5.3.3 *Juan Maldacena: Comments on emergent space-time*

Einstein looked at his equation

$$G_{\mu\nu} = T_{\mu\nu} \tag{1}$$

and he noticed that the left hand side is very beautiful and geometrical. On the other hand, the right hand side is related to the precise dynamics of matter and it depends on all the details of particle physics. Why isn't the right hand side as nice, beautiful and geometric as the left hand side?.

String theory partially solves this problem since in string theory there is no sharp distinction between matter and geometry. All excitations are described by different modes of a string. However, giving a stringy spacetime involves more than fixing the metric, it involves setting the values of all massive string modes. The classical string equations are given by the β functions of the two dimensional conformal field theory [1]

$$\beta_{g_i} = 0 \tag{2}$$

These equations unify gravity and matter dynamics. However, these are just the classical equations and one would like to find the full quantum equations that describe spacetime.

In order to understand the full structure of spacetime we need to go beyond perturbation theory. There are several ways of doing this depending on the asymptotic boundary conditions. The earliest and simplest examples are the "old matrix models" which describe strings in two or less dimensions [2]. We also have the BFSS matrix model which describes 11 dimensional flat space [3]. Another example is the gauge/gravity duality (AdS/CFT)[4, 5]. In all these examples we have a relation which says that an ordinary quantum mechanical system with no gravity is dual to a theory with gravity. Some of the dimensions of space are an emergent phenomenon, they are not present in the original theory but they appear in the semiclassical analysis of the dynamics.

In the gauge theory/gravity duality we have a relation of the form [5]

$$\Psi[ag] \sim e^{(a^D + \cdots)} Z_{Field\ theory}[g] \tag{3}$$

which relates the large a limit of the wavefunction of the universe to the field theory partition function, where a is the scale factor for the metric on a slice of the geometry.

Note that in this relation, the full stringy geometry near the boundary determines the field theory. It determines the lagrangian of the field theory. The full partition function is then equivalent to performing the full sum over interior stringy geometries. In the ordinary ADM parametrization we can think of the dynamical variables of 3+1 dimensional general relativity as given by 3-geometries. The analogous role in string theory is then played by the space of couplings in the field theory, since these are the quantities that the wavefunction depends on. By deforming the examples we know, it seems that it might be possible, in principle, to obtain any field theory we could imagine. In this way we see that the configuration space for a quantum spacetime seems to be related to the space of all possible field theories. This is a space which seems dauntingly large and hard to manage. So, in some sense, the wavefunction of the universe is the answer to all questions. At least all questions we can map to a field theory problem.

After many years of work on the subject there are some things that are not completely well understood. For example, it is not completely clear how locality emerges in the bulk. An important question is the following. What are the field theories that give rise to a macroscopic spacetime?. In other words, we want theories where there is a big separation of scales between the size of the geometry and the scale where the geometric description breaks down. Let us consider an AdS_4 space whose radius of curvature is much larger than the planck scale. Then the corresponding $2 + 1$ dimensional field theory has to have a number of degrees of freedom which goes as

$$c \sim \frac{R_{AdS}^2}{l_{Planck}^2} \tag{4}$$

In addition we need to require that all single particle or "single trace" operators with large spin should have a relatively large anomalous dimension. In other words, if we denote by Δ_{lowest} the lowest scaling dimension of operators with spin larger than two. Then we expect that the gravity description should fail at a distance scale given by

$$L \sim \frac{R_{AdS}}{\Delta_{lowest}} \tag{5}$$

It is natural to think that the converse might also be true. Namely, if we have a theory where all single trace higher spin operators have a large scaling dimension, then the gravity description would be good.

By the way, this implies that the dual of bosonic Yang Mills would have a radius of curvature comparable to the string scale since, experimentally, the gap between the mesons of spin one and spin larger than one is not very large.

One of the most interesting questions is how to describe the interior of black holes. The results in this area are suggesting that the interior geometry arises from an analytic continuation from the outside. Of course, we know that this is how we obtained the classical geometry in the first place. But the idea is that, even in a more precise description, perhaps the interior exists only as an analytic continuation [6]. A simple analogy that one could make here is the following. One can consider a simple gaussian matrix integral over $N \times N$ matrices [2]. By diagonalizing the matrix we can think in terms of eigenvalues. We can consider observables which are defined in the complex plane, the plane where the eigenvalues live. It turns out that in the large N limit the eigenvalues produce a cut on the plane and now these observables can be analytically continued to a second sheet. In the exact description the observables are defined on the plane, but in the large N approximation they can be defined on both sheets.

Faced with this situation the first reaction would be to say that the interior does not make sense. On the other hand we could ask the question: What is wrong with existence only as an approximate analytic continuation?. This might be good enough for the observers living in the interior, since they cannot make exact measurements anyway.

It seems that in order to make progress on this problem we might need to give up the requirement of a precise description and we might be forced to think about a framework, where even in principle, quantities are approximate.

One of the main puzzles in the emergence of space-time is the emergence of time. By a simple analogy with AdS/CFT people, have proposed a dS/CFT [7]. The idea is to replace the formula (3) by a similar looking formula except that the left hand side is the wavefunction of the universe in a lorentzian region, in a regime where it is peaked on a de-Sitter universe. Note that a given field theory is useful to compute a specific amplitude, but in order to compute probabilities we need to consider different field theories at once. For example, we should be able to vary the parameters defining the field theory. In AdS/CFT the way we fill the interior depends on the values of the parameters of the field theory. In this case this dependence translates into a dependence on the question we ask. So, for example, let us suppose that the de-Sitter ground state corresponds to a conformal field theory. If we are interested in filling this de-Sitter space with some density of particles, then we will need to add some operators in the field theory and these operators might modify the field theory in the IR. So they modify the most likely geometry in the past. So it is clear that in this framework, our existence will be part of the input. On the other hand, it is hard to see how constraining this is. In particular, empty de-Sitter space is favored by an exponentially large factor $e^{1/\Lambda}$. On the other hand, it is unclear that requiring our existence alone would beat this factor and produce the much less enthropic early universe that seems to have existed in our past.

Of course, dS/CFT suffers from the problem that we do not know a single example of the duality. Moreover, de-Sitter constructions based on string theory produce it only as a metastable state. In any case, some of the above remarks would also apply if we were to end up with a $\Lambda = 0$ supersymmetric universe in the far future. In that case, we might be able to have a dual description of the physics in such a cosmological $\Lambda = 0$ universe. It seems reasonable to think that these hypothetical dual descriptions would give us the amplitudes to end in particular configurations. In order to compute probabilities about the present we would have to sum over many different future outcomes.

In summary, precise dual descriptions are expected to exist only when the space-time has well defined stable asymptotics. In all other situations, we expect that the description of physics might be fundamentally imprecise. Let us hope that we will soon have a clear example of a description of a cosmological singularity.

Bibliography

[1] J. Polchinski, *String theory*, Cambridge University Press.
[2] For a review see: P. H. Ginsparg and G. W. Moore, arXiv:hep-th/9304011.
[3] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997) [arXiv:hep-th/9610043].
[4] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].
[5] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109]. E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].
[6] L. Fidkowski, V. Hubeny, M. Kleban and S. Shenker, JHEP **0402**, 014 (2004) [arXiv:hep-th/0306170].
[7] E. Witten, arXiv:hep-th/0106109. A. Strominger, JHEP **0110**, 034 (2001) [arXiv:hep-th/0106113].