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STUDY OF THE LASER MANIPULATION ON RELATIVISTIC ELECTRON BEAM FOR TERAHERTZ EMISSION*

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Abstract

Terahertz radiation plays an important role in cutting-edge scientific research. Terahertz radiation source based on relativistic electron beam can provide excellent terahertz radiation source. The performance of such radiation is closely related to the distribution of the electron beam. Therein, the laser modulation technology based on the undulator is widely used to manipulate the distribution of the electron beam, thereby manipulating the radiation characteristics, such as improving coherence, tuning spectrum and controlling pulse width. In this paper, we analytically discuss the effects of various non-ideal factors during the process of dual-laser difference frequency modulation, such as finite laser pulse width, laser frequency chirp, and electron beam phase space distribution distortion. This will help to further understand the laser modulation technology of relativistic electron beams in the terahertz band, thus promoting the development of terahertz photonic science.

INTRODUCTION

The THz radiation sources based on accelerator can meet the needs of further scientific research. Among them, the laser modulation technology via the undulator is widely used to manipulate the distribution of the electron beam, thereby controlling the radiation characteristics. The laser-electron interaction in the undulator has greatly promoted the development of manipulating relativistic electron beams and corresponding advanced radiation sources. Compared with the interaction [1] in vacuum, the existence of undulator greatly improves the energy coupling efficiency and this process is generally not affected by space charge force or phase space coupling, which is crucial for maintaining beam quality for further applications.

Some down-conversion schemes [2–5] can convert the target bunching frequency from the optical to the THz band for coherent radiation. They use two lasers with different wavelengths to modulate the energy of the electron beam respectively, thus forming the intensity envelope of the difference frequency in the THz band. The related experimental results show the periodic THz structure [3] in the electron beam. In addition, based on the laser heating regime, the slice energy modulation scheme [6] can obtain a high density modulation amplitude (the bunching factor can reach 0.4), and the scheme based on angular divergence modulation [7] reduces the power requirement of directly THz seed, and its bunching factor can exceed 0.4 with seed power of 1 kW, which improves the feasibility of using THz pulses as modulated lasers directly under existing mature technology. Moreover, the laser modulation method is also beneficial to expand the performance of the storage ring light source.

In this paper, the laser modulation technology for THz bunching is analyzed. The first is the general analysis of the dual-laser difference frequency modulation method, and focuses on the influence of various non-ideal factors on the modulation performance, including the energy chirp of the electron beam, the finite pulse length and frequency chirp of the laser . This will help to further understand the laser-modulated electron beams for THz radiation sources and promote the development of advanced THz integrated platforms.

DUAL-LASER DIFFERENCE FREQUENCY MODULATION

To realize the modulation in THz scale, two laser beams are used to modulate the electron beam in succession. Figure 1 shows the scheme, which mainly includes two undulators and a dispersion element, chicane. When the relativistic electron beam passes through the undulators and satisfies the resonance condition of the undulator with the laser, effective energy coupling can occur. With the different modulation wavelengths of the two lasers , the energy modulation with the frequency difference of two lasers can be formed in the longitudinal phase space of the electron beam, then the energy modulation is transformed into density modulation after the dispersion section, where the electron beam microbunching with periodic structure in THz band is obtained. This bunched electron beam can generate coherent THz radiation through multiple radiation modes.

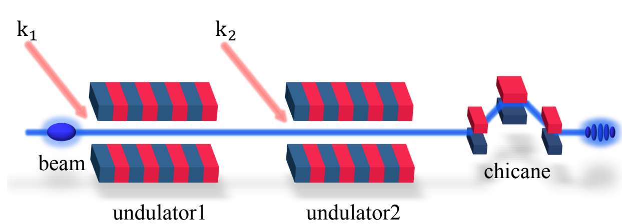

Figure 1: Modulation scheme.

The phase space evolution process of the electron beam is given below. Assuming that only the energy Gaussian distribution electron beam is considered, the initial distribution function of the beam is, $f_0(p) = N_0 \exp\left(-p^2/2\right)/\sqrt{2\pi}$ where N_0 is the number of electrons per unit length, $p =$

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$(E - E_0)/\sigma_E$ is the dimensionless energy deviation, and E_0 and σ_E are the average energy and energy spread.

The electron beam undergoes twice energy modulations in two undulators, and the laser wavenumbers are k_1 and k_2, respectively. Only the dispersion parameter R_{56} is considered in dispersion (for the terahertz band, the collective effect of this process can be ignored). The energy modulation and density modulation can be approximately expressed as, $p_1 = p + A_1 \sin\zeta + A_2 \sin(K\zeta + \phi)$ and $\zeta_1 = \zeta + Bp_1$, where $\zeta = k_1 z$ dimensionless longitudinal position, $K = k_2/k_1$, ϕ is the phase difference of two lasers, $B = k_1 R_{56}\sigma_E/E_0$ is the normalized dispersion intensity, $A_{1,2} = \Delta E_{1,2}/\sigma_E$ is the normalized energy modulation amplitude. At the exit of the dispersion section, the phase space distribution becomes,

$$f_f(\zeta_1, p_1) = \frac{N_0}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[p_1 - A_1 \sin(\zeta_1 - Bp_1) - A_2 \sin(K\zeta_1 - KBp_1 + \phi)\right]^2\right\}. \quad (1)$$

The density distribution of the electron beam is investigated by the integral of Eq. (1), $N(\zeta_1) = \int_{-\infty}^{+\infty} f_f(\zeta_1, p_1)\,\mathrm{d}p_1$. The Fourier component of the density distribution is given as the bucnhing factor, $b(\check{k}) = \left|\left\langle e^{-i\check{k}\zeta_1} N(\zeta_1)\right\rangle\right|/N_0$, where $\check{k} = k/k_1$ is the dimensionless wave number. The square brackets are averaged over ζ_1. The above formula is not zero only when $\check{k} = n + mK = a$ (where n and m are arbitrary integers), and the bucnhing factor can be written in the form of harmonic numbers,

$$b_{n,m} = \left|e^{-a^2B^2/2} J_n(-\mathrm{aB}A_1)\, J_m(-\mathrm{aB}A_2)\right|. \quad (2)$$

It can be found that by two lasers with similar wavelengths, for example, $\lambda_1 = 800\,\mathrm{nm}$, $\lambda_2 = 810\,\mathrm{nm}$ nm, the bunching wavelength $\lambda = 64\,\mu\mathrm{m}$ ($n = -1, m = 1$), which corresponds to the THz band.

According to the characteristics of Bessel function, it states that a larger bunching factors are more likely to appear in the case of $|n| = |m|$ and $A_1 = A_2$. Let $J_n(x)$ obtains the maximum value of y_n at $x = x_n$, the maximum value of the bunching factor is, $b_n^{\max} \cong y_n^2 e^{-x_n^2/2A^2}$. In the case of a given n, the maximal bunching factor is only related to the modulation amplitude, and when the modulation amplitude is large enough, $A \gg x_n$, the bunching factor tends to be saturated $b_n^{max} \cong y_n^2$. Taking $n = -1, m = 1, x_1 = 1.84, y_1 = 0.58$, $b_{-1,1}^{max} \cong 0.34$. Figure 2 shows the maximal bunching factor under different modulation amplitudes. When $A > 5$, the bunching factor almost reaches saturation.

The ideal case is considered in above analysis. In practice, electron beams and lasers can't maintain this situation. Hence, the influences of different non-ideal situations will be analyzed separately as follow.

Energy Chirp of Electron Beam

The linear energy chirp of initial longitudinal phase space of electron beam is considered,

$$f_0(p, \zeta) = \frac{N_0}{\sqrt{2\pi}} \exp\left[\frac{-(p - h\zeta)^2}{2}\right], \quad (3)$$

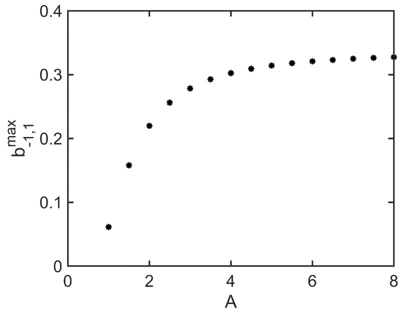

Figure 2: The relationship between the maximum bunching factor $b_{-1,1}^{max}$ and the modulation amplitude A.

where $h = \frac{dp}{d\zeta} = \frac{d\delta}{dz}\frac{1}{k_1\sigma_E/E_0}$ is the dimensionless chirp parameter of the electron beam. It can be found that the bunching factor is not zero only when the wavenumber satisfies the following formula,

$$\check{k} = \frac{n + mK}{1 + hB} = a' \quad (4)$$

the generalized harmonic number $a' = a/(1 + hB)$ is the same with the a in Eq. (2) when $h = 0$. The bunching factor is,

$$b_{n,m} = \left|e^{-a'^2B^2/2} J_n(-a'BA_1)\, J_m(-a'BA_2)\right|. \quad (5)$$

This formula is consistent with the Eq. (2), except for a different harmonic number. Therefore, the linear energy chirp does not affect the bunching effect. Moreover, the generalized harmonic number can be approximately linear with the chirp parameter (as shown in Fig. 3). This feature can be used to tune the modulation frequency.

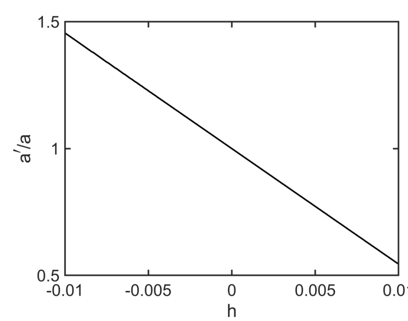

Figure 3: Harmonic number and energy chirp. The bunching factor remains maximum through the optimal B.

Finite Uulse Length of Laser

The previous simplified model assumes two infinitely long longitudinally uniform laser beams, which makes the laser-induced energy modulation perfect sinusoidal, so the spectrum of the modulated beam is bandwidthless in harmonics. This model can't be used for actual spectral attribute analysis. Therefore, the finite length of the laser pulse is considered here to explore its influence on the spectral distribution of the modulation electron.

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Here we only need to change the modulation amplitude parameter to a term related to the longitudinal coordinate, $A(z) = A_0 e^{-z^2/2\sigma_z^2}$, where $\sigma_z = c\sigma_t$ is the rms length of the Gaussian envelope of the laser electric field.

The bunching spectrum near the harmonics is,

$$b_{n,m}(\Delta k) \cong e^{-a^2B^2/2} \int_{-\infty}^{\infty} dz J_n \left(-aBA_{01} e^{-\frac{z^2}{2\sigma_{z1}^2}} \right) \times J_m \left(-aBA_{02} e^{-\frac{z^2}{2\sigma_{z2}^2}} \right) e^{-i\Delta kz} \quad (6)$$

where $\Delta k = k - ak_1$.

One is interested in $n = -1, m = 1$. And consider the simple case that the two modulation amplitudes $A_0 = A_{01} = A_{02}$ and the pulse length $\sigma_z = \sigma_{z1} = \sigma_{z2}$ are the same,

$$b_{-1,1}(\Delta k) = e^{-\xi^2/2} \int_{-\infty}^{\infty} dz J_1^2 \left(-\xi A_0 e^{-\frac{z^2}{2\sigma_{z1}^2}} \right) e^{-i\Delta kz}, \quad (7)$$

where $\xi = aB$. With infinite pulse length in Eq. (2)), it takes $\xi A_0 \approx 1.84$, so a modification is given due to finite pulse length, $\xi A_0 \approx 1.84(1 + \Delta c)$. Then,

$$b_{-1,1}(\Delta k) = e^{-[(1.84(1+\Delta c)/A_0]^2/2} \times \int_{-\infty}^{\infty} dz J_1^2 \left[-1.84(1 + \Delta c) e^{-\frac{z^2}{2\sigma_z^2}} \right] e^{-i\Delta kz} \quad (8)$$

Taking $A_0 = 3$ and $\sigma_z/c = 5$ ps, Fig. 4 shows that the finite laser length makes the spectrum continuously distributed and when Δc increases, a larger bunching factor can be obtained. Therefore, the finite laser length makes the optimal ξA_0 larger, which also means a larger dispersion parameter. It's because the modulation amplitude A_0 is the peak amplitude.

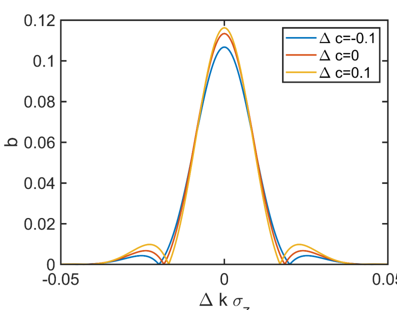

Figure 4: Bunching spectrum with finite laser pulse length.

Linear Frequency Chirp of Laser

Consider the linear chirp of laser frequency, $E_1 \propto e^{-i(k_1 z + \alpha_1 z^2)}$ and $E_2 \propto e^{-i(k_2 z + \alpha_2 z^2)}$. Then, set $n = -1, m = 1$, $\xi A_0 \approx 1.84(1 + \Delta c)$, $\alpha = \alpha_2 - \alpha_1$, it becomes,

$$b_{-1,1}(\Delta k) = e^{-[(1.84(1+\Delta c)/A_0]^2/2} \int_{-\infty}^{\infty} dz \times J_1^2 \left[-1.84(1 + \Delta c) e^{-\frac{z^2}{2\sigma_{z1}^2}} \right] e^{-i\Delta kz - i\alpha z^2} \quad (9)$$

The existence of frequency chirp brings the quadratic term related to the longitudinal coordinate.

Taking $A = 3$, $\sigma_z/c = 5$ ps, Fig. 5(a) shows the bunching spectra under different frequency chirps. It can be seen that the spectral width gradually increases with the increase of chirp parameters.

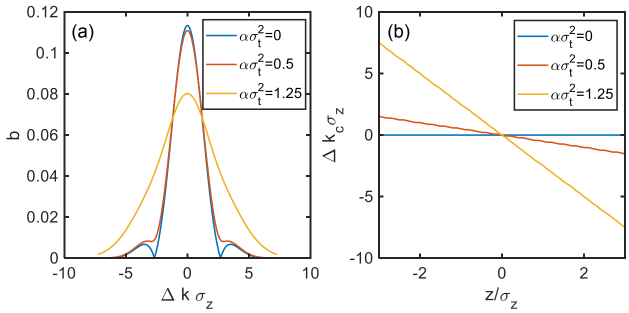

Figure 5: (a)Bunching spectrum of finite pulse length; (b)The slice wavenumber with different laser chirp rates.

The bunching factor of slice in $[z, z + \delta z]$ is calculated along the electron bunches. The central wavenumber offset for each slice is denoted as Δk_c. As shown in Fig. 5(b), It indicates that the laser difference frequency chirp $\alpha = \alpha_1 - \alpha_2$ is transferred to the bunching spectrum of the modulated electron beam.

Higher-order Energy Chirp of Electron Beam

The quadratic energy chirp is also considered,

$$f_0(p, z) = \frac{N_0}{\sqrt{2\pi}} \exp \left[\frac{-\left(p + q\zeta^2\right)^2}{2} \right] \quad (10)$$

where q is the dimensionless quadratic chirp parameter of the electron beam. The bunching factor is,

$$\mathrm{b}(\breve{k}) = e^{-i\breve{k}^2B^2/2} \sum_{n,m=-\infty}^{\infty} \int_{-\infty}^{\infty} dz J_n \left(-\breve{k}BA_1(z) \right) \times J_m \left(-\breve{k}BA_2(z) \right) e^{-i(k-ak_1)z} e^{i\breve{k}Bq\zeta^2} \quad (11)$$

The influence of the quadratic energy chirp is similar to that of the linear chirp of the laser frequency. So it will also cause the effect of spectral broadening, specifically, bringing linear chirp to the bunching frequency.

CONCLUSION

For the dual-laser difference frequency modulation scheme, we analyze the influence of various non-ideal factors on the THz bunching of electron beam, including energy chirp of eletron beam, finite pulse length and linear frequency chirp of laser. These will influence the characteristics of downstream THz radiation.

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