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OBSERVABLE QUANTITIES IN KINETICS OF ELECTROMAGNETIC FIELD IN MEDIUM

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The problem of parameters, which are necessary for nonequilibrium electromagnetic field description, is a key one for building the field kinetics whenever it is under discussion. This problem is investigated on the basis of the Bogolyubov reduced description method leading to the conclusion about the necessity of binary correlations in the minimal set of parameters taken into account in evolution equations. The corresponding theory can be built in terms of one-particle density matrices, Wigner distribution functions, and conventional simultaneous correlation functions of field operators. Rather different correlation functions are determined in measurements based on the coincidence scheme using quantum detectors and regarded as a basis for algebra of observables in quantum optics. Various approaches in theoretical and experimental research into field correlations are compared in the present paper.

1 Introduction

Quantum electromagnetic theory supposes using averages of operator dynamic variables for field state specification and experimental result analysis. A statistical operator gives the most general description of field states, but from the point of view of experiments only reduced description of electromagnetic fields is possible. The field kinetics embraces a number of physical theories such as electrodynamics of continuous media, radiation transfer theory, magnetic hydrodynamics, and quantum optics. In all the cases it is necessary to choose physical quantities providing an adequate picture of nonequilibrium processes after transfer to averages. In our previous investigations several approaches to the construction of kinetic equations have been outlined. Obviously, the choice depends on traditions and visibility of phenomenon description. All the methods can be connected due to relatively simple relations expressing their key quantities through one another. At the same time quantum optics introducing non-simultaneous correlation functions requires additional efforts for using the results obtained in the reduced description scheme.

2 Reduced description and necessary set of field parameters

The Bogolyubov reduced description method [1] can be a basis for the general consideration of the problem. Its starting point in this approach is a quantum Liouville equation for the statistical operator $\rho(t)$ of a system including electromagnetic field (subsystem f) and a medium (subsystem m)

$$\partial_t \rho(t) = -\frac{i}{\hbar}[\hat{H}, \rho(t)] \qquad (\hat{H} = \hat{H}_{\rm f} + \hat{H}_m). \tag{1}$$

The method is based on the functional hypothesis describing a structure of the operator $\rho(t)$ at long times

$$\rho(t) \xrightarrow[t \gg \tau_0]{} \rho(\gamma(t,\rho_0), \eta(t,\rho_0)) \qquad (\rho_0 \equiv \rho(t=0)) \tag{2}$$

where reduced description parameters of field $\gamma_\mu(t,\rho_0)$ and matter $\eta_a(t,\rho_0)$ are used. The complex of parameters $\gamma_\mu(t,\rho_0)$ is determined by the possibilities and traditions of experiments as well as by theoretical considerations. The development of the problem investigations has resulted in finding the main approximation for the statistical operator $\rho(\gamma,\eta)$, so called a quasi-equilibrium statistical operator $\rho_q(Z(\gamma), Z(\eta))$(though it describes states which are far from the equilibrium) defined by the relations

$$\rho_q(Z, Z_m) = \rho_{\rm f}(Z)\rho_m(Z_m);$$

$$\rho_{\rm f}(Z) = \exp\{\Phi(Z) - \sum_\mu Z_\mu \hat{\gamma}_\mu\}, \qquad {\rm Sp}_{\rm f}\rho_{\rm f}(Z) = 1, \quad {\rm Sp}_{\rm f}\rho_{\rm f}(Z(\gamma))\hat{\gamma}_\mu = \gamma_\mu;$$

$$\rho_m(X) = \exp\{\Omega(X) - \sum_a X_a \hat{\eta}_a\}; \qquad {\rm Sp}_m\rho_m(X) = 1, \qquad {\rm Sp}_m\rho_m(X(\eta))\hat{\eta}_a = \eta_a. \tag{3}$$

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Practically all the electrodynamics of continuous media operates with average values of electromagnetic field $E_n(x,t)$, $B_n(x,t)$ (hereafter we omit the dependence of reduced description parameters on an initial statistical operator ρ_0 of the system for simplicity). So corresponding operators $\hat{E}_n(x)$, $\hat{B}_n(x)$ are chosen as the operators $\hat{\gamma}_\mu$. In Coulomb gauge the following expressions can be used for them:

$$\hat{E}_n(x) = i\sum_{\alpha k}\frac{(2\pi\hbar\omega_k)^{1/2}}{V^{1/2}}e_{\alpha kn}(c_{\alpha k} - c^+_{\alpha,-k})e^{ikx},$$

$$\hat{B}_n(x) = i\sum_{\alpha k}\frac{(2\pi\hbar\omega_k)^{1/2}}{V^{1/2}}\varepsilon_{nlm}\tilde{k}_l e_{\alpha km}(c_{\alpha k} + c^+_{\alpha,-k})e^{ikx} \tag{4}$$

with standard notations of quantum electrodynamics. Nevertheless in paper [2] it has been pointed out that for this choice of operators $\hat{\gamma}_\mu$ the statistical operator $\rho_{\rm f}(Z)$ does not exist because of containing a linear form on Bose field operators $c_{\alpha k}$, $c^+_{\alpha k}$ in the exponent. The situation can be corrected with the availability of a quadratic form of the operators in the exponent; hence the statistical operator has a structure

$$\rho_{\rm f}(Z) = \exp\{\Phi(Z) - \sum_{\alpha k,\alpha' k'} Z^{\alpha\alpha'}_{kk'}c^+_{\alpha k}c_{\alpha' k'} - (\sum_{\alpha k,\alpha' k'}\tilde{Z}^{\alpha\alpha'}_{kk'}c^+_{\alpha k}c^+_{\alpha' k'} + \sum_{\alpha k} Z^\alpha_k c^+_{\alpha k} + h.c.)\}. \tag{5}$$

In fact, it means that the minimal set of reduced description parameters of electromagnetic field contains its binary fluctuations (correlations) besides its average strength though the average field can be absent.

3 Various approaches to constructing the kinetic equations with binary correlation account

In order to describe binary fluctuations of the field, normal $n^{\alpha\alpha'}_{kk'}(t) = {\rm Sp}\rho(t)c^+_{\alpha k}c_{\alpha' k'}$ and anomalous $\tilde{n}^{\alpha\alpha'}_{kk'}(t) = {\rm Sp}\rho(t)c_{\alpha k}c_{\alpha' k'}$ one-particle density matrices can be used (a T-1 theory) as well as connected with them normal ${\rm f}^{\alpha\alpha'}_k(x,t) = {\rm Sp}\rho(t)\hat{\rm f}^{\alpha\alpha'}_k(x)$ and anomalous $\tilde{\rm f}^{\alpha\alpha'}_k(x,t) = {\rm Sp}\rho(t)\hat{\tilde{\rm f}}^{\alpha\alpha'}_k(x)$ Wigner distribution functions (WDF) (a T-2 theory) where

$$\hat{\rm f}^{\alpha\alpha'}_k(x) = \sum_q c^+_{\alpha,k-q/2}c_{\alpha',k+q/2}e^{iqx}, \quad \hat{\tilde{\rm f}}^{\alpha\alpha'}_k(x) = \sum_q c_{\alpha,k+q/2}c_{\alpha',-k+q/2}e^{iqx}. \tag{6}$$

The anomalous density matrix and WDF are absent in a state with the statistical operator $\rho_{\rm f}(Z)$ at $\tilde{Z}^{\alpha\alpha'}_{kk'} = 0$ and average electromagnetic field equals zero at $Z^\alpha_k = 0$. The terminology "normal-anomalous" is connected with the concept of spontaneous break of symmetry, at that from this point of view a nonzero average field value is also a consequence of some symmetry breakdown.

Electromagnetic field fluctuations can be described also with average values of field operators

$$\langle E^x_n E^{x'}_l\rangle_t = {\rm Sp}\rho(t)\hat{E}_n(x)\hat{E}_l(x'), \quad \langle B^x_n B^{x'}_l\rangle_t = {\rm Sp}\rho(t)\hat{B}_n(x)\hat{B}_l(x'),$$

$$\langle E^x_n B^{x'}_l\rangle_t = \frac{1}{2}{\rm Sp}\rho(t)\{\hat{E}_n(x), \hat{B}_l(x')\}, \tag{7}$$

or corresponding correlation functions $(E^x_n E^{x'}_l)_t$, $(B^x_n B^{x'}_l)_t$, $(E^x_n B^{x'}_l)_t$ (a T-3 theory). The unique relationship of such theories proceeds from the formulas (5) and their consequence

$$c_{\alpha k} = (8\pi\omega_k\hbar V)^{-1/2}\int d^3x\{\hat{Z}_n(x)/k - \hat{E}_n(x)\}e^{ikx} \qquad (\hat{Z}_n(x) \equiv {\rm rot}_n\hat{B}(x)). \tag{8}$$

T-2 and T-3 theories allow describing a spatial behavior of the electromagnetic field in medium. Temporal equations for one-particle density matrices and Wigner distribution functions are called kinetic equations for photons in medium.

In the paper [3] a theory of T-1 type has been built for electromagnetic field in equilibrium plasma medium. The corresponding kinetic equation has a form

$$\partial_t g^{\alpha\alpha'}_{kk'} = i(\tilde{\omega}_k - \tilde{\omega}_{k'})g^{\alpha\alpha'}_{kk'} - (\nu_k + \nu_{k'})(g^{\alpha\alpha'}_{kk'} - n_k\delta_{\alpha\alpha'}\delta_{kk'}),$$

$$\partial_t x_{\alpha k} = -(i\tilde{\omega}_k + \nu_k)x_{\alpha k} + (\nu_k + i\omega_k\chi_k)x^*_{\alpha,-k} \tag{9}$$

where

$$g^{\alpha\alpha'}_{kk'}(t) = n^{\alpha\alpha'}_{kk'}(t) - x^*_{\alpha k}(t)x_{\alpha' k'}(t), \quad x_{\alpha k}(t) = {\rm Sp}\rho(t)c_{\alpha k};$$

$$\tilde{\omega}_k = \omega_k(1 - 2\pi\chi_k), \quad \nu_k = 2\pi\sigma_k \tag{10}$$

($\tilde{\omega}_k$ is a photon spectrum in the medium, n_k is Planck distribution with a medium temperature). The second equation (11) is actually a Maxwell equation with account of a material equation, at that σ_k and χ_k are conductivity and magnetic susceptibility of the medium expressed via the Green function of currents. In terms of WDF in the case of weak nonuniformity of a system state such kinetic equation takes the form

$$\partial_t \mathrm{f}_k^{\alpha\alpha'} = -\frac{\partial\tilde{\omega}_k}{\partial k_n}\frac{\partial \mathrm{f}_k^{\alpha\alpha'}}{\partial x_n} + \frac{1}{4}\frac{\partial^2\nu_k}{\partial k_n\partial k_l}\frac{\partial^2 \mathrm{f}_k^{\alpha\alpha'}}{\partial x_n\partial x_l} - 2\nu_k(\mathrm{f}_k^{\alpha\alpha'} - n_k\delta_{kk'}\delta_{\alpha\alpha'}). \tag{11}$$

4 Kinetics of electromagnetic field interacting with nonequilibrium system of emitters

In our previous papers (see [4]) electrodynamics in a medium consisting of two-level emitters has been built. Such a theory emerges in the course of research into the Dicke superfluorescence [5] on the basis of the Bogolyubov reduced description method. A standard approach lies in the framework of a theory of T-3 type. Emitter subsystem is regarded as non-uniform and it is convenient to describe it with a density of emitter energy

$$\hat{\varepsilon}(x) = \hbar\omega\sum_a \hat{r}_{az}\delta(x - x_a) \tag{12}$$

Operators of the reduced description parameters of the system in the developed theory are $\hat{\varepsilon}(x)$, $\hat{\gamma}_\mu$

$$\hat{\gamma}_\mu: \qquad \hat{E}_n^t(x), \quad \hat{B}_n(x), \quad \frac{1}{2}\{\hat{E}_n^t(x), \hat{E}_l^t(x')\}, \quad \frac{1}{2}\{\hat{E}_n^t(x), \hat{B}_l(x')\}, \quad \frac{1}{2}\{\hat{B}_n(x), \hat{B}_l(x')\}.$$

Maxwell equations in terms of averages has the form

$$\partial_t E_n(x,t) = c\,\mathrm{rot}_n B(x,t) - 4\pi J_n(x,\varepsilon(t),\gamma(t)), \qquad \partial_t B_n(x,t) = -c\,\mathrm{rot}_n E(x,t) \tag{13}$$

with a material equation

$$J_n(x,\varepsilon,\gamma) = \int dx'\sigma(x-x',\varepsilon(x))E_n(x') + c\int dx'\chi(x-x',\varepsilon(x))Z_n(x') \qquad (Z_n(x,t)\equiv \mathrm{rot}_n B(x,t)) \tag{14}$$

where Fourier transforms of functions $\sigma(x,\varepsilon)$, $\chi(x,\varepsilon)$ are conductivity $\sigma_k(\varepsilon)$ and magnetic susceptibility $\chi_k(\varepsilon)$ of the system. Equations for field correlations acquire the form

$$\begin{aligned}
\partial_t(E_n^x E_l^{x'}) &= c\,\mathrm{rot}_n(B^x E_l^{x'}) + c\,\mathrm{rot}'_l(E_n^x B^{x'}) - 4\pi(J_n^x E_l^{x'}) - 4\pi(E_n^x J_l^{x'}),\\
\partial_t(E_n^x B_l^{x'}) &= c\,\mathrm{rot}_n(B^x B_l^{x'}) - c\,\mathrm{rot}'_l(E_n^x E^{x'}) - 4\pi(J_n^x B_l^{x'}),\\
\partial_t(B_n^x E_l^{x'}) &= -c\,\mathrm{rot}_n(E^x E_l^{x'}) + c\,\mathrm{rot}'_l(B_n^x B^{x'}) - 4\pi(B_n^x J_l^{x'}),\\
\partial_t(B_n^x B_l^{x'}) &= -c\,\mathrm{rot}_n(E^x B_l^{x'}) - c\,\mathrm{rot}'_l(B_n^x E^{x'}).
\end{aligned} \tag{15}$$

Current-field correlations are expressed via field correlations by material equations obeying the Onsager principle.

5 Power fluxes in medium and correlation functions

In the theory of energy transfer power fluxes in medium is a problem of interest. An operator of power flux is given by the formula

$$\hat{q}_n(x) = \frac{c}{8\pi}\varepsilon_{nlm}\{\hat{E}_l(x), \hat{B}_m(x)\}. \tag{16}$$

According to the reduced description method, in the theory taking into account only binary field correlations exact averages of binary field functions are determined by the quasi-equilibrium field distribution. Therefore the power flux is expressed exactly via one-particle density matrix or WDF. For example, if the average field is absent, we come to an exact formula

$$q_n(x) = \frac{1}{V}\sum_{kq,\alpha\alpha'} n_{k-q/2,k+q/2}^{\alpha\alpha'}\varphi_n^{\alpha\alpha'}(k,q)e^{iqx} = \frac{1}{V}\sum_{k,\alpha\alpha'}\varphi_n^{\alpha\alpha'}(k,-i\frac{\partial}{\partial x})\mathrm{f}_k^{\alpha\alpha'}(x) \tag{17}$$

where the notation is used

$$\varphi_n^{\alpha\alpha'}(k,q) = (\delta_{nl}\delta_{ms} - \delta_{ml}\delta_{ns})\frac{\hbar c^2}{2}(k_1k_2)^{1/2}\{\tilde{k}_{1l}e^*_{\alpha k_1 s}e_{\alpha' k_2 m} + \tilde{k}_{2l}e^*_{\alpha k_1 m}e_{\alpha k_2 s}\}_{k_1=k-q/2,\ k_2=k+q/2}. \tag{18}$$

In a weakly non-uniform state this formula leads in a well-known elementary result

$$q_n(x) = \frac{1}{V}\sum_{k\alpha}\omega_k\hbar c\tilde{k}_n \mathrm{f}_k^{\alpha\alpha}(x). \tag{19}$$

6 Field correlation properties in quantum optics

The most general approach of quantum optics to the statistical properties of light is based on the technique of photon counting and the concept of an ideal quantum detector. Its operation analysis by Glauber [6] has led to the conclusion that the rate of photon counting is proportional to the local value of the field correlation function of the first order (the polarization properties are neglected)

$$p(t) \sim G^{1,1}(x,t;x,t) = \langle \hat{E}^{(-)}(x,t)\hat{E}^{(+)}(x,t)\rangle \equiv \mathrm{Sp}\{\rho\hat{E}^{(-)}(x,t)\hat{E}^{(+)}(x,t)\} \tag{20}$$

The quantum-statistical averaging is executed with the statistical operator of field $\hat{\rho}$ and deals with positive-frequency and negative-frequency parts of the electric field operator (4) in the interaction picture:

$$\hat{E}_n(x,t) = \hat{E}_n^{(+)}(x,t) + \hat{E}_n^{(-)}(x,t) \tag{21}$$

where

$$\hat{E}_n^{(+)}(x,t) \equiv i\sum_{k\alpha}\frac{(2\pi\hbar\omega_k)^{1/2}}{V^{1/2}}e_{\alpha kn}c_{\alpha k}e^{i(kx-\omega_k t)}, \qquad \hat{E}_n^{(-)}(x,t) = \hat{E}_n^{(+)}(x,t)^{\dagger}. \tag{22}$$

In quantum optics properties of the electromagnetic field are discussed in terms of correlation functions of the form

$$G^{ss'}_{n_1...n_s,l_1...l_{s'}}(y_1...y_s,y'_1...y'_{s'}) \equiv \mathrm{Sp}\{\rho\hat{E}^{(-)}_{n_1}(y_1)...\hat{E}^{(-)}_{n_s}(y_s)\hat{E}^{(+)}_{l_1}(y'_1)...\hat{E}^{(+)}_{l_{s'}}(y'_{s'})\} \tag{23}$$

where $y \equiv (x,t)$ [7]. Experiments of the second order (for example, the Hunbury-Brown-Twiss scheme based on photon detection coincidence) determine correlation functions of the type

$$G^{(2,2)}_{nl,ms}(y_1,y_2,y_3,y_4) \equiv \mathrm{Sp}\{\rho\hat{E}^{(-)}_n(y_1)\hat{E}^{(-)}_l(y_2)\hat{E}^{(+)}_m(y_3)\hat{E}^{(+)}_s(y_4)\} \tag{24}$$

Note that correlation functions of the first order can be expressed through the one-particle density matrix $n^{\alpha\alpha'}_{kk'}$ exactly

$$G^{(1,1)}_{nl}(x_1,t_1;x_2,t_2) = \frac{2\pi\hbar c}{V}\sum_{k_1\alpha_1k_2\alpha_2}\sqrt{k_1k_2}e^*_{\alpha_1k_1n}e_{\alpha_2k_2l}n^{\alpha_1\alpha_2}_{k_1k_2}e^{i(\omega_{k_1}t_1-k_1x_1)}e^{-i(\omega_{k_2}t_2-k_2x_2)} \tag{25}$$

and those of the second order can be expressed through it only approximately. The most interesting quantum correlation effects (such as photon antibunching, squeezing, sub-Poissonian statistics) are described with correlation functions concerning different time moments [7].

7 Conclusions

Kinetic theory of electromagnetic field in media has choosing a set of parameters describing nonequilibrium states of the field as a starting point with necessity. The minimal set of such parameters includes binary correlations of field amplitudes. The corresponding mathematical apparatus uses different structures of averages: one-particle density matrices, Wigner distribution functions, and conventional simultaneous correlation functions of field operators. All approaches can be connected with each other due to the possibility of expressing the main correlation parameters in various forms. The reduced description method elucidates the construction of kinetic equations in electrodynamics of continuous media (plasma, complex of two-level emitters) and radiation transfer theory. Simultaneous correlation functions the method deals with are insufficient for the description of time-correlation properties investigated by quantum optics.

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