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Numerical Modeling of the Interaction of Dark Atoms with Nuclei to Solve the Problem of Direct Dark Matter Search

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Abstract: The puzzle of the direct dark matter search can be resolved by examining the concept of «dark atoms», which consist of hypothetical stable lepton-like particles with a charge of $-2n$, where n is any natural number, bound to n nuclei of primordial helium. These «dark atoms», known as «XHe» (X-helium) atoms, remain undiscovered in experiments due to their neutral atom-like states. In this model, the positive results of the $DAMA/NaI$ and $DAMA/LIBRA$ experiments could be explained by the annual modulation of radiative capture of XHe atoms engaging in low-energy bound states with sodium nuclei. This specific phenomenon does not occur under the conditions of other underground experiments. The proposed solution to this puzzle involves establishing the existence of a low-energy bound state of «dark atoms» and nuclei while also considering the self-consistent influence of nuclear attraction and Coulomb repulsion. Resolving this complex issue, which has remained unsolved for the past 17 years, necessitates a systematic approach. To tackle this problem, numerical modeling is employed to uncover the fundamental processes behind the interaction of «dark atoms» with nuclei. To comprehend the essence of XHe's interaction with baryonic matter nuclei, a classical model is employed wherein quantum physics and nuclear size effects are progressively incorporated. A numerical model describing the interaction between XHe «dark atoms» and nuclei is developed through the continuous inclusion of realistic features of quantum mechanics in the initial classical three-body problem involving the X-particle, the helium nucleus, and the target nucleus. This approach yields a comprehensive numerical model that encompasses nuclear attraction and electromagnetic interaction between the «dark atom» and nuclei. Finally, this model aids in supporting the interpretation of the results obtained from direct underground dark matter experiments through the lens of the «dark atom» hypothesis.

Keywords: composite dark matter; stable charged particles; XHe; X-helium; «dark atoms»; low-energy bound state; nuclear interactions; Coulomb interaction

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1. Introduction

According to hypotheses that claim dark matter consists of particles, these particles are predicted beyond the framework of the Standard Model (SM) of elementary particle physics. In recent decades, theories have been developed that extend the Standard Model and propose various stable particles as candidates for dark matter—for example, supersymmetric (SUSY) particles and weakly interacting massive particles (WIMPs). These theories also have advantages in addressing the internal problems of the Standard Model [1,2].

However, the absence of experimental detection of such dark matter candidates stimulates research into a broader range of physics beyond the Standard Model. Particularly considering possible non-supersymmetric solutions aimed at reducing the divergence of the Higgs boson mass and explaining the physical nature of dark matter. Composite Higgs models, such as those based on the Walking Technicolor (WTC) hypothesis, offer such a solution. The application of the WTC hypothesis can also lead to a new approach to understanding dark matter and revealing its composite nature [3,4]. In particular, this approach assumes the existence of new stable, electrically charged particles. But it is known that dark matter does not emit electromagnetic radiation, while charged particles are sources of such radiation. Therefore, neutral weakly interacting elementary particles are usually considered as candidates for dark matter. However, if stable charged particles form composite neutral objects, they can also serve as dark matter of the universe. Studying such neutral objects may provide the opportunity to determine the properties of the charged stable particles from which they are composed.

Thus, Glashow proposed the existence of composite neutral dark matter, which is formed in the early stages of the universe's evolution as a tera-helium, consisting of new charged particles. Glashow's model is an extension of the Standard Model by introducing an additional symmetry group, $SU(2)$, which is associated with the presence of heavy partners of ordinary particles (tera-partners) [5]. This extension allows for the resolution of problems in the Standard Model, such as the violation of CP–parity in strong interactions and the problem of neutrino mass. However, even before the establishment of the necessary temperature of 25 keV, required for the formation of tera-helium and the annihilation and binding of incomplete annihilation products, all the free tera-electrons E^-, including those that make up tera-helium, are captured by primary helium. As a result, positively charged ions $(^4HeE^-)^+$ are formed, which prevents the implementation of this model [3].

In modern models of composite dark matter, the issues arising in Glashow's model can be resolved. For example, by introducing a new particle with a charge of -2, denoted as O^{--} (in general, the charge could be $-2n$, where n—is a natural number, then we will denote the particle as X), exceeding the number of its antiparticles, the problems of Glashow's model can be avoided. At a temperature of 100 keV, O^{--} forms a bound state with primary helium, which is called the OHe or «dark atom» [3]. In models where there are four or five generations of fermions, it is possible to form an excess of antiparticles [3]. In such cases, there can exist a stable state with a charge of -2, similar to tera-helium. Such a state can also bind with primary helium and form neutral OHe [3]. «Dark atoms» are considered as the promising candidates for the role of the composite dark matter, which satisfy all the criteria necessary to reproduce the cosmological data on which modern cosmology is based [3]. Namely, in spite of their nuclear interacting nature, the gas of «dark atoms» decouples from plasma and radiation before the stage of matter dominance and supports the development of gravitational instability. It provides conditions for the growth of small initial density fluctuations, leading to the formation of the large-scale structure of the Universe consistent with the data on the observed anisotropy of the cosmic microwave background (CMB) radiation [3,4]. Specifics of the «dark atom» nature of dark matter lead to a nontrivial «warmer than cold dark matter» scenario of structure formation, which needs special studies but in any case is compatible with the data of the precision cosmology [3]. The hypothesis of the existence of O-helium atoms can explain the contradictory results in direct dark matter search experiments. They are associated with the peculiarities of interaction between «dark atoms» and matter in underground detectors [6]. For instance, the positive results from the $DAMA/NaI$ and $DAMA/LIBRA$ experiments, indicating the detection of dark matter particles, contradict the negative results from other experiments such as $XENON100$, LUX, and $CDMS$ in their direct search for dark matter particles. In preliminary qualitative calculations, it was shown that «dark atoms» form low-energy bound states with nuclei of intermediate masses, excluding such a binding with nuclei of heavy elements [3]. In addition, taking into account the scalar and isoscalar nature of XHe, binding to such a low-energy «dark atom» nucleus state can go only due to an electric dipole $E1$ transition with a violation of isotopic invariance. This transition is proportional to the square of the relative velocity, and hence the temperature, and is therefore suppressed in cryogenic experiments [3]. All this explains the lack of positive results in other experiments, since their strategy is aimed at searching for the effects of the recoil nuclei, which may interpret effects of energy release as background events.

In the aforementioned Walking Technicolor (WTC) hypothesis, which is a composite Higgs model, there are mechanisms for particle mass generation and spontaneous symmetry breaking of the electroweak interaction. This model suggests the existence of a new type of interaction that binds a new type of quark [3,7]. Owing to the electroweak $SU(2)$ charge of the techni-particles WTC model provides the generation of an excess of $-2n$ charged stable techni-particles over their $+2n$ charged antiparticles. This excess is provided by sphaleron transitions, which establish its balance with baryon asymmetry. At the same time, the electric charge of the Universe is conserved, since the charge of the excessive $-2n$ charged particles is compensated by the charge of other stable charged particles, namely, by the excess of protons [8]. This correspondence between the excess of stable particles with a charge of $-2n$ and baryon asymmetry can explain the observed ratio of baryonic and dark matter densities [3,4]. Moreover, at the masses of these techno-particles in the 1 TeV range, their contribution to the total density corresponds to the observed dark matter. Since the mass of these heavy $-2n$ charged particles determines the mass of «dark atoms», therefore, they explain the observed density of all dark matter.

In WTC, the charge of «new» physics particles is not fixed. However, fractional charges are limited as free quarks are not observed. A major problem for scenarios involving hypothetical stable electrically charged particles is their absence in the surrounding matter. If such particles exist, they must be bound to ordinary matter and form anomalous isotopes with unusual Z/A ratios (number of protons to mass number). The main challenge for these scenarios is the suppression of an abundance of positively charged particles bound to electrons. They could behave like anomalous isotopes of hydrogen or helium. There are significant experimental constraints on such isotopes, particularly anomalous hydrogen, which strongly restrict the possibility of stable positively charged particles existing [3]. Thus, positively charged particles cannot be candidates for dark matter particles. The same problem arises if the model predicts the existence of stable particles with negative, odd charges. These particles combine with primordial helium to form ion-like systems with a charge of $+1$. Then, they undergo recombination with electrons to form atoms of anomalous hydrogen. Therefore, stable negatively charged particles can only have a charge of $-2n$ [3].

In this study, we consider a scenario where dark matter is composed of hypothetical stable particles, X, which evade detection in experiments, because they form neutral atom-like states called X-helium with primordial helium [3]. Since all these models also predict corresponding antiparticles with a charge of $+2n$, the cosmological scenario must provide a mechanism for their suppression. This may occur through a charge asymmetry associated with an excess of $-2n$ charged particles [3]. However, the overall electric neutrality of the Universe is maintained because the surplus charge is balanced out by an equivalent surplus of baryons with positive charge. As a result, the antiparticles with positive charge can undergo efficient annihilation in the early stages of the Universe. There are several models that predict such stable particles with a charge of $-2n$ [3].

2. *X*-helium Atoms

A «dark atom» refers to a system composed of $-2n$ charged particle (for $n = 1$, this corresponds to O^{--}) and n ^{4}He nuclei, which are bound together by the Coulomb force. The specific structure of this bound state is determined by the parameter $a \approx Z_\alpha Z_X \alpha A m_p R_{nHe}$, where α represents the fine structure constant, Z_X and Z_α denote the charge numbers of particle X and n nuclei of He, respectively, m_p represents the mass of a proton, A corresponds to the mass number of the n-nucleus of He, and R_{nHe} represents the radius of the corresponding nucleus. The physical meaning of the parameter a (not α) is related to the ratio of the Bohr radius of a «dark atom» to the radius of the n–α-particle nucleus. For example, for OHe, this parameter shows the ratio of the Bohr orbit for the α-particle in a «dark atom» to the radius of the helium nucleus. If the Bohr orbit of XHe turns out to be smaller than the size of the n-nucleus of helium, then the «dark atom» has the structure of Thomson's atom.

When the value of a falls within the range of $0 < a < 2$, the bound state exhibits a Bohr atom-like structure with a negatively charged particle at the core and a nucleus orbiting it in a manner similar to Bohr's model. On the other hand, when $2 < a < \infty$, the bound states resemble Thomson's atoms, where the nucleus vibrates around a heavy massive negatively charged particle.

If $Z_\alpha = 2$ and $Z_X = -2$, the model treats the α-particle as a point-like particle and travels along the Bohr radius. Consequently, the binding energy for OHe, involving the point-like charge of ^{4}He, can be calculated using the expression:

$$I_0 = \frac{Z_X^2 Z_\alpha^2 \alpha^2 m_{He}}{2} \approx 1.6 \text{ MeV}, \tag{1}$$

where m_{He} represents the mass of the α-particle.

In «dark atoms» OHe, the Bohr radius of rotation for He is equivalent to [3]:

$$R_b = \frac{\hbar c}{Z_X Z_\alpha m_{He} \alpha} \approx 2 \cdot 10^{-13} \text{ cm}. \tag{2}$$

According to the X-helium model, X can either exhibit lepton-like behavior or act as a distinct cluster of heavy quarks belonging to new families that have suppressed interaction with hadrons [3]. Furthermore, experimental evidence suggests that the minimum mass of multiply charged stable particles is approximately 1 TeV [9]. In this work, the mass of the X particle is always taken to be equal to 1 TeV.

The XHe hypothesis is notable for its simplicity, as it relies on only one parameter of «new» physics—the mass of the X particle. However, it necessitates a thorough understanding of known nuclear and atomic physics, which have not yet been applied to non-classical bound systems like «dark atoms» XHe [3]. Research focused on the active influence of this type of dark matter on nuclear transformations is crucial and leads to advancements in the nuclear physics of X-helium. This research is particularly important when assessing the quantitative role of «dark atoms» in primordial cosmological nucleosynthesis and stellar evolution [3].

The deceleration of cosmic XHe as it passes through terrestrial matter makes it challenging to directly detect dark matter particles using methods that rely on recoiling effects from WIMP–nucleus collisions. The parameters of X-helium atoms, both Thomson and Bohr, ensure effective slowing down of the flow of cosmic «dark atoms» to thermal energies and slow diffusion to the center of the Earth. Therefore, the search for dark matter particles using the effects of the recoil nucleus ([10]) is impossible in this case. And the interpretation of the results of the $DAMA$ experiment requires another explanation, which we propose and try to justify. When slow X-helium atoms interact with nuclei, they can form low-energy bindings between each other. Within the uncertainty range of nuclear physics parameters, there is a specific range where the binding energy in the OHe-Na system falls within the 2–4 keV interval [3]. When «dark atoms» are captured and enter this bound state, they release energy. This energy can be observed as an ionization signal in detectors like $DAMA$. The concentration of XHe in underground detectors is determined by the balance between the cosmic influx of dark matter atoms and their diffusion toward the center of the Earth. The presence of X-helium in the Earth's crust is quickly regulated through the dynamics of interactions between «dark atoms» and matter, considering the incoming cosmic XHe and changes in its flux. As a result, the capture rate of «dark atoms» should exhibit annual modulations, which would be reflected in the annual modulation of the ionization signal produced by these reactions. A significant consequence of this proposed explanation is the emergence of anomalous superheavy sodium isotopes in the detector material of $DAMA/NaI$ or $DAMA/LIBRA$. The mass of these isotopes is approximately an order of magnitude higher than that of regular isotopes of sodium. However, the occurrence of anomalous superheavy iodine isotopes is unlikely, as calculations indicate that it is not favorable for X-helium atoms to form low-energy bound states with these nuclei [3]. If the atoms of these unusual isotopes are not fully ionized, their ability to move is determined by atomic cross-sections and is approximately nine orders of magnitude lower than that for *O*He [3]. This means that they will remain in the detector. Thus, analyzing the material using mass spectroscopy can provide further confirmation of the presence of an *O*-helium component in the *DAMA* signal. The methods used for this analysis should consider the delicate nature of the bound states of *OHe-Na*, as their binding energy is only a few keV [3].

When dark matter atoms collide with matter, particularly in regions with higher concentrations of *X*He in the central part of the Galaxy, it can cause the excitation of *X*-helium. As a result, the excess of the positron annihilation line observed by *INTEGRAL* in the central part of the galaxy can be attributed to the creation of pairs emitted by the excited «dark atoms» *X*He that are formed through these collisions [3].

The primary issue with *X*He atoms is their potential strong interaction with matter. This arises from the unshielded nuclear attraction of helium to the nuclei of matter. This interaction has the potential to disrupt the bound system of dark matter atoms and result in the creation of abnormal isotopes. Strict experimental limitations on the concentration of these isotopes exist in terrestrial and marine environments [3]. To prevent an excessive production of abnormal isotopes, it is hypothesized that the effective interaction between *X*He and matter nuclei should have a shallow well and a barrier that hinders the fusion of *He* and/or *X* with the nucleus (see Figure 1). Facing the challenge of being a three-body problem, an exact analytical solution is absent. In order to evaluate the physical meaning of the proposed scenario with a potential barrier and a shallow well in the effective interaction potential, in this work, we developed our semiclassical approach, which is determined by the fact that the nontrivial nature of the problem requires, first of all, clarification of the possibility of implementing this scenario. Unlike ordinary atoms, in a dark atom, we have a leptonic core and a nuclear interacting shell. In this case, the usual approximations of atomic physics do not work. Therefore, we wanted to consistently approach a correct quantum mechanical description starting with a semiclassical model, which allows us to figure out the most essential points of the proposed dark atom scenario. Hence, this study puts forward a numerical approach to characterize the interaction between *X*He and matter nuclei, using a numerical model to reconstruct the form of the corresponding effective potential.

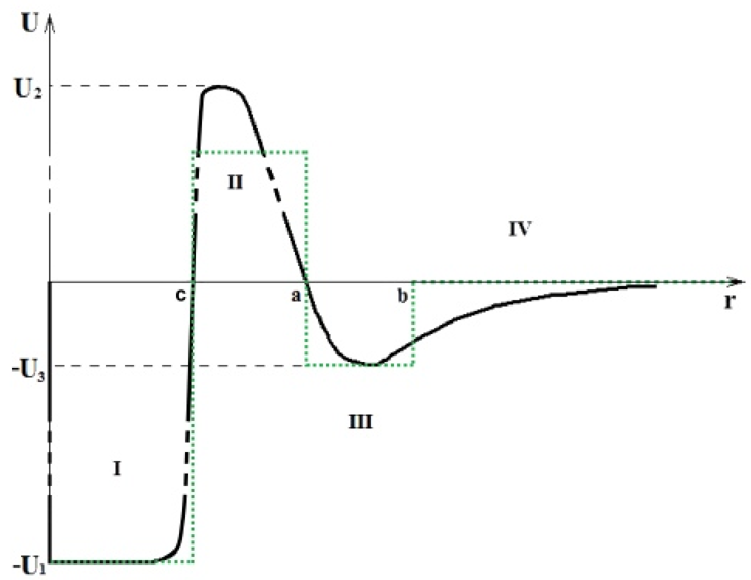

Figure 1. The interaction between *X*He and the nucleus of matter is characterized by the effective potential [3].

3. Bohr's Model

3.1. Simulation of O-Helium

The OHe system consists of two particles, namely the He nucleus and the O^{--} particle. These particles are bound together and considered point-like. In this system, a spherical coordinate system is centered at the O^{--} particle. The He nucleus moves randomly along the surface of a sphere with a radius equal to the atom's radius R_b. The He nucleus maintains a constant velocity known as the Bohr velocity V_α.

The speed of the α-particle moving along the Bohr orbit is equal:

$$V_\alpha = \frac{\hbar c^2}{m_{He} R_b} \approx 3 \cdot 10^4 \frac{\text{cm}}{\text{s}}. \tag{3}$$

Let us consider a numerical simulation scheme for the dynamic system of OHe:

(1) Within the OHe bound system, the α-particle is characterized by two independent degrees of freedom, specifically the polar angle, θ, and the azimuthal angle, ϕ. The initial values of these angles, θ_0 and ϕ_0, are used to compute the initial components of the α-particle's position vector, which is denoted as r_0.

(2) The variations in the polar angle, $d\theta$, and the azimuthal angle, $d\phi$, are calculated as the differences in angles while moving from the point r_{i-1} to the point r_i on the surface of a sphere within one iteration of time dt; here, the subscript i represents the iteration number:

$$d\theta = \left(\frac{V_\alpha dt}{R_b}\right)\left(2n - 1\right), \tag{4}$$

$$d\phi = \frac{\sqrt{\left(\frac{V_\alpha dt}{R_b}\right)^2 - \left(d\theta\right)^2}}{\cos(\theta)}\left(2n - 1\right), \tag{5}$$

where n is a random variable with a uniform distribution on the interval from 0 to 1.

(3) We verify the condition for the change in angles:

$$\left(d\theta\right)^2 + \left(\cos\theta d\phi\right)^2 \leq \left(\frac{V_\alpha dt}{R_b}\right)^2. \tag{6}$$

This condition is essential to ensure that the calculated trajectory of the alpha particle, based on the incremental angles $d\theta$ and $d\phi$, does not surpass the actual distance covered by the alpha particle on the sphere during time dt.

The positional components of the α-particle's vector, r, are computed for each time step, enabling the construction of its trajectory on the surface of sphere of the Bohr radius (see Figure 2). This sphere, with a radius of R_b, illustrates the α-particle's location through blue dots, representing its positions between dt time intervals. The density of these points on the sphere is determined by the number of iterations in the cycle.

3.2. OHe-Nucleus Coordinate System

Let us examine the OHe-nucleus system, which is composed of three charged, point-like particles. The coordinate system is positioned at the center of the target nucleus A. In the chosen reference system, O-helium moves in relation to the origin of coordinates. Herewith, the α-particle moves along a sphere with Bohr radius R_b; in the center of this sphere, there is a particle O^{--}. We introduce the position vector O^{--}, $\vec{r}$, and the position vector of the α-particle, $\vec{r}_\alpha$ (see Figure 3). In this scenario, $\vec{r}_\alpha$ is determined in the following manner:

$$\vec{r}_\alpha = \vec{r} + \vec{R}_b. \tag{7}$$

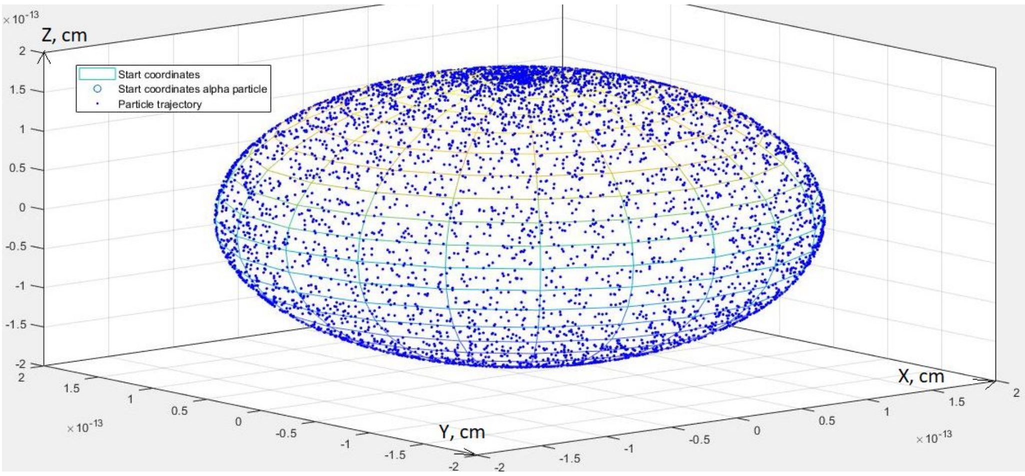

Figure 2. The spatial distribution of the $\alpha-$particle's coordinates within the orbit that corresponds to the ground state of the OHe system. Original authors' figure taken from [11].

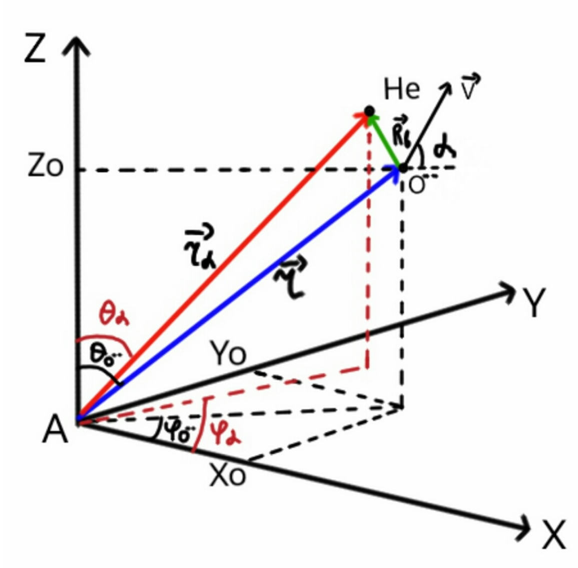

Figure 3. OHe−nucleus coordinate system. Original authors' figure taken from [11].

Our task is to consider the interaction of OHe with the nucleus by constructing a set of forces acting between all particles in the chosen coordinate system. We must take into account the electromagnetic forces acting between O^{--} and the nucleus, O^{--} and He, and He and the nucleus as well as the nuclear interaction between helium and the target nucleus. This problem is formulated as a three-body problem without an exact analytical solution. Thus, we propose a numerical approach to describe the listed interactions.

3.3. Simulation of the OHe-Nucleus System

The formulas for the Coulomb interaction between the α-particle and the target nucleus and between O^{--} and the target nucleus are as follows:

$$\vec{F}^{\,e}_{i\alpha} = \vec{F}^{\,e}_{i\alpha}(\vec{r}_{i\alpha}) = \frac{ZZ_\alpha e^2 \vec{r}_{i\alpha}}{r^3_{i\alpha}}, \quad (8)$$

$$\vec{F}^{\,e}_{iZO} = \vec{F}^{\,e}_{iZO}(\vec{r}_i) = \frac{ZZ_o e^2 \vec{r}_i}{r^3_i}, \quad (9)$$

where Z, Z_o are the charge numbers of the target nucleus and the O^{--} particle, respectively, and index i indicates the iteration number.

The nuclear interaction between the *He* nucleus and the target nucleus is determined by the nuclear force of the Woods–Saxon type, $\vec{F}_{i\alpha}^{N}$:

$$\vec{F}_{i\alpha}^{N} = -\frac{\frac{U_0}{p}\exp\left(\frac{r_{i\alpha} - R_Z}{p}\right)\frac{\vec{r}_{i\alpha}}{r_{i\alpha}}}{\left(1 + \exp\left(\frac{r_{i\alpha} - R_Z}{p}\right)\right)^2}, \tag{10}$$

where R_Z is the radius of the target nucleus, U_0 is the depth of the potential well, and $p \approx 0.55$ fm is a constant parameter.

The total force acting on the «dark atom» of OHe, $\vec{F}_{iSum}$, is calculated as follows:

$$\vec{F}_{iSum} = \vec{F}_{iZO}^{e} + \vec{F}_{i\alpha}, \tag{11}$$

where $\vec{F}_{i\alpha}$ is defined as the total force acting on the α-particle:

$$\vec{F}_{i\alpha} = \vec{F}_{i\alpha}^{e} + \vec{F}_{i\alpha}^{N}. \tag{12}$$

To calculate the total force acting on the «dark atom» OHe, we developed a numerical method based on the distance between objects. This method will utilize the described earlier model for describing the OHe system and apply it to the OHe-nucleus system for calculations.

(1) To begin, we will establish the initial conditions for the calculation. These conditions include the initial coordinates of O^{--}, denoted as $[x_0, y_0, z_0]$ or simply r_0, as well as the initial components of its velocity, denoted as $[V_{x_0}, V_{y_0}, V_{z_0}]$ or V_0, $(i = 0)$.

(2) Next, we determine the i-th value of the increment of the α-particle momentum at time interval dt, which is denoted as $d\vec{P}_{\alpha_i}$:

$$d\vec{P}_{i\alpha} = \vec{F}_{i\alpha}dt. \tag{13}$$

(3) The program checks for a condition to interrupt when the excess kinetic energy dT_i transferred to the α-particle exceeds the ionization potential of O-helium I_0. This condition leads to the destruction of the bound OHe system:

$$dT_i = \frac{d\vec{P}_{i\alpha}^{2}}{2m_\alpha} < I_0 \approx 1.6 \text{ MeV}. \tag{14}$$

(4) The new position of the particle O^{--} is calculated:

$$r_{i+1} = r_i + V_i dt \tag{15}$$

(5) Using the model for describing the system of the «dark atom» OHe and the new position of the O^{--} particle, the new position of the α-particle, $r_{\alpha_{i+1}}$, is calculated.

(6) During every iteration, the program computes the total force exerted on the OHe system, which is denoted as $\vec{F}_{isum}$.

(7) The program calculates the change in momentum $d\vec{P}_i$ of the OHe system, which is equal to the change in momentum of the particle O^{--} overall:

$$d\vec{P}_i = \vec{F}_{isum}dt. \tag{16}$$

(8) The program calculates the change in velocity $d\vec{V}_i$ of the particle O^{--} using the momentum increment $d\vec{P}_i$, and this velocity change is used for subsequent finding of the new speed, V_{i+1} by adding it to the old value of the O^{--} particle velocity:

$$d\vec{V}_i = \frac{d\vec{P}_i}{m_{O^{--}} + m_\alpha}. \tag{17}$$

Using the acquired data, the program restores movement trajectories of both the α-particle and O^{--} particle in the XY plane (see Figure 4). The target nucleus, specifically the Na nucleus, is depicted as a black circle in Figure 4, which displays the outcome of the program. The α-particle trajectory is represented by blue dots, while the O^{--} particle trajectory is shown as a red dashed line.

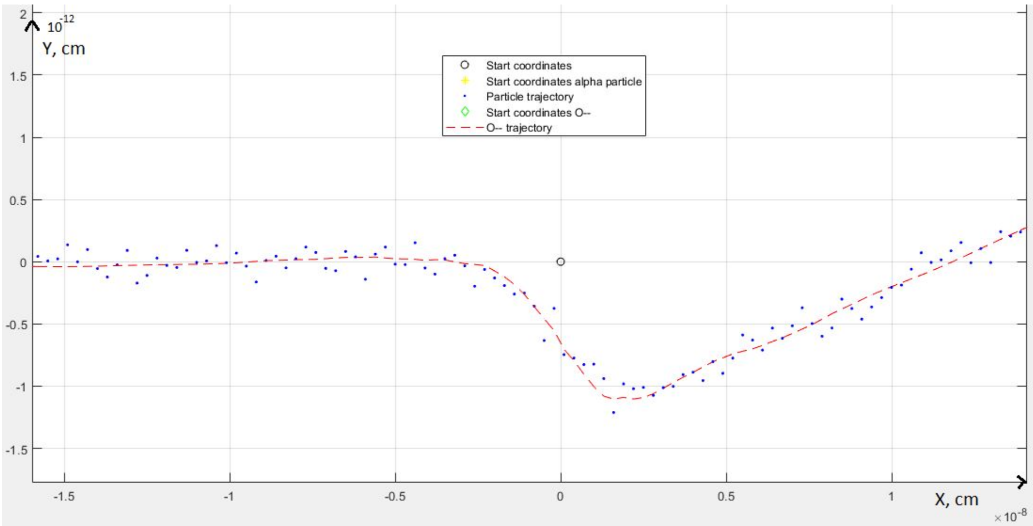

Figure 4. The movement trajectories followed by an $\alpha-$particle and the O^{--} in the XY plane. Original authors' figure taken from [11].

Figure 4 displays one of the outcomes of our simulation. The figure illustrates how the trajectory of O^{--} deviates from its initial path due to the Coulomb interaction between the He nucleus and the target nucleus. This deviation occurs because when rotating in Bohr's orbit, there are times when the He nucleus approaches the origin and experiences a stronger repulsion from the target nucleus compared to the attraction it receives from the O^{--} particle. Additionally, the figure shows that the trajectory of O^{--} undergoes oscillations. These oscillations are caused by an additional nuclear interaction between the α-particle and the nucleus, which leads to an attraction of the α-particle. As a result, as the α-particle approaches the nucleus, this force becomes stronger, causing further distortion in the trajectory of O^{--}.

3.4. Stark Effect in the OHe-Nucleus System

In our numerical Bohr model, we restrict the rotation orbit of He within the OHe atom, preventing its polarization. And we can observe the Coulomb repulsion between the «dark atom» and the target nucleus. Conversely, when an alternating external electric field is present, such as the one created by the target nucleus, we expect to see the Stark effect, which results in the polarization of XHe. We introduce the interaction dipole moment δ caused by the Stark effect to incorporate this effect into our semiclassical numerical model. By manually including δ in our numerical model, we can calculate the Stark force, which can be easily derived from the potential. This potential is determined by the same dipole moment. The appearance of δ is a result of the nuclear force and Coulomb force acting on the nHe nucleus from the outer nucleus. These forces are balanced by the Coulomb force between the particles of the dark matter atom. From this, we can derive an expression for δ:

$$\vec{\delta} = \frac{Z_\alpha \vec{E}}{Z_X 4/3\pi\rho} + \frac{\vec{F}^N_\alpha}{eZ_X 4/3\pi\rho}, \tag{18}$$

here, $\vec{E}$ represents the strength of the external electric field, and $\rho = \dfrac{Z_\alpha e}{4/3\pi R^3_{nHe}}$ denotes the charge density of the nHe nucleus.

In an external electric field, a moment of force acts on an electric dipole, which tends to rotate it so that the dipole moment turns along the direction of the field. The potential energy of a polarized XHe atom, the Stark potential, in an external electric field is equal to:

$$U_{St} = eZ_\alpha(\vec{E} \cdot \vec{\delta}). \tag{19}$$

In turn, the Stark force is calculated using the Stark potential as follows:

$$\vec{F}_{St} = -\operatorname{grad} U_{St}. \tag{20}$$

After adding the Stark force to the numerical model, using the obtained data, it is also possible to reconstruct the trajectories of the α-particle and the O^{--} particle for different impact parameters. For example, for zero and non-zero impact parameters, Figures 5 and 6 demonstrate the result of the program. The black circle shows the location of the target nucleus, for which we hereinafter take the sodium nucleus, since in the $DAMA/NaI$ or $DAMA/LIBRA$ experiment, the detector substance consists of NaI, and the results of our simulation may be useful for interpreting the results of this experiment. The blue dots show the trajectory of the α-particle, and the red dashed line shows the trajectory of the O^{--} particle in the XY plane.

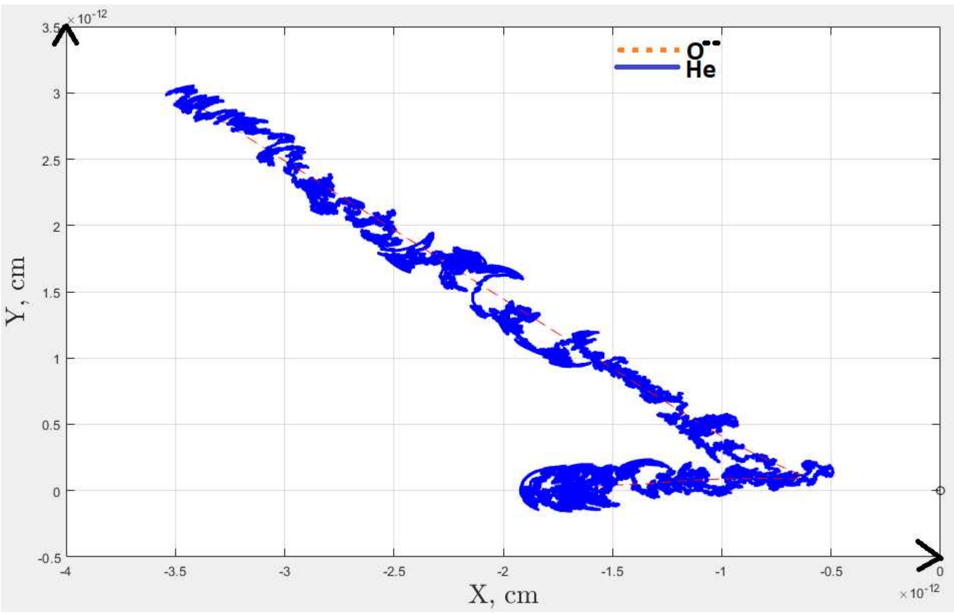

Figure 5. The movement trajectories followed by an $\alpha-$particle and the O^{--} particle in the XY plane for zero impact parameter.

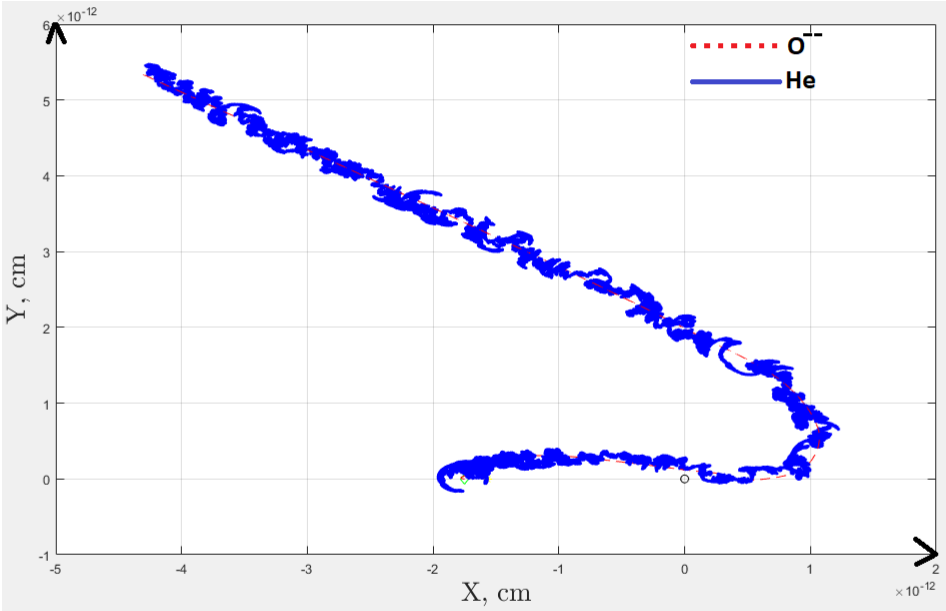

Figure 6. The movement trajectories followed by an $\alpha-$particle and the O^{--} particle in the XY plane for non-zero impact parameter.

1. Impact parameter $\beta = 0$ fm (see Figure 5).

The sum of the radii of the helium nucleus and the target nucleus Na, $R_{AHe} = 5.2$ fm. Examining the minimum distance between the α-particle and the target nucleus in Figure 5, $R_{\alpha_{min}} = 5.5$ fm, that is, the α-particle does not hit the target nucleus, and elastic scattering occurs.

The maximum value of the dipole moment is $\delta_{max} = 1.71 \cdot 10^{-12}$ cm.

The average value of the dipole moment is $\delta_{mean} = 8.97 \cdot 10^{-18}$ cm.

Figure 7 plots the dependence of the dipole moment on the distance between the O^{--} particle and the target nucleus Na at zero impact parameter. It can be seen that the closer the dark matter atom is to the target nucleus, the greater the polarization it experiences, which is consistent with the theory.

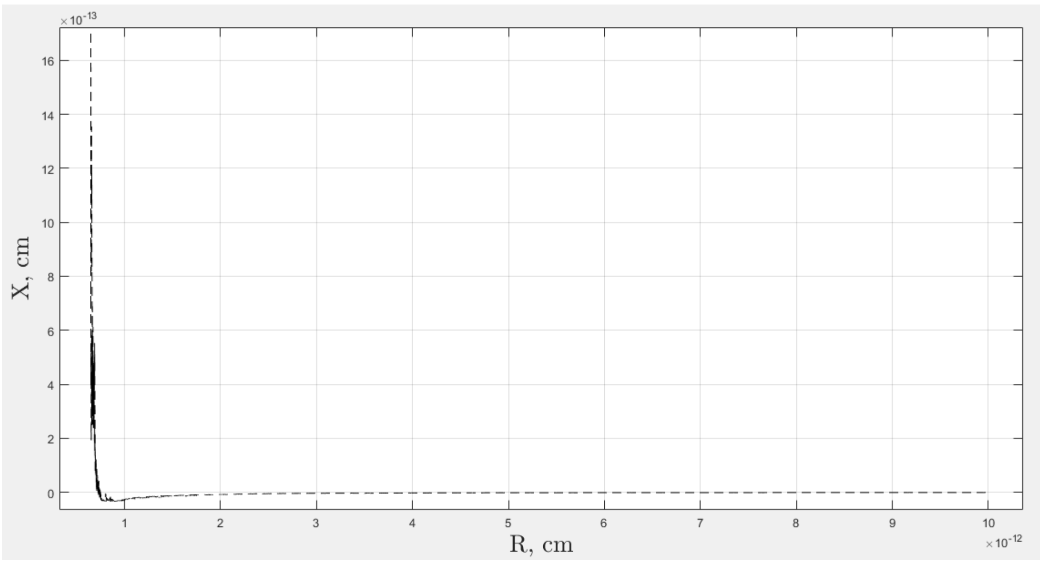

Figure 7. Dependence of the dipole moment on the distance between the O^{--} particle and the target nucleus Na (dot line) in the Bohr model at zero impact parameter.

Figure 8 plots the total interaction potential between OHe and the target nucleus Na depending on the O^{--} particle radius vector at zero impact parameter. It is clear from the figure that it qualitatively coincides with the theoretically expected form of the effective interaction potential between a dark matter atom and the nucleus of matter.

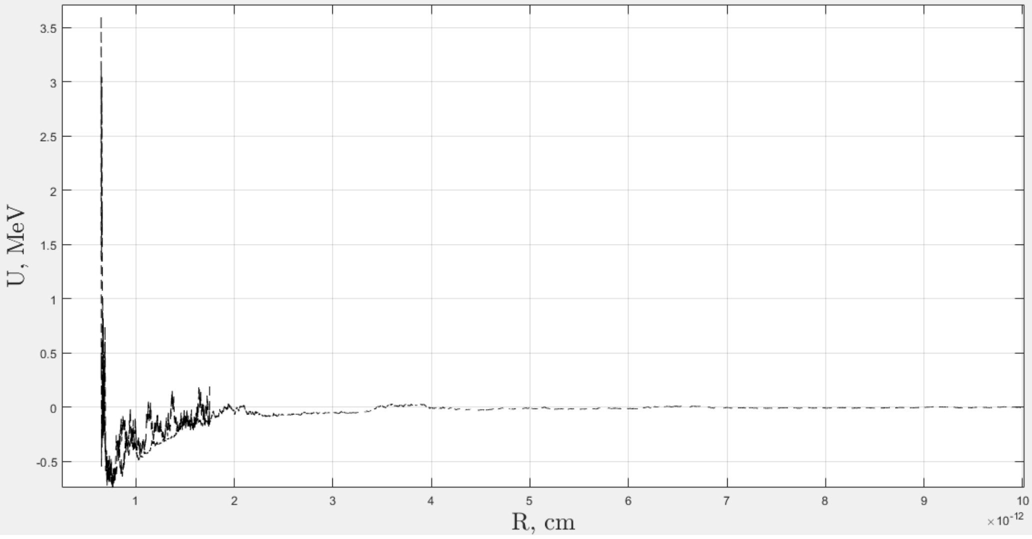

Figure 8. The total interaction potential between *O*He and the target nucleus *Na* (dot line) depending on the radius vector of the particle O^{--} at zero impact parameter.

2. Impact parameter $\beta = 1$ fm (see Figure 6).

The minimum distance between the α-particle and the target nucleus with a non-zero impact parameter $R_{\alpha_{min}} = 6.3$ fm; that is, elastic scattering also occurs in this case.

The maximum value of the dipole moment is $\delta_{max} = 7.31 \cdot 10^{-13}$ cm.

The average value of the dipole moment is $\delta_{mean} = 3.37 \cdot 10^{-15}$ cm.

Figure 9 plots the dependence of the dipole moment on the distance between the particle O^{--} and the target nucleus *Na* for a non-zero impact parameter. Again, we can see that the closer a «dark atom» is to the target nucleus, the more polarized it becomes.

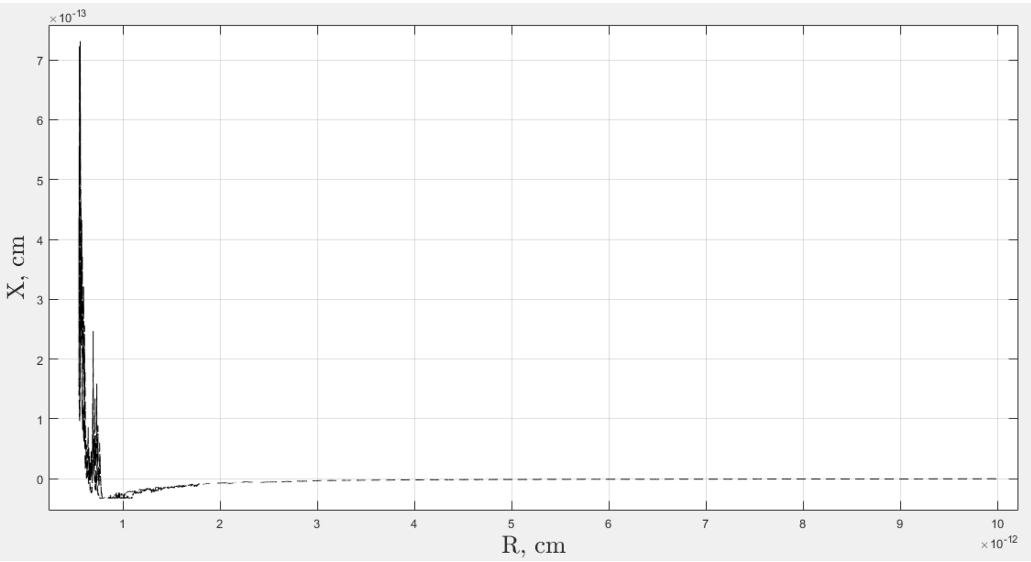

Figure 9. Dependence of the dipole moment on the distance between the particle O^{--} and the target nucleus *Na* (dot line) in the Bohr model for a non-zero impact parameter.

Figure 10 plots the total interaction potential between *O*He and the target nucleus *Na* depending on the particle radius vector O^{--} for a non-zero impact parameter. This total interaction potential also qualitatively coincides with the theoretically expected form of the effective interaction potential between «dark atom» and the nucleus of matter.

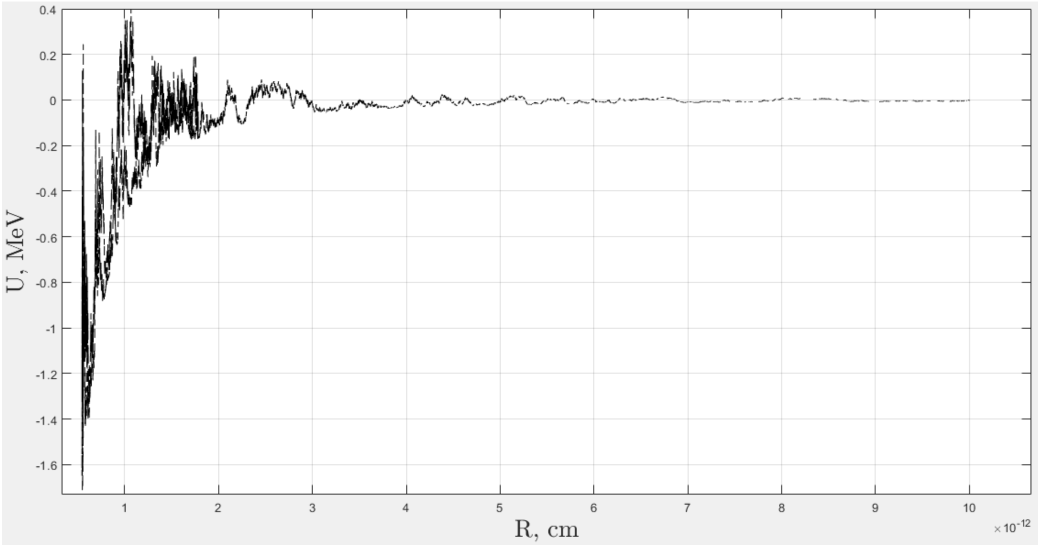

Figure 10. The total interaction potential between *O*He and the target nucleus *Na* (dot line) depending on the radius vector of the particle O^{--} for a non-zero impact parameter.

However, there are drawbacks to the Bohr atom model. For instance, our numerical model does not explicitly account for the Coulomb force between helium and O^{--}. Additionally, the rotation orbit of helium in the *O*He atom is manually fixed, which prevents its natural polarization due to the Stark effect. This necessitates the introduction of an analytical formula to calculate the dipole moment and the Stark force, leading to a decrease in the resulting accuracy. On the other hand, these issues can be resolved when considering the Thomson model of the atom. In this approach, helium is not treated as a point charge, moving randomly along a fixed Bohr orbit, but rather as a charged ball within which the O^{--} particle can oscillate. Furthermore, it is important to note that the case of -2 charged particles is just one specific scenario. The particle in the "new" physics we are examining can have a charge of $-2n$, forming *X*-helium «dark atoms» with n nuclei ^{4}He. These *X*-helium «dark atom» themselves, starting from $n = 2$, are Thomson atoms.

4. Thomson's Model

4.1. Simulation of X-helium

The *X*-helium «dark atom» comprises two interconnected components: the *n*-helium nucleus and the *X* particle. We establish a spherical coordinate system at the center of the charged *n*-helium nucleus, representing a charged sphere, within which resides the point particle *X*. When subjected to external disturbances, such as external forces causing a non-zero distance between the *n*-helium and *X*, the *X* particle initiates oscillations within the nHe nucleus (in reality, the nHe is significantly lighter than *X*, resulting in the nuclear drop oscillating around *X*).

The Coulomb interaction potential and corresponding force between the *n*-helium and *X* are expressed by the following equations:

$$U_{XHe}(R_{XHe}) = \begin{cases} -\dfrac{4e^2n^2}{R_{XHe}} & \text{for } R_{XHe} > R_{He}, \\ -\dfrac{4e^2n^2}{2R_{He}}\left(3 - \dfrac{R_{XHe}^2}{R_{He}^2}\right) & \text{for } R_{XHe} < R_{He}, \end{cases} \quad (21)$$

$$\vec{F}_{XHe}(R_{XHe}) = \begin{cases} -\dfrac{4e^2n^2}{R_{XHe}^3}\vec{R}_{XHe} & \text{for } R_{XHe} > R_{He}, \\ -\dfrac{4e^2n^2}{R_{He}^3}\vec{R}_{XHe} & \text{for } R_{XHe} < R_{He}, \end{cases} \quad (22)$$

where R_{XHe} is the distance between the X particle and the nHe nucleus, and R_{He} is the radius of the n-helium nucleus.

Let us consider a numerical simulation scheme for the dynamic XHe system.

(1) We will start with the following initial conditions: the particle X has an initial coordinate $\vec{R}_{0X} = 0$, and its initial velocity is set to the thermal speed in the medium, $V_{0_X} = \left(\frac{3kT}{M_{nuc}}\right)^{1/2}$. Here, M_{nuc} represents the mass of the target nucleus (e.g., Na), T is the temperature (assumed to be 25 degrees Celsius), and k is Boltzmann's constant. By using these initial conditions, we can calculate the initial force acting on the particle X.

(2) Next, we determine the increment of the components of the particle X's radius vector, denoted as dr_i, over a time interval dt:

$$dr_i = V_{i_X} dt. \tag{23}$$

(3) Using the calculated increment from step 2, we can determine the i + 1 value of the components of the radius vector for particle X, which is denoted as r_{i+1}:

$$r_{i+1} = r_i + dr_i. \tag{24}$$

(4) The program determines the force applied to the X particle, which is denoted as $\vec{F}_{iXHe}$. This force is then used to determine the momentum increment $d\vec{P}_i$ for particle X:

$$d\vec{P}_i = \vec{F}_{iXHe} dt. \tag{25}$$

(5) Using the momentum increment $d\vec{P}_i$, we can calculate the velocity increment $d\vec{V}_{i_X}$ for the X particle. This increment is added to the velocity from the previous iteration to find the new velocity for use in the next iteration:

$$d\vec{V}_{i_X} = \frac{d\vec{P}_i}{m_X}. \tag{26}$$

By analyzing the data, we can create a graph showing how the magnitude of particle X's radius vector changes over time. This graph, shown in Figure 11, demonstrates that particle X undergoes oscillations within the nucleus nHe with a period of approximately $2 \cdot 10^{-20}$ s. These oscillations occur because the Coulomb force between the nucleus nHe and particle X acts to bring particle X back toward the center of the nucleus and counteract any external disturbances. The fact that $R_{XHe} < 1$ fm indicates that the X-helium system is stable.

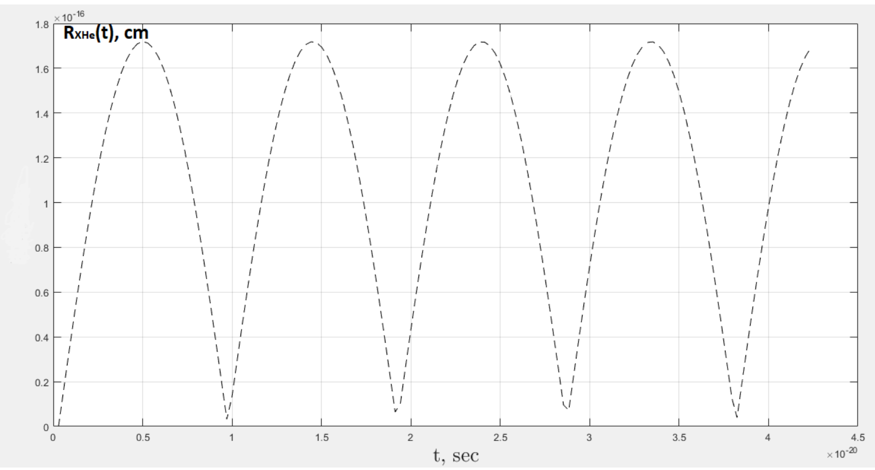

Figure 11. The relationship between the magnitude of the particle X's radius vector and time t (dot line), for $n = 1$. Original authors' figure taken from [12]

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4.2. Simulation of the XHe–Nucleus System

Simulation of the interaction of the XHe «dark atom» with the target nucleus occurs in the XHe–nucleus coordinate system, which is similar to the OHe–nucleus coordinate system described in Section 3.2 of this study. However, these systems differ in that the distance between the «dark atom» particles X and nHe is no longer fixed and is not equivalent to the Bohr radius. Therefore, the positions of the nHe particle, r_{He}, and the X particle, r, are determined independently. In this case, the distance between the particle X and nHe, r_{XHe}, is calculated by the formula:

$$\vec{r}_{XHe} = \vec{r}_{\alpha} - \vec{r} \tag{27}$$

The types of interactions acting in the XHe -nucleus system between particles are identical to the forces described in Section 3.3 of this study. However, two additional forces arise, which are equal in magnitude but opposite in direction. These forces represent the Coulomb force between X and nHe (refer to Formula (22)). The force acting on nHe is denoted as $\vec{F}_{i\alpha}^{XHe}$, while the force acting on X is denoted as $\vec{F}_{iX}^{XHe} = -\vec{F}_{i\alpha}^{XHe}$.

The total force to which particle X is subject, $\vec{F}_{iSum}^{X}$, is defined as follows:

$$\vec{F}_{iSum}^{X} = \vec{F}_{iZO}^{e} + \vec{F}_{iX}^{XHe}. \tag{28}$$

The total force acting on nHe, $\vec{F}_{i\alpha}$, is determined by the formula:

$$\vec{F}_{i\alpha} = \vec{F}_{i\alpha}^{e} + \vec{F}_{i\alpha}^{N} + \vec{F}_{i\alpha}^{XHe}. \tag{29}$$

To calculate the total force acting on the XHe «dark atom», we are developed a numerical method.

(1) The initial conditions we use include the initial coordinates X and nHe, represented by $\vec{r_0} = \vec{r}_{0\alpha}$, as well as their initial velocities. We set the initial velocities equal to the thermal speed in the medium, which is given by $V_{X_0} = V_{\alpha_0} = \left(\frac{3kT}{M_{nuc}}\right)^{1/2}$.

(2) The i-th value of the momentum increment nHe particle, $d\vec{P}_{i\alpha}$, and X particle, $d\vec{P}_{i}$, taken over the time interval dt is determined:

$$d\vec{P}_{i\alpha} = \vec{F}_{i\alpha} dt, \tag{30}$$

$$d\vec{P}_{i} = \vec{F}_{iSum}^{X} dt. \tag{31}$$

(3) By using the increment $d\vec{P}_{i\alpha}$ and $d\vec{P}_{i}$, we can determine the velocities of the nucleus nHe and X at the i + 1 time step. These velocities are denoted as $\vec{V}_{\alpha_{i+1}}$ and $\vec{V}_{X_{i+1}}$:

$$\vec{V}_{\alpha_{i+1}} = \vec{V}_{\alpha_i} + \frac{d\vec{P}_{i\alpha}}{m_{He}}, \tag{32}$$

$$\vec{V}_{X_{i+1}} = \vec{V}_{X_i} + \frac{d\vec{P}_{i}}{m_{X}}. \tag{33}$$

(4) Next, the i + 1 value of the radius vector for X and nHe can be calculated:

$$\vec{r}_{i+1} = \vec{r}_{i} + \vec{V}_{\alpha_{i+1}} dt, \tag{34}$$

$$\vec{r}_{\alpha_{i+1}} = \vec{r}_{\alpha_i} + \vec{V}_{X_{i+1}} dt, \tag{35}$$

(5) Then, the program calculates the total force acting on the X particle, represented by $\vec{F}_{iSum}^{X}$, as well as the total force acting on nHe, denoted as $\vec{F}_{i\alpha}$.

Based on the obtained data, we can plot the relationship between the radius vector of the particle X and the radius vector of the n-helium nucleus (see Figure 12). Additionally, we can plot the dependence of the total interaction potential between nHe and the target nucleus Na on r_α for $n = 1$ (see Figure 13).

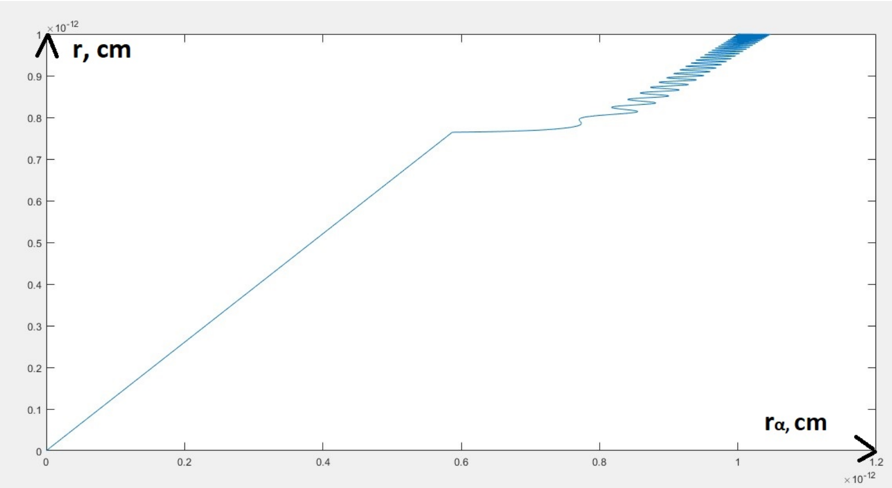

Figure 12. Dependence of r on r_α (blue line) in the Thomson model for $n = 1$. Original authors' figure taken from [12].

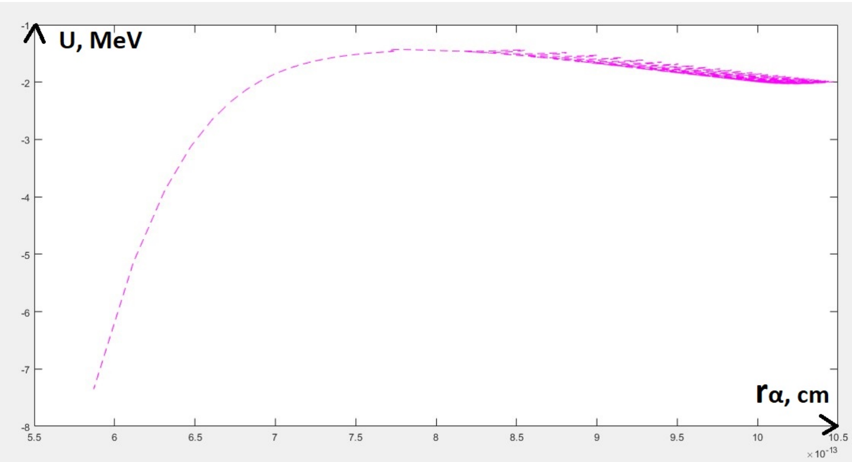

Figure 13. Dependence of the total interaction potential of nHe, at $n = 1$, with the target nucleus Na on r_α (pink dot line). Original authors' figure taken from [12].

From Figures 12 and 13, it is clear that the XHe system moves toward the target nucleus as a bound system. And the X particle is slightly ahead of the nHe nucleus, since the radius vector of the X particle at each point is less than the radius vector of the nucleus nHe (see Figure 12). The nHe nucleus flies behind oscillating; i.e., polarization of the «dark atom» is observed. It can be seen that at a sufficiently close distance from the target nucleus, the nuclear force between the n-helium nucleus and the target nucleus becomes quite strong. Therefore, it exceeds the Coulomb repulsion of nHe by the target nucleus and n-helium, rushing ahead of the X particle and penetrating the target nucleus (see Figure 13).

Following this, in our numerical model, we included the Coulomb force between the nHe and the target nucleus, as well as the Coulomb force between the X particle and the target nucleus, using a similar approach as in Formula (22). Namely, we introduced a condition that causes a change in the form of force when either the nHe or X particles penetrate the target nucleus.

From analyzing the trajectories obtained, we can identify two distinct scenarios for $n = 1$. When the impact parameter is zero, the XHe atom passes through the target nucleus Na; then, it reverses its direction and flies back in the opposite direction (see Figure 14).

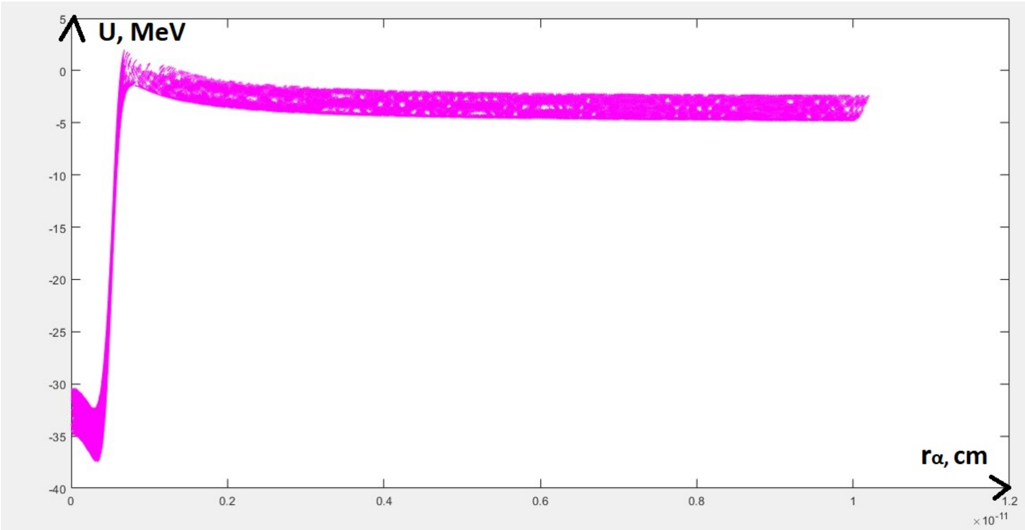

Figure 14. Dependence of the total interaction potential of nHe, for $n = 1$, with the target nucleus Na on r_α (pink line) at zero impact parameter. Original authors' figure taken from [12].

According to the theory, the interaction between slow X-helium atoms and nuclei can result in the formation of a low-energy bound state. When «dark atoms» are captured in this bound state, energy is released, which can be detected as an ionization signal in the $DAMA$ detector. Therefore, the low-energy bound state of XHe and the nucleus can be considered as a three-body oscillating system.

In Figures 15 and 16, we can observe that when the XHe atom with a non-zero impact parameter $\beta = 0.2 \cdot 10^{-13}$ cm collides with the target nucleus, a three-body oscillatory system is formed. The minimum distance between the n helium atom and the target nucleus Na in this case is $R_{\alpha_{min}} = 6.8 \cdot 10^{-16}$ cm.

In Figure 15, the black circle represents the target nucleus Na, while the yellow star and green diamond represent the initial positions of nHe and the X particle, respectively. The blue dots and red dotted line show the trajectories of nHe and the X particle, respectively.

The maximum value of the dipole moment is $\delta_{max} = 1.43 \cdot 10^{-12}$ cm.

The average value of the dipole moment is $\delta_{mean} = 5.39 \cdot 10^{-13}$ cm.

During trajectory analysis, it was discovered that changes in the mass of X do not have any effect on the outcome. The outcome is dependent on the impact parameter and the initial velocity of the system. However, regardless of their values, the cloud of particle coordinates always ends up inside the target nucleus. This is likely reasonable because a zero force balance for helium can only be achieved in the region where the nuclear and Coulomb forces are balanced. Outside of this region, nuclear force is minimal. As a result, Coulomb polarization occurs in XHe up to the boundary of the nucleus, while nuclear polarization only occurs inside.

When modeling using the Thomson atom approximation, several effects were observed. When the impact parameter is zero, the XHe atom passes through the target nucleus; then, it returns back and flies in the opposite direction. With a non-zero impact parameter, the XHe atom collides with the target nucleus, forming a vibrational system of three bodies. This is expected during the formation of a low-energy bound state of XHe with the nucleus. Additionally, in the Thomson model, the polarization of the «dark atom» automatically occurs due to the Stark effect, whereas in the Bohr model, it had to be manually introduced.

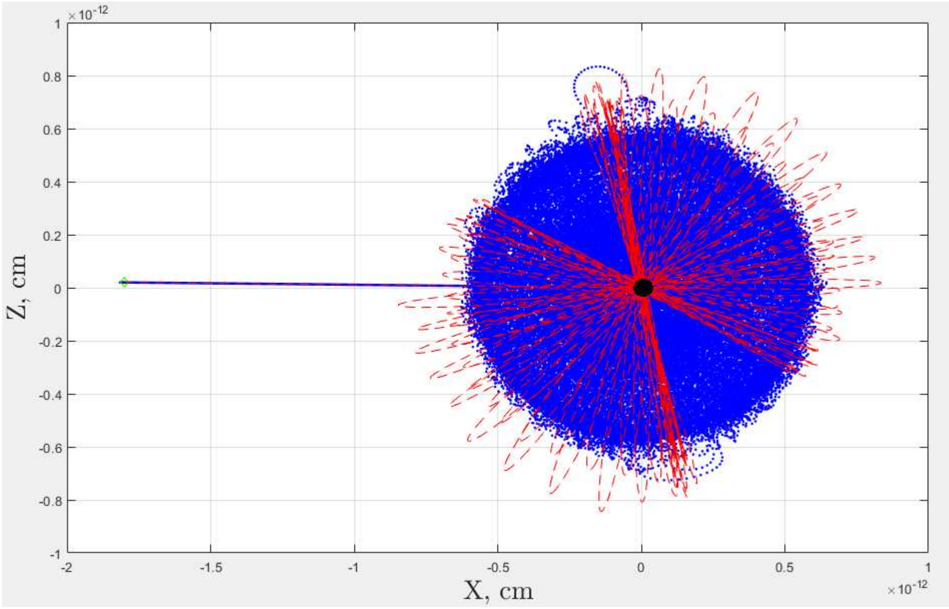

Figure 15. Trajectories of motion of a n-helium nucleus and X particle, for $n = 1$, in the XZ plane with a non-zero impact parameter in the Thomson model. Original authors' figure taken from [12].

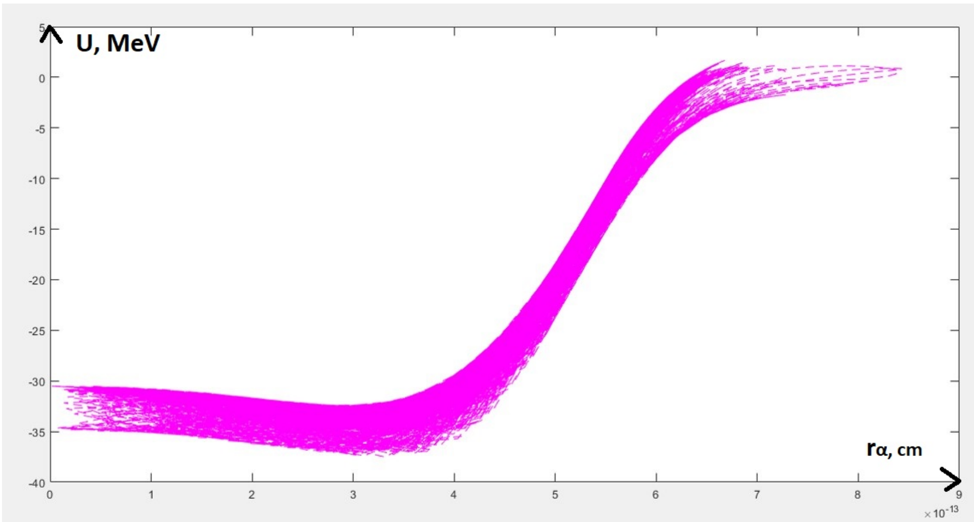

Figure 16. Dependence of the total interaction potential of nHe, for $n = 1$, with the target nucleus Na on r_α (pink dot line) for a non-zero impact parameter. Original authors' figure taken from [12].

However, the disadvantage of the Thomson approximation is that particle oscillations occur inside the target nucleus. And elastic scattering is not observed, the presence of which is very important for the existence of the XHe hypothesis. Herewith, in Bohr's atom approach, the opposite was true. In the Bohr model, the quantum mechanical connection between the particles of the «dark atom» is preserved. This quantum mechanical connection is expressed in the form of a fixed Bohr orbit of the α-particle, but this did not allow obtaining automatic polarization of the «dark atom». In Thomson's model, such a connection is lost. This happens because in this model, XHe represents two independent particles, between which the Coulomb force clearly acts, which can be introduced due to the automatically appearing polarization. That is, you can not introduce polarization «by hand», but in this case, the quantum-mechanical connection in XHe is lost. Or you can preserve this connection, but then you have to calculate the length of the dipole moment in an unnatural way, which affects the accuracy of the results. Therefore, in Thomson's model, it is necessary to introduce a connection between helium and the X particle. In addition, in order to more accurately describe the interaction of XHe with the nucleus of matter, which can lead to solving the problem of X-helium entering the nucleus and leading to the dominance of elastic interactions, it is also necessary to manually introduce the Stark force, as in the Bohr model, but already using the naturally obtained dipole moment length.

4.3. Stark Force in Thomson's Model

Since in the Thomson model, the radius vector of the nHe nucleus, r_{He}, and the X particle, r, are determined independently, the distance between X and nHe, r_{XHe}, is calculated as the difference between the radius vectors of these particles (see Formula (27)). But at the same time, r_{XHe} also determines the polarization of the «dark atom» in the Thomson model; therefore, the value of the dipole moment is equal to:

$$\delta = |\vec{r}_{XHe}|. \tag{36}$$

In order to add the Stark force to the Thomson model, in each i-th iteration, the iteration step for the nHe nucleus is determined, $h_i = |\vec{r}_{\alpha_{i+1}}| - |\vec{r}_{\alpha_i}|$, and using the calculated Coulomb force between nHe and the target nucleus of the substance, $\vec{F}^e_{i+1_\alpha}$, the potential and Stark force are calculated:

$$U_{St_{i+1}} = F^e_{i+1_\alpha} |\vec{r}_{XHe_{i+1}}|, \tag{37}$$

$$F_{i+1_{St}} = -\frac{U_{St_{i+1}} - U_{St_i}}{h_i}. \tag{38}$$

Consequently, the total force acting on nHe now consists of the nuclear force, the Coulomb force between X and nHe, the Coulomb force between nHe and the nucleus of the substance, and the Stark force.

To strengthen the connection between dark matter atom particles, we also changed the calculation of the $i + 1$ values of the velocities of the nHe nucleus and the X particle, $\vec{V}_{\alpha_{i+1}}$ and $\vec{V}_{X_{i+1}}$, by adding the following condition: if $|\vec{r}_{XHe}| < R_{He}$, i.e., if the X particle is inside nHe, where R_{He} is the radius of the n-helium nucleus, then the calculation is carried out using the following formulas:

$$\vec{V}_{\alpha_{i+1}} = \vec{V}_{\alpha_i} + \frac{d\vec{P}_{i\alpha}}{m_X + m_{He}}, \tag{39}$$

$$\vec{V}_{X_{i+1}} = \vec{V}_{X_i} + \frac{d\vec{P}_i}{m_X + m_{He}}. \tag{40}$$

Thus, the momentum increments of nHe, $d\vec{P}_{i\alpha}$, and particles X, $d\vec{P}_i$, are divided by the entire mass of XHe. If $|\vec{r}_{XHe}| > R_{He}$, i.e., if particle X is no longer inside nHe, then the increment in momentum is divided by the masses of the corresponding particles (see Formulas (32) and (33)).

From the analysis of trajectories in the Thomson model with the Stark force, for $n = 1$, i.e., when the X particle is an O^{--} particle, and the nHe nucleus is an α-particle, two characteristic cases can be distinguished independent of the impact parameter: elastic interaction and inelastic interaction, when the X particle enters the nucleus.

Elastic interaction with a non-zero impact parameter can be seen in Figure 17, which shows the trajectories of motion of nHe and X particles in the XZ plane during interaction with the Na nucleus.

In Figure 17, the blue line and the red dotted line show the trajectories of n- helium and the X particle during elastic interaction in the XZ plane, respectively. The green diamond shows the initial position of the «dark atom» particles. The figure shows that as a result of elastic scattering, XHe particles change their direction of motion to the opposite. The X-helium atom experiences greater polarization the closer it approaches the target nucleus.

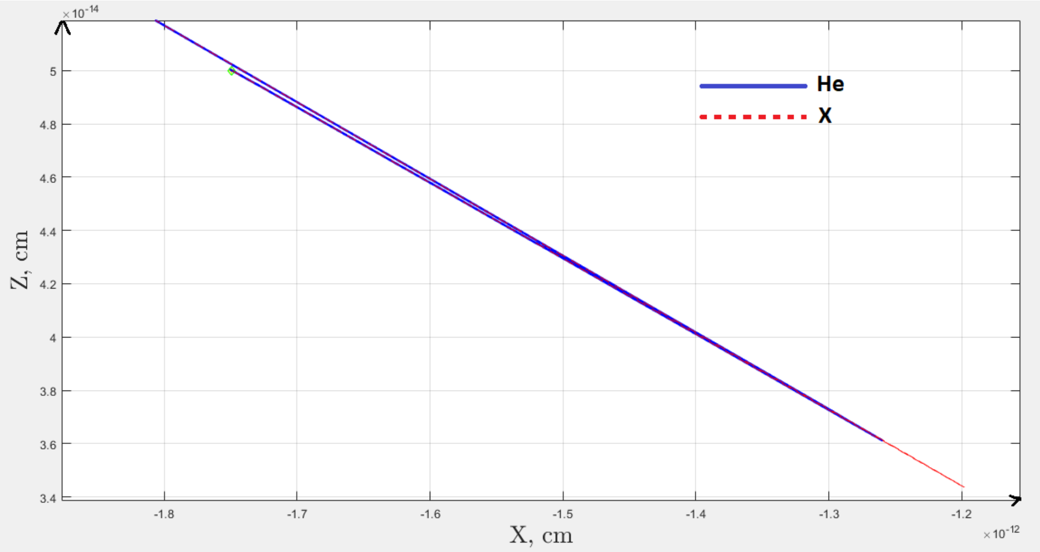

Figure 17. Trajectories of the nHe (blue line) and X particles (red dot line), for $n = 1$, in the XZ plane with a non-zero impact parameter, for elastic interaction.

The impact parameter and the minimum distance between the α-particle and the target nucleus, corresponding to the interaction shown in Figure 17, are equal to $\beta = 0.5 \cdot 10^{-13}$ cm , $R_{\alpha_{min}} = 1.3 \cdot 10^{-12}$ cm.

Figure 18 plots the dependence of the magnitude of the dipole moment modulus on the distance between the particle X and the target nucleus Na for elastic interaction and a non-zero impact parameter.

The maximum value of the dipole moment for the case in Figure 17 is $\delta_{max} = 9.08 \cdot 10^{-14}$ cm, while the average value of the dipole moment is $\delta_{mean} = 2.33 \cdot 10^{-15}$ cm (see Figure 18).

It is also possible to plot the dependence of various potentials corresponding to the forces acting between particles in the XHe–nucleus system, depending on the distance between nHe and the nucleus during elastic interaction (see Figure 19).

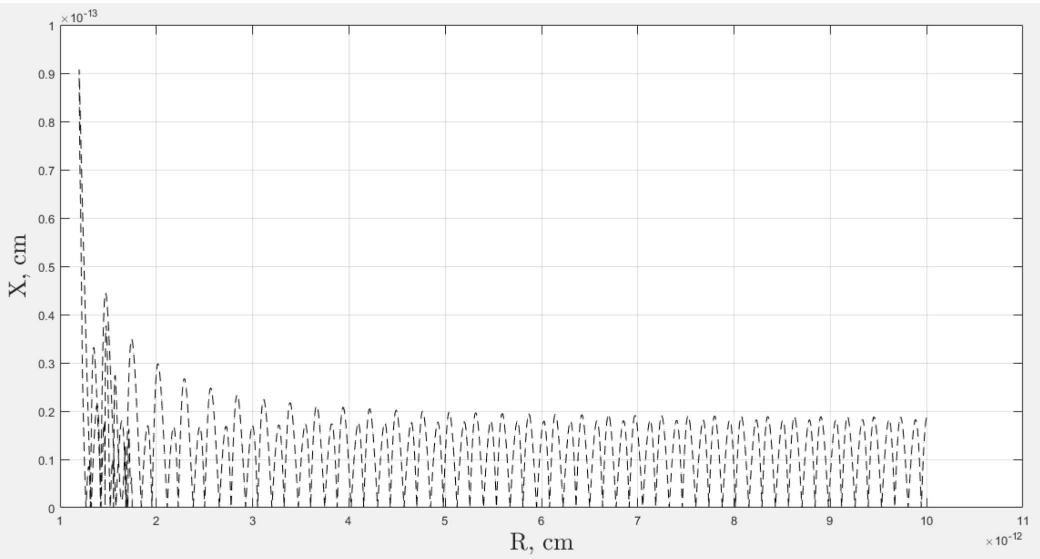

Figure 18. Dependence of the dipole moment modulus on the distance between the particle X and the target nucleus Na (dot line) in the Thomson model, for $n = 1$, with elastic scattering and a non-zero impact parameter.

In Figure 19, the blue circle shows the sum of the radii of the *n*-helium nucleus and the sodium target nucleus, while the red asterisk shows the initial position of the particles of the *X*He atom. The violet and red dotted lines show graphs of the dependence of the Coulomb potential between nHe and the nucleus and between X and the nucleus, respectively, on the distance between nHe and the nucleus. Red, blue and green solid lines show graphs of the dependence of the Stark potential, the total potential acting on *n*-helium, and the Coulomb potential between nHe and X, respectively, on the distance between nHe and the target nucleus. The black dotted line shows a graph of the total effective interaction potential between *X*He and the nucleus as a function of the distance between nHe and the nucleus.

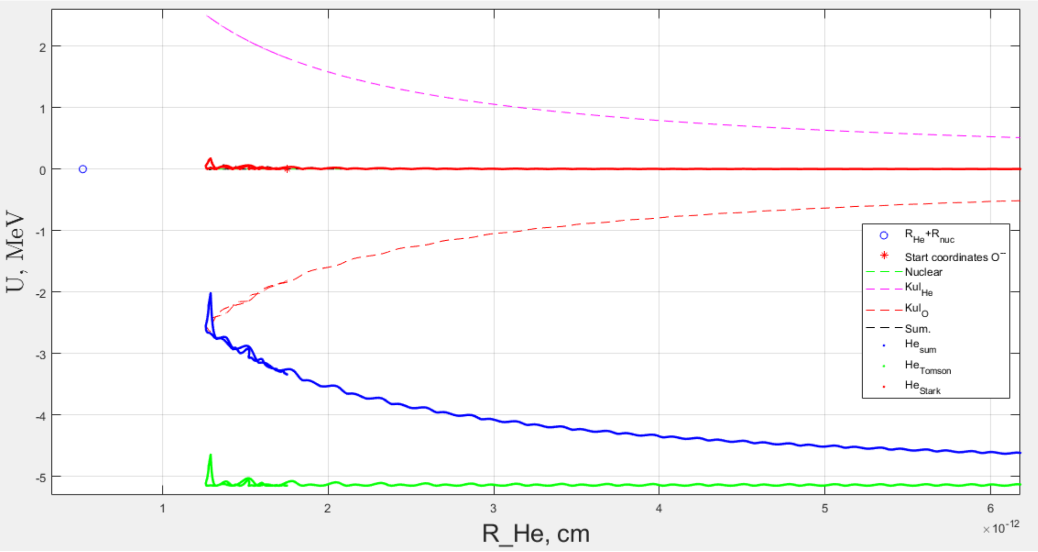

Figure 19. Dependence on the distance between nHe and the nucleus during elastic interaction, for $n = 1$: of the Coulomb potential between nHe and the nucleus (purple dotted line), between X and the nucleus (red dotted line), and between nHe and X (green solid line), of the Stark potential (red solid line), of the total potential acting on nHe (blue solid line), and of the total effective potential (black dotted line).

In Figure 19, one can see that the dark matter atom is weakly polarized, the Coulomb force between nHe and X remains approximately constant, nHe is repelled from the target nucleus stronger than X is attracted to it, and the «dark atom» experiences elastic scattering.

The result of inelastic interaction with a non-zero impact parameter can be seen in Figure 20, which shows the trajectories of motion of nHe and X particles in the XZ plane during interaction with the Na nucleus.

In Figure 20, the blue dots and the red dotted line show the trajectories of *n*-helium and the X particle during inelastic interaction in the XZ plane, respectively. The green diamond shows the initial position of the dark matter atom particles. The blue diamond shows the final position of the X particle. The black circle shows the origin of the coordinate system. From Figure 20, it is clear that as a result of inelastic scattering, the *X*He atom is destroyed and the X particle remains in the target nucleus, while the nHe nucleus flies further in the original direction.

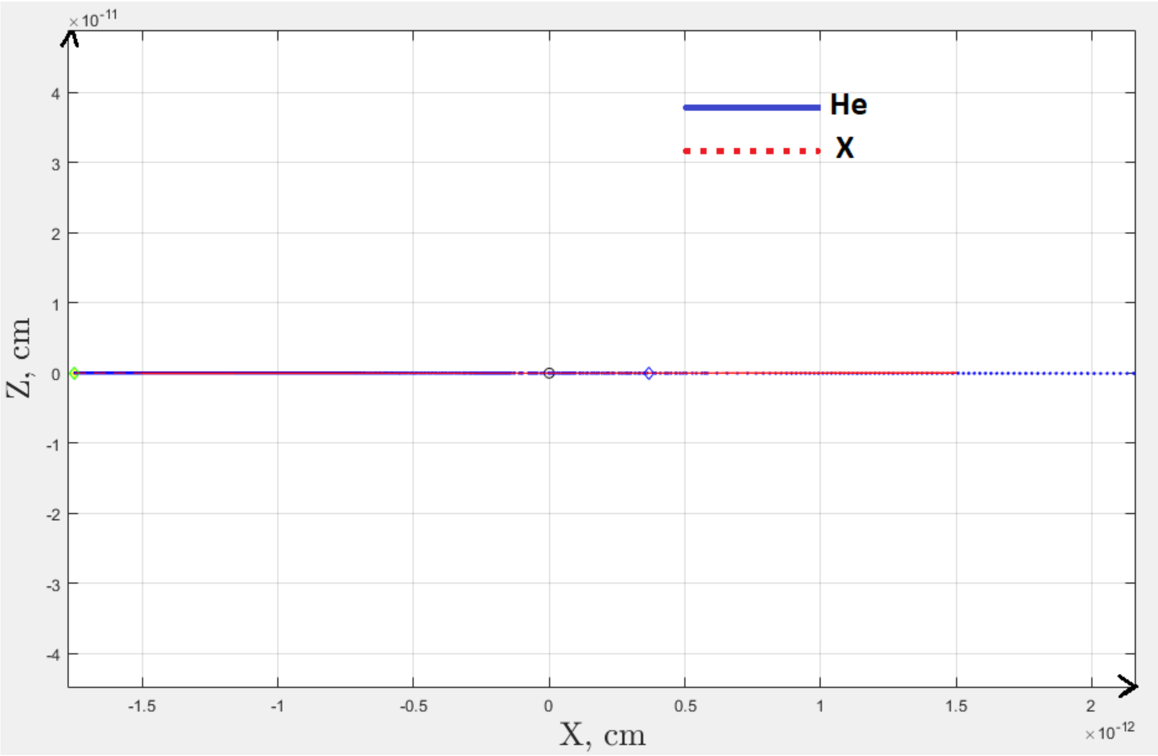

Figure 20. Trajectories of motion of the nHe (blue dots) and X particles (red dotted line), for $n = 1$, in the XZ plane with a non-zero impact parameter for inelastic interaction.

The impact parameter and the minimum distance between the α-particle and the target nucleus, corresponding to the interaction shown in Figure 20, are equal to $\beta = 0.3 \cdot 10^{-13}$ cm , $R_{\alpha_{min}} = 1.8 \cdot 10^{-15}$ cm.

Since, as a result of inelastic scattering and destruction of a «dark atom», the particle X oscillates around the target nucleus, and the helium nucleus flies away to infinity, the calculation of the length of the dipole moment, which is defined as the difference in the radius vectors of the particles of the «dark atom», over the entire interval is unphysical. Therefore, we present here the results calculated before the moment of nHe leaving the target nucleus: the maximum value of the dipole moment is $\delta_{max} = 5.59 \cdot 10^{-13}$ cm; and the average value of the dipole moment is $\delta_{mean} = 5.41 \cdot 10^{-14}$ cm.

It is also possible to plot the dependence of various potentials corresponding to the forces acting between particles in the XHe–nucleus system, depending on the distance between nHe and the nucleus during inelastic interaction (see Figure 21). The potential graphs in Figure 21 are labeled in the same way as in Figure 19.

From Figure 21, it is clear that particles of the XHe «dark atom» fall into the target nucleus. Afterwards, the X particle remains in the nucleus; this can be seen from the characteristic hole in the Coulomb potential between the X particle and the target nucleus. A dark matter atom is inelastically scattered due to the absence of an explicitly specified quantum–mechanical connection between the particles of the «dark atom» in the Thomson model. Therefore, despite the fact that the Stark and Coulomb force repels n-helium from the target nucleus, the particle X is attracted to it essentially as an independent particle. Consequently, it is possible to improve Thomson's model with the Stark force, getting rid of its shortcomings, only by explicitly adding to this model the quantum mechanical connection between the particles of the «dark atom».

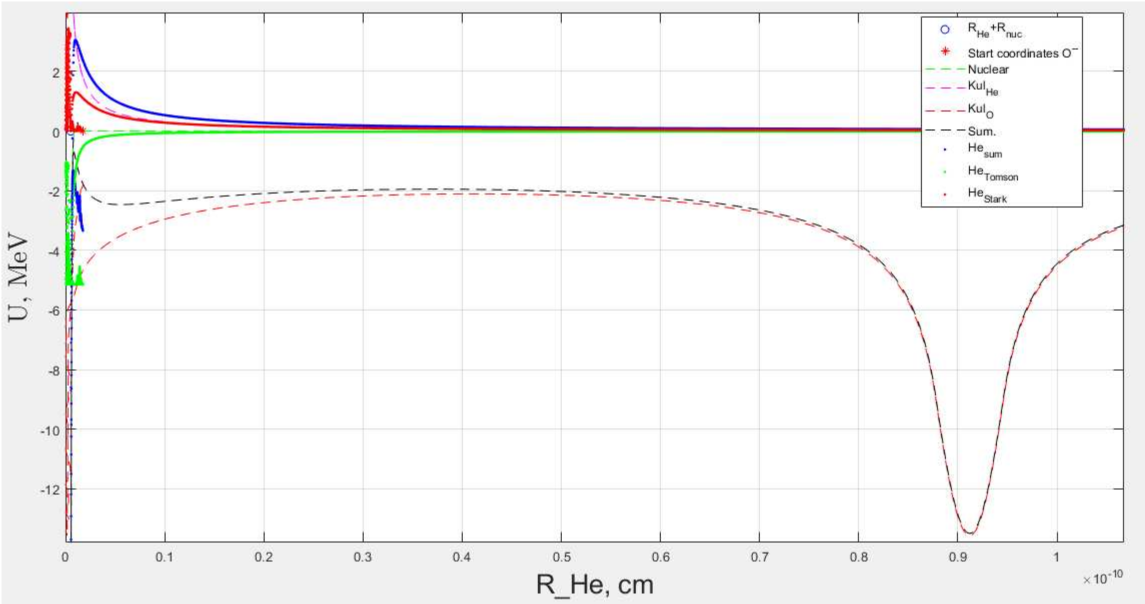

Figure 21. Dependence on the distance between nHe and the nucleus during inelastic interaction, for $n = 1$: of the Coulomb potential between nHe and the nucleus (purple dotted line), between X and the nucleus (red dotted line), and between nHe and X (green solid line), of the Stark potential (red solid line), of the total potential acting on nHe (blue solid line), and of the total effective potential (black dotted line).

5. Approach of Reconstructing of Interaction Potentials in the *X*He-Nucleus System

To reconstruct the effective potential of interaction between *X*He and the nucleus of matter, it is necessary to take into account all types of interaction existing in a given three-body system. We must take into account the electromagnetic interaction between the *X*He «dark atom» and the nucleus as well as the nuclear interaction between helium and the nucleus of the substance. And it is also necessary to take into account the Stark effect that occurs in an external, alternating electric field created by the nucleus of the substance, which leads to the polarization of *X*He.

5.1. Electric potential of X-helium

To obtain the electric potential, ϕ, created by the *X*He «dark atom» at a distance r from the center of the nHe nucleus, in which we place the nucleus of the substance, we need to solve the self-consistent Poisson equation for the test wave function $\psi = \frac{e^{-r/r_0}}{\sqrt{\pi r_0^{3/2}}}$, where r_0 is a free parameter:

$$\frac{1}{r}(\phi r)'' = -4\pi e\left(n_p + \frac{Z_X e^{-2r/r_0}}{\pi r_0^3}\right), \tag{41}$$

where en_p is the charge density of nHe:

$$en_p = \begin{cases} \dfrac{eZ_\alpha}{\frac{4}{3}\pi R_{nHe}^3} & \text{for } r < R_{nHe}, \\ 0 & \text{for } r > R_{nHe}. \end{cases} \tag{42}$$

Thus, by solving this equation, we obtained the potential created by the XHe atom, consisting of a finite nHe nucleus and an X particle, taking into account the screening effect of the X particle on the nHe nucleus:

$$\phi = \begin{cases} -eZ_X e^{-2r/r_0}\left(\frac{1}{r_0}+\frac{1}{r}\right) & \text{for } r > R_{nHe}, \\ -eZ_X e^{-2r/r_0}\left(\frac{1}{r_0}+\frac{1}{r}\right) + \frac{eZ_X}{r} + & \\ +\frac{eZ_\alpha}{R_{nHe}}\left(\frac{3}{2}-\frac{r^2}{2R_{nHe}^2}\right) & \text{for } r < R_{nHe}. \end{cases} \tag{43}$$

The potential energy of electrical interaction between XHe and the nucleus is equal to

$$U^e_{XHe} = eZ_A\phi, \tag{44}$$

where Z_A is the charge number of the nucleus of the substance.

5.2. Results of Reconstructing the Effective Interaction Potential in the XHe–Nucleus System with a Nuclear Force of the Woods–Saxon Type

Using the results of calculations, we created graphs of the dependence of the nuclear potential of the Woods–Saxon type, U^e_{XHe} (see Formula (44)), the Stark potential (recovered by calculating the dipole moment, see Formulas (18) and (19)), and the total interaction potential of a dark matter atom with the nucleus of matter on the distance between nHe and the nucleus of matter for $n = 1$, that is, for the OHe atom. The sodium nucleus was considered as the outer nucleus (see Figure 22).

Potentials are restored at the folllowing intervals: $[R_{NaHe}; 20R_{He}]$, where $R_{He} = 2.5$ fm is the root-mean-square radius of the helium nucleus, and $R_{NaHe} = 5.7$ fm is the sum of the root mean square radii of the helium nucleus and the Na nucleus, respectively.

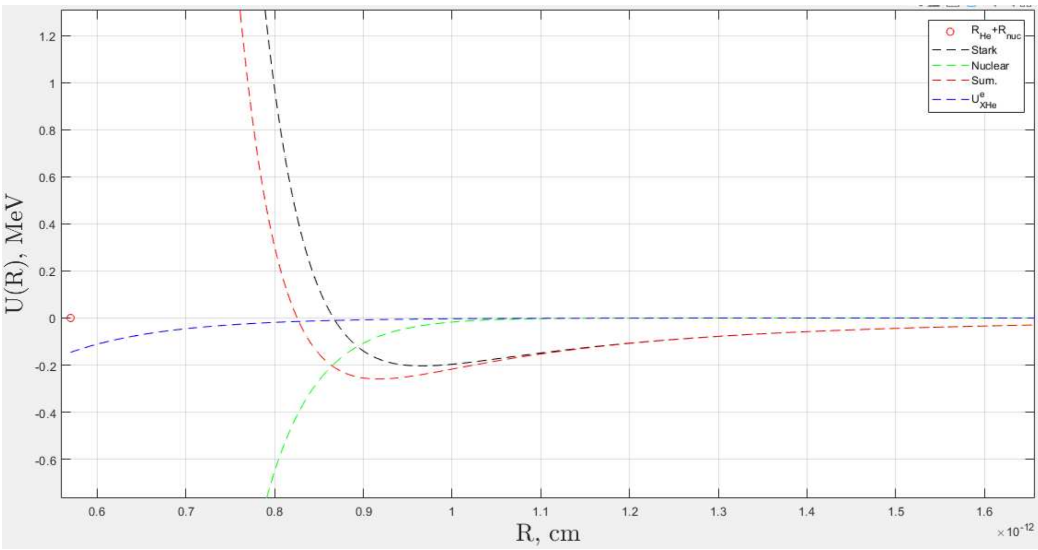

Figure 22. Dependence on the distance between He and the Na nucleus of the nuclear potential of the Woods–Saxon type (green dotted line), U^e_{XHe} (blue dotted line), of the Stark potential (gray dotted line) and of the total potential between OHe and the Na nucleus (red dotted line).

Figure 22 contains graphs with green, blue, gray and red dotted lines, showing the nuclear potential of the Woods–Saxon type, U^e_{XHe}, the Stark potential, and the total potential, respectively, depending on the distance between the helium nucleus of the OHe atom and the nucleus of matter. The red circle shows the sum of the root-mean-square radii of the helium nucleus and the nucleus of matter. In this figure, the total potential qualitatively repeats the expected shape of the effective potential of interaction between OHe and the nucleus of a substance. The total potential has a barrier, and you can also see the bound state well of the O-helium «dark atom» with a sodium nucleus with an energy of about 0.26 MeV.

Figure 23 plots the dependence of the dipole moment on the distance between the O^{--} particle and the target nucleus Na. From the figure, it can be seen that the closer the dark matter atom is to the nucleus of the substance, the greater the polarization, which is consistent with theory.

The maximum value of the dipole moment OHe when interacting with a sodium nucleus is $\delta_{max} = 4.96 \cdot 10^{-12}$ cm. The average value of the dipole moment OHe during interaction with a sodium nucleus is $\delta_{mean} = 1.05 \cdot 10^{-13}$ cm.

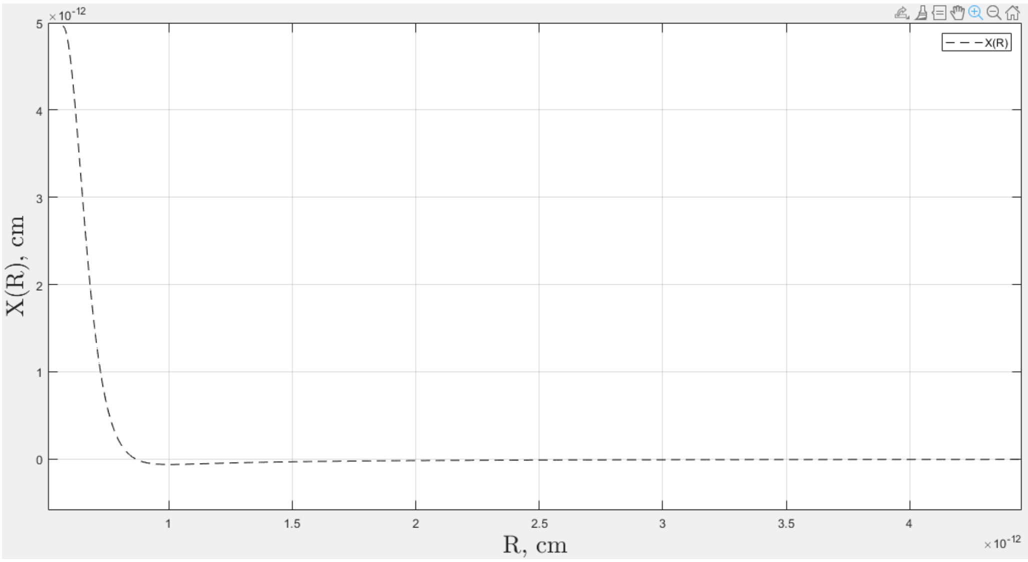

Figure 23. Dependence of the magnitude of the dipole moment on the distance between the O^{--} particle and the Na nucleus (dot line) when determining the nuclear interaction by a nuclear force of the Woods–Saxon type.

5.3. Results of Reconstructing the Effective Interaction Potential in the XHe–Nucleus System with a Nuclear Force Taking into Account the Non-Point Nature of Interacting Nuclei

Using the results obtained in the article [13], in our semiclassical approach to calculating the effective potential of interaction of XHe with the nucleus of matter, we replaced the Woods–Saxon nuclear potential, which generally does not take into account the finite sizes of interacting nuclei, with the nuclear potential, which is calculated according to Formula (23) from the article [13]. When deriving this formula, the densities of nucleons of interacting nuclei and the density of the binary nuclear system are taken into account. Due to the small overlap of nuclei, the spin interaction is neglected, and both nuclei are considered as spherical. As a result, the nuclear potential of interaction between helium and the sodium nucleus was calculated numerically using the previously mentioned formula. Next, using the difference scheme, the corresponding nuclear force was calculated. After that, using this nuclear force, we obtained the values of the dipole moment using Formula (18) of this work, with the help of which, in turn, the Stark potential was calculated.

Using the results of calculations, graphs were constructed of the dependence of the nuclear potential, taking into account the non-pointwise nature of interacting nuclei, U^{e}_{XHe}, the Stark potential, and the total potential of interaction of an OHe atom with the nucleus of a substance on the distance between He and the nucleus of a substance (see Figure 24). The sodium nucleus was considered as the outer nucleus.

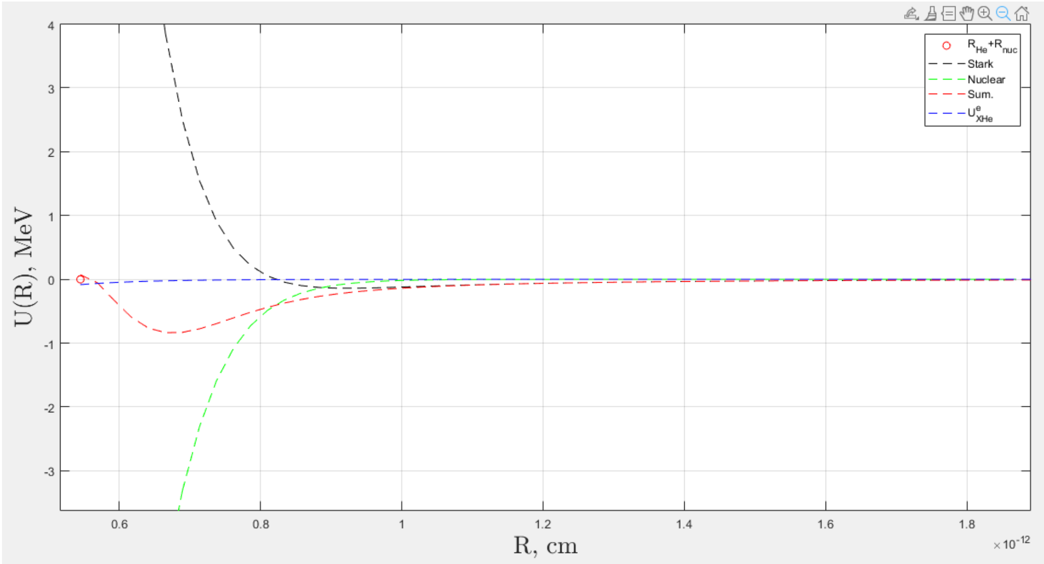

Figure 24. Dependence on the distance between *He* and the *Na* nucleus of the nuclear potential taking into account the non−point nature of interacting nuclei (green dotted line), U^e_{XHe} (blue dotted line), of the Stark potential (gray dotted line) and of the total potential between *O*He and the *Na* nucleus (red dotted line).

In Figure 24 the green, blue, gray and red dotted lines show graphs of the nuclear potential taking into account the non-pointwise nature of interacting nuclei, U^e_{XHe}, of the Stark potential and of the total potential, respectively, depending on the distance between the helium nucleus of the «dark atom» and the nucleus of matter. The red circle shows the sum of the radii of the helium nucleus and the nucleus of matter. In this figure, the total interaction potential has a well with a depth of about 0.85 MeV, but the shape of the effective interaction potential differs from the expected one.

Within the framework of the parameters uncertainty of nuclear physics, the binding energy in the *OHe*-*Na* system can fall within the 2–4 keV range [3]. This is a rather subtle phenomenon that may have been lost due to our semiclassical approach and therefore leading to a not quite accurate calculation of the dipole moment and, consequently, the Stark effect. Therefore, to improve the accuracy of the results of our calculation of the effective interaction potential, it is necessary to solve the Schrödinger equation for helium in the *OHe*–nucleus system in order to calculate the polarization of the «dark atom» using a quantum mechanical method and, thus, more accurately calculate the Stark potential.

Figure 25 plots the dependence of the dipole moment on the distance between the O^{--} particle and the target nucleus *Na*. This figure also shows that the *OHe* polarization increases with decreasing distance between the dark matter atom and the nucleus of substance, which is consistent with theory.

The maximum value of the dipole moment for interaction with the sodium nucleus is $\delta_{max} = 1.77 \cdot 10^{-12}$ cm. The average value of the dipole moment for interaction with the sodium nucleus is $\delta_{mean} = 6.41 \cdot 10^{-14}$ cm.

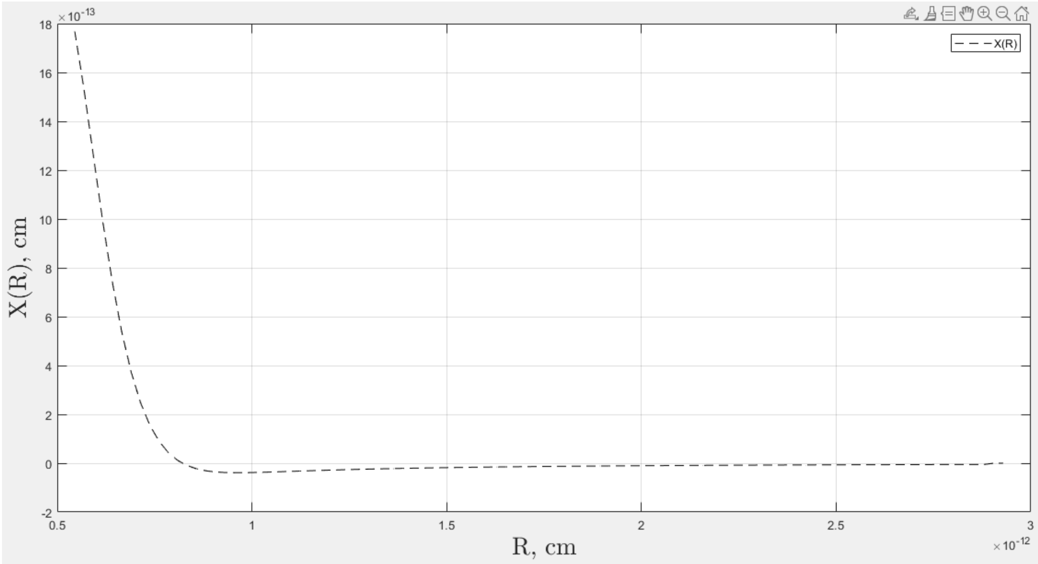

Figure 25. Dependence of the magnitude of the dipole moment on the distance between the O^{--} particle and the Na nucleus (dot line) when determining the nuclear interaction by the nuclear force, taking into account the non−pointwise nature of the interacting nuclei.

6. Conclusions

The lack of positive results of underground searches for WIMPs, as well as some problems regarding a simple dark matter description of the structure and evolution of galaxies [14,15] (as well as the data from GAIA DR3 catalogue), may posit the question of whether dark matter does exist at all and whether its physics and effects are worth studying. We can defend our position as follows: the Standard Model (SM) of elementary particles is incomplete and involves its well-motivated extensions beyond the Standard Model (BSM). Stable particles and forms of matter inevitably follow in the BSM models from the extension of the SM symmetry. The set of data of precision cosmology involves dark matter as the necessary element of the now standard cosmological paradigm. The problems related to the direct dark matter search or description of the galactic structure favor in our opinion more sophisticated but still physically well-motivated dark matter models. Here, we use such types of models, which are linked to the composite Higgs solution of the SM problem of Higgs boson mass divergence, to approach a solution for the puzzles of direct dark matter searches.

The study focuses on investigating the hypothesis of composite dark matter, where stable particles with a charge of $-2n$ combine with helium nuclei to form neutral atom-like states called XHe «dark atoms». These X-helium states can interact with ordinary matter nuclei, and the specifics of this interaction can provide explanations for certain experimental observations. It can be easily shown that in spite of their strong (nuclear) cross-section, XHe–XHe interactions do not lead to dissipation. Their number density is determined by the observed dark matter density and is so low that XHe gas becomes collisionless in the early universe soon after XHe is formed and on the scale of galaxies. Specifics of the formation of XHe and the dissipation of its density fluctuations leads dark atom cosmology to a Warmer than Cold Dark Matter scenario of structure formation, which needs special study but qualitatively is not excluded by the data of the precision cosmology (see e.g., ref. [3] for a review and details).

For the XHe dark matter atom model to be viable, it is necessary for repulsive interaction to occur at some distance between XHe and the nucleus. Solving this problem is crucial for the continued existence of the XHe hypothesis [3] . To prevent the excessive production of anomalous isotopes, it is assumed that there will be a barrier in the effective interaction potential between XHe and the nucleus, preventing fusion with the nucleus of matter. This problem is formulated as a three-body problem and lacks an exact analytical solution. Therefore, this study proposes a numerical approach to describe this interaction. The goal is to sequentially construct a numerical model that can reconstruct the shape of the corresponding effective potential. In this work, two semiclassical approaches are used: the approach of reconstructing particle trajectories, which includes the Bohr model and the Thomson model, and the approach of reconstructing the potential. These models describe a system of three charged particles interacting through Coulomb and nuclear forces.

In the Bohr atom approximation, elastic interaction predominates, and the effective potential of interaction between *O*He and the nucleus qualitatively coincides with the theoretically expected one: that is, there is a Coulomb barrier and potential well. However, there are drawbacks in this model, such as the absence of an explicit specification of the Coulomb force between helium and O^{--} and the fixed Bohr orbit of rotation for *He* in the *O*He atom, which prevents automatic polarization due to the Stark effect. On the other hand, the Thomson model of the atom solves these issues by considering the helium nucleus as a charged ball within which the O^{--} particle can oscillate. Additionally, the case of -2 charged particles is just one possibility, as the X particles being considered can have a charge of $-2n$ and form a «dark atom» with n nuclei ^{4}He, which themselves are Thomson atoms starting from $n = 2$. When modeling in the Thomson atom approximation, two distinct cases can be observed for $n = 1$ regardless of the impact parameter: elastic interaction and inelastic interaction when the O^{--} particle collides with the nucleus. In elastic scattering, the direction of motion for *O*He particles changes to the opposite direction, and the *O*-helium atom experiences greater polarization as it approaches the target nucleus. In inelastic scattering, the *O*He atom is destroyed, and the O^{--} particle remains in the target nucleus while the helium nucleus continues flying in its original direction.

In the Thomson model, the polarization of the «dark atom» occurs automatically due to the Stark effect, while in the Bohr model, it had to be manually included. However, the Bohr model preserves the quantum–mechanical connection between the «dark atom» particles, which is represented by the fixed Bohr orbit of the α-particle. On the contrary, in Thomson's model, this quantum mechanical connection is lost. But at the same time, in Bohr's model, this connection does not allow automatic polarization of the «dark atom». Due to the insufficient quantum mechanical connection between the particles of the «dark atom» in Thomson's model, the dark matter atom scatters inelastically. Despite the repulsion of helium from the nucleus by the Stark and Coulomb forces, O^{--} is attracted to it as an independent particle. This means that polarization cannot be manually introduced; however, in this case, the quantum–mechanical connection in *X*He is lost. Alternatively, this connection can be preserved, but then the length of the dipole moment needs to be calculated in an unnatural way, which affects the accuracy of the results. Therefore, Thomson's model requires a stricter quantum–mechanical connection between helium and the *X* particle.

In the semiclassical approximation of the potential restoration approach, the effective potential of interaction between *X*He and the nucleus of a substance exhibits the anticipated features: a Coulomb barrier and a bound state well in the *X*He–nucleus system. However, when considering the finite dimensions of the nucleus in the nuclear force for the *Na* nucleus, the shape of the effective interaction potential deviates significantly from what was expected. The existence of binding energy in the *O*He-*Na* system is a delicate effect that could have been lost since the Stark effect was calculated in a semiclassical way, which surely reduced the accuracy of the results obtained.

From the analysis of the length of the dipole moment, it is clear that the greater the polarization of a «dark atom», the closer it is to the nucleus of a substance, which is consistent with the theory. And the maximum possible value of the length of the dipole moment δ_{max} of a «dark atom» when interacting with the nucleus *Na* is equal to about 10^{-12} cm.

To enhance the precision of determining the effective interaction potential, it is crucial to adopt a quantum mechanical method in the future. This entails solving the Schrödinger equation for helium in the *X*He-nucleus system to compute the polarization of the dark matter atom in a quantum mechanical manner. By doing so, a more accurate evaluation of the Stark potential can be achieved.

Thus, in the future, it is planned to solve the Schrodinger equations in the XHe-nucleus system. Then, it is planned to calculate the dipole moments of the polarized XHe atom using the reconstructed wave functions of helium in an isolated «dark atom» and in the XHe-nucleus system. Afterwards, we plan to restore the Stark potential and construct an effective interaction potential of the XHe–nucleus system, in which, in addition to the Stark potential, the nuclear potential and the electromagnetic potential are also taken into account.

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