Tracking decay positrons in a magnetic field for muon microscope applications

C Young[1] and K M Kojima[2,3]

[1]Department of Engineering Physics, the University of British Columbia, 6224 Agricultural Road Vancouver, BC V6T 1Z1

[2]Centre for Molecular and Materials Science, TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T2A3, and

[3]Stewart Blusson Quantum Matter Institute, the University of British Columbia, 2355 East Mall, Vancouver, BC V6T 1Z4

E-mail: cysyoung@student.ubc.ca

E-mail: kojima@triumf.ca

Abstract. We present a theoretical calculation for a feasibility study of the Muon Microscope, which is intended to add positional resolution within the sample by tracking down the positron trajectories to its source positions. In the presence of a magnetic field, any positrons whose trajectories have components which are perpendicular to the magnetic field will start to move in a helical path due to the Lorentz force. Taking special relativity into account, we have analytically determined the trajectories of the positrons in a uniform magnetic field. We also evaluate more realistic cases, such as finite detector spatial resolutions, as well as the effect of positron scattering from the materials in its trajectory.

1. Introduction

Conventional muon spin relaxation (μSR) spectrometers consist of positron detectors, covering about 20% of the solid angle [1]. The detectors are in the shape of a plate or cylinder, most likely two layers of them to take the timing coincidence for reduction of noises and better identification of the particles. The size of the counters are in the order of 10 centimeters, and no spatial tracking of the positrons is anticipated.

The purpose of this paper is to evaluate what kind of spatial resolution could be achieved, if the positron detectors gain positional sensitivity. In high energy particle physics experiments, the product particles are tracked by the wire drift chambers [2], or more recently silicon trackers [3], where the position of the particles are detected and their tracks are reconstructed by the information from a large number of sensing wires or segmented/stripped silicon detectors. Traditional detectors used in μSR spectrometers are plastic scintillators with photo multipliers, but recent development in photo sensing detectors has enabled the use of scintillation fibers[4] and inexpensive silicon photo multipliers (SiPMs)[5], so that measurement of the positions are becoming possible keeping the high timing resolution necessary for μSR. This article starts with the ideal virtual detectors with perfect positional identification and no loss of the positron energy, and later assesses the effect of granularity of the detectors (e.g., thickness of the scintillation fibers), the distances of the detector arrangements, and the effect of materials in the track, such as cryostat walls and scintillation fibers themselves.

2. Theory

We have a uniform magnetic field in the z-direction, $\vec{B} = (0, 0, B)$ and a particle of charge q, rest mass m_0, position vector $\vec{r} = (x, y, z)$, velocity vector $\vec{v} = (v_x, v_y, v_z) = (dx/dt, dy/dt, dz/dt)$ and acceleration vector $\vec{a} = (a_x, a_y, a_z) = (d^2x/dt^2, d^2y/dt^2, d^2z/dt^2)$. Because the positron mass is m_0=0.5MeV/c^2, and the momentum end point of Michel decay is p_{max}=50MeV/c, special relativity has to be taken into account:

$$m = \gamma m_0$$

where γ is the Lorentz factor, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, c is the speed of light and v is the speed. The equation of motion is:

$$m\vec{a} = q(\vec{v} \times \vec{B}) = (qv_yB, -qv_xB, 0) \tag{1}$$

Integration of eq.1 is analytical under the condition that the speed of the particle does not change during its travel time between or passing through the detectors. Since the electrons from Michel decay are in the super relativistic regime ($\beta\gamma = p/Mc \gg 1$), their energy drop will be around the minimum ionization energy 2-3 MeV g/cm^2 [6], leading to the energy drop of 0.2-0.3 MeV from one layer of 1mm thick plastic scintillator. We may safely ignore the change in γ or the speed of the positrons for the super relativistic conditions with p_{max}=50 MeV/c.

The solution to this is:

$$x = Rcos(\alpha t - \delta) + x_c \tag{2}$$

$$y = -Rsin(\alpha t - \delta) + y_c \tag{3}$$

$$z = Ct + D \tag{4}$$

where $\alpha = \frac{qB}{m} = \frac{qB}{\gamma m_0}$, R, δ, C, D, x_c, y_c are constants.

It is clear from the equations above that the particle will follow helical motion through space, with radius R, phase shift δ, and center (x_c, y_c). By using the information from the few layers of position sensitive detectors, trajectories of the positrons may be tracked back to the $z = 0$ plane, which is the best guess for the sample position.

3. Experimental Set-up

We used G4Beamline [7] as our simulation tool. A width-less muon beam centered at $(x, y) = (0, 0)$ travels in the positive z direction, hitting a 1 mm thick target made of carbon which center is located at $z = 0$. The muon decay is setup in G4Beamline as the usual Michel decay mode.

The definition of the axis and the geometry of the detectors are shown in Fig.1. A muon beam (as indicated by the dark blue line) travels towards the detector (grey). The position sensitive detectors, which are all 1 mm thick, are located at $z = 100, 150$, and 200 mm (labeled $F1$, $F2$, and $F3$ respectively in the image below) and $y = -100, -150$ and -200 mm (labeled $D1, D2, D3$), both having dimensions of 100 x 100 mm^2, 150 x 150 mm^2, and 200 x 200 mm^2 respectively. We applied a uniform magnetic field in the z-direction.

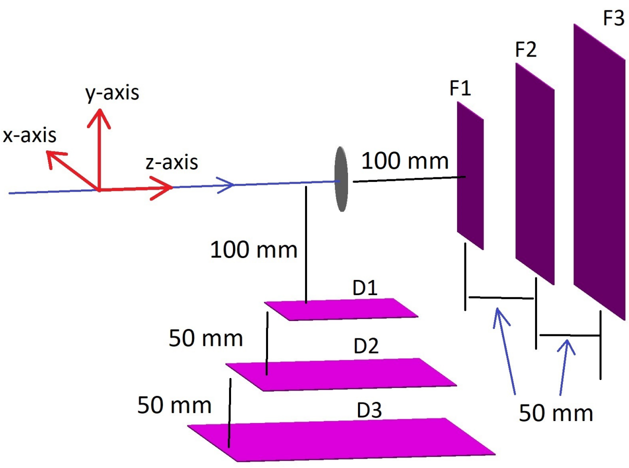

Figure 1: Experimental Set-up

4. Method of analysis

We have generated 25000 muons and recorded the position (x, y, z) of the positron hits on the detectors. The event ID's are carefully matched for the events. Once we have three sets of (x, y, z) points from the three detectors under the same event ID, we find the radius of the helix as follows: we project the helix (whose axis points in the $+z$ direction) in the xy-plane as a circle. There are three points on the circle $((x_i, y_i); i = 1, 2, 3)$ originating from the three detectors, and those make it possible to calculate the center (x_c, y_c) and the radius R. Three layers of detectors are necessary for the center and radius determination.

The next step is to reconstruct the evolution of the circular angle from the three points on the helix. If the point on the helix is defined by the points $(x(z), y(z), z)$, then the circular angle $\theta(z)$ is defined as:

$$\theta(z) = arctan\left(\frac{y(z) - y_c}{x(z) - x_c}\right) \tag{5}$$

It is crucial to recognize that the angle changes as a function of z at a constant rate. This is because the angular velocity $\alpha = \frac{qB}{\gamma m_0}$ and the z-direction velocity $v_z = \frac{dz}{dt}$ are constant under the assumption that γ is constant and scattering is ignored.

We can exploit the fact above by using the linear extrapolation formula using θ as a function of z:

$$\theta(z) - \theta_1 = k(z - z_1) \tag{6}$$

Where k is the slope and is equal to $\frac{\theta_3 - \theta_1}{z_3 - z_1}$ for points from detectors F/D1 and F/D3, i.e. (θ_1, z_1) and (θ_3, z_3). From there we can extrapolate back to $z = 0$ in order to find θ at $z = 0$. Once we have θ at $z = 0$ (lets call this value θ_0), we use the following formulae below to find the estimated position of the source at $z = 0$ (lets call it as (x_0, y_0)):

$$x_0 = x_c + Rcos(\theta_0) \tag{7}$$

$$y_0 = y_c + Rsin(\theta_0) \tag{8}$$

5. Results

5.1. Magnetic field dependence

As the first step, we confirm the accuracy of the helical approximation algorithm when the detectors have infinitely high resolution and do not scatter positrons. Since the beam in the simulation hits at (0,0), one can assess the accuracy of the extrapolations. Under the uniform magnetic field of 0.3 T, the (x, y) position estimated by the helical approximation of the positron trajectory is at $(0.0, 0.0) \pm 0.126$ mm where the error represents the standard deviation of the gaussian fit. It is significantly better than the quadratic approximation (the best theory for the zero-field trajectories) exercised on the three positional points from the three detectors, yielding $(0.0, 0.0) \pm 1.038$ mm (Fig.2a). We also compare the standard deviation (gaussian σ) as a function of the field (Fig.2b), and found that there is little field dependence in the accuracy for magnetic fields up to 0.5 T.

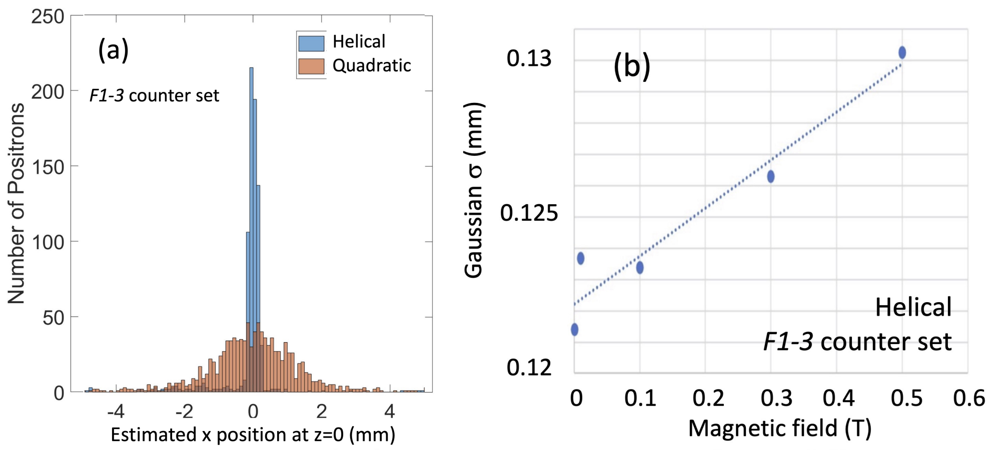

Figure 2: **a**: Helical vs Quadratic extrapolation for B = 0.3 T. **b**: Magnetic field dependence for the Helical Approximation standard deviations.

5.2. Detector position and pixel size dependence

For the Up-Down detector configuration $(D1, D2, D3)$, we find that the helical approximation algorithm is not as accurate as in the Forward-Backward configuration. In Fig.3a, we compare the estimated source position for the Forward detectors and Down detectors in B=0.3 T. The Down detectors estimates the beam position at $(0.0, 0.0) \pm 0.401$ mm, which is over three times worse than for the Forward detector.

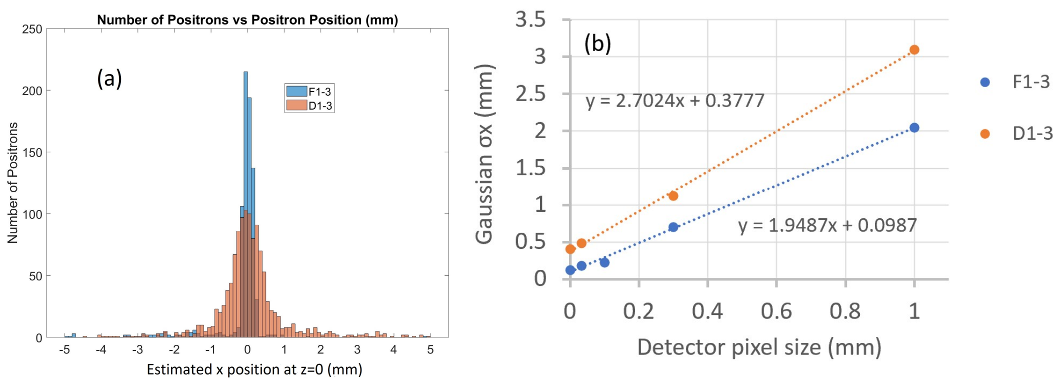

Figure 3: **a**: Positional estimate from the Forward-Backward vs Up-Down detector setup at B = 0.3 T. **b**: gaussian σ as a function of the detector pixel size.

Scintillation fibers have a finite width which would limit the spatial resolution possible for the hit positions. Because of the size of the optical detectors ($> 1 \times 1$ mm for SiPM) [5] and the scintillator fiber thickness necessary to gain enough photons, the width of ~1 mm is the practical lower limit for the positron detection. We rounded positional coordinates of the simulations to the detector granularity and plotted the positional estimate gaussian σ as the function of the detector pixel size in Fig. 3b. The reconstruction accuracy progressively worsened for poorer spatial resolution of the detectors.

We have checked the $D1-3$ detector model with 1/2 and 1/4 the scale (distances and sizes) and compared the results from the original size (Fig.4). The accuracy *improved* as the detectors are scaled down (see the inset in Fig.4). Also we found that the estimated center of the position is non-monotonically shifted by $\Delta x = 0.04, 0.314$ and 0.10 mm for the original, 1/2 and 1/4 scaled models, respectively.

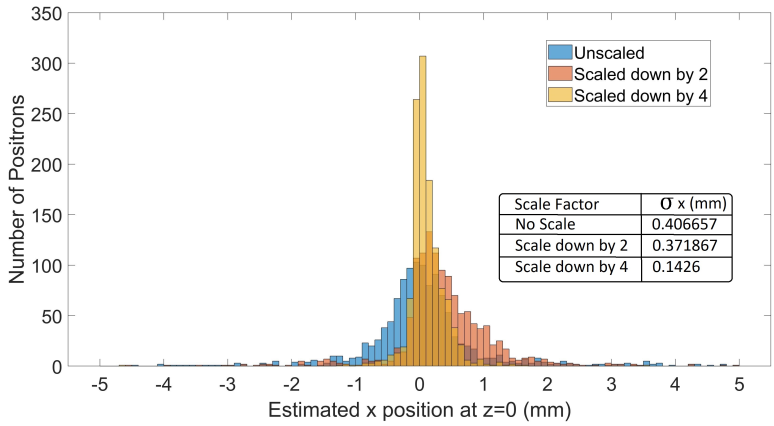

Figure 4: The estimated source position in the Up-Down geometry as a function of the scaling factor of the detector

5.3. Scattering from the cryostat wall and scintillators themselves

If we add a 1 mm stainless steel cryostat between the target and the first detector and compare it to the same setup but without the cryostat, we find that the positional accuracy significantly diminishes. The same is true for the self scattering of the scintillator fibers themselves (Fig.5).

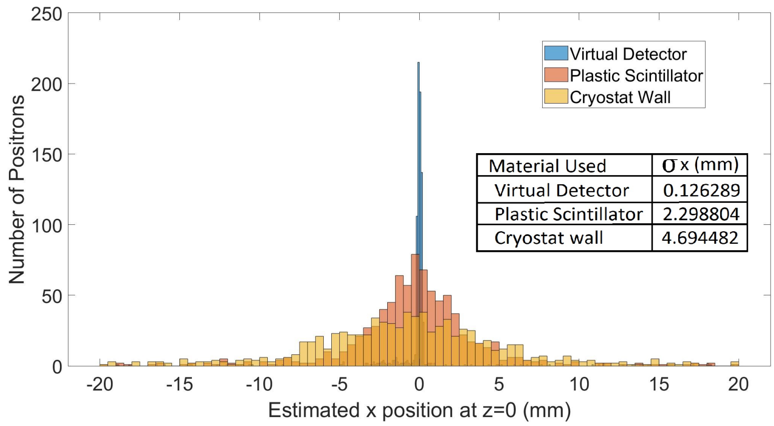

Figure 5: Estimated positrons with scatterings involved.

6. Discussions

As shown in our results, we find the significant factors for the high spatial resolution to be: (1) scattering due to the presence of a cryostat wall, and (2) pixel size of the detectors. These factors significantly diminish the accuracy in our simulations and indicate that the experiment would ideally take place in a vacuum in the absence of a cryostat. Ideally, our maximum accuracy would be 0.126 mm which may enable to measure the sample as small as that size. This will be a big advancement in the sample size required for μSR. It should also be noted that there is only a small correlation between the accuracy of the helical approximation algorithm and the magnetic field strength. However, there is a noticeable decrease in accuracy once we go down to 0.001 T. In this low-field regime, we should adopt a linear trajectory model for the electron motion, which is a simpler case than in finite field where the helical model is required.

One important finding from our simulations is that for the up-down detector geometry, the positional accuracy improves if we scale down our detector setup. We suspect that the scaled down models have more chances to pick up the low momentum positrons, whose trajectories have a smaller helical radius, which may have a better positional resolution in the x direction. The shift of the central position is probably the result from the helical motions, which are all in one rotational direction by the Lorentz force.

The scaling down of the detectors is practically difficult, because of the limitation of the scintillation fibers and the SiPM sizes. We recently found that a two dimensional silicon tracker called LGAD (Low Gain Avalanche Diode) with micrometer spatial resolution has been developed for the ATLAS high energy physics experiment at CERN [8]. This could be a perfect detector for the muon microscope.

When designing a realistic *muon microscope*, we have to (1) have the scintillation fibers (or any position sensitive detectors) as fine as possible and (2) avoid any walls in the positron trajectory. The requirement (2) suggests that the detector system should be in the vacuum sharing the cryostat vacuum for thermal insulation.

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