FIFTY YEARS THAT CHANGED OUR PHYSICS

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As seen through the Moriond meetings

1 Introduction

Going through the Proceedings of the Moriond meetings, from 1966 to 2016, we have a panoramic view of the revolution which took place in our understanding of the fundamental interactions. As we heard from J. Lefrançois, these meetings started as a gathering among a group of friends who loved physics and loved skiing. Through the years they became a major International Conference, still trying to keep part of the original spirit. The driving force behind this evolution has been our friend Jean Tran Than Van and this Conference is one of his many children. In 1966 Jean was fifty years younger and for a young man of that age fifty years is eternity. These years brought to each one of us their load of disappointments and sorrows, but also that of achievements and successes. We witnessed the birth and rise of the Standard Model and our greatest joy is to be able to say *I was there!*

Four years ago you discovered the last piece of this Model and, hopefully, this will open the way to new and greater discoveries. I wish all the young people in this room to be able to say, fifty years from now, *I was there!*

The twentieth century was the century of revolutions in Physics. Just to name a few:

Relativity, Special and General – Atoms and atomic theory – Radioactivity – The atomic nucleus – Quantum Mechanics – Particles and Fields – Gauge theories and Geometry.

Each one involved new physical concepts, new mathematical tools and new champions. Some of these revolutions were radical, others were conservative. Here, I will talk about the last two: Particles and Fields – Gauge theories and Geometry. They were conservative: things changed just enough so that they could remain the same. Yet, they influenced profoundly our way of looking at the fundamental laws of Nature. Not surprisingly, they were mostly rejected by the champions of the previous revolutions.

In this talk I will concentrate on the work done during the early Moriond meetings, essentially the 1960's and the early 1970's. I will cover: (i) the foundations of the Electroweak theory, (ii) the Renormalisation group and QCD and (iii) the importance of quantum anomalies.

2 The story

I start from the early Moriond meetings:
Moriond 1966: 20 Participants – Almost all talks were in French – The subjects were mostly of local interest, such as: Photoproduction, Electroproduction, The deuteron, Some projects for $e^+ - e^-$ physics.
Moriond 1967: 32 Participants – Still most talks were in French – The subjects expand to cover topics from the main stream of international research: Analyticity, Regge poles, Bootstrap, CP-violation, Quark model.
Moriond 1968: 39 Participants – Still most talks were in French – The subjects cover practically all the international research: Analyticity, Regge poles, CP-violation, Weak interactions, Current algebras.

As you can see, the science became almost immediately fully international. The point which took longer to follow was the language.

3 The scene

In the 1960's there were two main lines of research in theoretical high energy physics: (i) *The analytic S-matrix theory,* which was the dominant subject. (ii) *Symmetries and Current Algebras, Weak Interactions and CP-violation,* which were some secondary subjects. *Quantum Field Theory* was noticeable mainly by its absence. A totally marginal subject.

The title of this talk could have been: *Quantum Field Theory strikes back*, but, instead I want to show that our present edifice is built on results coming from all three lines of research and each one contributed its fair share to it.

• *The analytic S-matrix theory.* The suggestion to use the scattering matrix to formulate the theory of fundamental interactions goes back to John Archibald Wheeler (1911-2008) in 1937 and it was further expanded by Werner Karl Heisenberg (1901-1976) in 1943, but it is only in the late 1950's and early 1960's that it was developed as an alternative to quantum field theory. It is motivated by the fact that, in elementary particle physics, practically all information is obtained through scattering experiments, so one should formulate a theory in which the scattering amplitudes are the fundamental objects. This approach was developed by Geoffrey Foucar Chew, Stanley Mandelstam, Tullio Eugenio Regge (1931-2014) *et al.* The basic object is the S-matrix which is assumed to satisfy certain axioms. They include the obvious ones: *(i) Unitarity, (ii) Poincaré invariance, (iii) invariance under possible internal symmetries, (iv) crossing symmetry,* but also more specific ones, such as: *(v) Maximum analyticity.* This is the main *new* axiom in the theory. It can be formulated intuitively as the requirement that the Lorentz invariant functions representing a scattering amplitude are analytic functions of the external momenta with only those singularities imposed by unitarity. The trouble is that, although it is easy to give specific examples of this axiom, the precise mathematical formulation for the general case, as well as its self-consistency, have remained somehow vague. *(vi) Polynomial boundedness.* This axiom is necessary to write multi-dimensional dispersion relations. It states that the amplitude, considered as a function of several complex variables, is bounded at infinity by a polynomial at any complex direction. Although easy to formulate, it is not easy to study mathematically. In particular, it is not known under which conditions it may follow from the general axioms of quantum field theory.

The first dispersion relations in particle physics were written in the United States between 1955 and 1957. Soon afterwards, in 1958, Stanley Mandelstam extracted from the square diagram in ϕ^3 quantum field theory the analyticity properties of the two-particle elastic scattering amplitude as a function of two complex variables, energy and momentum transfer. He wrote a double dispersion relation which remained in the literature as *the Mandelstam representation.*

A second most influential result of this period is the one first obtained by Tullio Regge in 1959. He studied the properties of potential scattering using the Schrödinger equation in which one can compute, in terms of the potential V, the partial wave amplitudes $f_l(E)$, where E is the incident energy and $l = 0, 1, 2, ...$ the value of the angular momentum. Regge showed that there exists a unique function $F(J, E)$ with well defined analyticity properties in the complex J plane and which, when J takes integer values $J = 0, 1, 2, ...$, coincides with the partial wave amplitudes: $F(J, E)_{J=l} = f_l(E)$. The extension of this result to the general relativistic case is based on an assumption about the asymptotic behaviour of the amplitude for large J and uses a mathematical theorem by Carlson. It was done by André Martin, a regular contributor to the Moriond meetings. One of the practical results of Regge theory was the proof that the high energy behaviour of the scattering amplitude could exhibit a non-trivial dependence on the momentum transfer.

The Mandelstam representation on the one hand and the Regge behaviour on the other, became soon the corner stones of the analytic S-matrix theory. This approach, which from exaggerated heights of almost religious faith, fell into totally unjustified depths of oblivion, gave nevertheless rise to many fundamental concepts in high energy physics some of which I want to mention briefly here. This approach has been a constant subject in the Moriond meetings of the sixties and seventies.

1) Cutkosky unitarity relations. They are presented in Figure 1. It is not really a new concept but rather a method to compute amplitudes in a more efficient way and many complex QCD calculations done today are inspired from these relations.

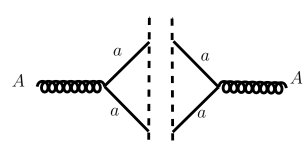

Figure 1 – The Cutkosky cutting rules: The imaginary part of a Feynman diagram can be computed by: (i) Cutting the diagram in all possible ways. (ii) For each cut replacing the cut propagators by the mass-shell delta functions. (iii) Integrating each time over the corresponding phase-space. (iv) Summing over all possible cuts. The value of the entire diagram can now be obtained by a dispersion relation.

2) Duality. Historically this concept was introduced in 1967 through the work of R. Dolen, D. Horn and C. Schmid. It is *the dual resonance model* shown schematically in Figure 2. When the momentum transfer t is sufficiently small, the amplitude can be written as a sum of contributions from s-channel resonances. Similarly, if s is sufficiently small, it may be written as a sum of t-channel resonances. Attention: We are not supposed to sum over s and t channel resonances, as we would if these were elementary particles in a field theory. The entire amplitude is given as a sum in either s, or t channel. There exist two equivalent descriptions of the same amplitude, hence the name ”duality”.

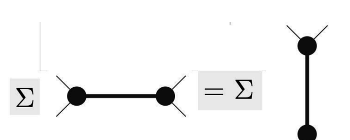

Figure 2 – The sum over s-channel resonance equals that over the t-channel ones.

This concept turned out to be fundamental in most modern developments of quantum field theory and string theory.

3) The Veneziano amplitude. This brings me to one of the most influential papers of the late sixties, the celebrated 1968 article by Gabriele Veneziano. The title is: *Construction of a crossing-symmetric, Regge-behaved amplitude for linearly rising trajectories.* Usually one starts from a dynamical model, for example a field theory, and computes the scattering amplitude. Veneziano did the opposite: He guessed a very special form and postulated it directly at the level of the amplitude, without asking the question of the existence of an underlying theory which would give rise to such an amplitude. The form is:

$$A(s,t) \sim \frac{\Gamma(-1+s/2)\Gamma(-1+t/2)}{\Gamma(-2+(s+t)/2)} \tag{1}$$

where the Γ functions are the ones which generalise the notion of the factorial. $\Gamma(n) = (n-1)!$ for positive integer n. They are analytic functions of their argument with simple poles for $n = 0, -1, -2, ...$ This extremely simple form has almost magic properties. It is crossing symmetric by construction, the singularities of the Γ functions reproduce a spectrum of particles lying on linearly rising Regge trajectories and it has the correct Regge asymptotic behaviour. Most important, it exhibits the property of duality: the sum of Regge exchanges in the cross channels gives rise to the resonances in the direct channel. This model was the basis of an entire branch of theoretical physics, that of dual models and string theories. Initially, it was meant to be a theory for hadronic physics and gave rise to phenomenological models such as *the Lund model,* but it was soon realised that it contains a version of quantum gravity. Out of its study emerged new fundamental concepts, such as supersymmetry, infinite dimensional algebras, or two dimensional conformal field theories, whose importance transcends the domain of high energy physics. Incidentally, let me point out that the Veneziano amplitude was written in the fall of 1968 and it was presented in the Moriond session of 1969.

- *Symmetries and Current Algebras, Weak Interactions and CP violation.* The subject is very vast and its review goes far beyond my competence. Let me just point out some milestones before coming to more recent developments.

1) Space-time and internal symmetries. The space-time symmetries go back to the dawn of science with the first direct reference found in Euclide's *Elements.* Internal symmetries have a much more recent origin. In 1932 Heisenberg introduced a kind of incomplete isospin symmetry for the nuclear forces. The complete picture is due to Nicholas Kemmer, who, in 1938, wrote down what we call today the Yukawa interaction[a], as:

$$\mathcal{L}_I = g\bar{\Psi}(x)\vec{\tau}\cdot\vec{\pi}(x)\gamma_5\Psi(x) \tag{2}$$

As a last step I want to mention a little known work of Fermi's: In 1951 he used a simple isospin argument in order to deduce the I=3/2 dominance in 140 MeV pion nucleon scattering. It was the first time that results of this kind were obtained in particle physics from symmetry principles alone, independently of any detailed dynamical model. From this time Symmetry became the name of the game. It entered the world of particle physics and proved to be the most profound and most powerful concept.

I have discussed elsewhere the history of gauge symmetries, so I will skip this part here.

2) Higher internal symmetries. With the discovery of strange particles it became obvious that isospin symmetry ought to be extended to a higher group. Going from $SU(2)$ to $SU(3)$

[a] Today we know that pions are pseudoscalars, but Kemmer did not know it. In fact in 1938 there was evidence for the existence of a charged Yukawa meson, although, as it turned out, it was the muon, but there was no evidence for a neutral one. Kemmer understood that isospin symmetry required such a neutral meson and in 1940 S. Sakata and Y. Tanikawa remarked that it could have escaped detection because it could decay into two photons with a very short life time. It is precisely what it turned out to do. So, π^0 is the first particle whose existence was predicted as a requirement for an internal symmetry.

sounds like a natural step but, in fact, it was not. $SU(2)$ is realised in a simple way with the nucleons belonging to the fundamental two-dimensional representation. The direct generalisation would yield a triplet (p, n, Λ). This scheme was first proposed in 1956 by Shoichi Sakata but it was disproved by the data. The correct solution was found independently by Murray Gell-Mann and Yuval Ne'eman in 1961 and it is known as *the Eightfold Way.* It assigns mesons and baryons in 8 and 10 dimensional representations of $SU(3)$ leaving the fundamental triplet empty. Its most important early success was the Gell-Mann – Okubo mass formula which predicted the existence of the Ω^-. This representation pattern, together with the proliferation of "elementary" particles, led some theorists to wonder whether we were not simply uncovering another layer of the onion. The *quark model* was proposed independently by Gell-Mann and George Zweig in 1964 and became soon quite popular. Since 1967 it has been a constant theme in many Moriond meetings. Incidentally, one of the main motivations behind the decision by Victor Weisskopf, then CERN's Director General, to approve the building of ISR, was precisely the discovery of quarks. This programme was realised in an unexpected way: ISR did support the evidence, first found at SLAC, for the existence of quarks, although it did not produce them as physical particles.

3) The Weak Interactions. Following an impressive series of experimental and theoretical investigations, the weak interaction hamiltonian took the current×current form with the current being a sum of a leptonic and a hadronic part in the $V-A$ combination:

$$H_I = \frac{G_F}{\sqrt{2}} J^\mu(x) J^\dagger_\mu(x) \quad ; \quad J^\mu(x) = l^\mu(x) + h^\mu(x) \tag{3}$$

This simple and elegant form, not only described all weak interaction phenomena known at the time, but also led to the discovery of several fundamental symmetry properties in particle physics, such as chiral symmetry. Its only drawback was the bad high energy behaviour. Already in 1936 Markus Fierz[?] computed the cross section for neutrino scattering and found that, at high energies, it increases with the neutrino energy:

$$d\sigma(\bar{\nu} + p \rightarrow n + e^+) = \frac{G_F^2}{2\pi^2} p_\nu^2 d\Omega \tag{4}$$

where p_ν is the neutrino momentum in the centre-of-mass system and $d\Omega$ is the element of the solid angle of the positron momentum. Similar conclusions were reached also by Heisenberg for the inelastic cross sections. It became immediately obvious that such a behaviour is unacceptable and this problem haunted weak interactions for many years. It finally led to the formulation of a new theory, *the Standard Model.*

4) Current Algebras. After discovering $SU(3)$, Gell-Mann proposed in 1964 the chiral Algebra of Currents. The symmetry group is assumed to be $U(3) \times U(3) \sim U(1) \times U(1) \times SU(3) \times SU(3)$. It has 18 conserved, or approximately conserved, currents out of which we can construct 18 charges, the generators of the group transformations. It is convenient to write them as follows: (i) Q_V for the vector $U(1)$. (ii) Q_A for the axial $U(1)$. (iii) Q^a_R for the right-hand $SU(3)$ and (iv) Q^a_L for the left-hand part, $a = 1, 2, ..., 8$. In the limit of exact symmetry they satisfy the commutation relations:

$$[Q^a_R \, , Q^b_R] = i f^{abc} Q^c_R \quad ; \quad [Q^a_L \, , \, Q^b_L] = i f^{abc} Q^c_L \quad ; \quad a, b, c = 1, 2, ..., 8 \tag{5}$$

with all other commutators vanishing. f^{abc} are the structure constants of $SU(3)$ and a sum over repeated indices is understood. It is instructive to see the fate of the various group factors: (i) The vector $U(1)$ remains as an exact symmetry and the corresponding conservation law is that of baryon number. (ii) The axial $U(1)$ puzzled people for a long time and it took some years before it was finally understood that, at the quantum level, the symmetry is broken by a phenomenon we shall see later, called "the axial anomaly". (iii) The $SU(3) \times SU(3)$ part is spontaneously broken as:

$$SU(3)_R \times SU(3)_L \ \rightarrow \ SU(3)_V \tag{6}$$

with $SU(3)_V$ being the diagonal subgroup of the chiral $SU(3)_R \times SU(3)_L$ which contains only vector currents. It is called "the flavour group". The corresponding Nambu-Goldstone bosons belong to the 0^- octet of flavour $SU(3)$. It was immediately realised that Current Algebra was a very powerful scheme and for several years it was an important research theme[b]. Its first success was the Adler-Weisberger relation obtained in 1965. It expresses the weak axial vector coupling in terms of the pion-nucleon scattering amplitude. A masterful application of the idea of P.C.A.C. (Partially Conserved Axial Current) giving direct experimental support to chiral symmetry.

4 Fermi's theory as a an effective field theory

Fermi's theory cannot be viewed as a fundamental theory because, in technical terms, it is *non-renormalisable.* In practical terms this means that, if we write any physical amplitude as a power series in the Fermi coupling constant G_F, every term in the expansion requires the introduction of a cut-off parameter Λ. In a renormalisable theory, such as quantum electrodynamics, there exists a well-defined prescription to take the limit $\Lambda \to \infty$ and obtain unambiguous results, but to a non-renormalisable theory the prescription does not apply. The cut-off must remain finite and its value determines the energy scale above which the theory cannot be trusted. This is the definition of *an effective theory.*

- *Where is the cut-off, or the vital importance of precision measurements.* Can we estimate an order of magnitude for the cut-off? A very simple method is the following: Ordinary dimensional analysis tells us that a physical quantity $\mathcal{A}$, for example a weak decay amplitude, can be written in a series expansion as:

$$\mathcal{A} = A_1 G_F \left(1 + \sum_{n=2}^{\infty} A_n (G_F \Lambda^2)^{n-1} \right) \tag{7}$$

where, in every order of the expansion, we have kept only the highest power in Λ. We see that the expression $g_{\text{eff}} = G_F \Lambda^2$ acts as an effective, dimensionless coupling constant. The expansion will become meaningless when $g_{\text{eff}} \sim 1$, which, for the numerical value of G_F, gives $\Lambda \sim 300$ GeV, a value which, for the accelerators of the 1960's, was essentially infinite.

It was B.L. Ioffe and E.P. Shabalin, from the Soviet Union, who first remarked that, in fact, one can do much better. Let us go back to the expansion (7) and consider also the sub-dominant terms in powers of Λ. We can rephrase their argument and write any physical quantity as a double expansion in g_{eff} and G_F:

$$\mathcal{A} = \sum_{n=0}^{\infty} A_n^{(0)} g_{\text{eff}}^n \ + \ G_F M^2 \sum_{n=0}^{\infty} A_n^{(1)} g_{\text{eff}}^n \ + \ (G_F M^2)^2 \sum_{n=0}^{\infty} A_n^{(2)} g_{\text{eff}}^n \ + \ \ldots \tag{8}$$

where the quantities $A_n^{(i)}$ may contain powers of the logarithm of Λ. M is some mass parameter, which, for a typical quantity in particle physics, is of the order of 1 GeV. The first series contains the terms with the maximum power of Λ for a given power of G_F, they are called *the leading divergences.* Similarly, the second series contains all the *next-to-leading divergences*, the third the *next-to-next-to-leading divergences*, etc. Following Ioffe and Shabalin, let us choose for $\mathcal{A}$ a quantity in strong interactions, for example the energy levels in a nucleus. The leading divergences represent the weak interaction corrections to this quantity. But weak interactions

[b] It was also a recurrent subject in the Moriond meetings, already from that of 1966.

violate parity and/or strangeness, therefore the high precision with which such effects are known to be absent in nuclear physics gives a much more stringent bound for Λ, of the order of 2-3 GeV. Similarly the next-to-leading divergences contribute to "forbidden" weak interaction processes, such as ΔS=2 transitions (the $K_L^0 - K_S^0$ mass difference), or $K_L^0 \to \mu^+\mu^-$ decays. Again, the precision measurements of such quantities give the same 2-3 GeV limit for Λ.

Let me make a digression at this point and remind that, for many people, the important problem of the time was that of the strong interactions. Once solved, it was believed that it would provide the cut-off for the weak interactions and any effort to look at the latter independently was pointless. The significance of the estimations presented in (7) and (8) was precisely that they used only the symmetry properties of the weak currents, in particular the Current Algebra relations (5). For example, the expression for the leading divergences can be written as:

$$A \sim \frac{G}{\sqrt{2}} \int d^4k \; e^{ikx} < a|T(J_\mu(x), J_\nu(0))|b > \frac{k^\mu k^\nu / m_W^2}{k^2 - m_W^2} \tag{9}$$

with m_W the mass of the intermediate vector boson. It is clear that this expression is divergent no matter what you assume for the strong interactions, as long as they satisfy the chiral symmetry (5).

5 Fighting the infinities

A cut-off as low as 2-3 GeV was clearly unacceptable. It meant that, at least for some processes, Fermi's theory should be corrected already at low energies. The fact that the Fermi theory was non-renormalisable was known since the early years, but I believe it is fair to say that it was the work of Ioffe and Shabalin which showed that the problem was not only mathematical but also physical. A long and painful struggle against the infinities started. Although it was fought by few people[c], it has been an epic battle given in two fronts: The first, the phenomenology front, aimed at finding the necessary modifications to the theory in order to eliminate the disastrous leading and next-to-leading divergences. The second, the field theory fronttried to find the conditions under which a quantum field theory involving massive, charged, vector bosons is renormalisable. It took the success in both fronts to solve the problem.

• *The Phenomenology front.* In a purely phenomenological approach the idea was to push the value of the cut-off beyond the reach of the experiments. The search went through several steps.

1) Early attempts. In the early attempts the effort was not focused on a particular physical problem, but aimed instead at eliminating the divergences, at least from physically measurable quantities. Some were very ingenious and a very incomplete list contains:

(i) The *physical* Hilbert space contains states with negative metric. The introduction of negative metric states is considered unacceptable because it implies violation of the unitarity condition. However, Tsung Dao Lee and Gian Carlo Wick observed that, if the corresponding "particles", in this case the weak vector bosons, are very short lived, the resulting unitarity violations could be confined into very short times and be undetectable.

(ii) The V-A form of the Fermi theory is an illusion and, in reality, the intermediate bosons mediating weak interactions are scalars. By a Fierz transformation, the effective Lagrangian could look like a vector theory for some processes. This way the theory is renormalisable, but at the price of loosing all insight into the fundamental role of the weak currents.

(iii) The coefficient of the divergent term may vanish accidentally. The idea was to compute the coefficient, for example in a loop expansion, for both the weak and the electromagnetic

[c]Most people doubted about the physical significance of the problem because of widespread mistrust towards field theory in general and higher order diagrams in particular. Since we had no theory, why bother about its higher order effects?

contributions. Setting it equal to zero gives an equation for the Cabibbo angle. The work by itself has today only a historical interest, but, as by-products, two interesting results emerged, summarised in the following two relations:

$$\tan\theta = \sqrt{\frac{m_d}{m_s}} \quad ; \quad \frac{|m_d - m_u|}{m_d + m_u} \sim \mathcal{O}(1) \tag{10}$$

where the masses are those of the three quarks. The first is in good agreement with experiment and relates the Cabibbo angle with the medium strong interactions which break $SU(3)$. The second, obtained by Cabibbo and Maiani, is more subtle: The prevailing philosophy was that isospin is an exact symmetry for strong interactions broken only by electromagnetic effects. In this case one would expect the mass difference in a doublet to be much smaller than the masses themselves. The second relation of (10) shows instead that isospin is badly broken in the quark masses and the approximate isospin symmetry in hadron physics is accidental, due to the very small values, in the hadronic mass scale, of m_u and m_d.

2) The leading divergences and the breaking of $SU(3) \times SU(3)$. The leading divergences in the series (8) raised the spectrum of strangeness and parity violation in strong interactions. The first step was to find the conditions under which this disaster could be avoided. The argument is based on the following observation: at the limit of exact $SU(3) \times SU(3)$ one can perform independent right- and left-handed rotations in flavour space and diagonalise whichever matrix would multiply the leading divergent term. As a result, any net effect should depend on the part of the interaction which breaks $SU(3) \times SU(3)$. In particular, one can prove that, under the assumption that the chiral $SU(3) \times SU(3)$ symmetry breaking term transforms as a member of the $(3, \bar{3}) \oplus (\bar{3}, 3)$ representation, the matrix multiplying the leading divergent term is diagonal in flavour space, *i.e.* it does not connect states with different quantum numbers, strangeness and/or parity. Therefore, all its effects could be absorbed in a redefinition of the parameters of the strong interactions and no strangeness or parity violation would be induced. This was first found for the one loop diagrams and then extended to all orders. This particular form of the symmetry breaking term has a simple interpretation in the formalism of the quark model: it corresponds to an explicit quark mass term and it was the favourite one to most theorists, so it was considered a welcome result.

3) The next-to leading divergences. Lepton-hadron symmetry - Charm. The solution of the leading divergence problem was found in the framework of the commonly accepted theory at that time. On the contrary, the next to leading divergences required a drastic modification, although, in retrospect, it is a quite natural one. It is the introduction of a fourth quark flavour, called *charm,* which implied a prediction for the existence of an entire class of new hadrons. The mechanism is well known, so I will not present it here.

• *The Field Theory front.* The main characters here are Martinus Justinus Godefriedus (known as "Tini") Veltman and his student Gerardus 't Hooft, but the subject received important contributions from many other physicists. Soon after the formulation of the Yang-Mills theories and the Intermediate Vector Boson (IVB) hypothesis (Schwinger 1957), several attempts were made to apply this formalism to the weak interactions (Schwinger 1957, S.A. Bludman 1958). The most important contribution from this period dates from 1961 and it is due to Sheldon Lee Glashow who was the first to introduce the idea of a mixing between the photon and the weak neutral boson with an angle which he called θ, (today it is called θ_W). None of these attempts addressed the question of renormalisation. In 1962 T.D. Lee and C.N. Yang studied the electrodynamics of massive charged vector bosons, (the *ξ-limiting formalism*) but no definite conclusion was reached.

The first important result on the Yang-Mills field theory was obtained by R.P. Feynman in 1963[d]. By an explicit computation of one loop diagrams he showed that the contribution of the

[d]Feynman was not interested in the Yang-Mills theory *per se*, he used it only as a laboratory for gravitation.

unphysical degrees of freedom, the longitudinal and scalar components of the vector field, does not cancel, contrary to what happens in QED. He postulated that the correct quantisation of the theory should include an additional unphysical field which should be added to every loop with a minus sign and conjectured that this trick should work to all orders. This result was shown systematically, using Feynman's path integral quantisation, by L.D. Faddeev and V.N. Popov in 1967 and the unphysical fields are called *Faddeev-Popov ghosts.*

Veltman was at CERN when he understood that the solution of the problems in weak interactions should be the Yang-Mills gauge theory. He reached this conclusion by studying the set of Ward identities which the weak currents should satisfy. In his direct way he decided to attack the problem frontally. He computed all the one loop divergences of the theory and looked whether and how they could cancel. He was using massive gauge bosons because, at that time, he was still unaware of the spontaneous breaking mechanism[e]. By the late sixties, when he had moved to Utrecht, he was joined by a young graduate student named Gerardus 't Hooft. 't Hooft's first papers, appeared in 1971 and marked the real turning point. From an obscure esoteric subject, gauge theories became all of a sudden the centre of attention. Immediately after 't Hooft and Veltman presented the first proof of the renormalisability and unitarity of spontaneously broken gauge theories[f].

6 Intermezzo

In the middle of the nineteen sixties two sets of papers appeared which were seemingly unrelated to anything else. They were largely ignored by the community.

• *The spontaneous symmetry breaking in the presence of gauge interactions.* It was the work of F. Englert and R. Brout ; P. Higgs ; G.S. Guralnik, C.R. Hagen and T.W.B. Kibble. They showed that the gauge bosons could acquire a mass by absorbing the Goldstone bosons which are expected in a spontaneous symmetry breaking. The resulting theory could have ably massive particles. These papers went unnoticed when first presented. The initial motivation appeared to be the breaking of flavour $SU(3)$ and no connection with weak interactions was made. The history of this phenomenon is very interesting but it has been presented in many places, so I will not elaborate on it here.

• *A model for leptons.* In 1967 S. Weinberg wrote the celebrated paper which contains the synthesis of Glashow's 1961 model of an electroweak theory with the Brout-Englert-Higgs mechanism of spontaneous symmetry breaking. He showed that the symmetry breaking was the origin of both, the separation of the weak and the electromagnetic interactions, as well as the fermion masses. The same model was presented the following year by A. Salam. The importance of this work does not need to be emphasised, but it was certainly not appreciated when first published. The reason is that it applied only to leptons and we were all strongly attached, and rightly so, to the idea of the universality of weak interactions. A model which seemed to break this universality was not taken seriously[g].

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[e]The calculations were quite involved and Veltman developed the first symbolic manipulation programme which he called *Scoonship*.

[f]Their proof was diagrammatic, but it was followed by a more formal one by B.W Lee and J. Zinn-Justin in 1972. The Ward identities for these theories were written by J.C. Taylor (1971) and A.A. Slavnov (1972). The final synthesis, which revealed the full geometrical origine of both the Yang-Mills fields and the Faddeev-Popov ghosts, was found between 1974 and 1976 by C.M. Becchi, A. Rouet and R. Stora, as well as by I. Tyutin. It is based on a new symmetry, called *BRST*, which has played an important role in the study of all gauge theories, including gravitation. It is a rather strange symmetry because it acts trivially on all physical observables.

[g]In the paper Weinberg conjectured that this theory could be renormalisable. It seems that he assigned this problem to a graduate student at MIT, but, apparently, nothing came out of it.

• *Back to Moriond.* Not surprisingly, none of these almost secret developments found their way in the early Moriond meetings. We know today that the decade of 1960 was an extraordinary one, but nobody at the time realised that a revolution was taking place. The first presentation of the electroweak gauge theory appears in Moriond 1973 but, since that time, we find regular dedicated sessions practically every year.

7 The renormalisation group and QCD

The second pillar of the Standard Model is QCD and its proposal went against every single prejudice we had developed in the sixties concerning the strong interactions. As we mentioned earlier, it was firmly believed that quantum field theory was irrelevant for such processes and only the S-matrix theory could give meaningful results[h]. If the electroweak theory was invented for essentially aesthetic reasons, QCD was found as a response to an experimental challenge. For many years the efforts to understand the nature of strong interactions were concentrated in the study of the experimental results from hadronic collisions. The resulting picture invariably appeared to be too complicated to allow for a simple interpretation. We understand now that this complexity should not be attributed to the fundamental interactions themselves, but is instead due to the fact that the objects we are dealing with, namely the hadrons, are themselves too complicated. It is as if we were trying to discover quantum electrodynamics by studying the interactions among complex molecules. The decisive progress came with the deep inelastic scattering experiments at SLAC, by the late sixties and early seventies. The surprising result was the appearance of scale invariance, a property which is very easy to understand using a naïve and wrong reasoning. If the nucleon is made out of constituents which interact with the electromagnetic field as point-like free particles, the scaling behaviour follows immediately. It is the well-known *parton model.* The trouble is that we cannot understand how free partons can bind so strongly inside the nucleons. As we all know today, the answer is *asymptotic freedom,* a surprising property of non-Abelian unbroken gauge theories. The tool here was the renormalisation group.

The RG equation was first written down by E.C.G. Stückelberg and A. Petermann in 1953. In today's notation it reads:

$$[M\frac{\partial}{M}+\beta\frac{\partial}{\partial\lambda}+\gamma_m m\frac{\partial}{\partial m}-n\gamma]\Gamma^{(2n)}(p_1,...,p_{2n};m,\lambda;M)=0 \tag{11}$$

where M is the subtraction point necessary to define the renormalisation procedure. β and the γ's are functions of the coupling constant and the other dimensionless parameters, such as m/M. They can be computed order-by-order in perturbation theory. M is an unphysical parameter and every M defines a different set of Green functions. The equation gives the condition under which all of them describe the same theory.

In 1970 C.G. Callan and K. Symanzik wrote an different equation, which was *the broken scale invariance Ward identity*:

$$\left[m_R\frac{\partial}{\partial m_R}+\beta\frac{\partial}{\partial\lambda_R}+n\gamma\right]\Gamma_R^{(2n)}=m_R^2\;\delta\;\Gamma_{\phi^2R}^{(2n)} \tag{12}$$

Here m is the physical mass. The equation involves a different set of dimensionless functions β, γ and δ, which depend only on the coupling constant. The meaning of this equation is the following: in four dimensions a renormalisable field theory has a formal scale invariance broken only by the mass parameters. At the quantum level this invariance is violated by the need of renormalisation. In the usual jargon, scale symmetry has anomalies. The C-S equation is the right way to incorporate the anomalies and gives the correct form of the Ward identities.

[h] It is remarkable that David Gross, one of the inventors of QCD, was a student of Geoffrey Chew at Berkeley and was trained as an S-matrix theorist.

As we just explained, each one of these two equations, (11) and (12), which look similar, expresses in fact different properties of the theory. They both appear to be of limited physical interest. Nobody cares much about the dependence on an unphysical parameter and nobody knows how to design an experiment in which the mass of a particle varies continuously.

Now that we explained in what these equations are different, we can turn into what they have in common. They both describe the response of the Green functions under a change of a dimensionful parameter. Therefore, by ordinary dimensional analysis we hope to be able to relate this response to that of the Green functions under the change of other dimensionful quantities. Of particular interest are, of course, quantities related to the incident energy because it can be varied in physical experiments. Already in 1954 M. Gell-Mann and F.E. Low understood that the renormalisation group equation could be used to study the asymptotic behaviour of scattering amplitudes but no physical results were obtained until the early seventies. The progress came from two fronts: (i) the study of phase transitions in statistical mechanics and (ii) the desire to give a field theoretic justification to the parton model. A central figure in both has been Kenneth Geddes Wilson. A student of Gell-Mann's, not only he was among the first persons to understand the physical meaning of the renormalisation group in describing the scaling properties of a theory, but also he developed many of the necessary tools to do it. They include the operator product expansion in field theory and the ϵ expansion in statistical mechanics[i].

The negative sign of the first term in the expansion of the β function of unbroken non-Abelian gauge theories was first discovered by 't Hooft in 1972, but he did not propose a concrete field theory model for strong interactions. It was rediscovered a few months later by David Politzer and, independently, David Gross and Franck Wilczek, who are credited with the discovery of QCD. Sydney Coleman and David Gross showed that there were no other asymptotically free theories. The important underlying assumption was confinement. Indeed, QCD has massless gluons and it is only by assuming confinement that we can explain their absence from the spectrum of physical states. Proving the property of confinement from the first principles of quantum field theory remains one of the great unsolved problems of theoretical physics.

With QCD we could reconstruct the successes of the parton model and go further and compute the small departures from exact scaling. We thus had two pictures to describe the composite structure of hadrons. The first, the parton model, is intuitively very simple. Hadrons, such as protons or neutrons, are made out of elementary constituents, the partons, interacting with the incident photon as quasi-free particles. The strong point of this picture is its simplicity. Among the weak points we can mention the lack of a theoretical basis for the free-particle assumption, the absence of a systematic way to estimate the corrections to this "zero-order" approximation and the difficulty to understand why partons are not kicked out of the hadron after they are hit by the virtual photon. The second picture is based on the property of non-Abelian gauge theories to be asymptotically free. It identifies the partons with quarks, anti-quarks and gluons. It explains the quasi-free particle behaviour and offers a consistent expansion scheme to compute the logarithmic corrections to scale invariance. Finally, through the property of the effective coupling constant to increase with distance, it provides for an intuitive, albeit neither rigorous nor quantitative, understanding of confinement. However, all this is achieved at the price of using a rather heavy formalism in which the simple intuitive picture of the photon interacting with individual point-like partons is lost. The combination of the two was achieved with the A.P.D.G.L. (Altarelli-Parisi, Dokshitzer-Gribov-Lipatov) equations. They extract out of QCD the evolution equations of the parton distribution functions and can be written at any desired order in the expansion. But in fact they are more than just a convenient reparametrisation. They help understanding and extending the limits of the validity of the expansion. In fact, most of the computations which provide the background estimations of the

[i] Wilson's ideas have influenced profoundly our views on the meaning of quantum field theory, but I shall not present them here. Let me only mention that he was also the first to propose the use of lattice simulations in order to study the strong coupling regime of QCD.

LHC experiments are based on these equations.

It is not necessary for me to show in this audience the results of these computations and their remarkable agreement with experiment. They establish the validity of the theory in the weak coupling regime. What is probably less well known is that the recent progress in the numerical lattice simulations has given impressive results also in the strong coupling regime, at least for the masses of the low lying hadron states.

QCD has been present in the Moriond meetings, albeit with a small delay. The first reference I found dates from 1976. For the early years the dominant theme was diffraction scattering. But by the late seventies it became a recurrent subject, often of dedicated sessions.

8 The importance of anomalies

The GIM mechanism implies that the quarks should form doublets of the weak $SU(2)$. The Kobayashi-Maskawa suggestion respects this structure. When later the b quark was discovered in FermiLab this was interpreted as a prediction for the existence of its partner, the t quark, prediction brilliantly verified by experiment. However, the mechanism does not tight together leptons and quarks, in spite of the fact that the title of the original GIM paper was *Weak interactions with lepton-hadron symmetry.* Such a symmetry is not implied by the requirement for the suppression of processes with FCNC. It is remarkable that such a symmetry is imposed by the mere mathematical consistency of the theory and it was discovered immediately after the renormalisability of general Yang-Mills theories, with or without spontaneous symmetry breaking, was proven.

In order to explain how the argument goes we have to remind an essential feature of gauge theories: their mathematical consistency is based on a set of complicated identities which relate the various Green functions of the theory. They are the Ward identities, or the Slavnov-Taylor identities we mentioned before. They translate the consequences of the symmetry of the theory to the level of the Green functions and they ensure the conservation of the symmetry currents. They are obtained by the standard Noether construction. However, there are cases where Quantum Mechanics brings an important complication. Going from the classical equations to the quantum theory involves a series of steps which often include a limiting procedure, for example the limit of some cut-off parameter going to infinity. This limit, although well defined, may not respect some of the symmetries of the classical equations. We call such symmetries *anomalous*[j], which means, in fact, that their consequences will not be satisfied in the quantum theory. The simplest example we shall use is that of quantum electrodynamics. In order to simplify the discussion we consider the case of a massless electron moving in an external electromagnetic field. It is easy to check, using the Dirac equation, that the theory admits two conserved currents:

$$\begin{aligned} J_\mu(x) &= \bar{\Psi}(x)\gamma_\mu\Psi(x) \quad &;\quad \partial^\mu J_\mu(x) = 0 \\ J^5_\mu(x) &= \bar{\Psi}(x)\gamma_\mu\gamma^5\Psi(x) \quad &;\quad \partial^\mu J^5_\mu(x) = 0 \end{aligned} \tag{13}$$

The two conserved currents of (13) correspond to the invariance of the massless Dirac equation under two sets of phase transformations, called *chiral transformations.* In the presence of a mass term the divergence of the axial current equals:

$$\partial^\mu J^5_\mu(x) = 2im_e\bar{\Psi}(x)\gamma^5\Psi(x) \equiv 2im_e J^5(x) \tag{14}$$

What we can prove is that the two conservation equations cannot be simultaneously satisfied in the quantum theory. The simplest way to obtain this result is to compute explicitly the triangle diagrams of Figure 3.

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[j] The term may be misleading, as it may give the impression that something contrary to common sense has happened.

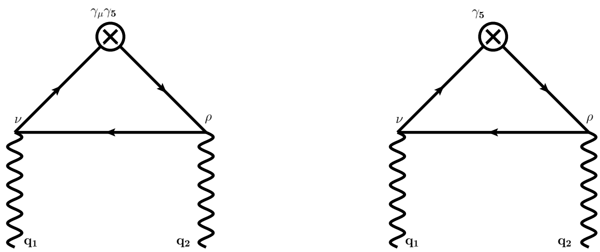

Figure 3 – The two lowest order diagrams contributing to the axial current Ward identity.

If we enforce the conservation of the vector current $J_\mu(x)$, the equation for the axial one becomes:

$$\partial^\mu J^5_\mu(x) = 2im_e J^5(x) + \frac{e^2}{8\pi^2}\epsilon_{\nu\rho\sigma\tau}F^{\nu\rho}(x)F^{\sigma\tau}(x) \tag{15}$$

where e is the charge of the electron. The second term in the r.h.s. of equation (15) is called *the axial anomaly*, which is a fancy way to say that the axial current of massless quantum electrodynamics is not conserved, contrary to what the classical equations of motion indicate.

This result has important physical consequences in particle physics but here we shall present only its implications for the electro-weak theory. For quantum electrodynamics the non-conservation of the axial current can be considered as a curiosity because this current does not play any direct physical role. However, in the electro-weak theory both vector and axial currents are important because of the non-conservation of parity in weak interactions. In proving the renormalisability of the gauge theory we need the full power of all its symmetries and, therefore, the conservation of all its currents. The axial anomaly breaks this conservation and the entire program collapses. As a result, the purely leptonic model, the one which was first constructed in 1967, is mathematically inconsistent.

The solution was first found in 1972 by Claude Bouchiat, J.I. and Philippe Meyer, as well as David Gross and Roman Jackiw. The important observation is that the anomaly is independent of the fermion mass. Every fermion of the theory, light or heavy, contributes the same amount and we must add all contributions in order to get the right answer. For the electroweak theory this means that we need both the leptons and the quarks. Each one will go around the triangle loop of Figure 3 and its contribution will depend only on the coefficients in the three vertices. A simple calculation shows that the total anomaly produced by the fermions of each family will be proportional to $\mathcal{A}$ given by:

$$\mathcal{A} = \sum_i Q_i \tag{16}$$

where the sum extends over all fermions in a given family and Q_i is the electric charge of the ith fermion. Since $\mathcal{A} = 0$ is a necessary condition for the mathematical consistency of the theory, we conclude that each family must contain the right amount of leptons and quarks to make the anomaly vanish. This condition is satisfied by the three colour model with charges 2/3 and -1/3, but also by other models such as the Han-Nambu model which assumes three quark doublets with integer charges given by (1,0), (1,0) and (0,-1). In fact, the anomaly cancellation condition (16) has a wider application. The Standard Model could have been invented after the Yang-Mills theory was written, much before the discovery of the quarks. At that time the "elementary" particles were thought to be the electron and its neutrino, the proton and the neutron, so we would have used one lepton and one hadron doublet. The condition (16) is satisfied. When quarks were discovered we changed from nucleons to quarks. The condition is again satisfied.

If to-morrow we find that our known leptons and/or quarks are composite, the new building blocks will be required to satisfy this condition again. Since the contribution of a chiral fermion to the anomaly is independent of its mass, it must be the same no matter which mass scale we are using to compute it.

The moral of the story is that families must be complete. The title of the GIM paper was correct. Thus, the discovery of a new lepton, the tau, implied the existence of two new quarks, the b and the t, prediction which was again verified experimentally.

The above discussion was confined to the $SU(2) \times U(1)$ gauge theory but the principle of anomaly cancellation should be imposed in any gauge theory in order to ensure mathematical consistency. This includes models of strong interactions and grand-unified theories. H. Georgi and S.L. Glashow found the generalisation of the anomaly equation (16) for a gauge theory based on any Lie algebra. It takes a surprisingly simple form:

$$\mathcal{A}_{abc} = \mathrm{T}r\left(\gamma^5\{\Gamma_a, \Gamma_b\}\Gamma_c\right) \tag{17}$$

where Γ_a denotes the Hermitian matrix which determines the coupling of the gauge field W_a^μ to the fermions through the interaction $\bar{\Psi}\gamma_\mu\Gamma_a\Psi W_a^\mu$. As we see, Γ_a may include a γ^5. Georgi and Glashow showed that the anomaly is always a positive multiplet of $\mathcal{A}_{abc}$, so this quantity should vanish identically for all values of the Lie algebra indices a, b and c.

Since gauge theories are believed to describe all fundamental interactions, the anomaly cancellation condition plays an important role not only in the framework of the Standard Model, but also in all modern attempts to go beyond, from grand unified theories to superstrings. It is remarkable that this seemingly obscure higher order effect dictates to a certain extent the structure of the world.

9 Epilogue

This short review cannot cover fifty years of theoretical high energy physics[k]. If you go through the Proceedings of the Moriond meetings you will get a much more complete picture of the evolution of our discipline. You will be impressed, as I was, by the wide coverage, (even subjects still remote from experiment, such as string theory, are present), but also by the reactivity which was often very fast. Many subjects are presented months after they were invented. The selection I presented here is necessarily partial and personal. Obviously, this is not *The End of History.* It is not even the end of the story. The Moriond meetings will continue. It is only the end of the time which was allocated to me.

[k]For lack of time I left out many important subjects and the few I discussed I did so only superficially. At this stage a list of references will not be very useful. I promise a more complete review which I will put in the arXiv.

2.
Heavy Flavours