Comparison of an Alternate Solenoid and Long Solenoid Cooling Channels

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Abstract

Performances of 3 cooling channels are compared: (A) FOFO, (B) alternate solenoid channel with additional correcting coils of small radius, (C) long solenoid channel with single or double field flip. Realistic input file obtained by the same precooling part with 16 GeV proton beam and carbon target is used in all the cases. After an optimization, muon/proton ratio after the cooling is about 0.1-0.12 being maximal in FOFO channel. The double field flip channel allows to reach minimal transverse emittance 2.1 mm, single flip and (B)-channels provide 3.6-3.7 mm, and FOFO – 5.4 mm

1 Introduction

The baseline model of a cooling system for the neutrino factory presumes an usage of 201 MHz accelerating cavities inserted into solenoids of about 70 cm radius [1]. A serious problem at the cooling is large momentum spread of muon beam after a bunching that is typically [2]

$$(pc)_{min}=120 \text{ MeV}; \quad (pc)_{center}=180 \text{ MeV}; \quad (pc)_{max}=240 \text{ MeV}.$$

Several possibilities to decrease the spread have been considered in [3]-[6] but they are not integrated yet in the cooling system.

At such momentum spread, many particles in FOFO cooling channel can fall in resonance regions having betatron phase advance per cell about π, 2π, etc. Beam emittance blow up and particle loss by scrapping are possible at such conditions, and a special choice of the lattice parameters is required to avoid these. An universal way is a shortening of the cell to decrease phase advance of all the particles below π. However, the ratio (field on the coil)/(central field) grows very fast with this, and a large central field as required for an effective cooling is really unachievable.

One way around this problem is an insertion of small correcting solenoids into an alternate solenoid channel of big radius to suppress the dangerous resonances. It allows to decrease the mentioned ratio by factor about 1.5 and to get proportionally less beam emittance [7].

A more radical way is refusal from alternate solenoid scheme and a usage for the cooling long solenoids with unifirm field and single field flip between. With an approptiate field flip section, it solves the poroblem of transverse motion and provides a suppression of the beam angular momentum after the cooling. However, there is rather strong longitudinal perturbation at the field flip that is the most serios problem in this case. A double flip channel allows to partially solve this problem [8].

The purpose of this report is to compare performances of these 3 cooling channels *at the same field on the coils* taken as 8 T. The same input file obtained in [2] by simulation of a precooling part is used in all the cases. A matching with considered cooling channel is performed by simple scaling of transverse coordinate/momentum ratio, if needed.

2 FOFO cooling channel

Magnetic field of an alternate FOFO channel is shaped by coils of big radius - typically 70 cm, that is comparable with a period length. At such conditions the field on the solenoid axis is almost sinusoidal because higher harmonics of the field decrease very fast with distance from the coils. Maximal and minimal β-functions of the periodic channel with sinusoidal field on the axis are plotted in Fig.1 in dependence on reduced particle momentum pc/B_0L, where $2L$ is the lattice period, B_0 is amplitude of the field. It is seen that there are a lot of betatron resonances in such a system resulting an appearance wide stop bands on the plot. The motion is stable only at

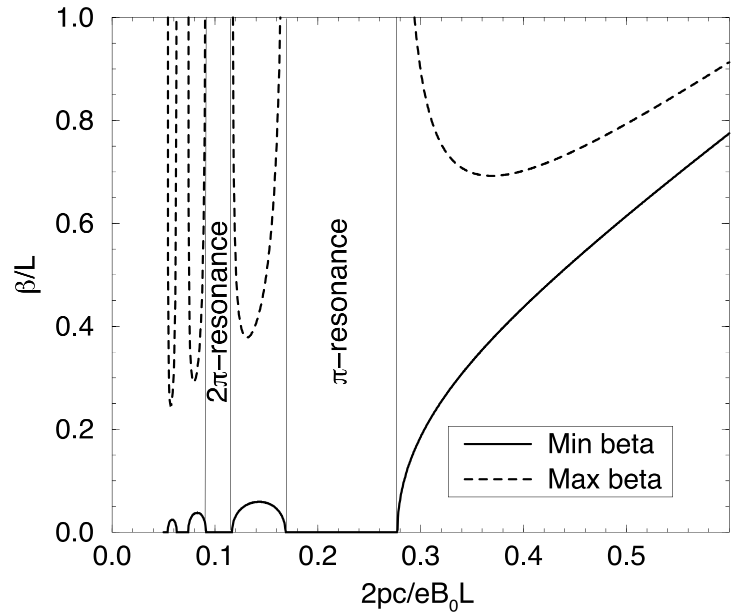


Figure 1: Minimal and maximal β-functions of a solenoid with silusoidal axial magnetic field.

$$pc > 0.14 e B_0 L, \quad \text{or} \quad 0.084 B_0 L > pc > 0.059 B_0 L, \quad \text{etc.}$$

Because ratio p_{max}/p_{min} is about 2 in the beginning of the cooling, only 1st region of stability is available imposing requirements for the lattice parameters:

$$B_0 L \simeq 6(pc/e)_{min}$$

that is 2.4 T-m at $p_{min} = 120$ MeV/c. At such conditions, β-function of particles with central momentum oscillates in the range:

$$0.6L < \beta < 0.8L.$$

According to baseline scenario β_{min} should be about 35 cm to get required beam emittance after the cooling. It gives the following parameters of the lattice: $L \simeq 0.6$ m, $B_0 \simeq 4$ T. However, at solenoid radius 70 cm, magnetic field on the coil exceeds 30 T in this case that is inadmissible, of course.

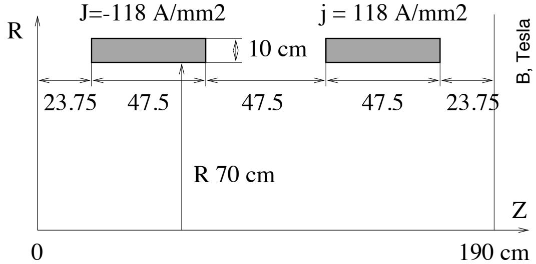


Figure 2: Schematic of the FOFO cell.

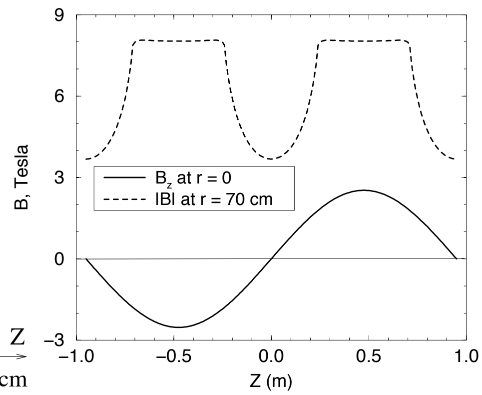


Figure 3: Magnetic field of the FOFO cell.

It depends very strongly on the period length and reasonably reduces at $L = 95$ cm. A possible coil configuration and corresponding field map of such a system are shown in Figs. 2 and 3. In this case, field on the coils reaches 8 T at $B_0 = 2.67$ T and $\beta_{min} = 54$ cm.

3 Alternate solenoid channel with correcting coils

Performances of the alternate solenoid cooling channel can be improved by a suppression of the resonances. A channel with $|B| = const$ and instantaneous field flip is resonance free providing unmodulated β-function

$$\beta = 2pc/e|B|.$$

It means that the main direction can be using of "rectangular" axial magnetic field with as fast field flip as possible. By the reasons mentioned above, such a flip cannot be produced by coils of radius like 70 cm. Fortunately, r.m.s. beam radius σ is significantly less allowing to use coils of radius 15-20 cm, that is (3-4)σ, typically. Schematic of the cooling cell satisfying these requirements is shown in Fig.4, and its magnetic field is plotted in Fig.5. Correcting coils of 18 cm radius accelerates the field flip by factor about 4 and essentially weaken all the resonances. Optimization of their length and current density

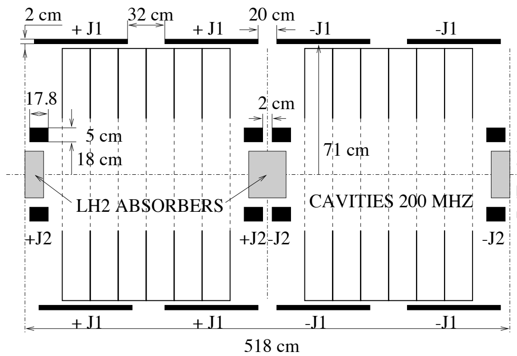


Figure 4: Schematic of the cell with correcting coils.

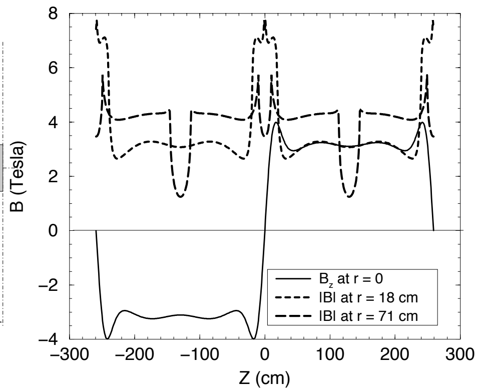


Figure 5: Magnetic field of the cell with correcting coils.

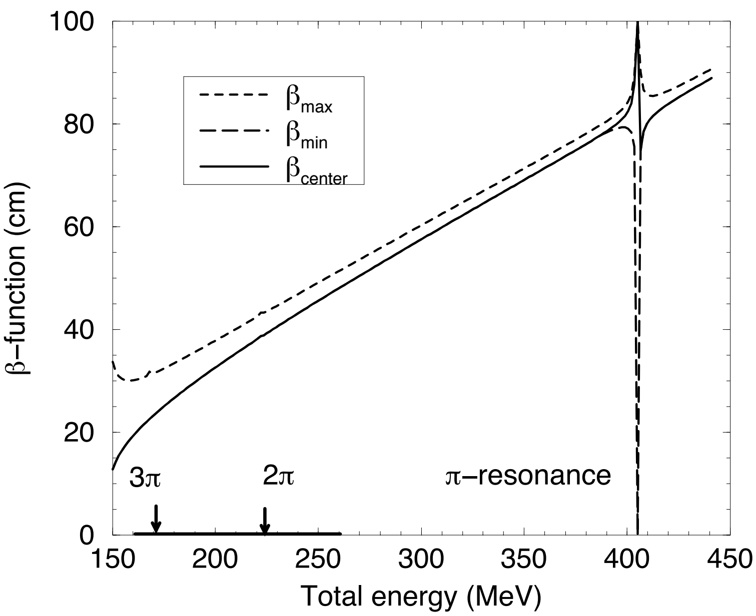


Figure 6: Minimal and maximal β-functions of a solenoid with correcting coils.

as well as spacer length (32 cm) allows to fully suppress the most dangerous resonances. The results are represented in Fig.6 where minimal and maximal β-functions of this channel in dependence on muon total energy are plotted. Solid line is β-function at the center of small solenoid where the absorber is located. Horizontal line shows a range occupied by the beam in the beginning of the cooling. Positions of linear resonances corresponding to phase advances 2π and 3π per half-cell are shown too, though they are totally suppressed. π-resonance is enough weakened, and only higher resonances located at $E < 150$ MeV cause modulation of β-function which is about 35 cm for central energy.

4 Long solenoid channel

More radical way is a usage for the cooling 2 long solenoids with uniform magnetic field and single fast field flip between them as is schematically shown in Fig.7. The single flip is necessary and sufficient to get the beam with zero angular momentum after the cooling [8]. With an appropriate matching, there is no resonances and beam envelope modulation in such a system.

Dynamics of the cooling in a long solenoid differs from alternate solenoid as illustrated in Fig.8. Larmor radius of a particle decays two times faster at the cooling whereas its Larmor center is almost immovable (there is slow diffusion because of scattering). As a result, emittance of the beam decreases in the beginning of solenoid and slowly grows later having a minimum. The field flip should be performed at the point of minimum. R.m.s. Larmor radius is much less then r.m.s. radius of Larmor centers in this point. The

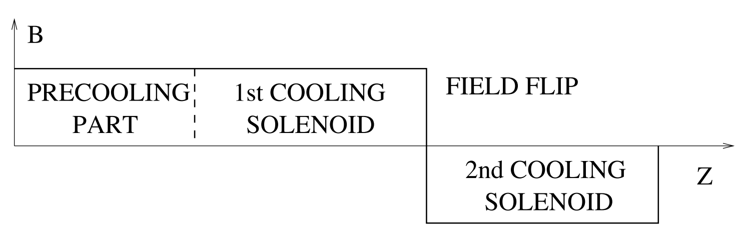


Figure 7: Magnetic field in a long solenoid single field flip channel (schematically).

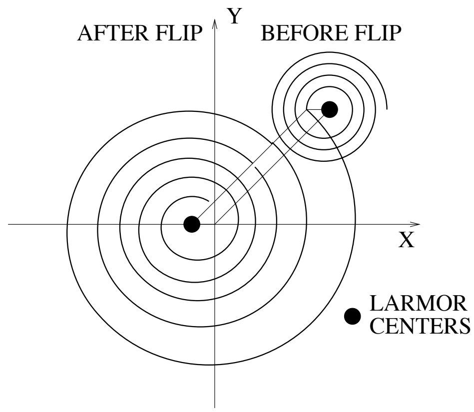


Figure 8: Cooling in a long solenoid single field flip channel (schematically).

flip produces an exchange:

$$\text{Larmor radius} \Longleftrightarrow \text{Radius of Larmor center.}$$

Therefore Larmor centers are about 0 in the 2nd solenoid, and all the particles rotate around the solenoid axis by spiral trajectories providing fast decrease the beam radius and transverse momentum. Final emittance of the beam can be estimated by formulae [8]:

$$\varepsilon_{out} = \frac{\varepsilon_{eq}}{2}\left(a + \frac{1}{a}\right), \quad a = \sqrt{\ln(b \ln b)}, \quad b = \frac{4\varepsilon_{in}}{\varepsilon_{eq}},$$

where ε_{eq} is the equilibrium emittance in perfect alternate solenoid channel with the same magnetic field and $|B| = const$. Typical value of a is about 2 corresponding $\varepsilon_{out}/\varepsilon_{eq} = 1.25$ that is a loss of the long solenoid channel in comparison with alternate solenoid one. But magnetic field on the coil of real alternate solenoid channel exceeds the field on its axis at least by factor 2. Therefore, if the channels with the same field on coils are compared, the long solenoid system looks better: $\varepsilon_{out}/\varepsilon_{eq} \simeq 0.7$.

So the technical problems are minimized in the long solenoid channel being concentrated actually in the field flip section which should provide the beam transformation as described above. Schematic of the section for the field flip ± 7 T is represented in Fig.9, and its magnetic field is plotted in Fig.10. Maximal field on the coils is 8 T in this design. Trajectories of several muons of different energy with zero initial transverse momentum are shown

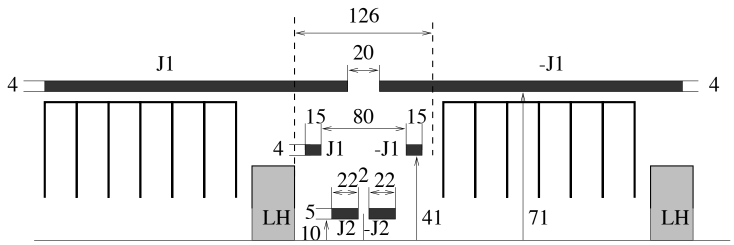


Figure 9: Schematic of the field flip section.

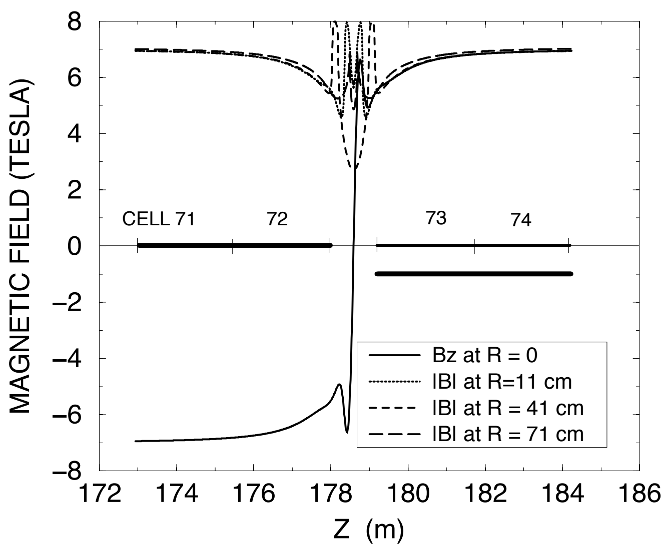


Figure 10: Magnetic field of the field flip section.

in Fig.11 (r vs z). After a perfect (instantaneous) field flip the particles should rotate around the solenoid axis corresponding $r = const$. Really there is a modulation $\Delta r/r$ about 3-5% that is not bad.

A serious problem of such a channel is longitudinal perturbation. Any field flip changes transverse momentum of a particle p_t and - by this - its longitudinal velocity, producing a mismatching with accelerating field. It is easy to see that, at reference energy E_{ref}, equilibrium energy of the particle

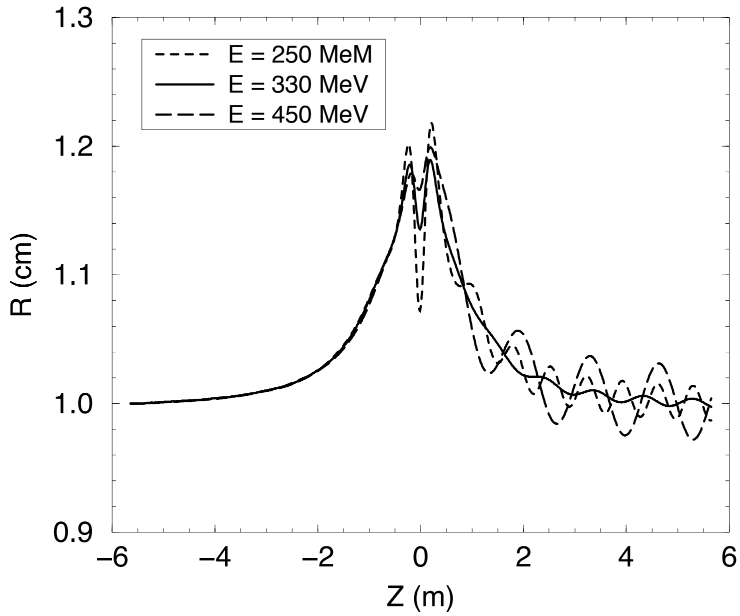


Figure 11: Trajectories of particles in the field flip section.

depends on its transverse momentum as

$$E_0 = E_{ref}\sqrt{(1 + \left(\frac{p_t}{mc}\right)^2}.$$

Before the flip r.m.s. transverse momentum is relatively small, but just after that it strongly increases producing

$$E_0 \simeq E_{ref}\sqrt{1 + \left(\frac{eBr}{mc^2}\right)^2}.$$

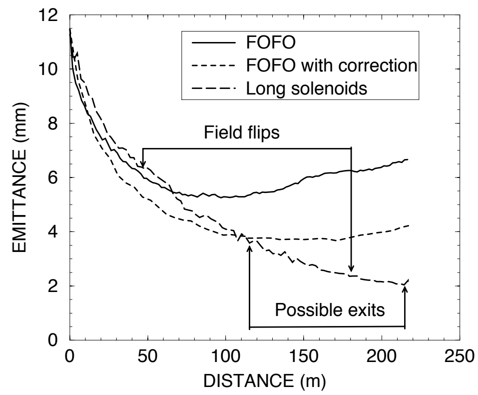
For example, at $B = 5$ T and $r = 10$ cm we have $E_0 \simeq 1.7E_{ref}$. Therefore it is better to perform the field flip in relatively small magnetic field like 2-3 T, and adiabatically increase it later to 5-7 T to get additional decrease of transverse emittance.

5 Simulation

A simulation was done to compare performances of 3 cooling channels described above. The cell schematic and magnetic field of FOFO channel of 190 cm length are represented in Figs. 2 and 3. This channel wil be indicated later by symbol A. The cell and field of alternate solenoid channel with correcting coils are shown in Figs. 4 and 5 (channel B). The long solenoid channel C consists of 3 parts. 1st one is 3 Tesla solenoid of about 50 m length followed by the field flip section. After the flip, the field increases adiabatically from -3 to -7 Tesla (65 m). It is a possible end of the cooler providing zero angular momentum beam out the solenoid. Another option is a continuation of 7 T solenoid 100 m more to get an additional cooling. 2nd field flip is required in this case to suppress angular momentum which appears in this part again (this most complicated section is shown in Figs. 9 and 10). Maximal magnetic field on the coils of any channel is about 8 T.

In all the cases the same input file is used. It is obtained by pre-cooling part described in [2] and includs: carbon target followed by 16 GeV proton driver; decay channel of 50 m length; 200 MeV induction linac of 100 m length; minicooling by 2.6 m of liquid hydrogen; and buncher of 15 m length. Muons in the bucket have energy from 160 to 260 MeV at the central energy 200 MeV (the bucket is non-symmetric). Slow acceleration at the cooling (0.8 - 1 MeV/m) is used as an alternative of an emittance exchange to reduce the particle loss due to an increase of longitudinal emittance. The reference energy of the long solenoid channel is decreased on 15 MeV at each field flip to partly compensate decrease of average longitudinal velocity of the particles. Accelerating frequency is 201 MHz, average accelerating gradient is 12 MeV/m (15 MeV/m in cavities), synchronous phase is 25.7^0.

Normalized r.m.s. emittance of the beam vs distance is plotted in Fig.12 for each cooling channel. In channel A, it reaches about 5.4 mm after 80 m of the cooling, and slowly increases later because of subsequent acceleration which leads to a growth of the equilibrium emittance. Of course, the cooling should be terminated in this point. In channel B, the emittance minimum 3.8 mm is achieved after about 115 m of the cooling. Because of features of channel C, there are only 2 possible termination points where the beam angular momentum is 0 after the exit from the solenoid. 1st one is placed on the distance of 115 m where the beam emittance is 3.6 mm. However it significantly exceeds the equilibrium emittance of this channel allowing to



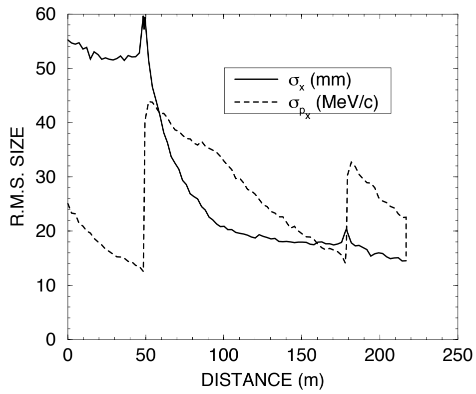


Figure 12: Evolution of r.m.s. transverse emittance at the cooling.

Figure 13: Evolution of the beam size and transverse momentum in the long solenoid cooling channel.

continue the cooling. It requires the 2nd field flip at the point 180 m, and exit should be done after 217 m of the cooling. The beam emittance is 2.1 mm in this point.

R.m.s. beam radius and transverse momentum decay like emittance in the alternate solenoid channels A and B. Their behavior in channel C is more complicated as is shown in Fig.13. A fast decrease of the beam radius appears only after 1st field flip. Transverse momentum decays more or less uniformly but blowing up after any field flip and down after exit from the channel.

The beam intensity is plotted in Fig.14 in terms of muon/proton ratio. Initial value is about 0.2, however only about 0.135 muons per proton are in the FR bucket just after the bunching. Therefore there is essential particle loss just in the beginning of any channel. Additional loss is caused by longitudinal heating by absorbers and at the field flips. In the poins where transverse emittance is minimal, the muon/proton ratio is:

A: 0.117; B: 0.095; C: 0.107 and 0.100

It corresponding to transmission 70-87% if only the particles in the bucket are taken into account. Longitudinal distribution of the paricles is non-Gaussian; nevertheless r.m.s. emittance maybe useful for comparison of the channels. It is at the same points:

A: 62 mm; B: 67 mm; C: 66 mm and 84 mm

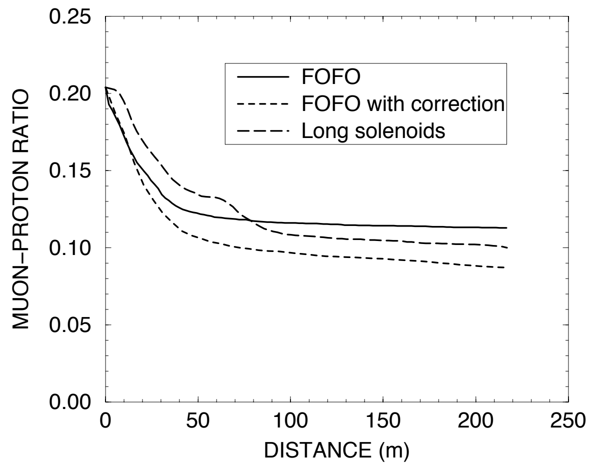


Figure 14: Muon/proton ratio at the cooling.

6 Conclusion

Because of numerous and strong betatron resonances, FOFO channel is capable to operate at relatively small axial field like 2.5 T providing β-function about 60 cm. Achievable transverse emittance without the beam scrapping is about 5.4 mm at the channel length of about 80 m.

At the same field on the coils (8 T), alternate soleniod channel with correcting coils of small radius provides β-function about 35 cm and the beam emittance 3.8 mm at the length 115 m.

Two options of the long solenoid channel are possible. In the short one (115 m, single field flip) the beam emittance is about 3.6 mm. In the long version (217 m, double field flip) ahievable emittance is 2.1 mm.

The best transmission has FOFO channel providing 0.117 μ/p with considered precooling part. The ratio is less (0.095) for the channel with correctors, probably because of lower frequency of the field flips. Long solenoid channel provides the ratio 0.107 or 0.100 in dependence on the channel length.

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