

The flavour symmetry S_3 and the neutrino mass matrix with two texture zeroes

F. González Canales* and **A. Mondragón****

Instituto de Física, Universidad Nacional Autónoma de México, 04510, México D.F., México

E-mail: *ffelix@fisica.unam.mx, **mondra@fisica.unam.mx

Abstract. In this work, discuss neutrino masses and mixings in the framework of a minimal S_3 symmetric extension of the Standard Model. In this model, the mass matrices of all fermions take the same generic form with two texture zeroes. The mass matrices of the neutrinos and charged leptons are re-parameterized in terms of their eigenvalues, then the neutrino mixing matrix, V_{PMNS} , is computed and exact, explicit analytical expressions for the neutrino mixing angles as functions of the masses of neutrinos and charged leptons are obtained in excellent agreement with the latest experimental data. We also compute the branching ratios of some selected flavour-changing neutral current (FCNC) processes, as well as the contribution of the exchange of neutral flavour-changing scalars to the anomaly of the magnetic moment of the muon, as functions of the masses of charged leptons and the neutral Higgs bosons. We find that the $S_3 \times Z_2$ flavour symmetry and the strong mass hierarchy of the charged leptons strongly suppress the FCNC processes in the leptonic sector, well below the present experimental bounds by many orders of magnitude. The contribution of FCNC's to the anomaly of the muon's magnetic moment is small, but not negligible.

1. Introduction

The observation of flavour oscillations of solar, atmospheric, reactor, and accelerator neutrinos established that they have non-vanishing masses and mix among themselves, much like the quarks do [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. This discovery brought out very forcibly the need of extending the Standard Model (SM) in order to accomodate in the theory the new data on neutrino physics in a consistent way that would allow for a unified and systematic treatment of the observed hierarchy of masses and mixings of all fermions. At the same time, the number of free parameters in the extended form of the SM had to be drastically reduced in order to give predictive power to the theory. These two seemingly contradictory demands are met by a flavour symmetry under which the families transform in a non-trivial fashion. The observed pattern of neutrino mixing and, in particular, the non vanishing and sizeable value of the reactor mixings angle strongly suggest a flavour permutational symmetry.

The result of a combined analysis of all available neutrino oscillation data, including the recent results from long-baseline $\nu_\mu \rightarrow \nu_e$ searches at the Tokai to Kamioka (T2K) and Main Injector Neutrino Oscillation Search (MINOS) experiments, give the following values the difference of the squared neutrino masses and the mixing angles in the lepton mixing matrix, U_{PMNS} , at 1σ

confidence level [21]:

$$\Delta m_{21}^2 = 7.59_{-0.18}^{+0.20} \times 10^{-5} \text{ eV}^2, \quad (1)$$

$$\Delta m_{31}^2 = \begin{cases} -2.40_{-0.09}^{+0.08} \times 10^{-3} \text{ eV}^2, & (m_{\nu_2} > m_{\nu_1} > m_{\nu_3}). \\ +2.50_{-0.16}^{+0.09} \times 10^{-3} \text{ eV}^2, & (m_{\nu_3} > m_{\nu_2} > m_{\nu_1}). \end{cases}$$

$$\sin^2 \theta_{12}^l = 0.312_{-0.015}^{+0.017}, \quad \sin^2 \theta_{23}^l = \begin{cases} 0.52 \pm 0.06, \\ 0.52_{-0.07}^{+0.06} \end{cases}, \quad \sin^2 \theta_{13}^l = \begin{cases} 0.016_{-0.006}^{+0.008}, \\ 0.013_{-0.005}^{+0.007} \end{cases}, \quad (2)$$

the upper (lower) row corresponds to inverted (normal) neutrino mass hierarchy.

In the last ten years, important theoretical advances have been made in the understanding of the mechanisms for the mass fermion generation and flavour mixing. A phenomenologically and theoretically meaningful approach for reducing the number of free parameters in the Standard Model is the imposition of texture zeroes [22, 23] and/or flavour symmetries. For a recent review of flavour symmetry models see [24]. Also, certain texture zeroes may be obtained from a flavour symmetry [25, 26]. In the case of the Minimal S_3 -Invariant Extension of the Standard Model [27, 28, 29, 30, 31, 32, 33], the concept of flavour and generations is extended to the Higgs sector in such a way that all the matter fields – Higgs, quarks, and lepton fields, including the right-handed neutrino fields – have three species and therefore transform under the flavour symmetry group as the three dimensional representation $1 \oplus 2$ of the permutational group S_3 . A model with more than one Higgs $SU(2)$ doublet has tree level flavour changing neutral currents whose exchange may give rise to lepton flavour violating processes and may also contribute to the anomalous magnetic moment of the muon. An effective test of the phenomenological success of the model is obtained by verifying that all flavour changing neutral current processes and the magnetic anomaly of the muon, computed in the S_3 -Invariant extended form of the Standard Model, agree with the experimental values.

2. The Minimal S_3 -invariant Extension of the Standard Model

In the Standard Model analogous fermions in different generations have identical couplings to all gauge bosons of the strong, weak and electromagnetic interactions. Prior to the introduction of the Higgs boson and mass terms, the Lagrangian is chiral and invariant with respect to permutations of the left and right fermionic fields.

The six possible permutations of three objects (f_1, f_2, f_3) are elements of the permutational group S_3 . This is the discrete, non-Abelian group with the smallest number of elements. The three-dimensional real representation is not an irreducible representation of S_3 . It can be decomposed into the direct sum of a doublet f_D and a singlet f_s , where

$$f_s = \frac{1}{\sqrt{3}}(f_1 + f_2 + f_3), \quad f_D^T = \left(\frac{1}{\sqrt{2}}(f_1 - f_2), \frac{1}{\sqrt{6}}(f_1 + f_2 - 2f_3) \right). \quad (3)$$

The direct product of two doublets $\mathbf{p}_D^T = (p_{D1}, p_{D2})$ and $\mathbf{q}_D^T = (q_{D1}, q_{D2})$ may be decomposed into the direct sum of two singlets \mathbf{r}_s and $\mathbf{r}_{s'}$, and one doublet \mathbf{r}_D^T where

$$\mathbf{r}_s = p_{D1}q_{D1} + p_{D2}q_{D2}, \quad \mathbf{r}_{s'} = p_{D1}q_{D2} - p_{D2}q_{D1}, \quad (4)$$

$$\mathbf{r}_D^T = (r_{D1}, r_{D2}) = (p_{D1}q_{D2} + p_{D2}q_{D1}, p_{D1}q_{D1} - p_{D2}q_{D2}). \quad (5)$$

The antisymmetric singlet $\mathbf{r}_{s'}$ is not invariant under S_3 .

Since the Standard Model has only one Higgs $SU(2)_L$ doublet, which can only be an S_3 singlet, it can only give mass to the quark or charged lepton in the S_3 singlet representation, one in each family, without breaking the S_3 symmetry.

Hence, in order to impose S_3 as a fundamental symmetry, unbroken at the Fermi scale, we are led to extend the Higgs sector of the theory. The quark, lepton and Higgs fields are

$$Q^T = (u_L, d_L), \quad u_R, \quad d_R, \quad L^T = (\nu_L, e_L), \quad e_R, \quad \nu_R \quad \text{and} \quad H, \quad (6)$$

in an obvious notation. All of these fields have three species, and we assume that each one forms a reducible representation $\mathbf{1}_S \oplus \mathbf{2}$. The doublets carry capital indices I and J , which run from 1 to 2, and the singlets are denoted by Q_3 , u_{3R} , d_{3R} , L_3 , e_{3R} , ν_{3R} and H_S . Note that the subscript 3 denotes the singlet representation and not the third generation. The most general renormalizable Yukawa interactions of this model are given by

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}, \quad (7)$$

where

$$\begin{aligned} \mathcal{L}_{Y_D} = & -Y_1^d \bar{Q}_I H_S d_{IR} - Y_3^d \bar{Q}_3 H_S d_{3R} - Y_2^d [\bar{Q}_I \kappa_{IJ} H_1 d_{JR} + \bar{Q}_I \eta_{IJ} H_2 d_{JR}] \\ & - Y_4^d \bar{Q}_3 H_I d_{IR} - Y_5^d \bar{Q}_I H_I d_{3R} + \text{h.c.}, \end{aligned} \quad (8)$$

$$\begin{aligned} \mathcal{L}_{Y_U} = & -Y_1^u \bar{Q}_I (i\sigma_2) H_S^* u_{IR} - Y_3^u \bar{Q}_3 (i\sigma_2) H_S^* u_{3R} - Y_4^u \bar{Q}_3 (i\sigma_2) H_I^* u_{IR} \\ & - Y_2^u [\bar{Q}_I \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \bar{Q}_I \eta_{IJ} (i\sigma_2) H_2^* u_{JR}] - Y_5^u \bar{Q}_I (i\sigma_2) H_I^* u_{3R} + \text{h.c.}, \end{aligned} \quad (9)$$

$$\begin{aligned} \mathcal{L}_{Y_E} = & -Y_1^e \bar{L}_I H_S e_{IR} - Y_3^e \bar{L}_3 H_S e_{3R} - Y_2^e [\bar{L}_I \kappa_{IJ} H_1 e_{JR} + \bar{L}_I \eta_{IJ} H_2 e_{JR}] \\ & - Y_4^e \bar{L}_3 H_I e_{IR} - Y_5^e \bar{L}_I H_I e_{3R} + \text{h.c.}, \end{aligned} \quad (10)$$

$$\begin{aligned} \mathcal{L}_{Y_\nu} = & -Y_1^\nu \bar{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \bar{L}_3 (i\sigma_2) H_S^* \nu_{3R} - Y_4^\nu \bar{L}_3 (i\sigma_2) H_I^* \nu_{IR} \\ & - Y_2^\nu [\bar{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \bar{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR}] - Y_5^\nu \bar{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{h.c.}, \end{aligned} \quad (11)$$

and

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (12)$$

Furthermore, we add to the Lagrangian the Majorana mass terms for the right-handed neutrinos

$$\mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_2 \nu_{3R}^T C \nu_{3R} - M_3 \nu_{3R}^T C \nu_{3R}. \quad (13)$$

Due to the presence of three Higgs fields, the Higgs potential $V_H(H_S, H_D)$ is more complicated than that of the Standard Model. This potential was analyzed by Pakvasa and Sugawara [34] who found that in addition to the S_3 symmetry, it has a permutational symmetry S'_2 : $H_1 \leftrightarrow H_2$, which is not a subgroup of the flavour group S_3 . In this communication, we will assume that the vacuum respects the accidental S'_2 symmetry of the Higgs potential and that $\langle H_1 \rangle = \langle H_2 \rangle$. With these assumptions, the Yukawa interactions, eqs. (8)-(11) yield mass matrices, for all fermions in the theory, of the general form [27]

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}. \quad (14)$$

The left-handed Majorana neutrinos ν_L naturally acquire their small masses through the see-saw mechanism type I of the form

$$\mathbf{M}_\nu = \mathbf{M}_{\nu_D} \tilde{\mathbf{M}}_R^{-1} (\mathbf{M}_{\nu_D})^T, \quad (15)$$

—	+
H_S, ν_{3R}	$H_I, L_3, L_I, e_{3R}, e_{IR}, \nu_{IR}$

Table 1. Z_2 assignment in the leptonic sector.

where \mathbf{M}_{ν_D} and $\tilde{\mathbf{M}}_R$ denote the Dirac and right handed Majorana neutrino mass matrices, respectively.

In principle, all entries in the mass matrices can be complex since there is no restriction coming from the flavour symmetry S_3 . The mass matrices are diagonalized by bi-unitary transformations as

$$\begin{aligned} U_{d(u,e)L}^\dagger \mathbf{M}_{d(u,e)} U_{d(u,e)R} &= \text{diag}(m_{d(u,e)}, m_{s(c,\mu)}, m_{b(t,\tau)}), \\ U_\nu^T \mathbf{M}_\nu U_\nu &= \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \end{aligned} \quad (16)$$

The entries in the diagonal matrices may be complex, so the physical masses are their absolute values.

The quark and lepton flavor mixing matrices, V_{PMNS} and V_{CKM} , arise from the mismatch between diagonalization of the mass matrices of u and d type quarks and the diagonalization of the mass matrices of charged leptons and left-handed neutrinos, respectively,

$$V_{CKM} = U_{uL}^\dagger U_{dL}, \quad V_{PMNS} = U_{eL}^\dagger U_\nu K, \quad (17)$$

where K is the diagonal matrix of the Majorana phase factors. Therefore, in order to obtain the unitary matrices appearing in eq. (17) and get predictions for the flavor mixing angles and CP violating phases, we should specify the mass matrices.

3. The mass matrices in the leptonic sector and Z_2 symmetry

A further reduction of the number of parameters in the leptonic sector may be achieved by means of an Abelian Z_2 symmetry. A possible set of charge assignments of Z_2 , compatible with the experimental data on masses and mixings in the leptonic sector is given in Table 1.

These Z_2 assignments forbid the following Yukawa couplings

$$Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0. \quad (18)$$

Therefore, the corresponding entries in the mass matrices vanish, *i.e.*, $\mu_1^e = \mu_3^e = 0$ and $\mu_1^\nu = \mu_5^\nu = 0$.

3.1. The mass matrix of the charged leptons

The mass matrix of the charged leptons takes the form

$$M_e = m_\tau \begin{pmatrix} \tilde{\mu}_2 & \tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_2 & -\tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_4 & \tilde{\mu}_4 & 0 \end{pmatrix}. \quad (19)$$

The unitary matrix U_{eL} that enters in the definition of the mixing matrix, V_{PMNS} , is calculated from

$$U_{eL}^\dagger M_e M_e^\dagger U_{eL} = \text{diag}(m_e^2, m_\mu^2, m_\tau^2), \quad (20)$$

where m_e , m_μ and m_τ are the masses of the charged leptons, and

$$\frac{M_e M_e^\dagger}{m_\tau^2} = \begin{pmatrix} 2|\tilde{\mu}_2|^2 + |\tilde{\mu}_5|^2 & |\tilde{\mu}_5|^2 & 2|\tilde{\mu}_2||\tilde{\mu}_4|e^{-i\delta_e} \\ |\tilde{\mu}_5|^2 & 2|\tilde{\mu}_2|^2 + |\tilde{\mu}_5|^2 & 0 \\ 2|\tilde{\mu}_2||\tilde{\mu}_4|e^{i\delta_e} & 0 & 2|\tilde{\mu}_4|^2 \end{pmatrix}. \quad (21)$$

Notice that this matrix has only one non-ignorable phase factor [29].

Once $M_e M_e^\dagger$ has been reparametrized in terms of the charged lepton masses, it is straightforward to compute M_e and U_{eL} also as functions of the charged lepton masses [29]. The resulting expression for M_e , written to order $(m_\mu m_e/m_\tau^2)^2$ and $x^4 = (m_e/m_\mu)^4$ is

$$M_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & 0 \end{pmatrix}. \quad (22)$$

This approximation is numerically exact up to order 10^{-9} in units of the τ mass. Notice that this matrix has no free parameters other than the Dirac phase δ_e .

The unitary matrix U_{eL} that diagonalizes $M_e M_e^\dagger$ and enters in the definition of the neutrino mixing matrix V_{PMNS} may be written as

$$U_{eL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_e} \end{pmatrix} \begin{pmatrix} O_{11} & -O_{12} & O_{13} \\ -O_{21} & O_{22} & O_{23} \\ -O_{31} & -O_{32} & O_{33} \end{pmatrix}, \quad (23)$$

where the orthogonal matrix \mathbf{O}_{eL} in the right hand side of eq. (23), written to the same order of magnitude as M_e , is

$$\mathbf{O}_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} x \frac{(1+2\tilde{m}_\mu^2+4x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -\frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} x \frac{(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & \frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} (1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2) & -x \frac{(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2) \sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{\sqrt{1+x^2}\tilde{m}_e\tilde{m}_\mu}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix}, \quad (24)$$

where, as before, $\tilde{m}_\mu = m_\mu/m_\tau$, $\tilde{m}_e = m_e/m_\tau$ and $x = m_e/m_\mu$.

The mass values of the charged lepton masses [35]:

$$m_e = 0.51099891 \text{ MeV}, \quad m_\mu = 105.658367 \text{ MeV}, \quad \text{and} \quad m_\tau = 1776.82 \text{ MeV}, \quad (25)$$

we obtain

$$\mathbf{O}_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} \frac{\tilde{m}_e}{\tilde{m}_\mu} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \frac{\tilde{m}_e}{\tilde{m}_\mu} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -1 & -\frac{\tilde{m}_e}{\tilde{m}_\mu} & 0 \end{pmatrix} + \mathcal{O}(10^{-5}), \quad (26)$$

which can be written in following form:

$$\mathbf{O}_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{\tilde{m}_e}{\tilde{m}_\mu} & -1 & 0 \\ 1 & -\frac{\tilde{m}_e}{\tilde{m}_\mu} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (27)$$

The second matrix on the right side is a numerically exact up to order $\mathcal{O}(10^{-7})$ in units of the τ mass.

3.2. The mass matrix of the neutrinos

In the minimal S_3 -invariant extension of Standard Model, the Yukawa interactions and the $S_3 \times Z_2$ flavour symmetry yield a mass matrix for the Dirac neutrinos of the form

$$\mathbf{M}_{\nu_D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}. \quad (28)$$

In principle, all entries in the mass matrix \mathbf{M}_{ν_D} can be complex, since there is no restriction coming from the $S_3 \times Z_2$ flavour symmetry.

The mass of the left-handed Majorana neutrinos, \mathbf{M}_ν , are generated by the see-saw mechanism type I,

$$\mathbf{M}_\nu = \mathbf{M}_{\nu_D} \tilde{\mathbf{M}}_R^{-1} (\mathbf{M}_{\nu_D})^T, \quad (29)$$

where $\tilde{\mathbf{M}}_R$ is the mass matrix of the right-handed neutrinos, which we take to be real and diagonal but non-degenerate

$$\tilde{\mathbf{M}}_R = \text{diag}(M_1, M_2, M_3). \quad (30)$$

Then, the mass matrix \mathbf{M}_ν takes the form

$$\mathbf{M}_\nu = \begin{pmatrix} \frac{2(\mu_2^\nu)^2}{\bar{M}} & \frac{2\lambda(\mu_2^\nu)^2}{\bar{M}} & \frac{2\mu_2^\nu\mu_4^\nu}{\bar{M}} \\ \frac{2\lambda(\mu_2^\nu)^2}{\bar{M}} & \frac{2(\mu_2^\nu)^2}{\bar{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\bar{M}} \\ \frac{2\mu_2^\nu\mu_4^\nu}{\bar{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\bar{M}} & \frac{2(\mu_4^\nu)^2}{\bar{M}} + \frac{(\mu_3^\nu)^2}{M_3} \end{pmatrix} \quad (31)$$

where

$$\lambda = \frac{1}{2} \left(\frac{M_2 - M_1}{M_1 + M_2} \right) \quad \text{and} \quad \bar{M} = 2 \frac{M_1 M_2}{M_2 + M_1} \quad (32)$$

When the first two right-handed neutrino masses are equal, the parameter λ vanishes and we recover the expression for \mathbf{M}_ν given in Kubo *et al* [27].

As we have assumed the the right-handed neutrino mass matrix $\tilde{\mathbf{M}}_R$ to be real, the complex symmetric neutrino mass matrix \mathbf{M}_ν has only three independent phase factor that come from the parameters μ_2 , μ_3 and μ_4 . Here, to simplify the analysis we will consider the case when $\arg\{\mu_2^\nu\} = \arg\{\mu_3^\nu\}$ o $2\arg\{\mu_4^\nu\} = \arg\left\{2\frac{(\mu_4^\nu)^2}{\bar{M}} + \frac{(\mu_3^\nu)^2}{M_3}\right\}$. The general case, with three independent phase factors, will be considered in detail elsewhere.

In the case considered here, the phase factors may be taken out of \mathbf{M}_ν as

$$\mathbf{M}_\nu = Q \overline{\mathbf{M}}_\nu Q \quad (33)$$

where

$$Q = \begin{pmatrix} e^{i\phi_2} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_4} \end{pmatrix} \quad \text{and} \quad \overline{\mathbf{M}}_\nu = \begin{pmatrix} a & d & b \\ d & a & e \\ b & e & c \end{pmatrix} \quad (34)$$

with $\phi_2 = \arg \{\mu_2^\nu\}$, $\phi_4 = \arg \{\mu_4^\nu\}$, $a = \frac{2|\mu_2^\nu|^2}{|\overline{M}|}$, $b = \frac{2|\mu_4^\nu||\mu_2^\nu|}{|\overline{M}|}$, $c = \frac{2|\mu_4^\nu|^2}{\overline{M}} + \frac{|\mu_3^\nu|^2}{M_3}$, $d = \frac{2|\lambda||\mu_2^\nu|^2}{|\overline{M}|}$ and $e = \frac{2|\mu_2^\nu||\mu_4^\nu||\lambda|}{|\overline{M}|}$.

The real symmetric matrix $\overline{\mathbf{M}}_\nu$ may be brought to diagonal form by means of a similarity transformation with an orthogonal matrix \mathbf{O}_ν as

$$\overline{\mathbf{M}}_\nu = \mathbf{O}_\nu \text{diag} \{m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\} \mathbf{O}_\nu^T, \quad (35)$$

the columns in \mathbf{O}_ν are the normalized eigenvectors of $\overline{\mathbf{M}}_\nu$.

In order to compute \mathbf{O}_ν we notice that the diagonalization of $\overline{\mathbf{M}}_\nu$ is equivalent to the diagonalization of a mass matrix $\widehat{\mathbf{M}}$ with two texture zeroes.

First define a new mass matrix $\overline{\mathbf{M}}_\nu'$ obtained from $\overline{\mathbf{M}}_\nu$ by a $\frac{\pi}{4}$ rotation

$$\mathcal{U}_{\frac{\pi}{4}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}, \quad (36)$$

through the similarity transformation $\overline{\mathbf{M}}_\nu' = \mathcal{U}_{\frac{\pi}{4}}^\dagger \overline{\mathbf{M}}_\nu \mathcal{U}_{\frac{\pi}{4}}$. Then, the matrix $\overline{\mathbf{M}}_\nu'$ can be written in the following form:

$$\overline{\mathbf{M}}_\nu' = \mu_0 \mathbb{I}_{3 \times 3} + \widehat{\mathbf{M}} \quad (37)$$

where $\mathbb{I}_{3 \times 3}$ is the identity matrix,

$$\mu_0 = a - d = \frac{2|\mu_2^\nu|^2}{|\overline{M}|} (1 - |\lambda|) \quad \text{and} \quad \widehat{\mathbf{M}} = \begin{pmatrix} 0 & A & 0 \\ A & B & C \\ 0 & C & D \end{pmatrix} \quad (38)$$

with $A = \frac{b-e}{\sqrt{2}}$, $B = c + d - a$, $C = \frac{b+e}{\sqrt{2}}$ and $D = 2d$. As mentioned before the diagonalization of $\overline{\mathbf{M}}_\nu$ is reduced to diagonalization of the real symmetric matrix $\widehat{\mathbf{M}}$, which is a matrix with two texture zeroes of class I [25].

In fact, we can obtain a more general result by means of the similarity transformation, $\mathbf{M}' = \mathcal{R}_{\frac{\pi}{4}}^\dagger \mathbf{M} \mathcal{R}_{\frac{\pi}{4}}$, to applied to the general form of the mass matrices given in eq. (14), *i. e.*,

$$\mathbf{M}' = \mathcal{R}_{\frac{\pi}{4}}^\dagger \mathbf{M} \mathcal{R}_{\frac{\pi}{4}} = \widetilde{\mu} \mathbb{I}_{3 \times 3} + \widetilde{M} \quad (39)$$

where $\widetilde{\mu} = \mu_2 - \mu_1$,

$$\mathcal{R}_{\frac{\pi}{4}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \widetilde{M} = \begin{pmatrix} 0 & \mu_2 & 0 \\ \mu_2 & 2\mu_1 & \sqrt{2}\mu_5 \\ 0 & \sqrt{2}\mu_4 & \mu_3 + \mu_1 - \mu_2 \end{pmatrix}. \quad (40)$$

The basis in which the neutrino mass matrix takes the form (40) is a basis in the space of fermion flavors in which the mass matrices have the same form with two texture zeros for all fermions in the theory. However, this property does not imply special relations among mass eigenvalues and flavor mixing parameters. [36].

It is well known [37, 38] that in the hadronic sector the masses of quarks may be obtained from a matrix with two texture zeroes which successfully reproduces the strong mass hierarchy of up and down type quarks. Also, the numerical values of the quark mixing angles determined in this framework are in good agreement with the experimental data [26]. Additionally, in a unified treatment in which the mass matrices of all fermions have a similar form with two texture zeroes class I and a normal hierarchy, the numerical values obtained for masses and mixing of the neutrinos are in very good agreement with all available experimental data [26, 23]. Therefore, we expect that the mixing matrix $PMNS$ that will be obtained from the mass matrices \mathbf{M}_ν and M_e could reproduce the current experimental data of the masses and mixings in the leptonic sector of the theory.

As in the case of the charged leptons, the matrices \mathbf{M}_ν and U_ν can be reparametrized in terms of the neutrino masses. For this we use the information that we already have about the diagonalization of a matrix with two texture zeroes of class I [25, 26, 37, 38]. Then, the mass matrix \mathbf{M}_ν for a Normal [Inverted] hierarchy in the mass spectrum takes the form

$$\mathbf{M}_\nu^{N[I]} = \begin{pmatrix} \mu_0 + d & d & \frac{1}{\sqrt{2}} (C^{N[I]} + A^{N[I]}) \\ d & \mu_0 + d & \frac{1}{\sqrt{2}} (C^{N[I]} - A^{N[I]}) \\ \frac{1}{\sqrt{2}} (C^{N[I]} + A^{N[I]}) & \frac{1}{\sqrt{2}} (C^{N[I]} - A^{N[I]}) & m_{\nu_1} + m_{\nu_2} + m_{\nu_3} - 2(\mu_0 + d) \end{pmatrix} \quad (41)$$

where

$$A^{N[I]} = \sqrt{\frac{(m_{\nu_2} - \mu_0)(m_{\nu_{3[1]}} - \mu_0)(\mu_0 - m_{\nu_{1[3]}})}{2d}}, \quad (42)$$

$$C^{N[I]} = \sqrt{\frac{(2d + \mu_0 - m_{\nu_1})(2d + \mu_0 - m_{\nu_{2[3]}})(m_{\nu_{3[2]}} - \mu_0 - 2d)}{2d}}. \quad (43)$$

Also, the values allowed for the parameters μ_0 and $2d + \mu_0$ are in the following ranges

$$m_{\nu_{2[1]}} > \mu_0 > m_{\nu_{1[3]}} \quad \text{and} \quad m_{\nu_{3[2]}} > 2d + \mu_0 > m_{\nu_{2[1]}}. \quad (44)$$

Now, the unitary matrix U_ν takes the following form:

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_\nu} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} O_{11}^{N[I]} & O_{12}^{N[I]} & O_{13}^{N[I]} \\ O_{21}^{N[I]} & O_{22}^{N[I]} & O_{23}^{N[I]} \\ O_{31}^{N[I]} & O_{32}^{N[I]} & O_{33}^{N[I]} \end{pmatrix} \quad (45)$$

where $\delta_\nu = \phi_2 - \phi_4$ and $\mathbb{O}^{N[I]}$ is the matrix that diagonalizes $\widehat{\mathbf{M}}$. Explicit expressions for the elements of the orthogonal matrix $\mathbb{O}^{N[I]}$ reparametrized in terms of the neutrino masses are

given in table 2, but in order to avoid a confused notation the dependence of $O_{ij}^{N[I]}$ on neutrinos masses is given in terms of the expresions:

$$\begin{aligned} \mathcal{D}_1^{N[I]} &= 2d(m_{\nu_2} - m_{\nu_1}) (m_{\nu_{3[1]}} - m_{\nu_{1[3]}}), & \mathcal{D}_2^{N[I]} &= 2d(m_{\nu_2} - m_{\nu_1}) (m_{\nu_{3[2]}} - m_{\nu_{2[3]}}), \\ \mathcal{D}_3^{N[I]} &= 2d(m_{\nu_{3[1]}} - m_{\nu_{1[3]}}) (m_{\nu_{3[2]}} - m_{\nu_{2[3]}}), & f_1 &= (2d + \mu_0 - m_{\nu_1}), \\ f_2^{N[I]} &= [-1] (2d + \mu_0 - m_{\nu_2}), & f_3^{N[I]} &= [-1] (m_{\nu_3} - \mu_0 - 2d). \end{aligned} \quad (46)$$

see table 2. The superscripts N and I denote the normal and inverted hierarchy respectively.

4. The neutrino mixing matrix

The neutrino mixing matrix V_{PMNS} , is the product $U_{eL}^\dagger U_\nu K$, where K is the diagonal matrix of the Majorana phase factors, defined by $K = \text{diag}(1, e^{i\alpha}, e^{i\beta})$. Now, with the help of eqs. (23), (27) and (45), we obtain the theoretical expression of the elements of the lepton mixing matrix, V_{PMNS}^{th} . This expression has the following form:

$$V_{PMNS}^{th} = \begin{pmatrix} V_{e1}^{th} & V_{e2}^{th} e^{i\alpha} & V_{e3}^{th} e^{i\beta} \\ V_{\mu 1}^{th} & V_{\mu 2}^{th} e^{i\alpha} & V_{\mu 3}^{th} e^{i\beta} \\ V_{\tau 1}^{th} & V_{\tau 2}^{th} e^{i\alpha} & V_{\tau 3}^{th} e^{i\beta} \end{pmatrix} \quad (47)$$

where

$$\begin{aligned} V_{e1}^{th} &= \frac{\tilde{m}_e}{\tilde{m}_\mu} O_{11}^{N[I]} - O_{21}^{N[I]} e^{i\delta_l}, & V_{e2}^{th} &= \frac{\tilde{m}_e}{\tilde{m}_\mu} O_{12}^{N[I]} - O_{22}^{N[I]} e^{i\delta_l}, \\ V_{e3}^{th} &= \frac{\tilde{m}_e}{\tilde{m}_\mu} O_{13}^{N[I]} - O_{23}^{N[I]} e^{i\delta_l}, & V_{\mu 1}^{th} &= -O_{11}^{N[I]} - \frac{\tilde{m}_e}{\tilde{m}_\mu} O_{21}^{N[I]} e^{i\delta_l}, \\ V_{\mu 2}^{th} &= -O_{12}^{N[I]} - \frac{\tilde{m}_e}{\tilde{m}_\mu} O_{22}^{N[I]} e^{i\delta_l}, & V_{\mu 3}^{th} &= -O_{13}^{N[I]} - \frac{\tilde{m}_e}{\tilde{m}_\mu} O_{23}^{N[I]} e^{i\delta_l}, \\ V_{\tau 1}^{th} &= O_{31}^{N[I]}, & V_{\tau 2}^{th} &= O_{32}^{N[I]}, & V_{\tau 3}^{th} &= O_{33}^{N[I]} \end{aligned} \quad (48)$$

with $\delta_l = \delta_\nu - \delta_e$, the elements O^e and $O^{N[I]}$ in the eqs. (48) are given in the eq. (24) and the table 2, respectively.

4.1. The Mixing Angles

In the standard PDG parametrization, the entries in the lepton mixing matrix are parametrized in terms of the mixing angles and phases. Thus, the mixing angles are related to the observable moduli of lepton V_{PMNS} through the relations:

$$\sin^2 \theta_{12}^l = \frac{|V_{e2}|^2}{1 - |V_{e3}|^2}, \quad \sin^2 \theta_{23}^l = \frac{|V_{\mu 3}|^2}{1 - |V_{e3}|^2}, \quad \sin^2 \theta_{13}^l = |V_{e3}|^2. \quad (49)$$

Then, theoretical expression for the lepton mixing angles as functions of the lepton mass ratios are readily obtained when the theoretical expressions for the modulii of the entries in the $PMNS$ mixing matrix, given in eqs. (48), are substituted for $|V_{ij}|$ in the right hand side of eqs. (49).

$$\sin^2 \theta_{12}^l = \frac{\left(\frac{\tilde{m}_e}{\tilde{m}_\mu}\right)^2 \left(O_{12}^{N[I]}\right)^2 + \left(O_{22}^{N[I]}\right)^2 - 2\frac{\tilde{m}_e}{\tilde{m}_\mu} O_{12}^{N[I]} O_{22}^{N[I]} \cos \delta_l}{1 - \left(\frac{\tilde{m}_e}{\tilde{m}_\mu}\right)^2 \left(O_{13}^{N[I]}\right)^2 - \left(O_{23}^{N[I]}\right)^2 + 2\frac{\tilde{m}_e}{\tilde{m}_\mu} O_{13}^{N[I]} O_{23}^{N[I]} \cos \delta_l} \quad (50)$$

Set	Elements O_{ij}
I	$O_{11}^{N[I]} = -\sqrt{\frac{[-1](m_{\nu_3}-\mu_0)(m_{\nu_2}-\mu_0)f_1}{\mathcal{D}_1^{N[I}}}$ $O_{13}^{N[I]} = \sqrt{\frac{[-1](\mu_0-m_{\nu_1})(m_{\nu_2}-\mu_0)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}}$ $O_{22}^{N[I]} = \sqrt{\frac{2d(m_{\nu_2}-\mu_0)f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}}$ $O_{31}^{N[I]} = -\sqrt{\frac{[-1](\mu_0-m_{\nu_1})f_2^{N[I]}f_3^{N[I]}}{\mathcal{D}_1^{N[I]}}}$ $O_{33}^{N[I]} = [-1]\sqrt{\frac{(m_{\nu_3}-\mu_0)f_1f_2^{N[I]}}{\mathcal{D}_1^{N[I]}}}$ $O_{12}^{N[I]} = \sqrt{\frac{(m_{\nu_3[1]}-\mu_0)(\mu_0-m_{\nu_1[3]})f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}}$ $O_{21}^{N[I]} = \sqrt{\frac{[-1]2d(\mu_0-m_{\nu_1})f_1}{\mathcal{D}_1^{N[I]}}}$ $O_{23}^{N[I]} = \sqrt{\frac{[-1]2d(m_{\nu_3}-\mu_0)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}}$ $O_{32}^{N[I]} = -1[+1]\sqrt{\frac{(m_{\nu_2}-\mu_0)f_1f_3^{N[I]}}{\mathcal{D}_1^{N[I]}}}$
II	$O_{11}^{N[I]} = \sqrt{\frac{[-1](m_{\nu_3}-\mu_0)(m_{\nu_2}-\mu_0)f_1}{\mathcal{D}_1^{N[I}}}$ $O_{13}^{N[I]} = \sqrt{\frac{[-1](\mu_0-m_{\nu_1})(m_{\nu_2}-\mu_0)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}}$ $O_{22}^{N[I]} = \sqrt{\frac{2d(m_{\nu_2}-\mu_0)f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}}$ $O_{31}^{N[I]} = -1[+1]\sqrt{\frac{[-1](\mu_0-m_{\nu_1})f_2^{N[I]}f_3^{N[I]}}{\mathcal{D}_1^{N[I]}}}$ $O_{33}^{N[I]} = [-1]\sqrt{\frac{(m_{\nu_3}-\mu_0)f_1f_2^{N[I]}}{\mathcal{D}_1^{N[I]}}}$ $O_{12}^{N[I]} = -\sqrt{\frac{(m_{\nu_3[1]}-\mu_0)(\mu_0-m_{\nu_1[3]})f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}}$ $O_{21}^{N[I]} = \sqrt{\frac{[-1]2d(\mu_0-m_{\nu_1})f_1}{\mathcal{D}_1^{N[I]}}}$ $O_{23}^{N[I]} = \sqrt{\frac{[-1]2d(m_{\nu_3}-\mu_0)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}}$ $O_{32}^{N[I]} = -\sqrt{\frac{(m_{\nu_2}-\mu_0)f_1f_3^{N[I]}}{\mathcal{D}_1^{N[I]}}}$
III	$O_{11}^{N[I]} = \sqrt{\frac{[-1](m_{\nu_3}-\mu_0)(m_{\nu_2}-\mu_0)f_1}{\mathcal{D}_1^{N[I}}}$ $O_{13}^{N[I]} = -\sqrt{\frac{[-1](\mu_0-m_{\nu_1})(m_{\nu_2}-\mu_0)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}}$ $O_{22}^{N[I]} = \sqrt{\frac{2d(m_{\nu_2}-\mu_0)f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}}$ $O_{31}^{N[I]} = -\sqrt{\frac{[-1](\mu_0-m_{\nu_1})f_2^{N[I]}f_3^{N[I]}}{\mathcal{D}_1^{N[I]}}}$ $O_{33}^{N[I]} = -\sqrt{\frac{(m_{\nu_3}-\mu_0)f_1f_2^{N[I]}}{\mathcal{D}_1^{N[I]}}}$ $O_{12}^{N[I]} = \sqrt{\frac{(m_{\nu_3[1]}-\mu_0)(\mu_0-m_{\nu_1[3]})f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}}$ $O_{21}^{N[I]} = \sqrt{\frac{[-1]2d(\mu_0-m_{\nu_1})f_1}{\mathcal{D}_1^{N[I]}}}$ $O_{23}^{N[I]} = \sqrt{\frac{[-1]2d(m_{\nu_3}-\mu_0)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}}$ $O_{32}^{N[I]} = \sqrt{\frac{(m_{\nu_2}-\mu_0)f_1f_3^{N[I]}}{\mathcal{D}_1^{N[I]}}}$

Table 2. Sets of $O_{ij}^{N[I]}$. The superscripts N and I denote the normal and inverted hierarchy respectively.

$$\sin^2 \theta_{23}^l = \frac{\left(O_{13}^{N[I]}\right)^2 + \left(\frac{\tilde{m}_e}{\tilde{m}_\mu}\right)^2 \left(O_{23}^{N[I]}\right)^2 + 2\frac{\tilde{m}_e}{\tilde{m}_\mu} O_{13}^{N[I]} O_{23}^{N[I]} \cos \delta_l}{1 - \left(\frac{\tilde{m}_e}{\tilde{m}_\mu}\right)^2 \left(O_{13}^{N[I]}\right)^2 - \left(O_{23}^{N[I]}\right)^2 + 2\frac{\tilde{m}_e}{\tilde{m}_\mu} O_{13}^{N[I]} O_{23}^{N[I]} \cos \delta_l} \quad (51)$$

$$\sin^2 \theta_{13}^l = \left(\frac{\tilde{m}_e}{\tilde{m}_\mu}\right)^2 \left(O_{13}^{N[I]}\right)^2 + \left(O_{23}^{N[I]}\right)^2 - 2\frac{\tilde{m}_e}{\tilde{m}_\mu} O_{13}^{N[I]} O_{23}^{N[I]} \cos \delta_l \quad (52)$$

where, as before, $\tilde{m}_\mu = m_\mu/m_\tau$ and $\tilde{m}_e = m_e/m_\tau$.

In a first, preliminary analysis for the mixing angle θ_{13}^l and for an inverted neutrino mass hierarchy ($m_{\nu_2} > m_{\nu_1} > m_{\nu_3}$) the eq. (52) takes the form:

$$\sin^2 \theta_{13}^l \approx \frac{(\mu_0 + 2d - m_{\nu_3})(\mu_0 - m_{\nu_3})}{(m_{\nu_1} - m_{\nu_3})(m_{\nu_2} - m_{\nu_3})}. \quad (53)$$

Now, with the following values for neutrino masses $m_{\nu_2} = 0.056\text{eV}$, $m_{\nu_1} = 0.053\text{eV}$ and $m_{\nu_3} = 0.048\text{eV}$, and the parameters values $\delta_l = \pi/2$, $\mu_0 = 0.049$ and $d = 8 \times 10^{-5}$ we get $\sin^2 \theta_{13}^l \approx 0.029 \rightarrow \theta_{13}^l \approx 9.8^\circ$. In a more complete analysis, where will be make a χ^2 fit of the exact theoretical expressions for the modulii of the entries of the lepton mixing matrix $|(V_{PMNS}^{th})_{ij}|$ to the experimental values (for example the values given by Gonzalez-Garcia [14]) will be considered in detail elsewhere.

5. Flavour Changing Neutral Currents (FCNC)

Models with more than one Higgs $SU(2)$ doublet have tree level flavour changing neutral currents. In the Minimal S_3 -invariant Extension of the Standard Model considered here, there is one Higgs $SU(2)$ doublet per generation coupling to all fermions. The flavour changing Yukawa couplings may be written in a flavour labelled, symmetry adapted weak basis as

$$\begin{aligned} \mathcal{L}_Y^{\text{FCNC}} = & (\bar{E}_{aL} Y_{ab}^{ES} E_{bR} + \bar{U}_{aL} Y_{ab}^{US} U_{bR} + \bar{D}_{aL} Y_{ab}^{DS} D_{bR}) H_S^0 \\ & + (\bar{E}_{aL} Y_{ab}^{E1} E_{bR} + \bar{U}_{aL} Y_{ab}^{U1} U_{bR} + \bar{D}_{aL} Y_{ab}^{D1} D_{bR}) H_1^0 + \\ & (\bar{E}_{aL} Y_{ab}^{E2} E_{bR} + \bar{U}_{aL} Y_{ab}^{U2} U_{bR} + \bar{D}_{aL} Y_{ab}^{D2} D_{bR}) H_2^0 + \text{h.c.} \end{aligned} \quad (54)$$

The Yukawa couplings of immediate physical interest in the computation of the flavour changing neutral currents are those defined in the mass basis, according to $\tilde{Y}_m^{EI} = U_{eL}^\dagger Y_w^{EI} U_{eR}$, where U_{eL} and U_{eR} are the matrices that diagonalize the charged lepton mass matrix defined in eqs. (16). We obtain [30]

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1} \begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m \quad \text{and} \quad \tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m, \quad (55)$$

where $\tilde{m}_\mu = 5.94 \times 10^{-2}$, $\tilde{m}_e = 2.876 \times 10^{-4}$ and $x = m_e/m_\mu = 4.84 \times 10^{-3}$. All the non-diagonal elements are responsible for tree-level FCNC processes. If the S'_2 symmetry in the Higgs sector is preserved [34], $\langle H_1^0 \rangle = \langle H_2^0 \rangle = v$.

The amplitude of the flavour violating process $\mu \rightarrow 3e$, is proportional to $\tilde{Y}_{\mu e}^E \tilde{Y}_{ee}^E$ [39]. Then, the leptonic branching ratio,

$$Br(\mu \rightarrow 3e) = \frac{\Gamma(\mu \rightarrow 3e)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} \quad \text{and} \quad \Gamma(\mu \rightarrow 3e) \approx \frac{m_\mu^5}{3 \times 2^{10} \pi^3} \frac{\left(Y_{\mu e}^{1,2} Y_{ee}^{1,2}\right)^2}{M_{H_{1,2}}^4}, \quad (56)$$

which is the dominant term, and the well known expression for $\Gamma(\mu \rightarrow e\nu\bar{\nu})$ [40], give

$$Br(\mu \rightarrow 3e) \approx 2(2 + \tan^2 \beta)^2 \left(\frac{m_e m_\mu}{m_\tau^2}\right)^2 \left(\frac{m_\tau}{M_H}\right)^4, \quad (57)$$

Table 3. Leptonic FCNC processes, calculated with $M_{H_{1,2}} \sim 120 \text{ GeV}$.

FCNC processes	Theoretical BR	Experimental upper bound BR	References
$\tau \rightarrow 3\mu$	8.43×10^{-14}	2×10^{-7}	B. Aubert <i>et. al.</i> [42]
$\tau \rightarrow \mu e^+ e^-$	3.15×10^{-17}	2.7×10^{-7}	B. Aubert <i>et. al.</i> [42]
$\tau \rightarrow \mu\gamma$	9.24×10^{-15}	6.8×10^{-8}	B. Aubert <i>et. al.</i> [43]
$\tau \rightarrow e\gamma$	5.22×10^{-16}	1.1×10^{-11}	B. Aubert <i>et. al.</i> [44]
$\mu \rightarrow 3e$	2.53×10^{-16}	1×10^{-12}	U. Bellgardt <i>et al.</i> [41]
$\mu \rightarrow e\gamma$	2.42×10^{-20}	1.2×10^{-11}	M. L. Brooks <i>et al.</i> [45]

taking for $M_H \approx 120 \text{ GeV}$ and $\tan\beta = 1$ we obtain $Br(\mu \rightarrow 3e) = 2.53 \times 10^{-16}$, well below the experimental upper bound for this process, which is 1×10^{-12} [41].

Similar computations give the numerical estimates of the branching ratios for some others flavour violating processes in the leptonic sector. These results, and the corresponding experimental upper bounds are shown in Table 3. In all cases considered, the theoretical estimations made in the framework of the minimal S_3 -invariant extension of the SM are well below the experimental upper bounds [30].

6. Muon anomalous magnetic moment

In the minimal S_3 -invariant extension of the Standard Model we are considering here, we have three Higgs $SU(2)$ doublets, one in the singlet and the other two in the doublet representations of the S_3 flavour group. The Z_2 symmetry decouples the charged leptons from the Higgs boson in the S_3 singlet representation. Therefore, in the leading order of perturbation theory there are two neutral scalars and two neutral pseudoscalars whose exchange will contribute to the anomalous magnetic moment of the muon. Since the heavier generations have larger flavour-changing couplings, the largest contribution comes from the heaviest charged leptons coupled to the lightest of the neutral Higgs bosons.

A straightforward computation gives

$$\delta a_\mu^{(H)} = \frac{Y_{\mu\tau} Y_{\tau\mu}}{16\pi^2} \frac{m_\mu m_\tau}{M_H^2} \left(\log \left(\frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right). \quad (58)$$

With the help of (55) we may write $\delta a_\mu^{(H)}$ as

$$\delta a_\mu^{(H)} = \frac{m_\tau^2}{(246 \text{ GeV})^2} \frac{(2 + \tan^2\beta)}{32\pi^2} \frac{m_\mu^2}{M_H^2} \left(\log \left(\frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right), \quad (59)$$

Taking again $M_H = 120 \text{ GeV}$ and the upper bound for $\tan\beta = 14$ gives an estimate of the largest possible contribution of the FCNC to the anomaly of the muon's magnetic moment $\delta a_\mu^{(H)} \approx 1.7 \times 10^{-10}$. This number has to be compared with the difference between the experimental value and the Standard Model prediction for the anomaly of the muon's magnetic moment [46]

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (28.7 \pm 9.1) \times 10^{-10}, \quad (60)$$

which means

$$\frac{\delta a_\mu^{(H)}}{\Delta a_\mu} \approx 0.06. \quad (61)$$

Hence, the contribution of the flavour changing neutral currents to the anomaly of the magnetic moment of the muon is smaller than or of the order of 6% of the discrepancy between the experimental value and the Standard Model prediction.

7. Conclusions

A well defined structure of the Yukawa couplings is obtained, which permits the calculations of mass and mixings matrices for quark and leptons in a unified way. A further reduction of redundant parameters is achieved in the leptonic sector by introducing a Z_2 symmetry. The flavour symmetry group $S_3 \times Z_2$ relates the neutrino mass spectrum and mixings. This allowed us to compute the neutrino mixing matrix, V_{PMNS} , explicitly in terms of the masses of the charged leptons and neutrinos and one phase δ_l . In this model, we obtained a general result for mass matrices of all fermions in the theory. This result is that through of a similarity transformation we can write all mass matrices of fermions in the same generic form with two texture zeros. We also found that V_{PMNS} has three CP-violating phases, namely, one Dirac phase $\delta_l = \delta_\nu - \delta_e$ and two Majorana phases, α and β , which are functions of the neutrino and lepton masses. The numerical values of the reactor, θ_{13} , mixing angle is determined by the masses of the neutrinos which have an inverted hierarchy, with the values $|m_{\nu_2}| = 0.056$ eV, $|m_{\nu_1}| = 0.053$ eV and $|m_{\nu_3}| = 0.048$ eV, obtained $\theta_{13}^l \approx 9.8^\circ$ in agreement with the latest analysis of the experimental data on neutrino oscillations and mixings. We also obtained explicit expressions for the matrices of the Yukawa couplings of the lepton sector parametrized in terms of the charged lepton masses and the VEV's of the neutral Higgs bosons in the S_3 -doublet representation. These Yukawa matrices are closely related to the fermion mass matrices and have a structure of small and very small entries reflecting the observed charged lepton mass hierarchy. With the help of the Yukawa matrices, we computed the branching ratios of a number of FCNC processes and found that the branching ratios of all FCNC processes considered here are strongly suppressed by powers of the small mass ratios m_e/m_τ and m_μ/m_τ , and by the ratio $(m_\tau/M_{H_{1,2}})^4$, where $M_{H_{1,2}}$ is the mass of the neutral Higgs bosons in the S_3 -doublet. Taking for $M_{H_{1,2}}$ a very conservative value ($M_{H_{1,2}} \approx 120$ GeV), we found that the numerical values of the branching ratios of the FCNC in the leptonic sector are well below the corresponding experimental upper bounds by many orders of magnitude. It has already been argued that small FCNC processes mediating non-standard quark-neutrino interactions could be important in the theoretical description of the gravitational core collapse and shock generation in the explosion stage of a supernova [50, 51, 52]. Finally, the contribution of the flavour changing neutral currents to the anomalous magnetic moment of the muon is small but non-negligible, and it is compatible with the best, state of the art measurements and theoretical computations.

8. Acknowledgements

We thank Prof. Jens Erler and Dr. Genaro Toledo-Sánchez for helpful discussions about $g - 2$. This work was partially supported by DGAPA-UNAM under contract PAPIIT-IN113712-3.

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