argued that such a singularity comes from an element of the theory that bears no relevance to physicality.

In 1956, Pirani pointed out that all of the confusion associated with the relativistic gravitational wave theory coming from the choice of coordinate systems can be resolved by reformulating the gravitational waves in terms of manifestly observable Riemannian curvature tensor. This idea is unfortunately based on the assumption that Einstein's general theory of relativity, which reduces the effect of gravitation to Riemann's curvature tensor over spacetime, is valid.

The year after, Feynman presented a thought experiment that suggests that the gravitational wave carries energy by generating heat.

Remark Feynman rightly questioned the validity of force field as it creates a diverging energy at the location where a charge is. Yet, unfortunately he was using the concept of heat, of which nature we do not have a coherent idea. Thermodynamics is a seriously complex and challenging branch of theoretical physics that is based upon statistical methods, which themselves complicate the situation significantly. Statistical method is a serious question in thermodynamics.

9.4. Experimental results
Here is a short history of the experimental results on gravitational wave research.

(i) In 1969 and 1970, Joseph Weber built the world first gravitational wave detector and detected gravitational waves from the Milky Way's galactic center. This result predicted that the life of our galaxy is to be much shorter than its inferred age.
(ii) In 1974, Hulse-Taylor discovered the first binary pulsar. Their pulsar showed a gradual decay of the orbital period which agreed with the momentum and energy loss in the gravitational radiation as predicted by the GTR.
(iii) Despite the apparent setback of Weber's usage of his gravitational wave detector, further development was conducted along this line. In the 1970's, Forward and Weiss made the first notable detector which was followed by the construction of CEO600, LIGO and Virgo.
(iv) In 2015, LIGO made the first detection of gravitational waves. It was inferred that the signal, dubbed GW150914, was coming from the merger of two black holes with masses $36^{+5}_{-4}M\odot$ and $29^{+4}_{-4}M\odot$ resulting in a $62^{+4}_{-4}M\odot$ black hole. LIGO concluded that the detected gravitational wave carried roughly three solar masses or about 5×10^{47}joules.

As Hulse-Taylor's result suggests, the gravitational waves are to carry momentum and energy. Classical wave dynamics asserts that waves do not carry any momentum as the motion of the wave is nothing but the motion of local vibrations. Momentum of a motion comes from the motion of real (not hypothetical) masses. Moreover, as we have discussed in the foregoing, the concept of energy is invalid. The work needed to accelerate from $m0$ to mv is not always mv^2. It depends upon the way we accelerate.

There seems to be some confusion about detecting gravitational vibration and gravitational wave. In the case of the EM wave, we have never observed EM waves in the way we observe water waves or string waves. All we observed at our location is the vibration of the strength of EM force from a distance, caused by the vibration of the source, which is a charge.

In the case of classical waves, vibration at the source, i.e. oscillator, creates the wave in the surrounding medium and the wave travels through the medium. Despite some fundamental mathematical and categorical differences that make this theory more difficult, this theory is as scientifically legitimate as particle dynamics. This is what is know as classical wave mechanics.

However, it is important to understand that what should be called *radiative waves* are not waves in this wave mechanical sense as there is no medium which would facilitate the wave motion. The effect of the transmission of the vibration at the source is received at the receiving end through an action at a distance mechanism and these two waves are of entirely different categories.

10. Theory of quantum gravity: general review

10.1. EM waves versus gravitational waves

10.1.1. EM waves are not physical but modal waves In Maxwell's EM field theory, the two EM wave equations were obtained simultaneously as follows:

$$\frac{\partial}{\partial t}\nabla \times \mathbf{H} = -\frac{1}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2} = -c\nabla \times \nabla \times \mathbf{E}\frac{\partial^2 \mathbf{E}}{\partial t^2} \Longrightarrow \nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{73}$$

Similarly we have

$$\nabla^2 \mathbf{H} = \frac{1}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2}. \tag{74}$$

In Maxwell's EM theory, the interaction between **E** and **H** in general came from the current vector **J** as the generalized Ampère's law:

$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}. \tag{75}$$

Maxwell's EM wave equation is derived under the assumption that $\mathbf{J} = 0$. This equation shows that the speed of EM wave in vacuum is c. However, according to this theory, the speed of EM wave is c only when the wave is created in vacuum without current.

When given basic calculational rules and Maxwell's EM field axioms, it is mathematical trivia to derive the EM wave equations. In the process of doing so, no physics is involved either as **E** and **H** are not physical entities. They are modal entities. Electric and magnetic wave equations are not equations of physical reality. Instead they are wave equations of counter-factual modality.

Maxwell later showed that accelerating charges generate electromagnetic waves. However such charges create current **J** and then, according to Maxwell, there is no reason to think that the EM waves created in this way move with speed c. So there is something bizarre about Maxwell's theory of EM waves.

Remark Radio engineers say that when we have a closed circuit, it generates an EM wave whose frequency is one cycle of the current.

In general, using any function of space and time, one can define a function that represents a wave in the space-time. This is mathematical trivia. Waves can liberally appear outside of physics. This simply means that the concept of waves transcends physics. Physical waves are very special, limited waves. EM waves are good examples of waves that are not physical but modal waves as the EM fields are not physical entities. They are counter-factual modal waves.

10.1.2. Gravitational waves of Heaviside-Poincaré In the case of gravitational waves, we do not see the claimed analogy to EM waves. There is only one gravitational field **G**. It seems that the only analogy comes from that acceleration creates modal waves. But this analogy is very weak. For example, there is no such thing as the continuum flow (current) of masses in dynamics. It exists in fluid dynamics but fluid dynamics and dynamics are of entirely different categories. They are entirely different theories. Moreover the claimed speed **c** of the gravitational waves comes not from the nature of the mass but from the Lorentz transformation, which has no relevance to physics as it causes contradictions. This makes the entire project of gravitational wave theory highly questionable. One cannot build a theory by just relying on weak and shaky analogies to some other theory that itself is already questionable.

If one wants to develop a STR version of the gravitational wave theory, at least one issue must not be ignored. Namely, STR rejects gravitation as gravitation is acceleration and thus violates the principle of relativity. However, this fundamental principle is being ignored while discussing the STR dynamics.

Before the authors presented in [2] the PPP (power-line power pole paradox), discussion on the inconsistency of STR kinematics was limited only to the twin paradox, which was completely off the point. The twin paradox involves acceleration, which is excluded in the special theory of relativity kinematics. So, it has nothing to do with STR kinematics.

10.2. Feynman's QED and gravitational wave theory

Considering that Feynman went back to the action-at-a-distance electrodynamics, it is surprising that the possibility of an action at a distance gravitational wave theory was seemingly never considered. This might open up a door to an alternative theory of gravitational waves and its quantization.

Mathematics does not deal with time as its natural flow is beyond the domain of mathematics. In physics though time is a reality that has to be dealt with. Although time describes changes like speed, Einstein and Lorentz made time change with respect to speed. This appears to be a wrong direction to take. Dirac's reversible time, as in QED, is yet another problematic issue. As all physical evolution takes place in terms of time, time cannot be changed by physical processes.

Remark Physics claims that, despite all of the criticism, the experimental physics empirically verifies the claims made by theoretical physics, such as quantum physics and relativity theory. Due to the technical difficulty and high cost, experimental verification of the theory of relativity, which is logically invalid, is impossible to carry out. Very few attempts to verify this theory experimentally were made at s great public cost and nothing is has been verified in the process. When it comes to quantum physics, particles are too small to be picked up and measured in the way we perform measurements in classical physics. All we do is to use trajectories that particles left, which are not supposed to be there in lieu of the uncertainty principle, and interpret the experiment by applying theoretical formulas. Of course, the applied formulas are always verified. This is why QED is a most experimentally verified theory.

10.3. Gravitational wave theory based on the general theory of relativity

10.3.1. Principle of equivalence Einstein thought that if an accelerating reference frame can be reduced to an inertial frame, in which acceleration induces a gravitational field, then it is possible to treat accelerating frames as inertial frames inside the theory of relativity which rejects reference frames that are under acceleration and does so for a legitimate reason. He called this the *principle of equivalence*.

This argument is logically flawed. As we showed in Section 4, one cannot even move a single reference frame on pain of the power-pole power-line paradox (PPP paradox).

Let's consider Dingle's clock paradox. Dingle was aware that STR cannot involve acceleration. Hence, he considered two already synchronized clocks moving towards each other with constant speed and posed the question whether each clock sees the other moving slower. Physics responded by using a paradox that involves the forbidden acceleration: it was argued that if a twin takes off and comes back, this twin is the one who is younger. However, for a twin to depart and come back, he has to accelerate. So, this is not the issue that Dingle was discussing. We presented a closely related, correct example to counter this. Namely, even if we admit acceleration, we still have a problem. Assume two twins take off and come back and meet again completely symmetrically in route and time, do they find each other younger?

Without realizing this fundamental problem with the idea of relatively moving reference frames, Einstein went on to build his special theory of relativity in which two reference frames move relative to each other with constant speed. The reason for this constant speed restriction by Einstein was because the concept of mutually accelerating reference frames contradicts the third law of dynamics.

Without being aware of this problem and, more fundamentally, of PPP, Einstein re-introduced accelerating frames as inertial frames with induced gravitational field in the following way: Assume a spaceship is in inertial motion in our reference frame. Furthermore, a force accelerates this spaceship with rate α. A body m in the spaceship experiences a force $\mathbf{f}$ which is due to the acceleration of the spaceship which makes the body m move with an acceleration of rate $\mathbf{a}$ in the frame of the spaceship. Putting aside what the force $\mathbf{f}$ is, this means $\mathbf{f} = m\mathbf{a}$. Then from our perspective, m in the space ship experiences the acceleration with rate $\alpha + \mathbf{a}$. So, m will experience $\mathbf{f} = m(\alpha + \mathbf{a})$. Therefore,

$$\mathbf{f} - m\alpha = m\mathbf{a} \tag{76}$$

This means that from our perspective the acceleration α on the spaceship induces an additional force $-m\alpha$ on m, which he called inertial force, upon the mass m and the equation (76) yields the force m experiences in the accelerating spaceship. This Einstein referred to as the second law in the accelerating frame of the spaceship. According to Einstein, upon the modification of $\mathbf{f}$ to $\mathbf{f} - m\alpha$, the second law is conserved under the choice of accelerating reference frames.

Einstein's argument is however irrelevant as the underlying inertial reference frame is invalid on pain of PPP. The PPP paradox rejects even the inertial frames, thereby upending any hopes for the theory of relativity. Below we list some of the issues that need to be discussed in relation to this. In [2], we presented a thorough analysis of the above mentioned idea of Einstein. Here we will present a short version of it:

(i) According to the special theory of relativity, relativistic addition of speeds $v \oplus v'$ is not classical addition. So, how can the addition of accelerations be the same as classical addition?

(ii) The inertial force is also closely bound up with the issue of *fictitious force* on a mass inside an orbiting object. Fictitious force is not reality. The reason why we have a problem with the fictitious force for an orbiting spaceship is because an orbiting spaceship is under centripetal acceleration. Hence, it is not an inertial frame. The creation of fictitious force called centrifugal force does not make the orbiting spaceship an inertial reference frame.

Even more fundamentally, in the theory of dynamics of Newton theoretically there is no such thing as a spaceship (or a train). *All physical bodies must be reduced to point masses as otherwise, we cannot define their motion, nor can we apply a force (which is a pointed arrow) upon them.* For this reason, in physics, there is no such thing as a body inside a spaceship.

Furthermore, regarding the gravitational field that Einstein introduced to an accelerating space, the concept of the force field, in general, violates the action-reaction law and, in turn, the principle of relativity. Moreover, such a gravitational field has no source for the gravitational forces spreading all over the space. This is yet another violation of the third law, albeit in a different sense.

All of this means that it is impossible to have a consistent theory of accelerating reference frames. This however is not surprising. In the above and in our other writings, we already showed that, regardless of whether the frame is inertial or accelerating, a reference frame that moves another frame immediately leads to the power-line-power-pole paradox (PPP). This means that the conceptual setting of relatively moving reference frames is untenable.

10.3.2. Locality of the principle of equivalence and the general space-time coordinate system
The equivalence principle appears to apply to only local scenarios such as spaceships or trains as accelerating frames. This is obviously not prescribed by the theory of relativity. If the spaceship, as discussed above, is a reference frame then the reference frame of the spaceship itself, in which the spaceship is stationary, must be the reference frame of the spaceship.

Unaware of the problem that the equivalence principle is false, Einstein made a move to represent all accelerating reference frames as local inertial frames in which acceleration is replaced by the induced gravitational field. What is missing in this picture is that, as we mentioned above, this gravitational field spreads all over the universe. It is not just a local field. Nevertheless, this wrong idea lead Einstein to consider the general coordinate system upon which all accelerating frames are treated as local inertial frames with an induced gravitational field. Henceforth, Einstein moved on to develop the concept of general absolute reference frames to which we will turn in what follows.

Remark In dynamics, we do not consider a space like a spaceship or a train. Newton told us that we have no mathematical way to represent such an object. We have no mathematical way to describe the motion of such an object. This is why Newton reduced such objects to point masses.

For considering the cosmos with time as a most fundamental coordinate, Einstein assumed that the whole cosmos is occupied by a fluid whose molecules are clocks of any variety. This fluid can flow freely, provided that there is no turbulence, so that neighboring molecules always have an almost equal speed and the velocity of the flow is a continuous function.

Remark This means that Einstein assumed a universal time and a universal space in which these clocks move. So, this sea of clocks does not define universal time and universal space. To the contrary, the existence of such a sea of clocks assumes the existence of a universal time and universal space. We have come around a big circle.

Remark There is one more serious conceptual issue that is deeply associated with Einstein's view of universal spacetime. The infinitely many clocks that Einstein used to fill the absolute space with also have their own internal time and space in order to be able to function as clocks. Einstein's theory of time and space depends upon this hidden time and space.

Einstein proceed with his description of universal space-time as follows: Each clock is allocated three coordinates (x_1, x_2, x_3) in such a manner that:

(i) No two clocks will have the same coordinates.
(ii) Neighboring clocks have neighboring coordinates, therefore, coordinates are also continuous with respect to spatial displacement.
(iii) The coordinates of each clock remain the same through time. As time elapses at each clock, its readings assumed to increase but the rate of increase is not necessarily uniform as compared with a local standard clock. No attempt is made to synchronize distant clocks; neighboring clocks are assumed to be sufficiently synchronized so that the clocks' readings are continuous with respect to spatial displacement.
(iv) The reading of a clock will be denoted by x_0.

It is unfortunate that this paradigm is not possible for the reasons outlined below:

a) No clock has any specific coordinate as it is not a point object. Newton reduced a physical mass to a point mass to give a specific position in the space at each time. This was possible as he classically assumed no internal physical structure in a physical mass. Unfortunately, clocks are complex engineering objects with a complex internal physical structure. The observed time using these clocks depends upon the internal physics of the system of such clocks.
b) In the continuum, there is no such thing as a point next to another point. So, the concept of neighboring points is invalid. There is no such thing as a real number next to a given real number. This is because in between any two real numbers one can always find a real number. As we pointed out in the foregoing, this property is called the density of the set of real numbers.

c) As pointed out in b), there is no such thing as the coordinate of a clock in this setting. Each clock occupies continuously (thus infinitely) many points in the space. It is by itself a complex infinitary physical structure. Furthermore, in 1) Einstein assumes the fluid of clocks and in 3) he says that the coordinate of each clock remains the same. These statements are contradictory.

There are also further issues:

(1) Upon what time and space the mechanics of such point clocks are defined? Each clock is a physical system and so, it is operating in a spacetime that is not the same as the spacetime defined by the clock. This is to say that the spacetime(x_0, x_1, x_2, x_3) does not define the internal dynamics of the clock at (x_0, x_1, x_2, x_3). Moreover, where is the clock that governs the spacetime in which this clock operates? According to the general theory of relativity, the time of this spacetime (x_0, x_1, x_2, x_3) and that of the spacetime in which this clock operates are not the same and how much they are synchronized depends upon the location of the clock that defines the spacetime that defines the clock. This problem is bound up with a more general problem associated with the instrumentalist view of time as clocks. This view falls into the following vicious circle: The clock that is supposed to define time must operate, as a dynamical system, upon some time and space. Then, how these time and space are supposed to be defined? For Newton time was not to be defined by instruments.

(2) It is a common sense among researchers in dynamical system theory that time has a special status and different from all other coordinates of the system. This is in agreement with the idea of Newton in his classical dynamics. He said that time, unlike other coordinates, has a natural flow that moves forward only. This makes it impossible to consider time as reading of clocks.

(3) As we explained in Section 7.2, countable infinity and uncountable infinity are entirely different things. There are at most countably many clocks in this universe. No matter how closely we put clocks together, we cannot form a continuum of clocks. No matter how one puts countably infinite particles together, he will not make mathematical continuum. This is mathematically the same problem as the problem of photons, which are supposed to exist for each frequency: as the frequency has a continuous spectrum, there must be uncountably infinite particles called photons. Countably infinite points will never form a real continuum. In her book *Lost in Mathematics*, Hossenfelder complains that physics today is lost in the too heavy usage of mathematics. The authors of this paper made incisive analysis of the current theories of physics and showed exactly where they abused mathematics. See, e.g. [1, 2, 3, 4]. The mathematics is not the problem. It is the lack of understanding of mathematics.

(4) What does it mean to be sufficiently synchronized? The concept of synchronization presupposes external, absolute time which contradicts the concept of relativism. Here, we have to check time of each clock at precisely the same moment in absolute time.

10.3.3. Minkowskian local frame Suppose at a point P in a gravitational field, which is a fluid of infinitely many clocks, a freely falling non-rotational (relative to distant stars) local inertial frame is constructed. We further assume that the axioms of the special theory of relativity are valid within this frame as it is supposed to be an inertial reference frame. So, we can set up a Cartesian coordinate system *(Px,Py,Pz)* at this point P. Furthermore, we can distribute clocks over the frame, all synchronized to the clock at P. Using this frame and the clocks, events occurring in the vicinity of P over a suitably restricted time period can be allocated space-time coordinates (t, x, y, z).

Remark (1) Rotation is a dynamical concept that is defined in terms of time. If time is defined in terms of a sea of clocks, then how is it possible to define the rotation of a local inertial frame which apparently also is a sea of clocks? We may replace clocks by atomic clocks. Then it is not a rotation but a vibration. Still, we have the same problem. (2) As we assumed that the universe is a sea of clocks that are not overall synchronized, such a coordinate system (Px, Py, Pz) is not universal. It is a local coordinate system around P. (3) It is not quite clear why the time period must be restricted.

Now suppose that in this local inertial frame a pair of neighborhood events have space-time coordinates (t, x, y, z) and $(t + dt, x + dx, y + dy, z + dz)$. Then, $d(\tau)$ such that

$$(d(\tau))^2 = (dt)^2 - (1/c^2)((dx)^2 + (dy)^2 + (dz)^2) \tag{77}$$

is Lorentz invariant. This is called *Minkowski metric*. The Minkowski distance between an event A and the event "distant observer notices event A" is 0. Such a metric does not seem to be useful in physics.

Remark Lorentz transformation is irrelevant to theoretical physics as the claim by Lorentz that this transformation maps wave equations to wave equations is false. False is also Einstein's claim that all equational axioms of Maxwell are Lorentz invariant. So, Minkowski distance as well is irrelevant to physics. Although it is invariant under the Lorentz transformation, it is not invariant under the time dilation (TD) and length contraction (LC).

Remark The Lorentz transformation maps a time into a different time depending upon the target location. This alone is enough to dismiss this coordinate transformation, which came from the wrong interpretation of the Michelson-Morley experiment.

Remark The remark above is related to the issue of an internal conflict between the role of the Lorentz transformation and TD+LC within the special theory of relativity. Way before this, already the Minkowski metric had an issue with the mathematical concept of metric (distance) on a geometric space. All distances on geometric spaces must satisfy the basic axioms of metric spaces in topology. Minkowski's metric does not. So, the claim that LT conserves the Minkowski metric means nothing for physics.

For a larger scale (temporal, as well as spatial) issues, it is necessary to use the general reference frames. If x_i are space-time coordinates relative to such a general frame, transformations of the form $x_i' = \pi(x_0, x_1, x_2, x_3)$ must exist relating (x_0, x_1, x_2, x_3) to (t, x, y, z) [the local inertial frame] such that

$$\begin{aligned} t = \theta(x_0, x_1, x_2, x_3), \quad & x = \pi(x_0, x_1, x_2, x_3), \\ y = \psi(x_0, x_1, x_2, x_3), \quad & z = \gamma(x_0, x_1, x_2, x_3). \end{aligned} \tag{78}$$

Then, if x_i are subjected to increments dx_i, the corresponding increments in t, x, y, z will be given by

$$dx = (\partial(\theta)/\partial(x_0))dx_0 + (\partial(\pi)/\partial(x_1))dx_1 + (\partial(\psi)/\partial(x_2))dx_2 + (\partial(\gamma)/\partial(x_3))dx_3 \tag{79}$$

etc. and the substitution in the equation of proper time interval

$$(d(\gamma))^2 = (dt)^2 - (1/c^2)((dx)^2 + (dy)^2 + (dz)^2) \tag{80}$$

will result in an expression $d\tau^2$ which is quadratic in the increments dx, i.e. whose terms will either involve squares of the dx_i or product of two different dx_i. Thus

$$d\tau^2 = \sum_{i=0}^{3}\sum_{j=0}^{3}(g_{ij})dx_i dx_j \tag{81}$$

where the coefficients (g_{ij}) will be the functions of x_i. In general, a continuum in which the interval between neighboring points is given by a quadratic form like (81) is called a *Riemannian space* and the quadratic form like (81) is called its *metric*. Thus, the space-time continuum is a four-dimensional Riemannian space whose interval is everywhere identified with the proper time interval between neighboring events in a local inertial frame.

Remark The physical relevance of (81) above is in question. The Minkowski distance is invariant under the Lorentz transformation, which itself has no relevance to physics. Furthermore, this distance is not invariant under TD and LC, which are in the domain of relativistic physics (and unfortunately they are inconsistent.

Remark In the theory of relativity, [speed], which is defined as the ratio between [distance] and [time], alters [distance] and [time] through TD+LC or the Lorentz transformation. So, the metric of a space-time is determined in terms of the [speed] of the reference frame, which itself is determined by [time] and [distance]. There is an obvious contradiction here.

It should be clear now that the claim that the space-time of the general theory of relativity is locally Minkowskian is false.

11. Geodesics

11.1. General equation of geodesics

When we express a linear function of one variable on 2D space, this function becomes a straight line graph. The coefficient of the first order variable is the slope of the line. This idea was extensively exploited by train companies to visualize train operation on a 2D space where one coordinate is the time coordinate and the other coordinate is the location coordinate in terms of the distance from the start point expressed at the distance from the origin station. It is called "operational diagram" and this simple concept was given the name of 4D spacetime. In the 4D spacetime all constant speed 3D motions should be just straight lines and the slope of the line is the constant speed 3D motion.

So, in 4D spacetime geometry of 3D motions, the Euclidean geometric distance between two points $P_1(t_1, x_1, y_1, z_1)$ and $P_2(t_2, x_2, y_2, z_2)$ in the 4D spacetime is:

$$\overline{P_1P_2} = \sqrt{(t_1-t_2)^2+(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}. \tag{82}$$

The slope of the line segment P_1P_2 is given as

$$((x_1-x_2)/(t_1-t_2),\ (y_1-y_2)/(t_1-t_2),\ (z_1-z_2)/(t_1-t_2))\,. \tag{83}$$

This changes in the general theory of relativity: the distance, as per Minkowski, between P_1 and P_2 is

$$\widetilde{P_1P_2} = \sqrt{(t_1-t_2)^2-(1/c^2)\left\{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2\right\}}. \tag{84}$$

It can be shown that this distance, which we shall call *the Minkowski distance* is smaller than the Euclidean distance along any curve connecting P_1 and P_2 in the 4D space. Clearly, $\widetilde{P_1P_2} < \overline{P_1P_2}$.

But for any path l_{P_1,P_2} between P_1 and P_2, $\overline{P_1P_2} \leq \overline{l_{P_1,P_2}}$, where $\overline{l_{P_1,P_2}}$ is the Euclidean distance of l_{P_1,P_2}.

From this, relativism concludes that the 4D spacetime with the Minkowski metric is not a Euclidean space but a curved Riemannian space. Nonetheless, under the guidance of Hilbert's school at Göttingen, Einstein continued to develop his theory as follows: Under the equivalence principle,

$$d\tau^2 = \sum_{i=0}^{3}\sum_{j=0}^{3}(g_{ij})dx_i dx_j = (dt)^2 - (1/c)((dx)^2 + (dy)^2 + (dz)^2). \tag{85}$$

From this, Einstein concludes that the free fall in the gravitational field is a geodesic [in the 4D spacetime]. This is called the *geodesic principle*. Under this principle we obtain equations of motion for bodies falling freely in a gravitational field.

Remark Einstein's principle of equivalence is false and the geodesic principle is a consequence of this principle.

Using Riemannian geometry for the general metric $d\tau^2 = \sum_{i=0}^{3}\sum_{j=0}^{3}(g_{ij})dx_i dx_j$, we can show the following general equation of a geodesic

$$\frac{d}{d\tau}\left(\sum_{j=0}^{3} g_{ij}\frac{dx_j}{d\tau}\right) = \frac{1}{2}\sum_{j=0}^{3}\sum_{k=0}^{3}\frac{\partial g_{ik}}{\partial x_i}\frac{dx_j}{d\tau}\frac{dx_k}{d\tau} \qquad (i = 0, 1, 2, 3). \tag{86}$$

For the metric

$$(d(\tau))^2 = (dt)^2 - (1/c)((dx)^2 + (dy)^2 + (dz)^2), \tag{87}$$

these equations reduce to

$$\frac{d^2x}{d\tau^2} = \frac{d^2y}{d\tau^2} = \frac{d^2z}{d\tau^2} = \frac{d^2t}{d\tau^2} = 0. \tag{88}$$

This is equivalent to

$$x = \frac{dx}{d\tau}t + a, \quad y = \frac{dy}{d\tau}t + b, \quad z = \frac{dz}{d\tau}t + c \tag{89}$$

But as we mentioned above, this Minkowski metric is irrelevant to physics because the Lorentz transformation is invalid in physics. A physical transformation that tells us that time is determined by location and speed, and a metric that is invariant under such a transformation has no role in physics. This transformation was induced from the wrong interpretation of the Michelson-Morley experiment.

Remark Historically, this is a trend in physics that started with Maxwell's EM field theory which used mathematics to conceal the ontologically questionable concept of EM fields, which are not physical reality but counter-factual modalities. After this, the theory of electromagnetism became detached from physical reality and we were presented with the concept of EM waves, which we still do not know what they are. As EM fields are modal entities, one thing we can say about them is that the EM waves associated with those fields are not physical reality but modal waves. We still do not have any explanation as to what it means to observe modal waves.

11.2. Light bending revisited

According to the general theory of geodesics, light coming from a distant star passing near our sun has a geodesic (4D) which bends near the sun due to the gravity of the sun. As it is bending in the 4D spacetime, this bending cannot be expressed graphically. But when we drop the time bending, the light path bends in our 3D path. This 3D bending is a phenomenon that takes place in our 3D Euclidean space. As we are in this 3D space, in theory, we will not be able to observe this bending. This was what Gamow warned us about long time ago. Gumow said that, unless we are in the position of Newton out of the universe observing from the shoulder of God, we will not be able to observe this bending.

One of the new obstacles in modern physics is that as theory gets very complex, the chance of making fatal errors at the stage of theory development dramatically increases. As experiments get more complex and delicate, the challenge of setting them and the chance of interpreting them wrongly dramatically increase as well. We have been seeing serious cases of this. The danger comes form both fronts, namely theory and experiment.

11.3. Momentum and energy in the general theory of relativity

According to the general theory of relativity, the momentum-energy tensor T_{ij} which describes the distribution of mass, energy and momentum, yields the corresponding g_{ij} enabling us to calculate the geodesics of the system. It was Schwarzschild who first obtained an exact solution to Einstein's equation for spherically symmetric field. He used the solutions to calculate the motion of a planet in the field of a sun.

In Section 3 we have discussed that relativistic energy $e = mc^2$, where m is the relativistic mass, is flawed. This m is a relativistic adjustment of classical mass of Newton so that the conservation of momentum holds under relativistic collision. As the special theory of relativity is invalid on pain of fundamental contradictions, this adjustment is invalid as well. Indeed, the relativistic energy-momentum relation of Einstein

$$e^2 = c^2p^2 + m_0^2c^4 \tag{90}$$

is false as $e = mc^2$ is false.

We have shown that the special theory of relativity deduces $e = 0$ instead of $e = mc^2$. The correct derivation follows the following steps: (1) $p = mv$. (2) $f = dp/dt = 0$ as p is constant. (3) So, in the end $e = 0$.

More fundamentally, the concept of energy is logically flawed even classically. This is because the work needed to accelerate from $m0$ to mv is not necessarily $mv^2/2$. It depends upon how we accelerate.

Although momentum is a legitimate ontological concept, energy is not. The latter is a modal concept as it is a possibility to do work. In the abstract domain of mathematics, by dropping these conceptual issues one can consider such things as the energy-momentum tensor. But this is not physics. What makes theoretical physics transcends pure mathematics is the ontology.

The conflict between energy and momentum affected theoretical physics since Newton started physics, which he rightly called natural philosophy. He rejected energy and went for momentum, while Leibniz used energy. This issue reappeared in the renormalization problem of QED. Going against the mainstream, Feynman chose momentum and the Lagrangian over energy and the Hamiltonian. However, his motive for this was not because of the obscurity of the concept of energy but because he realized that the concept of force field is invalid causing the self-energy problem. To avoid this problem he had to explicitly deal with time; for him, time was not just an agent to bring change. This is how he chose the Lagrangian.

12. Relativistic gravitational waves revisited

After showing that the project of the general theory of relativity is physically and logically flawed, let us go back to the modern day gravitational wave theory which is based upon the concept of gravity, which comes from the general theory of relativity.

12.1. The problem of singularity

Einstein and Rosen discovered that in full generality of the general theory of relativity, solutions of gravitational wave equations have a singularity. Robertson argued that such a singularity comes from the element of the theory which has no relevance to physics. This is an interesting development. At least Robertson agreed that the general theory of relativity is governed by physically irrelevant mathematical elements. Now the open question is if isolating such undesirable elements will yield a consistent, physically meaningful theory. Another question to be asked is that if a theory predicts several things and one of them is invalid, shall we accept the rest of the predictions.

12.2. Source of the confusion: Riemann's curvature tensor

In 1956, Pirani pointed out that all of these confusion associated with the relativistic gravitational wave theory coming from the choice of coordinate systems can be resolved by reformulating the gravitational waves in terms of manifestly observable Riemann curvature tensor. Again, this observation is off. Such opinion should follow only after resolving all fundamental logical and mathematical inconsistencies of the general theory of relativity. It is not the matter of the tensor. It is the matter of the momentum-energy which the tensor is to represent.

This problem has a precedence in the EM field theory of Maxwell. This theory, which was passed down to quantum electrodynamics (QED), is inconsistent. The Lorentz force contradicts the second law. From this contradiction anything such as QED can follow, thereby making QED meaningless.

12.3. Feynman on energy of gravitational waves

In 1957, Feynman presented a thought experiment which suggests that the gravitational wave carries energy by generating heat using gravitational waves. On the other hand though, he adopted energy, and not momentum, in order to deal with the divergence problem of QED.

12.4. Momentum-carrying waves

In wave mechanics, no water wave, which moves through water, will carry momentum as there is no momentum in such waves. No mass moves in the direction of the motion of water waves. It is the local vibration of the medium which moves to the direction of the waves. No earthquake will move water as tsunami from the epicentre to thousands of kilometers away with the speed of 500km/h. As we keep stressing, there is no such thing as EM waves as EM fields are not physical reality. More fundamentally, energy is not a legitimate physical concept, as we discussed above. The work needed to accelerate from $m0$ to mv is not necessarily $mv^2/2$. It depends upon how we accelerate.

The idea of waves carrying momentum arose as the result of the theory of relativity. This led to the wrong equation of momentum-energy, which related momentum and energy. This in turn led to the wrong conclusion that as waves carry energy, while they carry momentum. Energy is a counterfactual modality and momentum is a physical reality. They are of different categories.

12.5. Observable Riemannian curvature tensor

As already pointed out above, Einstein and Rosen discovered that in full generality of GTR, solutions of gravitational wave equations have a singularity. In 1956, Pirani pointed out that all of these confusions associated with the relativistic gravitational waves are coming from the choice of coordinate systems and can be resolved by reformulating the gravitational waves in terms of manifestly observable Riemann curvature tensor.

The problem here is not the technical nature. The question is what did Pirani mean by "observable". Is it being observable in the local Minkowski space or in the absolute space? From the point of view of mathematics, it must be in the absolute frame as the absolute space curvature is defined by the energy-momentum tensor.

What about the actual measurement? We have our own local Minkowski space, according to Einstein and contingent upon the assumption that the relativity inside this local space is consistent and valid. So, can legitimize our observation in this local space? There are two issues here. First, Mnkowski local spacetime theory is inconsistent. Second, the Riemann tensor defines the curvature of the absolute space. How can local observation of local space lead us to the global curvature of the absolute space?

12.6. Red shift

It has been accepted by the mainstream physics community that the electromagnetic wave shifts towards the red spectrum due to the relative velocities of the source and the observer. The argument goes as follows: Assume we emit a light beam upward from the floor to the ceiling. Due to the downward acceleration of the room, by the time the light reaches the ceiling the ceiling is moving faster than the source on the floor was when the light left it. In other words, the receiver at the ceiling is approaching the source (to be precise: where it was when the light left). Therefore, we should expect the blue shift due to the Doppler effect. And the observer in the room will notice the blue shift. This will make the observer notice the downward acceleration. This contradicts the equivalence principle which says that the free falling body will not notice its free falling. So, there must be a red shift due to the light moving upwards against the gravitation to compensate for this blue shift. This is how Einstein obtained the red shift.

This argument is flawed however. It is not the observer in the room who sees the ceiling falling towards where the floor used to be. It is an outside observer who will see that the ceiling is falling towards where the floor used to be. Einstein was not aware that the inside observer is also subjected to the same acceleration due to the gravity. All of this confusion is consistent with the fact that Einstein's argument for the principle of equivalence is fundamentally flawed, as we discussed in Section 10.3.1.

13. Quantum gravity theory

13.1. General issues

Despite many fundamental errors in quantum electrodynamics, there is a strong urge to build a theory of quantum gravity. It appears that there is little interest in learning from the experiences of QED, which went wrong as we discussed in this paper and in e.g. [3]. Quantizing the gravitational field theory is just as problematic as the electromagnetic field theory of Maxwell. One might be advised to consider the classical wave mechanics. This theory assumes a continuum wave medium which is foreign to classical gravitational dynamics and it is completely disjoint from Newtonian point mass dynamics. One can mathematize the situation of applying force to a point mass in Newton's world, but we have no idea what it really means to apply a force to a unit area (or volume). This difficulty comes from differences in the mathematical structure. In geometry, a point and an area (or a volume) are of entirely different categories. Reflecting upon this, Newton limited his theory to just point masses. The issue here for wave mechanics is that force is a vector that acts only on a point. It is a serious issue for theory of quantum gravity. Maybe Robinson's infinitesimal calculus (non-standard analysis) will shed some light to this difficult and fundamental problem [6]. Unfortunately, recently the authors discovered that the would-be revolutionary theory of infinitesimals by Robinson fell short of the expectation. The following contradiction is its undoing: For $f(x) = x^2$

$$f'(x) = \frac{f(x+dx) - f(x)}{dx} = \frac{(x+dx)^2 - x^2}{dx} = \frac{2xdx + (dx)^2}{dx} = 2x + dx = 2x \tag{91}$$

Here dx has two mutually contradicting assumptions: one is $dx \neq 0$ and the other is $dx = 0$. In short, we do have infinitesimals but they do not support even derivatives. So, infinitesimal calculus, despite the impressive work of Robinson, is useless for mathematics. Robinson successfully created infinitesimals, which are positive numbers smaller than all positive real numbers, but they have no place to fit into in calculus (mathematical analysis).

Once theoretical physics stepped into fluid dynamics and quantum electrodynamics, it reached the point where mathematics is not just a language for physics but a most essential element of physics. After all, physics is not just working on an idealized point. It is working on geometric continuum structures. The mathematics needed for physics is not the mathematics but the conceptually advanced pure mathematics. What physics should be concerned with is the dynamic analysis of physical structure. It demands a solid new mathematics.

13.2. Quantization of gravitational waves

Considering the fact that in QED we started with the quantization of EM waves as photons, which are rest mass zero particles that keep moving with constant speed c, maybe the first thing to do in this direction would be to obtain the quanta of the gravitational wave.

However, we have some trouble with Einstein's photon. Namely, how is it possible that a particle that never stops, that always moves with speed c has a rest mass. Having a rest mass of 0 and having no rest mass at all are entirely different categories. Let us make this argument more concrete. Consider $x^2 + 1 = 0$. This equation has no real solution. Does this mean $x = 0$ in a theory where we do not consider imaginary numbers? Mathematicians will say that it means that there is no solution to this equation.

Einstein's reasoning for the rest mass 0 for the photon was because the photon moves with speed c, which makes its relativistic energy diverge. As discussed in Section 3, Einstein thought that the rest mass of the photon is 0 as he thought that $0/0$ can be any number. But this immediately leads to mathematical contradictions.

The reason why QED appeared to be the most verified theory is mostly because it dealt with extremely small particles moving with very fast speed. This makes it impossible to do what we are used to do in experiments. It is clear that almost all QED experiments were "measured" by applying theoretical formulas to trajectories, which are not supposed to be there according to the theory's uncertainty principle. Certainly the formulas used to measure are all verified. According to probability theory, there is no such thing as experimental verification or refutation of a probabilistic theory as such is a theory of probabilistic prediction and the relative frequency converges only at the limit.

The bottom line is that instead of working on projects based on wrong ideas, we should go back to the source of those problems and correct them. The first thing we must do is to understand the invalidity of the theory of relativity and do something about it. Correcting this theory would lead to the correction of quantum mechanics and, eventually, the theory of gravitation.

13.3. Renormalization problem

Tomonaga-Schwinger renormalization The reason why QED managed to find solutions to the problem of renormalization is because they treated the issue as a pure mathematical issue, as did Tomonaga. Tomonaga showed that for problems in QED, the diverging terms are always linear or logarithmic. So, he managed to show for these problems how to solve the diverging term problem. As Dyson showed, Feynman's solution to this problem, in which he used Kramer's cutoff, is a special case of the general solution of Tomonaga.

In the theory of quantum gravity, apparently this is not the case. But is this the weakness of this theory? We do not know. We do not think that the solution presented by Tomonaga is intrinsic. It was just a matter of luck that Tomonaga found that the diverging problems in QED are caused by the linear or logarithmic terms.

In the end the renormalization issue for QED is closely related to the issue of the electromagnetic mass, which appeared in the classical EM theory as closely studied by Thomson and Poincaré. So, it had some ontological connection to reality. We do not see this relation in the theory of quantum gravity. The difference here is that in the case of QED, it was that the EM force field added extra mass to the charges. In the case of the gravitational field theory, there is only one force, gravity.

All of this is to say that, contrary to the development of the theory of renormalization in QED, there is no ontological explanation of the problem of renormalization in the quantum gravity theory. It is a pure mathematical theory that has little relevance to physics.

13.4. Gravitons and galaxies

Despite all of these difficulties and negative perspectives emanating from the mentioned conceptual and mathematical incoherences, dedicated quantum gravity research still holds high hopes for a project of building a successful theory of quantum gravity as a unification of the gravitational theory and quantum mechanics: one for the understanding of large scale physics and the other for the micro scale physics. In order for such a theory to be possible, we must first take care of the problems in quantum mechanics.

From an empirical point of view, this appears however to be difficult. Astronomical objects are way too large for us to perform experiment on in the way we perform experiments for the world of our size-range. All we can do is to observe the far way stars relying upon EM waves. Basically we know nothing about EM waves and light except that they are not physical reality. They are modal waves that travel through the modal fantasy of EM fields. Also, the very theory that governs the physics of the Doppler effect of EM waves (light) is logically inconsistent, as we mentioned in this paper. Moreover, the only theoretical tool that gives us the power to conclude something about distant stars is the Doppler effect. Despite so much trust we have in this effect, the logical reality is that we have no understanding of the connection between the frequency shift and the energy conservation. Frequency shift means the shift of energy and there seems to be no clear explanation of this observed energy shift and the total energy conservation. We are therefore not sure if our calculating the speed of a distant star through the Doppler effect is trustworthy. On top of it, once we get this speed correctly, we have to translate it to the relativistic one, if we use relativism, and clearly there is a vicious circularity in this argument. This is the problem of v in the gamma factor being the classical speed. More fundamentally, the gamma factor is the element of the special theory of relativity that we showed is inconsistent.

The world of quantum particles, on the other hand, is way too small for the world of our size perform experiments on. This is why, in von Neumann's quantum mechanics, we finished up with the probabilistic theory with uncertainty.

The fact is that neither astronomical scale physics nor micro-scale physics can be handled by the traditional empirical method any more.

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