

# DIFFERENT WAYS FOR GRAVITON MASS EVALUATIONS

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A graviton detection is an extremely hard problem<sup>1</sup> but if a graviton exists then it relatively easier to evaluate a graviton mass. In February 2016 the LIGO & VIRGO collaboration reported the discovery of gravitational waves in merging black holes, therefore, the team confirmed GR predictions about an existence of black holes and gravitational waves in the strong gravitational field limit. Moreover, in their papers the joint LIGO & VIRGO team presented an upper limit on graviton mass such as  $m_g < 1.2 \times 10^{-22}$  eV<sup>2</sup> analyzing gravitational wave signal as it was suggested earlier<sup>3</sup>. So, the authors concluded that their observational data do not show any violation of classical general relativity. We show that an analysis of bright star trajectories could constrain graviton mass with a comparable accuracy with accuracy reached with gravitational wave interferometers and the estimate is consistent with the one obtained by the LIGO & VIRGO collaboration. This analysis gives an opportunity to treat observations of bright stars near the Galactic Center as a useful tool to obtain constraints on the fundamental gravity law such as modifications of the Newton gravity law in a weak field approximation. In that way, based on a potential reconstruction at the Galactic Center we obtain bounds on a graviton mass.

## 1 Introduction

In spite of a great success of a general relativity (GR) development in a more than a century we know only a few cases where we really need a strong gravitational field approximation to describe a physical reality. If we speak about observable manifestations of black hole features we need models with a strong gravitational field to describe 1) inspiraling, merging and ring down stage of binary black hole evolution; and 2) shapes of shadows around black holes. Perhaps, very soon observers will need GR corrections and later a full GR approach to fit observational data for bright stars near the Galactic Center. Assuming that a radiation in a spectral line is emitted from a region near a black hole horizon, it was found (and after that it was observed the X-ray  $K_\alpha$ -line) that an observed shape of the spectral line can be an important indicator of a strong gravitational field near a black hole, moreover, one can evaluate a black hole spin analyzing a spectral line structure<sup>4,5</sup>. Another phenomenon, where one really needs a strong gravitational field approach, is a shadow formation started since papers<sup>6,7,8</sup> (see also calculations of shadows for different cases<sup>9,10,11,12,13,14,15</sup> and recent reviews on the subject<sup>16,17</sup>). The problem is connected with attempts to resolve the smallest spot at the Galactic Center with VLBI interferometry in mm-

band<sup>18</sup>. Simulations show that in general cases shadows (dark spots in the sky) are surrounded by bright images<sup>7,8</sup>. As it was noted earlier, observations of bright star trajectories near the Galactic Center could provide an efficient tool to evaluate a gravitational potential in particular, analyzing these trajectories one can obtain constraints on parameters of black hole and stellar cluster<sup>19</sup> and on parameters of dark matter distribution<sup>20,21</sup>.

Two groups of astronomers with VLT and Keck telescopes observe stars near the Galactic Center, see papers<sup>22,23,24</sup> and references therein. An analysis of S2 like star trajectories gives an opportunity to obtain stringent constraints on alternative theories of gravity, including  $R^n$  theory which is a generalization of the classical GR and  $n = 1$  corresponds to GR<sup>25,26</sup> (there are also stringent constraints from Solar system data<sup>27</sup>), and Yukawa gravity<sup>28</sup>. In the paper we describe a procedure to obtain a graviton mass constraint from analysis of trajectories of bright stars at the Galactic Center.

## 2 Gravity Theories with Massive Graviton

A gravity theory with massive graviton was introduced in M. Fierz and W. Pauli<sup>29</sup>. However, some unexpected properties of such theories have been found such as van Dam–Veltman–Zakharov (vDVZ) discontinuity and a presence of ghosts (and related instabilities) and other pathologies from quantum field theory point of view<sup>30</sup>. However, there is a significant progress to overcome such problem and build a consistent theory without Boulware – Deser ghosts<sup>31,32</sup>. Here, we will not discuss theoretical aspects of massive gravity theory and we will consider only observational features of such an approach. There are different suggestions to evaluate a graviton mass, some of them are rather exotic and based on hardly verified assumptions<sup>32,33</sup>. Systematics of proposed experiments and observations is not well investigated, moreover, some weaknesses of the proposals for a graviton mass evaluation are pointed out in the review<sup>32</sup>.

## 3 Graviton Mass Estimate from Gravitational Wave Signal

If a graviton has a mass  $m_g$ , then in this case a speed of gravitational wave propagation could differ from  $c$  and we have a dispersion relation<sup>3,34</sup>

$$\frac{v_g^2}{c^2} = 1 - \frac{m_g^2 c^4}{E^2}, \quad (1)$$

where  $E$  is a graviton energy. Gravitons with different energies propagate with different velocities. Assume that we have gravitational waves and electromagnetic waves from the same source (from supernovae explosion, for instance). In this case we have<sup>3,34</sup>

$$1 - \frac{v_g}{c} = 5 \times 10^{-17} \left( \frac{200 \text{ Mpc}}{D} \right) \left( \frac{\Delta t}{1 \text{ s}} \right), \quad (2)$$

where  $\Delta t = \Delta t_a - (1 + z)\Delta t_e$  is the time difference, where  $\Delta t_a$  and  $\Delta t_e$  are the differences in arrival time and emission time of the two signals, respectively, and  $z$  is the redshift of the source. Usually  $\Delta t_e$  is unknown, however, one could find an upper limit for  $\Delta t_e$  (for instance from a theoretical model), therefore, one could evaluate  $1 - \frac{v_g}{c}$ , therefore,  $m_g$ . Following papers<sup>3,34</sup> and assuming that the frequency of gravitational wave is  $\nu$  and  $h\nu \gg m_g c^2$  ( $h$  is Planck's constant), therefore, we have  $\frac{v_g}{c} \approx 1 - \frac{1}{2} \frac{h}{\lambda_g \nu}$ , where  $\lambda_g = \frac{h}{m_g c}$  or  $\lambda_g \approx \frac{1}{2} \frac{1}{\sqrt{1 - v_g/c}}$ . If one has an upper limit for  $1 - v_g/c$ , it can be re-written as a lower limit for  $\lambda_g$ , as the following expression<sup>3,34</sup>

$$\lambda_g = 3 \times 10^{12} \text{ km} \left( \frac{200 \text{ Mpc}}{D} \frac{\nu}{100 \text{ Hz}} \right) \left( \frac{1}{\nu \Delta t} \right). \quad (3)$$

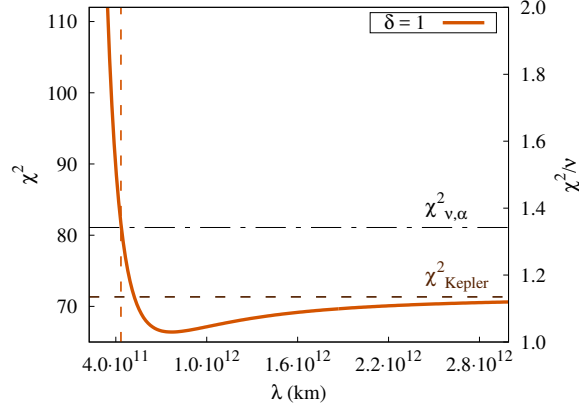


Figure 1 –  $\chi^2$  (solid lines) as a function of Yukawa range of interaction  $\lambda$ . The values  $\lambda_g < \lambda_x$  can be excluded with 90% probability.

It is a lucky case if one observe electromagnetic and gravitational radiation from the same source. But even in the case if only gravitational radiation has been detected as it was noted<sup>3</sup> because gravitational wave signal with a massive graviton will be different from signal for a graviton with a vanishing mass and in this case for  $D \approx 200\text{Mpc}$ ,  $\nu \approx 100\text{Hz}$ ,  $\nu\Delta t \sim \rho^{-1} \approx 0.1$ . The result is  $\lambda_g > 10^{13}$  km. Based on ideas expressed in<sup>3,34</sup>, the LIGO/VIRGO collaboration obtained the same estimate for the Compton wavelength of a massive graviton<sup>2,35,36</sup>.

#### 4 Graviton Mass Estimates from Trajectories of Bright Stars near the Galactic Center

We use a modification of the Newtonian potential corresponding to a massive graviton case<sup>3,34</sup>:

$$V(r) = -\frac{GM}{(1+\delta)r} \left[ 1 + \delta e^{-\left(\frac{r}{\lambda}\right)} \right], \quad (4)$$

where  $\delta$  is a universal constant (we put  $\delta = 1$ ). In our previous studies<sup>28</sup> we found constraints on parameters of Yukawa gravity. As it was described in papers<sup>37,38</sup> we used observational data from NTT/VLT<sup>22</sup>. If we wish to find a limiting value for  $\lambda_x$ , so that  $\lambda > \lambda_x$  with a probability  $P = 1 - \alpha$  (where we select  $\alpha = 0.1$ ) normalized  $\chi^2$  depending on  $\lambda_x$  has to be equal to the threshold depending on degree of freedom  $\nu$  and parameter  $\alpha$  or in other words,  $\chi^2(\lambda_x) = \chi^2_{\nu,\alpha}$ . Computing these quantities we obtain  $\lambda_x = 2900 \text{ AU} \approx 4.3 \times 10^{11} \text{ km}$ . Now we obtain the upper limit on a graviton mass and we could claim that with a probability  $P = 0.9$ , a graviton mass should be less than  $m_g = 2.9 \times 10^{-21} \text{ eV}$  (since  $m_g = h c / \lambda_x$ ) in the case of  $\delta = 1$ <sup>37,38,39,40</sup>, see also Fig. 1 ( the plot is adopted from the paper<sup>38</sup> ).

#### 5 Conclusions

As it was noted earlier, our graviton mass estimate is slightly greater than estimate with LIGO interferometer, however, a) our estimate was obtained in independent way with other observational data; b) our estimate is consistent with LIGO's one; c) our estimate will definitely improved with forthcoming facilities such as GRAVITY, E-ELT and TMT because more precise observations of bright star orbits will give an opportunity to reconstruct a gravitational potential at the Galactic Center in a more accurate way, therefore, one can expect a better estimates for  $\lambda$  parameter and a graviton mass. However, such a progress will be not very rapid because of an exponential dependence of a potential on  $\lambda$ .

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## References

1. F. Dyson, *Intern. J. Mod. Phys. A* **28** 1330041 (2013).
2. B. P. Abbott *et al.*, *Phys. Rev. Lett.* **116**, 061102 (2016).
3. C. Will, *Phys. Rev. D* **57**, 2061 (1998).
4. A.C. Fabian *et al.*, *Mon. Not. R. Astron. Soc.* **238** 729 (1989).
5. Y. Tanaka *et al.*, *Nature* **375**, 659 (1995).
6. S. Chandrasekhar, *Mathematical Theory of Black Holes* (Clarendon Press, Oxford, 1983).
7. H. Falcke, F. Melia and E. Agol, *Astrophys. J.* **528**, L13 (2000).
8. F. Melia and H. Falcke, *Ann. Rev. Astron. Astrophys.* **39** 309 (2001).
9. A. F. Zakharov *et al.*, *Astron. & Astrophys.* **442**, 795 (2005).
10. A. F. Zakharov *et al.*, *New Astron.* **10**, 479 (2005).
11. F. De Paolis *et al.*, *Gen. Rel. Grav.* **43**, 977 (2010).
12. A. F. Zakharov *et al.*, *New Astron. Rev.* **56**, 64 (2012).
13. A. F. Zakharov, *Phys. Rev. D* **90**, 062007 (2014).
14. A. F. Zakharov, *J. Astrophys. Astr.* **36**, 539 (2015).
15. A. F. Zakharov, in *Gravitation, Astrophysics, and Cosmology*, eds. V. N. Melnikov and J.-P. Hsu, p. 176 (World Scientific, Singapore, 2016).
16. H. Falcke and S. B. Markoff, *Class. Quan. Grav.* **30**, 244003 (2013).
17. T. Johannsen, *Class. Quan. Grav.* **33**, 113001 (2016).
18. S. S. Doeleman *et al.*, *Nature* **455**, 78 (2008).
19. A. A. Nucita *et al.*, *Publ. Astron. Soc. Pac.* **119**, 349 (2007).
20. A. F. Zakharov A F *et al.* *Phys. Rev. D* **76**, 062001 (2007).
21. A.F. Zakharov *et al.*, *Space Sci. Rev.* **48**, 301 (2009).
22. S. Gillessen *et al.*, *Astrophys. J.* **707**, L114 (2009).
23. S. Gillessen *et al.*, *Nature* **481**, 51 (2012).
24. L. Meyer *et al.*, *Science* **338**, 84 (2012).
25. D. Borka *et al.*, *Phys. Rev. D* **85**, 124004 (2012).
26. A. F. Zakharov *et al.*, *Adv. Space Res.* **54**, 1108 (2014).
27. A. F. Zakharov *et al.*, *Phys. Rev. D* **74**, 107101 (2006).
28. D. Borka *et al.*, *J. Cosmol. Astropart. Phys.* JCAP **11**, 050 (2013).
29. M. Fierz and W. Pauli, *Proc. R. Soc. London Ser. A* **173**, 211 (1939).
30. D. G. Boulware and S. Deser, *Phys. Rev. D* **6**, 3368 (1972).
31. C. de Rham, G. Gabadadze and A.G. Tolley, *Phys. Rev. Lett.* **106**, (2011).
32. C. de Rham *et al.*, *Preprint* arXiv:1606.08462v1 [astro-ph.CO].
33. A. S. Goldhaber and M. M. Nieto, *Rev. Mod. Phys.* **82** 939 (2010).
34. C. Will, *Liv. Rev. Relat.* **17**, 4 (2014).
35. B. P. Abbott *et al.*, LIGO Document P1500213-v27 (*Preprint* arXiv:1602.03841).
36. B. P. Abbott *et al.*, *Phys. Rev. X* **6** 041015 (2016); LIGO-P1600088 (*Preprint* arXiv:1606.04856 [gr-qc]).
37. A. F. Zakharov *et al.*, *J. Cosmol. Astropart. Phys.* JCAP **05**, 045 (2016).
38. A. F. Zakharov *et al.*, *EPJ Web of Conferences* **125**, 01011 (2016).
39. A. F. Zakharov *et al.*, *EPJ Web of Conferences* **138**, 010010 (2017).
40. A. F. Zakharov *et al.*, *Journal of Physics: Conference Series* **798**, 012081 (2017).