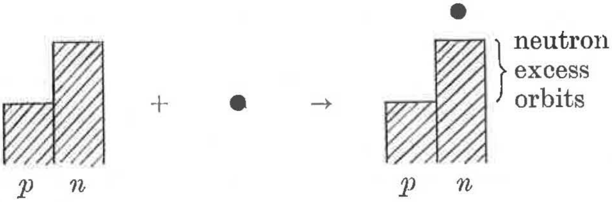
Analogs, Anti-Analogs and so forth

Two states in different nuclei with the same nucleon number A are said to be isobaric-spin analogs or simply *isobaric analogs* if they belong to the same isobaric-spin multiplet. Examples of such multiplets have been known in light nuclei since the early days of nuclear physics; the $T = 1$ triad formed by the $J = 0^+$ ground state of C^{14} ($T_z = +1$), the 0^+ excited state at 2.14 MeV in N^{14} ($T_z = 0$) and the 0^+ ground state of O^{14} ($T_z = +1$) is particularly familiar.

Before 1961, it was believed (without adequate analysis of the situation) that Coulomb forces, which do not saturate and whose influence on energies is proportional to Z^2, should be so important in heavy nuclei that the notion of isobaric analog states would lose all usefulness. One of the most fruitful developments in nuclear physics in the past decade has been the experimental discovery that this expectation is totally off the mark. It neglects a vital aspect of the situation. It is true that Coulomb forces are nonsaturating because of their long range; however, another important consequence of long range is the occurrence of large energy shifts with very little mixing of states. In other words, the diagonal effects of the Coulomb interaction are vastly greater than the off-diagonal. An additional fact that was not fully understood is that a large neutron excess favors conservation of T. The point here is that, for a heavy nucleus with a large neutron excess, all of the protons in low-lying states are in orbits completely filled with neutrons. The effect of the isobaric-spin raising operator on such a state is then given by $T_+\psi = 0$, since T_+ simply replaces each proton in turn by a neutron in the same state and the resulting neutrons have nowhere to go. Thus ψ has $T = T_z = \frac{1}{2}(N - Z)$. The erroneous notion therefore persisted that analog states in heavy nuclei should be spread over many MeV in regions of high level density and therefore there was no point in looking for them. It has been found in hundreds of experiments that the low-lying bound states of all known stable nuclei (Z, N) have sharply defined isobaric analogs in the neighboring nucleus $(Z + 1, N - 1)$. The observed widths range downwards from 200 keV (in the heaviest nuclei).

Given the existence of sharp analog states as an unexpected bounty of nature, we can either exploit them or try to understand them. The vast majority of isobaric-analog studies have followed the former course—that of exploitation. It is still not entirely clear to what extent the sharpness of the isobaric-analog resonances necessarily implies goodness of T. Multiplet relations can survive large symmetry breaking. It is likewise uncertain to what extent charge-dependent nuclear interactions conspire with the Coulomb interactions to produce the T mixing. We shall return to the problem of understanding isobaric analog states—their widths and isobaric-spin purity—in a future note.

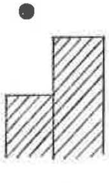
Certain aspects of the exploitation of isobaric analogs in nuclear spectroscopy have been discussed by Feshbach and Kerman in an earlier contribution to Comments.[1] The most useful of such applications has been to studies of the isobaric analogs of bound states formed by (d, p) stripping reactions. The result has been a detailed interlocking study of neutron and proton parentage relations between levels of neighboring nuclei whose mass numbers differ by one unit. Consider then the addition of a neutron in orbit j to the nucleus $(Z, N - 1)$. This process may be realized in a (d, p) reaction with the core in its ground state. It is represented by Fig. 1.



$$|\psi_0\rangle \quad + \quad (\nu)_j \quad \rightarrow \quad |\psi_j^\nu\rangle$$

FIGURE 1

The crucial point is that the corresponding single-proton excitation $|\psi_j^\pi\rangle$ in the nucleus $(Z + 1, N - 1)$ (Fig. 2) cannot be a nuclear



$$|\psi_j^\pi\rangle$$

FIGURE 2

eigenstate since it does not have good isobaric spin. This is clear from its isobaric-spin structure:

$$| \psi_j^\pi \rangle = \phi_{T_0}^{T_z=T_0} \chi_{1/2}^{-1/2}$$

with $T_0 = \frac{1}{2}(N + Z - 1)$, where it is assumed that the "core" state $| \psi_0 \rangle$ has pure isobaric spin T_0. Thus the single-proton state has two T-components,

$$| \psi_A \rangle = P_{T_>} | \psi_j^\pi \rangle ,$$

$$| \psi_{\bar{A}} \rangle = P_{T_<} | \psi_j^\pi \rangle ,$$

where $T_> = T_0 + 1/2$, $T_< = T_0 - 1/2$, and P is the corresponding projection operator. These states are represented symbolically in Fig. 3.

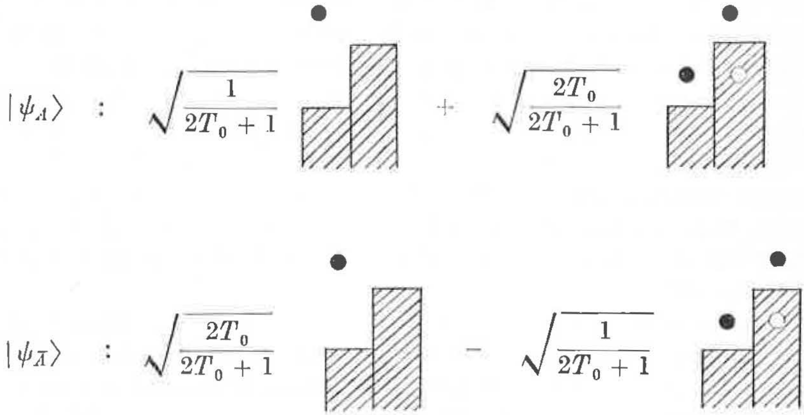


FIGURE 3

The $T_>$ component of the single-proton excitation has $T = T_0 + 1/2$, $T_z = T_0 - 1/2$; it is the isobaric analog of the single-neutron state $| \psi_j^\nu \rangle$,

$$| \psi_A \rangle = T_- | \psi_j^\nu \rangle .$$

The $T_<$ component $\psi_{\bar{A}}$ is often referred to as the anti-analog of $| \psi^\nu \rangle$. The situation is illustrated in Fig. 4.

$| \psi_A \rangle$ and $| \psi^\nu \rangle$ (members of an isobaric-spin multiplet with $T = T_>$) differ only in the replacement of a neutron by a proton in the same state; their energy difference (apart from the neutron–proton mass difference and possible charge-dependent nuclear contributions) is thus the Coulomb energy of a proton with the density distribution of an excess neutron in the field of the core nucleus (Z, N). We shall discuss

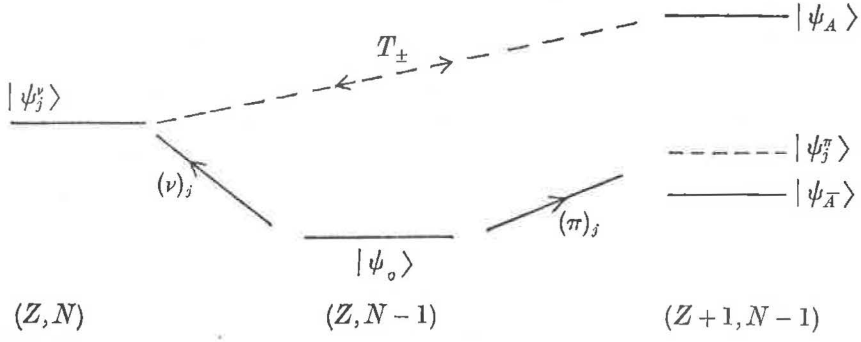


FIGURE 4

what can be learned from such Coulomb energy differences in a later contribution. The energy difference between $|\psi_A\rangle$ and $|\psi_{\bar{A}}\rangle$ has nothing to do with Coulomb effects. It is a symmetry-energy due to the fact that states of the same particle structure (in the sense of Fig. 3) but different isobaric spin necessarily have different space symmetry; nuclear forces are stronger in states of higher space symmetry. Present indications are that the energy differences $\Delta E = E_A - E_{\bar{A}}$ increase slowly with neutron excess from a value of ~ 3 MeV for $A = 40$ $(N - Z = 2)$ to about 7 MeV for $A = 90$ $(N - Z = 10)$ and 10 or 11 MeV around $A = 208$ $(N - Z \sim 40)$.

In nuclei with $A \gtrsim 60$, the analogs $|\psi_A\rangle$ of low-lying single-neutron bound states $|\psi^{\nu}\rangle$ lie well above the threshold for proton emission in the nucleus $(Z + 1, N - 1)$. They can therefore be formed as resonances in proton-induced reactions on the target nucleus $|\psi_0\rangle$, $(Z, N - 1)$. Such studies have proved enormously fruitful. Elastic proton scattering through isobaric analog resonances has provided an independent means of calibrating the conventional (d, p) analysis of the single-neutron parent analogs $|\psi^{\nu}\rangle$. Inelastic proton scattering to various excited states $|\psi_i\rangle$ $(i \neq 0)$ of the core has provided the first systematic source of direct information on parentage relations involving excited states of the core: $(p, p'\gamma)$ correlation experiments promise to yield detailed parentage decompositions (including relative phases) in situations where several different orbits j can contribute to the parentage relation between states $|\psi_0\rangle$ and $|\psi^{\nu}\rangle$. It is clear, however, that studies of the isobaric analog $|\psi_A\rangle$ yield information on *neutron parentage only*. All information concerning proton parentage not already implied through charge independence by the facts about neutron parentage is contained in the anti-analog. The anti-analogs of low-lying bound states are invariably either bound or inaccessible to proton resonance formation because of the Coulomb barrier. Thus anti-analogs must be studied by direct reactions such as (*d*, *n*) or (He^3, *d*).

In conclusion, it is amusing that the furore over analogs has narrowed the field in a long-standing dispute over terminology. No nuclear physicist talks now of "*i*-spin" or "*T*-spin" and an occasional "isotopic-spin" is regarded as a slip of the tongue. It seemed for a while that isobaric-spin had become standard usage but a recent book and a conference started a new crusade for "iso-spin". To coin a phrase, we must await the verdict of history.

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2. For general reading on this subject, we suggest:
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