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TREE-LEVEL UNITARITY IN THE PRESENCE OF WARPED GEOMETRIES*

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Perturbative unitarity for $W_{\rm L}^+W_{\rm L}^- \to W_{\rm L}^+W_{\rm L}^-$ scattering is discussed within the Randall–Sundrum theory with two 3-branes. It is shown that the exchange of massive 4D Kaluza–Klein gravitons leads to amplitudes growing linearly with the CM energy squared. If the curvature, m_0, of the metric is too small, the gravitational contributions lead to a violation of unitarity. However, m_0 must be small enough relative to the 5D Planck mass, $M_{\rm Pl\,5}$, to avoid quantum gravitational effects. As a result, we find that there is only a small range of $m_0/M_{\rm Pl}$ for which the model is consistent. The width of the window is determined by the size of the 4D cutoff of the theory. The extension of the Randall–Sundrum scenario to include curvature-Higgs mixing is also considered and limits on the mixing parameter are determined.

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1. Introduction

The Standard Model (SM) of electroweak interactions describes successfully almost all existing experimental data, however the model suffers from many theoretical drawbacks. One of these is the hierarchy problem: namely, the SM cannot consistently accommodate the weak energy scale $\mathcal{O}(1~{\rm TeV})$

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and a much higher scale such as the Planck mass scale $\mathcal{O}(10^{19}$ GeV). Therefore, it is commonly believed that the SM is only an effective theory emerging as the low-energy limit of some more fundamental high-scale theory that presumably could contain gravitational interactions. In the last few years there have been many models proposed that involve extra dimensions. These models have received tremendous attention since they could provide a solution to the hierarchy problem. One of the most attractive attempts has been formulated by Randall and Sundrum (RS) [1], who postulated a 5D universe with two 4D surfaces ("3-branes"). All the SM particles and forces with the exception of gravity are assumed to be confined to one of those 3-branes called the visible brane. Gravity lives on the visible brane, on the second brane (the "hidden brane") and in the bulk. All mass scales in the 5D theory are of order of the Planck mass. By placing the SM fields on the visible brane, all the order Planck mass terms are rescaled by an exponential suppression factor (the "warp factor") $\Omega_0 \equiv e^{-m_0 b_0/2}$, which reduces them down to the weak scale $\mathcal{O}(1$ TeV) on the visible brane without any severe fine tuning. To achieve the necessary suppression, one needs $m_0 b_0/2 \sim 35$. This is a great improvement compared to the original problem of accommodating both the weak and the Planck scale within a single theory.

The model is defined by the 5D action:

$$S = -\int d^4x\, dy \sqrt{-\widehat{g}} \left(\frac{\widehat{R}}{\epsilon^2} + \Lambda \right) + \int d^4x\, \sqrt{-g_{\rm hid}} (\mathcal{L}_{\rm hid} - V_{\rm hid}) + \int d^4x\, \sqrt{-g_{\rm vis}} (\mathcal{L}_{\rm vis} - V_{\rm vis}) . \quad (1)$$

In order to obtain a consistent solution to the Einstein's equations corresponding to a low-energy effective theory that is flat, the branes must have equal but opposite cosmological constants and these must be precisely related to the bulk cosmological constant; $V_{\rm hid} = -V_{\rm vis} = \frac{12 m_0}{\epsilon^2}$ and $\Lambda = -\frac{12 m_0^2}{\epsilon^2}$. Then the following metric is a solution of the Einstein's equations:

$$\widehat{g}_{\widehat{\mu}\widehat{\nu}}(x,y) = \left(\begin{array}{c|c} e^{-2 m_0 b_0 |y|} \eta_{\mu\nu} & 0 \\ \hline 0 & -b_0^2 \end{array} \right) . \quad (2)$$

After an expansion around the background metric we obtain the following non-standard gravity-matter interactions

$$\mathcal{L}_{\rm int} = -\frac{1}{\widehat{\Lambda}_W} \sum_{n \neq 0} h^n_{\mu\nu} T^{\mu\nu} - \frac{\phi_0}{\Lambda_\phi} T^\mu_\mu \,, \quad (3)$$

where $h^n_{\mu\nu}(x)$ are the Kaluza–Klein (KK) modes of the graviton field $h_{\mu\nu}(x,y)$,

$\phi_0(x) \equiv \sqrt{6}M_{\rm Pl}e^{-m_0(b_0+b(x))/2}$ is the radion field, $\widehat{\Lambda}_W \simeq \sqrt{2}M_{\rm Pl}\Omega_0$, where $\Omega_0 = e^{-m_0 b_0/2}$, and $\Lambda_\phi = \sqrt{3}\widehat{\Lambda}_W$.

The key advantage of the RS model is a possibility of solving the hierarchy problem. In particular, the 4D electro-weak scale is given in terms of the $\mathcal{O}(M_{\rm Pl})$ 5D Higgs vev, $\widehat{v}$, by the following relation:

$$v_0 = \Omega_0\widehat{v} = e^{-m_0 b_0/2}\widehat{v} \sim 1 \text{ TeV} \quad \text{for} \quad m_0 b_0/2 \sim 35\,. \tag{4}$$

The RS model can be naturally extended (for details see [2] and [3]) by including mixing between gravitational end electroweak degrees of freedom:

$$S_\xi = \xi \int d^4x\sqrt{g_{\rm vis}}R(g_{\rm vis})H^\dagger H\,, \tag{5}$$

where $R(g_{\rm vis})$ is the Ricci scalar for the metric induced on the visible brane. The action (5) leads to the following exotic kinetic terms for the Higgs boson (h_0) and radion (ϕ_0) fields:

$$\mathcal{L} = -\frac{1}{2}\left\{1+6\gamma^2\xi\right\}\phi_0\Box\phi_0 - \frac{1}{2}\phi_0 m_{\phi_0}^2\phi_0 - \frac{1}{2}h_0(\Box + m_{h_0}^2)h_0 - 6\gamma\xi\phi_0\Box h_0\,, \tag{6}$$

where $\gamma \equiv v_0/\Lambda_\phi$.

The states that diagonalize both the kinetic energy and mass terms and have canonical normalization are h and ϕ:

$$\begin{aligned} h_0 &= \left(\cos\theta - \frac{6\xi\gamma}{Z}\sin\theta\right)h + \left(\sin\theta + \frac{6\xi\gamma}{Z}\cos\theta\right)\phi \equiv dh + c\phi\,,\\ \phi_0 &= -\cos\theta\frac{\phi}{Z} + \sin\theta\frac{h}{Z} \equiv a\phi + bh\,, \end{aligned}$$

where the mixing angle θ is defined through

$$\tan 2\theta \equiv 12\gamma\xi Z\frac{m_{h_0}^2}{m_{\phi_0}^2 - m_{h_0}^2(Z^2 - 36\xi^2\gamma^2)}\,, \tag{7}$$

for $Z^2 \equiv 1+6\xi\gamma^2(1-6\xi)$. The diagonal Higgs and radion masses are denoted by m_h and m_ϕ, respectively.

It is useful to note that in order to have $Z^2 > 0$, ξ must lie in the region:

$$\xi_{\rm min} \equiv \frac{1}{12}\left(1 - \sqrt{1+\frac{4}{\gamma^2}}\right) \le \xi \le \frac{1}{12}\left(1 + \sqrt{1+\frac{4}{\gamma^2}}\right) \equiv \xi_{\rm max}\,. \tag{8}$$

Also needed are the vector boson couplings to scalars. After adopting the diagonal basis for the Higgs boson and radion, these couplings *relative to SM strength* are $g_{VVh} = d + \gamma b$ and $g_{VV\phi} = c + \gamma a$ ($V = W, Z$).

2. Vector boson scattering

Let us begin by reviewing the limit on the Higgs-boson mass in the SM from requiring that $W_{\rm L}^+W_{\rm L}^- \to W_{\rm L}^+W_{\rm L}^-$ scattering be unitary at high energy. The constraint arises when we consider the elastic scattering of longitudinally polarized W bosons. The amplitude can be decomposed into partial wave contributions: $T(s,\cos\theta) = 16\pi\sum_J(2J+1)a_J(s)P_J(\cos\theta)$, where $a_J(s) = \frac{1}{32\pi}\int_{-1}^{1}T(s,\cos\theta)P_J(\cos\theta)d\cos\theta$. In the SM, the partial wave amplitudes take the form $a_J = A_J\left(\frac{s}{m_W^2}\right)^2 + B_J\left(\frac{s}{m_W^2}\right) + C_J$, where s is the center of mass energy squared. Contributions that are divergent in the limit $s \to \infty$ appear only for $J = 0,1$ and 2. The A-terms vanish by the virtue of gauge invariance, while, as is very well known, the B-term for $J = 1$ and 0 ($B_2 = 0$) arising from gauge interaction diagrams is canceled by Higgs-boson exchange diagrams. In the high-energy limit, the result is that a_J asymptotes to an $m_{\rm H}$-dependent constant. Imposing the unitarity limit of $|\mathbf{Re}\, a_J| < 1/2$ implies the Lee–Quigg–Thacker bound [4] for the Higgs boson mass: $m_{\rm H} \lesssim 870$ GeV.

We will now show that within the RS model and its extensions the unitarity requirement yields important constraints on model parameters when gravitational contributions to $W_{\rm L}^+W_{\rm L}^- \to W_{\rm L}^+W_{\rm L}^-$ scattering are included. The various contributions to the amplitude are given in Table I. From the table, we see that the SM high-energy divergent contributions are confined to $J = 0,1$ and 2; however, $J = 2$ contributions cancel between the t-channel and contact terms, so that effectively the SM contributes to $J = 1$ and 0 only. Further, in the SM limit where $R^2 = 1$, all the SM contributions cancel at $\mathcal{O}(s^2)$ and $\mathcal{O}(s^1)$. Note that the leading $\mathcal{O}(s)$ t-channel G exchange does not yield a well-defined partial-wave amplitude as the corresponding integral over $\cos\theta$ diverges at $\cos\theta = \pm 1$. However, since the graviton being exchanged is massive the full t-channel contribution leads to finite partial-wave decomposition with well-defined a_J.

It is important to note that even though graviton exchange leads to the same divergence as the SM vector boson and the contact interactions, *i.e.* $\propto s$, its angular dependence is different ($J = 2$ *vs* $J = 1$); therefore, the graviton cannot replace the Higgs boson and restore correct high-energy unitary behavior of the amplitude.

As is well known, the cancellation of the $\mathcal{O}(s^2)$ contributions in Table I between the contact term and s- and t-channel gauge-boson exchange diagrams is guaranteed by gauge invariance. Even more remarkable is the cancellation of the most divergent graviton exchange terms. Indeed, a naive power counting shows that the graviton exchange can yield terms at $\mathcal{O}(s^5)$, while the actual calculation shows that only the linear term $\propto s$ survives;

TABLE I

The leading contributions to the $W_L^+W_L^- \to W_L^+W_L^-$ amplitude for $R^2 \equiv g_{VVh}^2 + g_{VV\phi}^2 = 1 - \gamma^2(1-6\xi)^2/Z^2$. G denotes a single KK graviton.

diagram	$\mathcal{O}(\frac{s^2}{v^4})$	$\mathcal{O}(\frac{s^1}{v^2})$
γ, Z s-channel	$-\frac{s^2}{g^2v^4}4\cos\theta$	$-\frac{s}{v^2}\cos\theta$
γ, Z t-channel	$-\frac{s^2}{g^2v^4}(-3+2\cos\theta+\cos^2\theta)$	$-\frac{s}{v^2}\frac{3}{2}(1-5\cos\theta)$
$WWWW$ contact	$-\frac{s^2}{g^2v^4}(3-6\cos\theta-\cos^2\theta)$	$-\frac{s}{v^2}2(-1+3\cos\theta)$
G s-channel	0	$-\frac{s}{24\hat{\Lambda}_W^2}(-1+3\cos^2\theta)$
G t-channel	0	$-\frac{s}{24\hat{\Lambda}_W^2}\frac{13+10\cos\theta+\cos^2\theta}{-1+\cos\theta}$
(h-ϕ) s-channel	0	$-\frac{s}{v^2}R^2$
(h–ϕ) t-channel	0	$-\frac{s}{v^2}\frac{-1+\cos\theta}{2}R^2$

all the potentially divergent terms $\mathcal{O}(s^5, s^4, s^3, s^2)$ cancel. The mechanism behind the cancellation is as follows. In the high-energy region the graviton propagator behaves as k^2. The graviton couples to the energy-momentum tensor $T_{\mu\nu}$, so the amplitude for a single graviton exchange is of the form $T_{\mu\nu}D^{\mu\nu,\alpha\beta}T_{\alpha\beta}$. Since the energy-momentum tensor is conserved, $k^\mu T_{\mu\nu} = 0$, all the contributions from the numerator of the graviton propagator which are proportional to the momentum do not contribute. (Note that for this argument to apply, all the external particles must be on their mass shell.) This removes two potential powers of s in the amplitude. In order to understand the disappearance of two additional powers of s, let us calculate the energy-momentum tensor for the final state consisting of a pair of longitudinal W bosons. A direct calculation in the asymptotic region yields

$$\langle 0|T^{\mu\nu}|W_L^+W_L^-\rangle \propto s\,. \tag{9}$$

Note that the factor $1/m_W^2$, which comes from the vector boson polarization vectors has been canceled by two powers of m_W coming from on-shellness of the longitudinal vector bosons, *i.e.* m_W^2 replaces s which originate from $T^{\mu\nu}$[1]. In short, when the two vertices are contracted with the propagator of the virtual graviton, four potential powers of s disappear leading to a single power of s [2]. These arguments apply equally to s- and t-channel diagrams.

[1] For transverse W's the energy-momentum tensor would also behave as s, however this time in accordance with naive power counting. Note that for transverse vector bosons the polarization vector does not provide any additional power of momentum. Consequently, for graviton exchange, in contrast to gauge theories, amplitudes grow as s both for longitudinal and transverse polarization of the vector bosons involved. For an illustration of a calculation for transverse vector boson polarizations, see [5].

[2] For fermions in either initial or final state the amplitude behaves as $s^{1/2}$.

For the divergent ($\propto s$) and constant terms we obtain the following contributions from graviton, SM vector bosons and ϕ–h exchange for the $J = 2$, 1 and 0 partial waves:

$$
\begin{aligned}
a_2 &= -\frac{1}{192\pi\hat{\Lambda}_W^2}\left\{\left[91 + 30\log\left(\frac{m_G^2}{s}\right)\right]s + \left[241 + 210\log\left(\frac{m_G^2}{s}\right)\right]m_G^2 \right. \\
&\qquad \left. +32g^2v^2\right\} + \mathcal{O}(s^{-1})\,, \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
a_1 &= -\frac{1}{384\pi\hat{\Lambda}_W^2}\left\{\left[73 + 36\log\left(\frac{m_G^2}{s}\right)\right]s + 36\left[1 + 3\log\left(\frac{m_G^2}{s}\right)\right]m_G^2 \right. \\
&\qquad \left. +37g^2v^2\right\} + \frac{1}{32\pi}\left[\frac{s}{v^2}(1-R^2) + R^2g^2 + \frac{12\cos^2\theta_W - 1}{2\cos^2\theta_W}g^2\right] + \mathcal{O}(s^{-1})\,, \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
a_0 &= -\frac{1}{384\pi\Lambda_W^2}\left\{\left[11 + 12\log\left(\frac{m_G^2}{s}\right)\right]s - \left[10 - 12\log\left(\frac{m_G^2}{s}\right)\right]m_G^2 \right. \\
&\qquad \left. +19g^2v^2\right\} + \frac{1}{32\pi}\left[\frac{s}{v^2}(1-R^2) + R^2g^2 - 4\frac{\overline{m}_{\rm scal}^2}{v^2}\right] + \mathcal{O}(s^{-1})\,, \qquad (12)
\end{aligned}
$$

where $\overline{m}_{\rm scal}^2 = g_{VVh}^2 m_h^2 + g_{VV\phi} m_\phi^2$. We note that $1 - R^2 = \gamma^2(1-6\xi)^2/Z^2$ deviates from 0 as a consequence of the non-orthogonality of the "rotation" which diagonalizes (with canonical normalization) the Higgs-radion sector. Only in the conformal limit of $\xi = 1/6$ does the scalar kinetic mixing have no physical consequences. Eqs. (10)–(12) show that radion mixing gives rise to incomplete cancellation of the s^1/v^2 terms[3]. Also note that unitarity limits are sensitive to the weighted Higgs-radion mass: $\overline{m}_{\rm scal}^2 = g_{vvh}^2 m_h^2 + g_{vv\phi}^2 m_\phi^2$.

It should be emphasized that the mixing effects are not directly related to the particular gravitational background involved. Thus, in this respect, the RS model serves only as a possible illustration. In contrast, the spectrum of heavy gravitons is an intrinsic feature of the warped nature of the background and directly impacts the unitarity analysis.

To analyze the consequences of unitarity constraints, it is crucial to keep in mind that we are dealing with a cutoff theory that should only be reliable, in particular unitary, for energies up to some fraction of the cutoff. Constraints on the theory will arise when the increasing s/v^2 terms lead to a violation of unitarity for energies below the cutoff scale. To illustrate this in more detail, let us first consider what Eq. (12) implies if KK graviton

[3] This was also noticed in [6].

exchanges are neglected. The leading terms for a_0 are:

$$a_0 = \frac{1}{32\pi}\left[f(s) + g^2R^2 - 4\frac{\overline{m}^2_{\rm scal}}{v^2} + \text{ graviton contributions }\right], \qquad (13)$$

where [3,6] $f(s) = \frac{s}{v^2}(1-R^2) = -\frac{s}{\Lambda_\phi^2}\left[\frac{(1-6\xi)}{Z}\right]^2$, with $Z^2 \equiv 1 + 6\xi\gamma^2(1-6\xi)$. For $\xi = \xi_{\rm max}, \xi_{\rm min}$ [see Eq. (8)] the leading contribution f varies between 10 and 100 (100 and 700) for $m_h = 0.1$ TeV and $m_\phi = 0.2 \div 1.0$ TeV if $\Lambda_\phi = 2$ TeV (5 TeV) for $s = (2\Lambda_\phi/3)^2$, while, at the same time, R remains within a reasonable perturbative range of $1 \div 2$. As a result, the range of ξ for which the theory is unitary for s up to a cutoff scale of $\mathcal{O}(\Lambda_\phi^2)$ is limited.

In Fig. 1, we plot the boundaries of the ξ ranges consistent with unitarity, $|\text{Re } a_{0,1}| < 1/2$, for $s < s^{\rm max} = \widehat{\Lambda}_W^2$. In order to separate the radion effect, the graviton contribution is neglected in the figure. The vertical boundaries in Fig. 1 are those from the unitarity limit; they are essentially independent of m_ϕ. The hour-glass-shaped boundaries are from requiring theoretical consistency of the model. The exact bounds on ξ depend on the value of $s^{\rm max}$. If the higher cutoff of $s^{\rm max} = \Lambda_\phi^2$ is employed, the restrictions are more severe. It is worth noticing that for low m_h, the mass-dependent terms are not very important. It is only as m_h approaches the SM unitarity limit that the mass-dependent terms become a really significant factor.

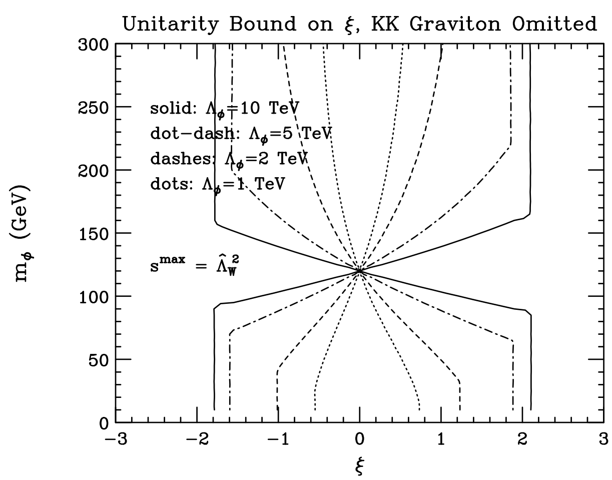

Fig. 1. For a variety of Λ_ϕ values, we plot the bounds on ξ as a function of m_ϕ coming from theoretical consistency and unitarity when KK gravitons are omitted. We have taken $m_h = 120$ GeV and used $s^{\rm max} = \widehat{\Lambda}_W^2$ for the effective theory. The vertical boundaries are those arising from the unitarity constraints. The hour-glass-shaped portions of the boundary arise from theoretical consistency of the model.

Let us now discuss pure KK graviton effects. The relevant parameters here are $\widehat{\Lambda}_W \simeq \sqrt{2} M_{\rm Pl} \Omega_0$, $m_n = m_0 x_n \Omega_0$, and $\Lambda_\phi = \sqrt{6} M_{\rm Pl} \Omega_0 = \sqrt{3} \widehat{\Lambda}_W$, where m_n is the mass of the n-th graviton KK mode and the x_n are the zeroes of the Bessel function J_1 ($x_1 \sim 3.8$, $x_2 \sim 7.0$). To solve the hierarchy problem, $\Omega_0 M_{\rm Pl} = e^{-m_0 b_0/2} M_{\rm Pl}$ should be of order of TeV. A useful relation following from the above equations is:

$$m_n = x_n \frac{m_0}{M_{\rm Pl}} \frac{\Lambda_\phi}{\sqrt{6}}\,. \tag{14}$$

If the ratio $m_0/M_{\rm Pl}$ was known then all the KK masses would be determined and therefore our predictions for a_J would be unique. However, to set the scale of m_0 independently of b_0 ($m_0 b_0 \simeq 70$ is required by the hierarchy problem) requires additional arguments. For our discussion, the most important restriction is that reliability of the RS approach requires [1] $m_0/M_{\rm Pl} \ll 1$ in order that the ratio of the bulk curvature m_0 to $M_{\rm Pl\,5}$, $\frac{m_0}{M_{\rm Pl\,5}} \sim \left[\frac{m_0}{M_{\rm Pl}}\right]^{2/3}$, is small.

Summing over all KK excitations with $2m_n < \sqrt{s^{\rm max}}$, [4] keeping only the m_G^2-dependent terms in $a_{0,1,2}$ in Eqs. (10), (11) and (12), we obtain the results for the partial wave amplitudes shown in Fig. 2. When $m_0/M_{\rm Pl}$ is small, Eq. (14) implies that one is summing over a very large number of KK excitations. As $m_0/M_{\rm Pl}$ increases, the number summed over slowly decreases. One finds a requirement of $m_0/M_{\rm Pl} \gtrsim 0.05$ for $s^{\rm max} = \Lambda_\phi^2$ and of $\gtrsim 0.01$ for $s^{\rm max} = \widehat{\Lambda}_W^2$. These *lower bounds* on $m_0/M_{\rm Pl}$ are quite strong and interesting given that the RS model should not be trusted [1] for $m_0/M_{\rm Pl} > 0.1$. Using $s^{\rm max} = \widehat{\Lambda}_W^2$, only the range $0.01 < m_0/M_{\rm Pl} < 0.1$ is consistent with both bounds. An alternative view is also possible. Namely, if $m_0/M_{\rm Pl}$ is known, then the perturbative unitarity argument can be adopted to *determine* the cutoff of the effective 4D theory.

Now, let us combine graviton KK and Higgs–radion exchanges. The effects are, of course, most dramatic if the $m_0/M_{\rm Pl}$ values are chosen near the lowest values for which unitarity is satisfied with KK graviton contributions only. So, for $s^{\rm max} = \Lambda_\phi^2$, we employ $m_0/M_{\rm Pl} = 0.070$ and for $s^{\rm max} = \widehat{\Lambda}_W^2$, we use $m_0/M_{\rm Pl} = 0.019$. The limits on ξ for $\Lambda_\phi = 2$ TeV and 5 TeV are displayed in Fig. 3, where $a_{0,1,2}$ are all required to be within unitarity limits.

[4] This choice of the highest KK modes included in the sum is somewhat arbitrary. However, it turns out that the major part of the result comes from the lowest KK states so that the precise point at which the KK sum is terminated is not very relevant.

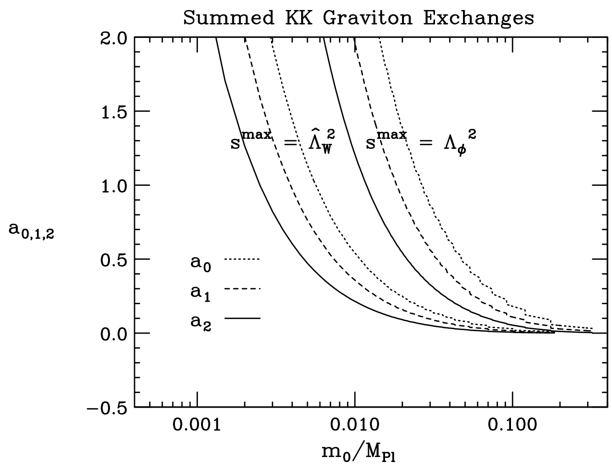

Fig. 2. The amplitudes $a_{0,1,2}(s = \Lambda_\phi^2)$ and $a_{0,1,2}(s = \widehat{\Lambda}_W^2)$ are plotted as functions of $m_0/M_{\rm Pl}$, after summing: $a_J(s) = \sum_{n, 2m_n < \sqrt{s}} a_J(m_G = m_n, s)$. The plotted values of a_i terminate when $m_0/M_{\rm Pl}$ is such that $2m_1$ exceeds $\Lambda_\phi = \sqrt{3}\widehat{\Lambda}_W$ or $\widehat{\Lambda}_W$, for the two respective s values above. Higgs–radion terms are not included.

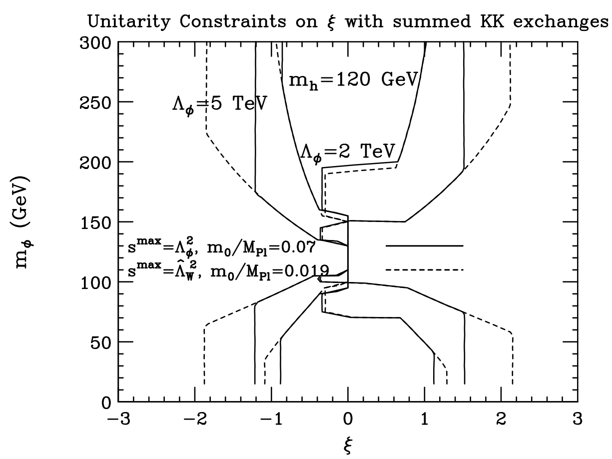

Fig. 3. The unitarity limits on ξ after summing over all KK graviton excitations with $2m_n < \sqrt{s}$.

3. Discussion and conclusions

We have discussed perturbative unitarity for $W_L^+ W_L^- \to W_L^+ W_L^-$ within the Randall–Sundrum theory with two 3-branes and shown that the exchange of massive 4D Kaluza–Klein gravitons leads to amplitudes growing linearly with the CM energy squared. We have found that the gravitational contributions can cause a violation of unitarity for s below the cutoff of the theory if the curvature, m_0, of the RS background metric is too small. On the other hand, m_0, must be small enough relative to the 5D Planck mass, $M_{\mathrm{Pl}\,5}$ in order to avoid quantum gravitational effects. Consequently, there is only a small range of m_0/M_{Pl} for which the model is fully consistent. The width of the window is determined by the size of the 4D cutoff of the theory. The extension of the Randall–Sundrum scenario to include the curvature-Higgs mixing was also considered. We found that the limits on ξ derived from unitarity could be altered by the inclusion of the full tower of KK graviton exchanges for certain choices of m_0/M_{Pl}.

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