PION-PION INTERACTION AND ANTINUCLEON ANNIHILATION

G. Pinski and E. C. G. Sudarshan

University of Rochester, Rochester, New York

K. T. Manhantappa

Harvard University, Cambridge, Massachusetts

(presented by E. C. G. Sudarshan)

The systematic experimental investigations of the Goldhaber groups at Berkeley on the antinucleon annihilation process makes it worthwhile to work out detailed theoretical predictions for various annihilation mechanisms. In view of the large multiplicity of strongly interacting particles involved it is advantageous to work within a statistical framework and incorporate specific dynamical assumptions into this framework. Considerable progress in computations has resulted from the use of a manifestly covariant formulation of the statistical theory. Earlier calculations relating to the antinucleon annihilation process had as their main aim the determination of the relative probabilities for various annihilation channels with different multiplicities; and they used hence the simplest covariant statistical model in which the transition matrix element was assumed constant for all configurations allowed by selection rules. The average pion-pion angular correlation predicted by this model is in excellent agreement with experiment, though the physical interpretation of the length (mass) parameter entering the theory was not clear.

It is necessary to refine this simple statistical theory for two main reasons : firstly the recent progress in the understanding of the electromagnetic structure of the nucleon and a point of view regarding the phenomenology of pion interactions (of which Chew has been the most eloquent advocate) necessitate the examination of the role of pion-pion interaction in antinucleon annihilation. Secondly, the experimental results show that the angular correlation of pions with like charges is different from the angular correlation of pions with unlike charges.

A pion-pion resonance may be incorporated within the statistical framework in a straightforward fashion by a covariant analogue of the Breit-Wigner one-level formula; we write for the differential transition rate to an allowed configuration $\{p_i\}$ an expression which involves a resonance function of the effective mass of pairs of particles. The simplest such expression is

$$1+\lambda^2\sum_{i>j}\delta[(p_i+p_j)^2-\mu^2]$$

where λ^2 and μ^2 are two constants with the dimensions of the square of a mass. This modified expression for the transition rate will affect the predictions and, in general, the details will depend on the resonance parameters one has introduced. In addition to the " mechanical " parameters one has also to specify the isotopic spin characteristics; and to be completely general one must include multiple resonances as well the possibility of larger resonating groups of particles.

To make any tangible progress it is necessary to sacrifice full generality and explore the quantitative predictions for specific resonant models as well as to isolate general features which are not sensitively model-dependent. We have undertaken a systematic investigation of this type, the principal results of which to date are listed below. An effort has been made to formulate the problem so as to circumvent extensive numerical computation; and, as usual, computation of covariant quantities simplifies the problem considerably.

We have chosen for study the currently fashionable pion-pion resonance in the $T = 1$ state, with a " resonance mass " $\mu = 4$ (in units of the pion mass). Rather than ask for the pion-pion angular correlation (which is a " non-covariant " quantity) we calculate the probability distribution of the effective mass squared

$$M^2 = (p_1^0+p_2^0)^2-(\mathbf{p}_1+\mathbf{p}_2)^2$$

of a pair of pions of known charge (++ or +−). For the actual computation of this it is necessary to specify the value of the constant λ^2 characterizing the resonance. We have computed this probability for the two extreme cases of $\lambda^2 = 0$ and $\lambda^2 = \infty$. These results were all computed using a desk calculator; and this is made possible by the use of several recurrence relations that we have derived in this connection and the covariant nature of the quantities that we calculate. The isotopic spin weights were computed by reducing it to a standard but elementary counting problem.

Plots of the effective squared mass distributions (see Fig. 1) show the following features : the nonresonant ($\lambda = 0$) case exhibits a broad distribution in M^2 and is the same irrespective of whether like (++) or unlike (+−) pions are selected, or an average is taken. For the resonant case the like (++) pion M^2 distribution is still relatively broad (though quantitatively different from the nonresonant case); but the unlike (+−) pion M^2 distribution is sharply peaked at a value of M slightly higher than 4. And this effect is true even for the averaged M^2 distribution taken without regard to the pion charges. These qualitative features are easy to understand : the pion pairs tend to come out with the resonant effective mass but are constrained by energy-momentum conservation. The two-like-charged pion M^2 distribution does not get any contribution from resonant pairs since we have taken a $T = 1$ resonance. But what is remarkable is the sharpness of the peak for $M \sim 4$ even for a half-width of a pion mass for the resonance.

These results thus show that the M^2 distributions constitute a very efficient test for pion-pion resonances and are more suitable than the pion-pion angular correlations. From a study of the numerical results we are led to assert that a reasonably sharp two-pion resonance in any isotopic spin state and at any effective mass *must* show up in the M^2 distributions; and that these distributions provide a more sensitive determination of the resonance parameter μ than is afforded by nucleon structure and pion-nucleon scattering phenomenology.

So far we have not made any statements regarding the multiplicity. The present model with λ (in pion mass units) and the statistical parameter κ (for reducing the phase space integrals to a common dimension, and roughly corresponding to the Fermi radius) both being equal to unity gives the multiplicity of 5.7 provided double and triple resonant pairs are taken into account. This latter modification only sharpens the resonance peaks in the M^2 distribution even further.

The picture of antinucleon annihilation that emerges is thus quite similar to the idea first propounded by Pomeranchuk, who suggested that the free " spreading " of pions in a hydrodynamical model begins only when the pions are just " separated " by the range of pion-pion interactions. This model which was originally suggested for other multiple meson production processes, is particularly relevant for antinucleon annihilation. And the agreement of the pion multiplicity in those cases where there is one kaon pair with the predictions of the simple statistical model again seems to be a qualitative indication of the importance of a pion-pion resonance since the average pion energy is the same in this case as in the non-kaon channel.

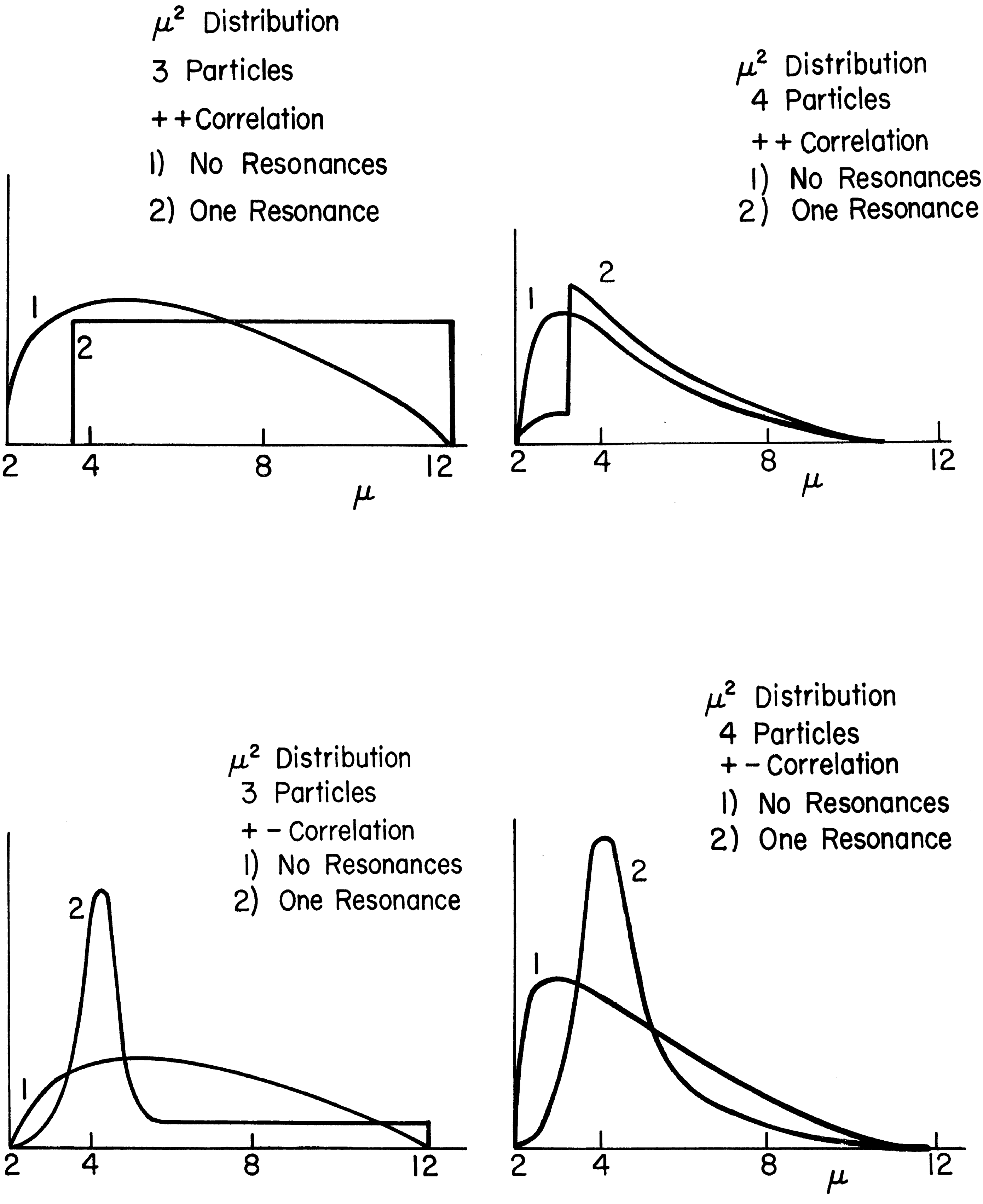

Fig. 1 The M^2-distributions for the resonant and non-resonant cases for various multiplicities.

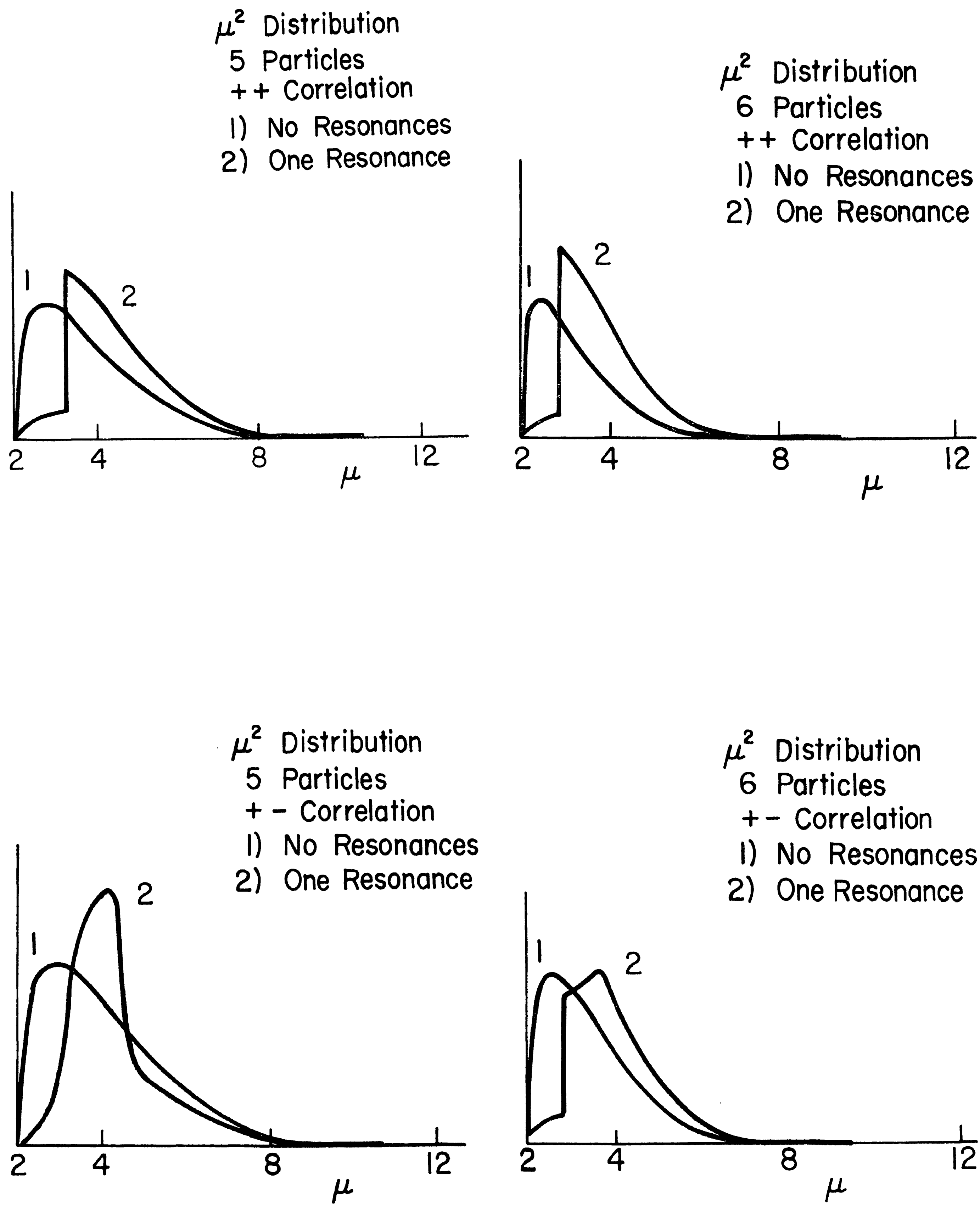

Fig. 1 *(cont.)* The M^2-distributions for the resonant and non-resonant cases for various multiplicities.

DISCUSSION

WENZEL: Is the correlation that you predict for the like pions in the right direction?

SUDARSHAN: It qualitatively tends to make two pions come out with momenta that are very well correlated. At the moment, however, we have not computed $\langle \cos \theta \rangle$. We have no specific view as to the origin of the resonance. It may be a ρ_0 meson as Sakurai and others would like it to be or it may be just a " plain old resonance " or both. I also want to remark that the same method can also be used for a three-pion resonance. All you have to do is to calculate the effective mass distribution for three pions.

WATAGHIN: Have you made use in your calculations of the interaction volume which is used in some of these statistical formulations?

SUDARSHAN: Perhaps I should have mentioned this before. We do not have a volume in the covariant model because in the covariant model the dimensions are a little different. In the three dimensional case, people write the phase space as d^3p and multiply it by an Ω_0, so that this becomes dimensionless so that you can compare transition probabilities. In our model we have d^4p with a delta function after it and the delta function takes off two powers of the momentum and therefore the quantity we have is the square of the cube root of a volume, or an area. Therefore we should really write down κ^2 in front, where κ is a parameter, the hidden parameter I said we used. So any real comparison is a bit " dishonest ".

PRODUCTION OF PARTICLE BEAMS AT VERY HIGH ENERGIES

S. D. Drell

Stanford University, Stanford, California

We consider the production of beams of very high energy strongly interacting nuclear particles. By studying the transition amplitudes near their poles corresponding to " real " one-particle intermediate states we show that photons are very effective in initiating collimated beams of high energy charged pions and *K*-mesons, (anti-)nucleons, etc. A photon of energy k, incident on a nuclear target, produces a high energy charged pion, say, of energy $\omega_q > \frac{1}{2}k$ in a forward cone of opening angle $\theta_{\frac{1}{2}} \sim \mu/\omega_q$ with a cross section (that is reduced by roughly the fine structure constant, $\alpha = 1/137$, from geometric), $1/\mu^2 = 20$ mb. For nucleon initiated processes, on the other hand, although one avoids the fine structure constant, the statistical model predicts that very high energy secondaries emerge in only a very small fraction of the collisions.

This result is of significance for predicting and comparing yields from very high energy electron and proton accelerators. Experiments which can be performed on existing machines are proposed to check the validity of this " pole analysis " which, as applied here to general inelastic processes, is an extension of the work of Chew, Goebel, and Chew and Low [1]. In addition, feasible coincidence experiments are proposed to measure the π-π scattering length and the (γ-3π) coupling strength.