

The Muon $g-2$ Experiment at Fermilab: Precision Measurements and Nonlinear Beam Dynamics

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On behalf of the Muon $g-2$ Collaboration

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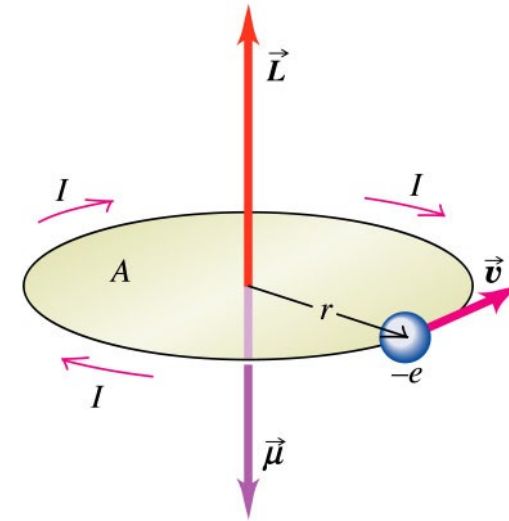
Office of
Science

The UNM Mechanical Engineering Spring Seminar Series – February 23, 2024

Muon Anomalous Magnetic Dipole Moment (a_μ)

$$\mu = g \frac{e}{2m} s \quad \text{Gyromagnetic ratio equation}$$

Classical: $g = 1$



Dirac Equation: $g = 2$

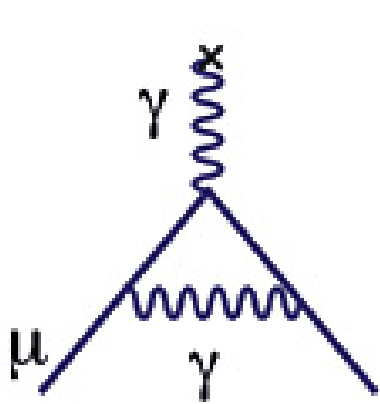
$$i \left(\partial_\mu - ieA_\mu(x) \right) \gamma^\mu \psi(x) = m\psi(x)$$

Interactions w/ quantum foam: $g > 2$

$$a_\mu = \frac{g-2}{2}$$

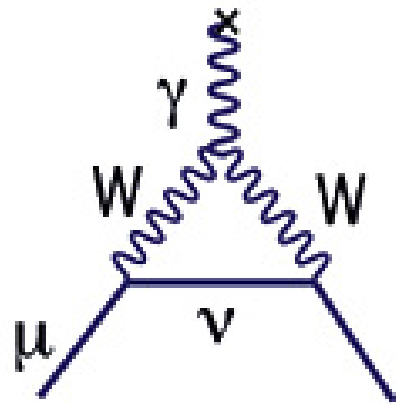
Muon anomaly

Contributions to a_μ in the Standard Model



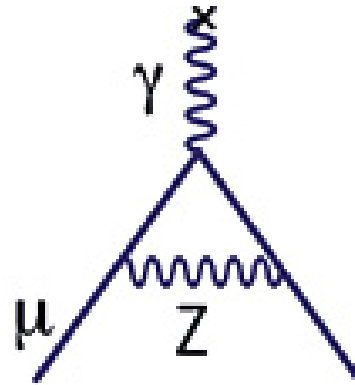
QED

QED incl. 4-loops + 5-loops
 $a_\mu = 116\,584\,718.86 \times 10^{-11}$
 $\delta a_\mu = 0.03 \times 10^{-11}$



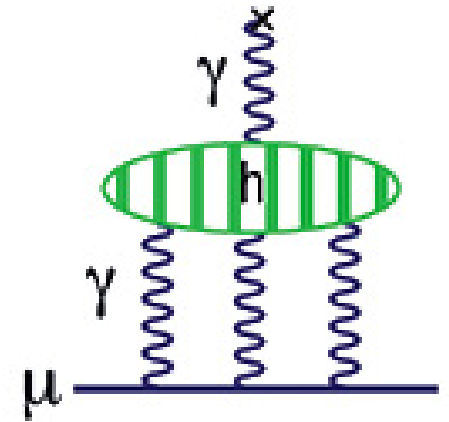
electroweak

Weak to 2-loops
 $a_\mu = 153.6 \times 10^{-11}$
 $\delta a_\mu = 1.1 \times 10^{-11}$



LO hadronic

Hadronic LO VP
 $a_\mu = 6\,894.6 \times 10^{-11}$
 $\delta a_\mu = 32.5 \times 10^{-11}$



hadronic LbL

Hadronic LbL
 $a_\mu = 103.4 \times 10^{-11}$
 $\delta a_\mu = 28.8 \times 10^{-11}$

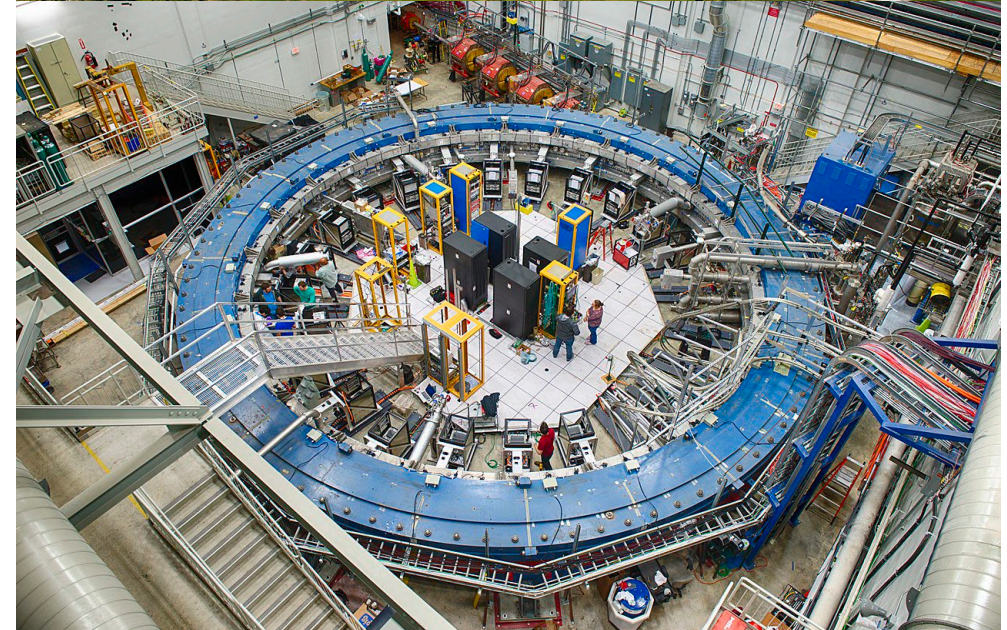
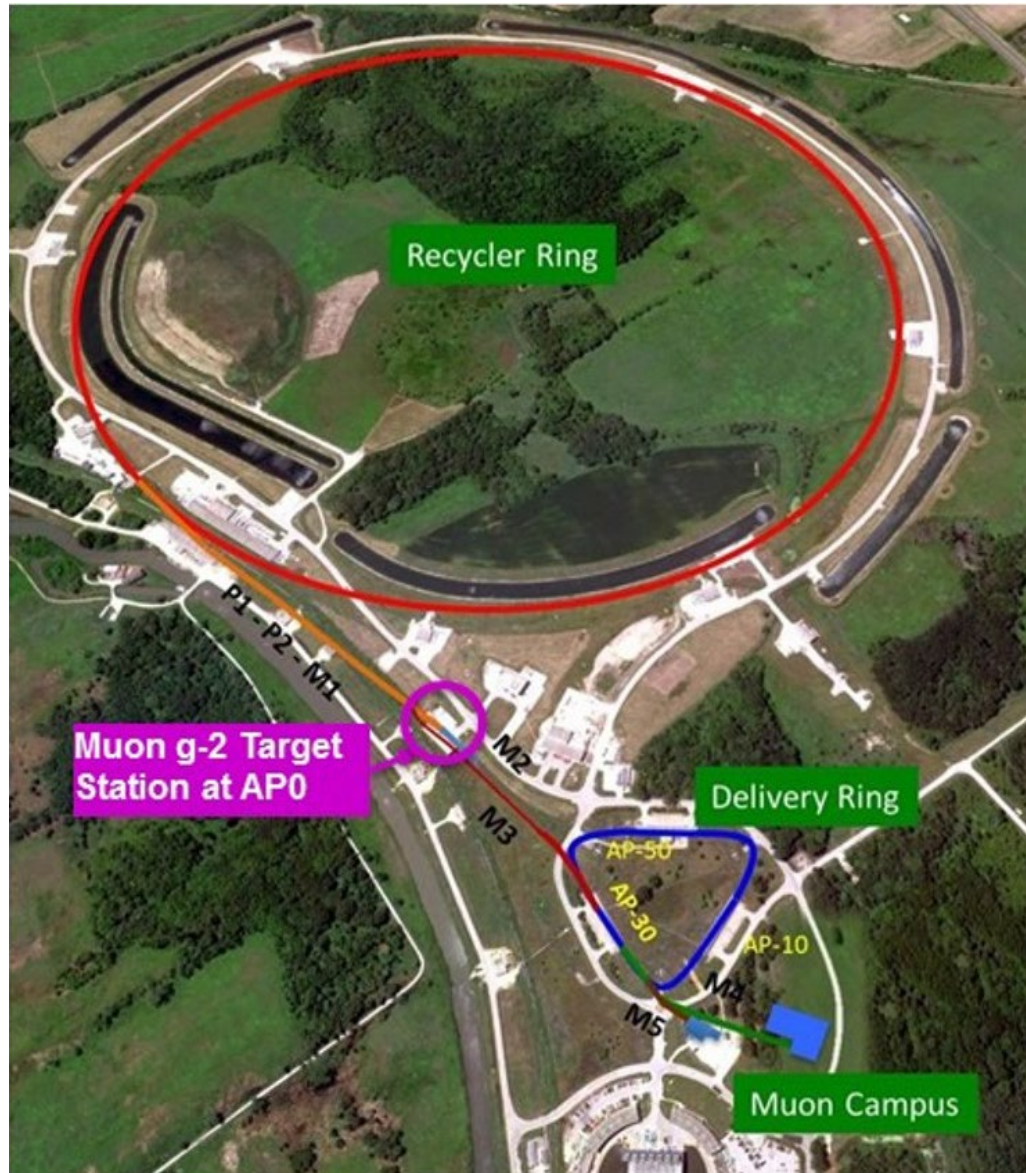
a_μ (FNAL) = 0.00 116 592 055(24) [203 ppb]

a_μ (Exp) = 0.00 116 592 059(22) [190 ppb]

a_μ (Th) = 0.00 116 591 810(43) [370 ppb] (Review by Keshavarzi, 2022)

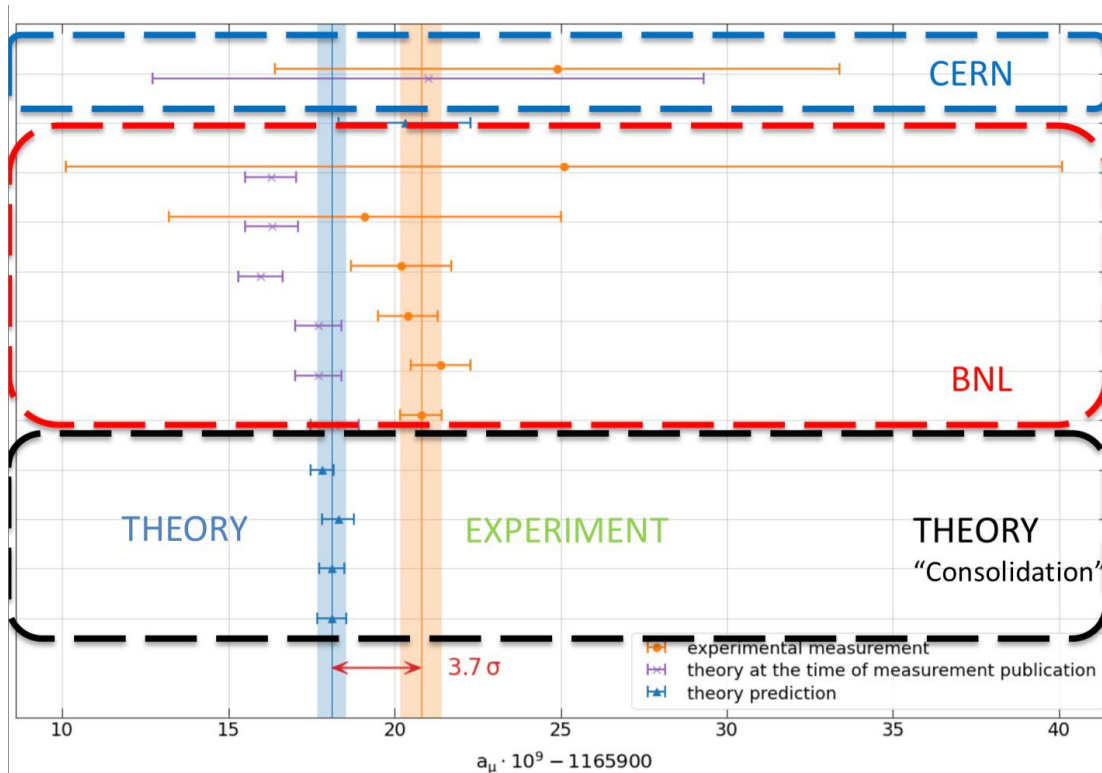
- Dark matter
- Supersymmetry (SUSY)
- Extra dimensions
- Additional Higgs bosons

Introduction

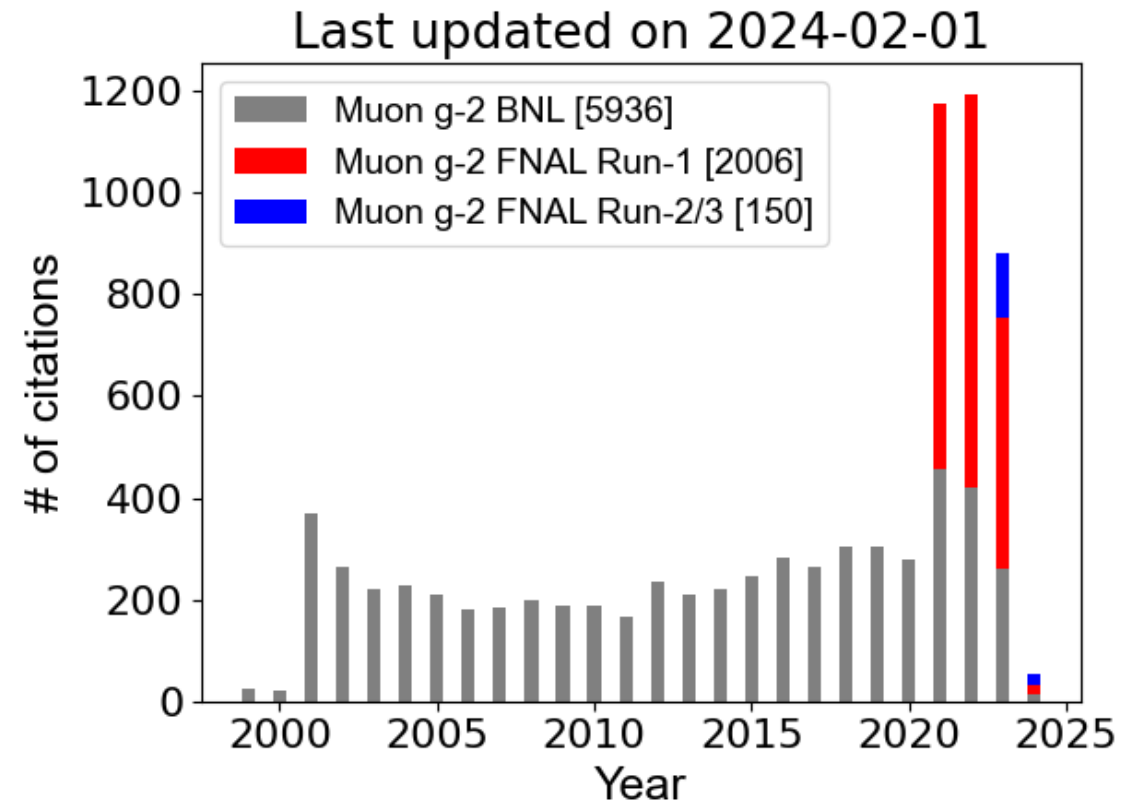


The History and Relevance of Muon g-2

The history of the Muon g-2 experiments has its roots in the series of experiments at CERN

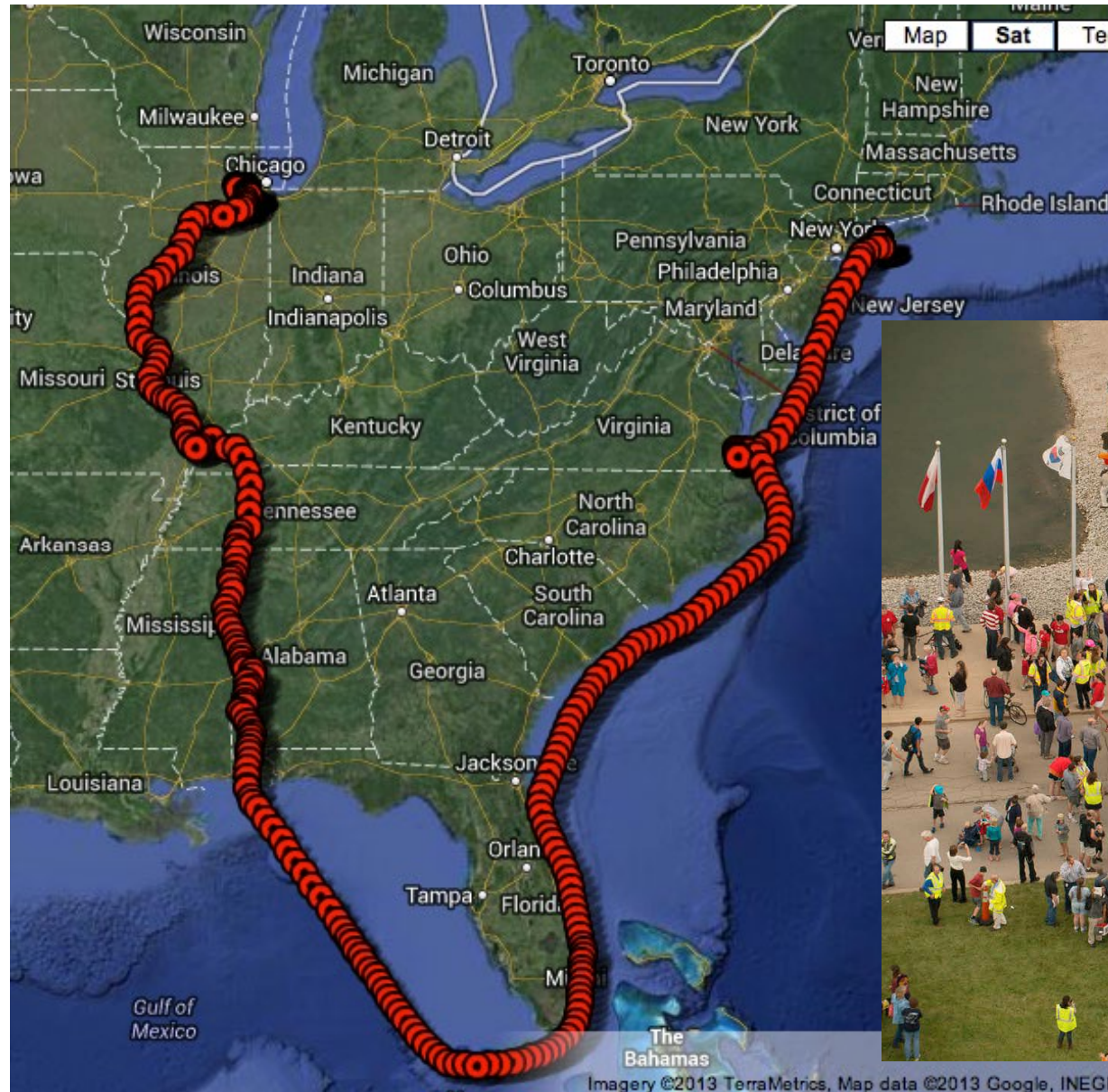


Citation tracker

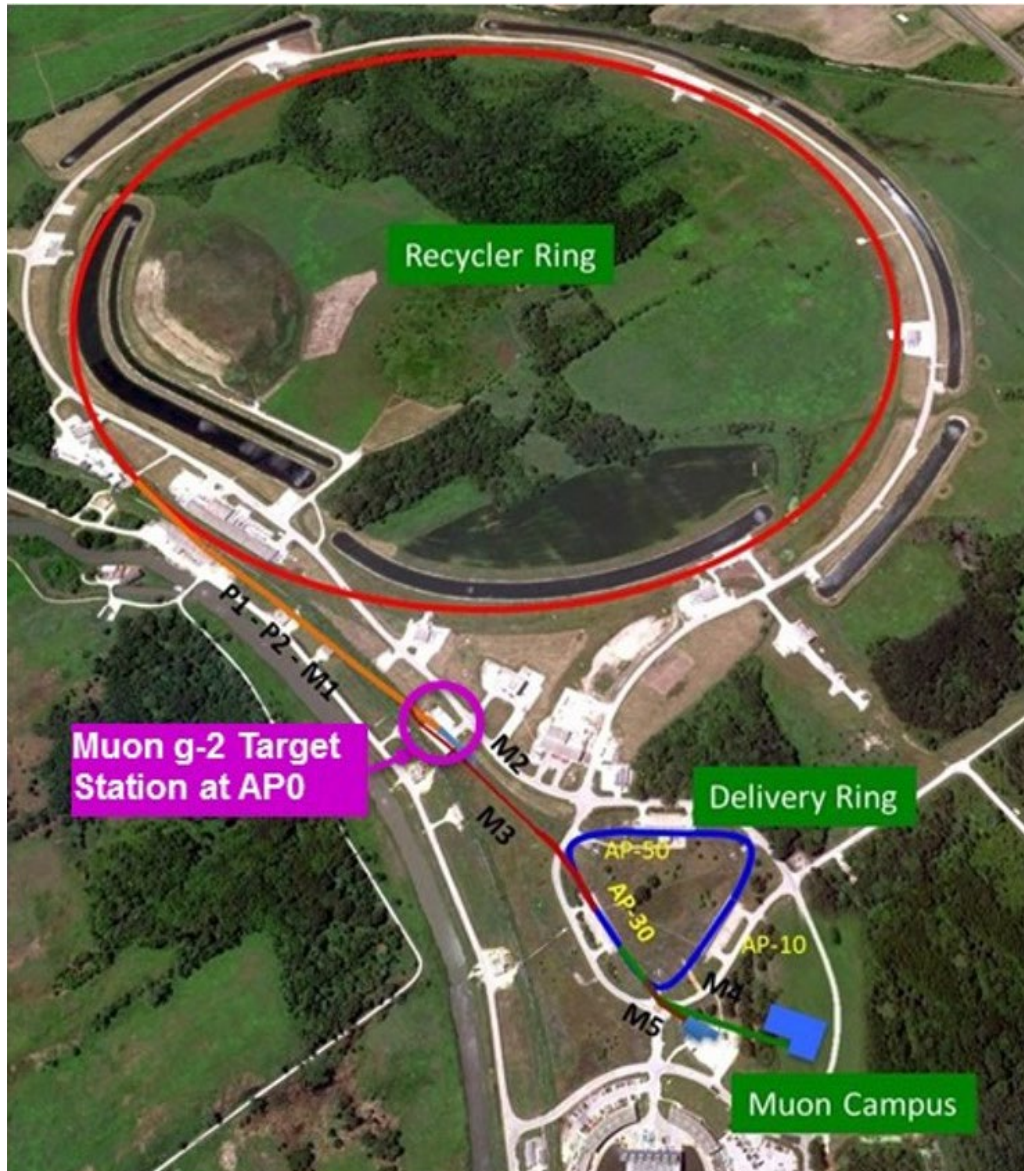


The Great Move

2013: The Great Move



The Muon g-2 Experiment at Fermilab (E989)



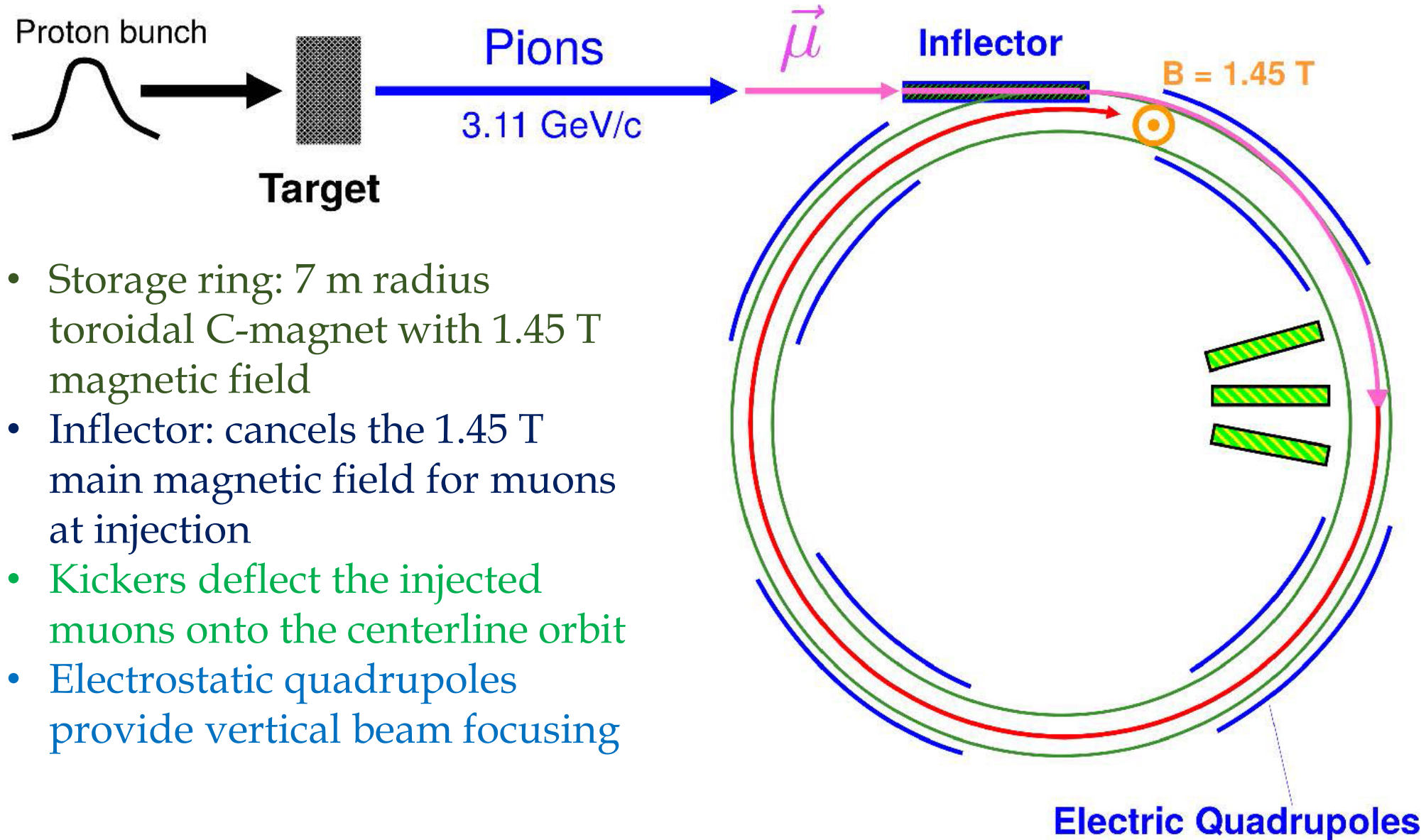
Improvements over the Muon g-2 Experiment at BNL (E821):

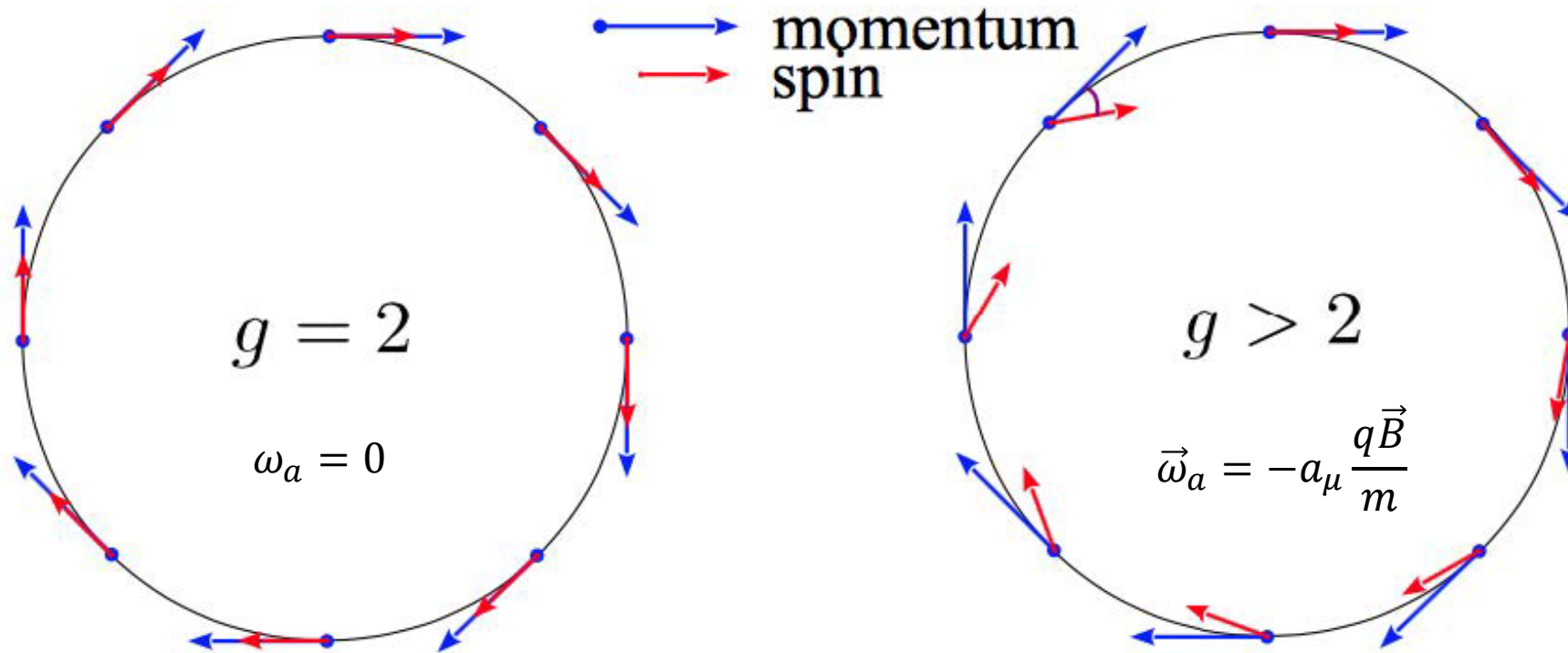
- More muons, delivered more often to the storage ring
- Improved muon storage function
- Better beam dynamics modeling
- Higher field uniformity and better field monitoring
- Reduced spin precession frequency systematics

Technical design projection:

- ~20x more data
- ~3x reduction of systematic errors

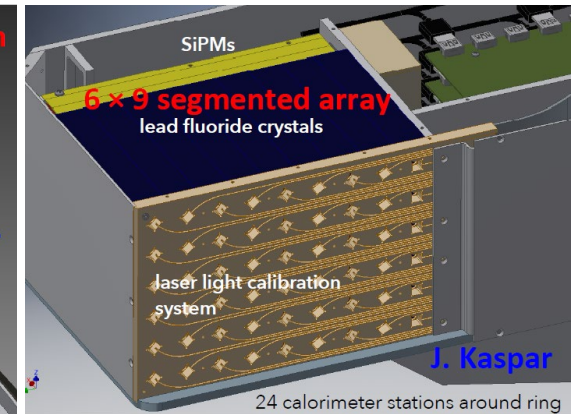
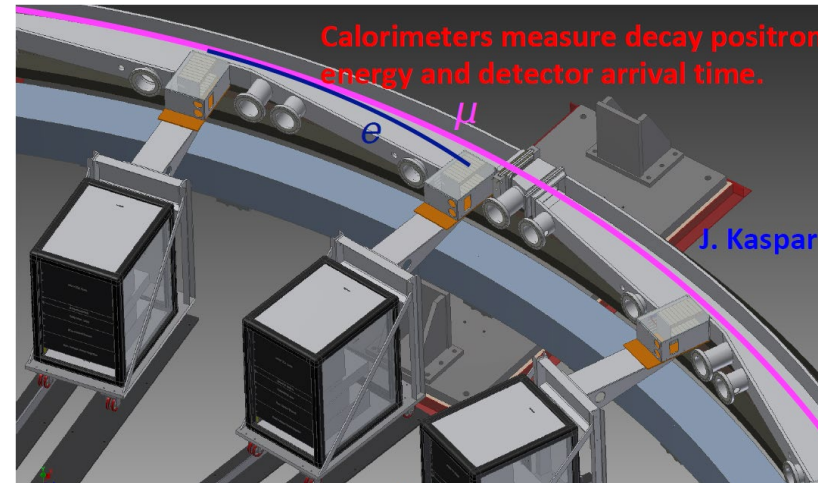
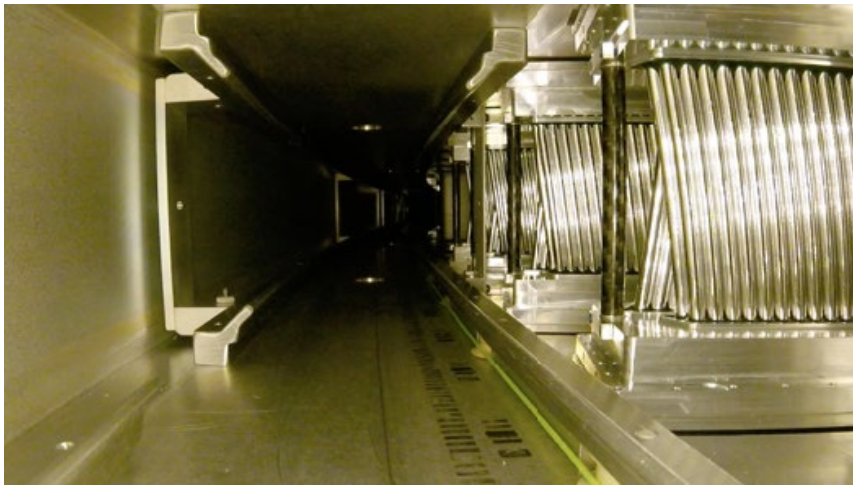
The Muon g-2 Storage Ring



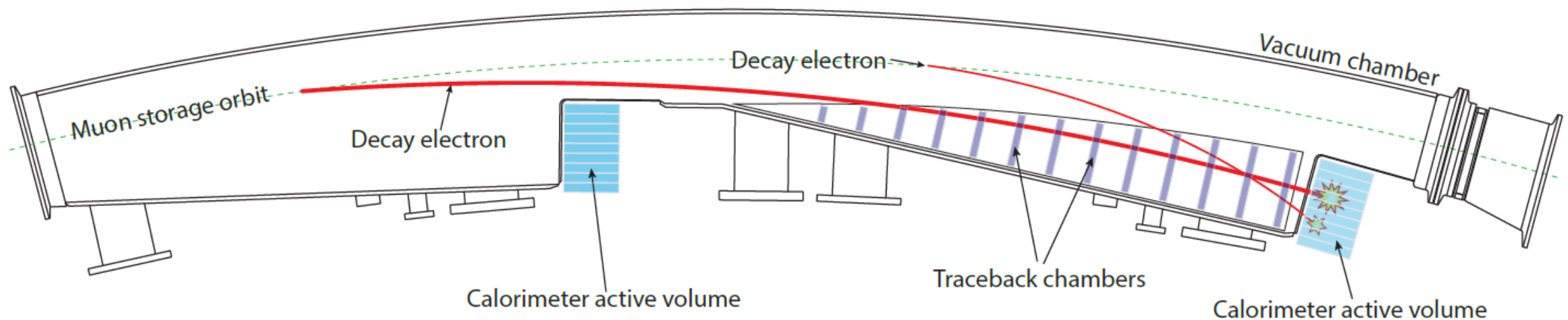


If $g=2$, the angle between the magnetic moment and the momentum does not change.
If $g>2$, the angle between the magnetic moment and the momentum changes linearly.

Measurement of Muon Spin Precession

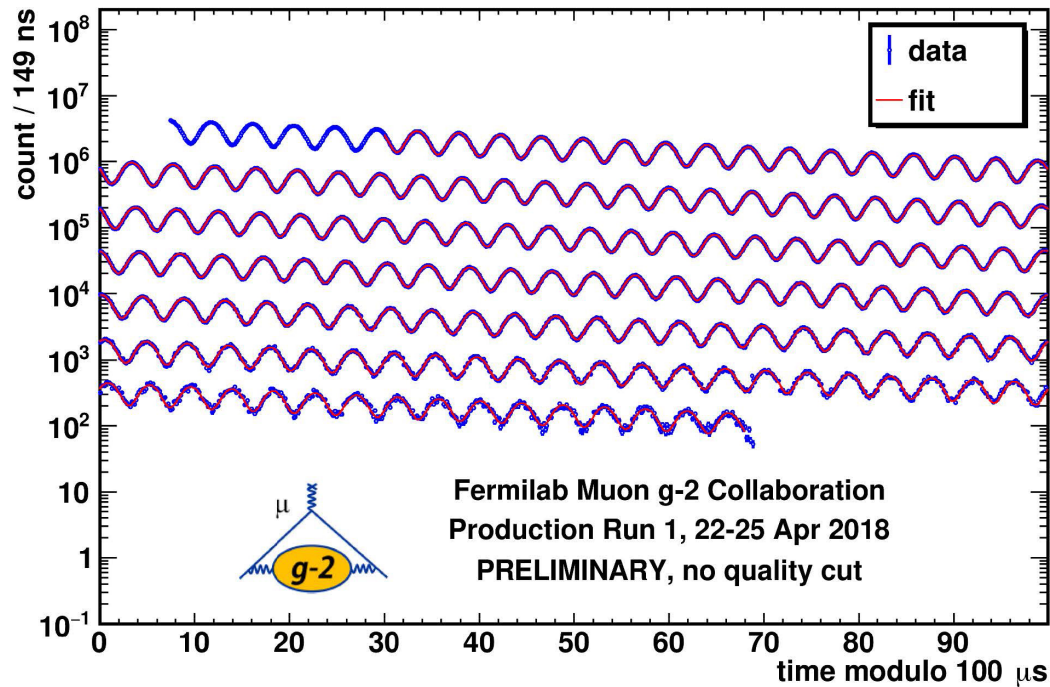


Crystals are 25×25×140 mm



Straw trackers: reconstruct decay e^+ trajectories
Calorimeters: detect decay e^+ energy and arrival times

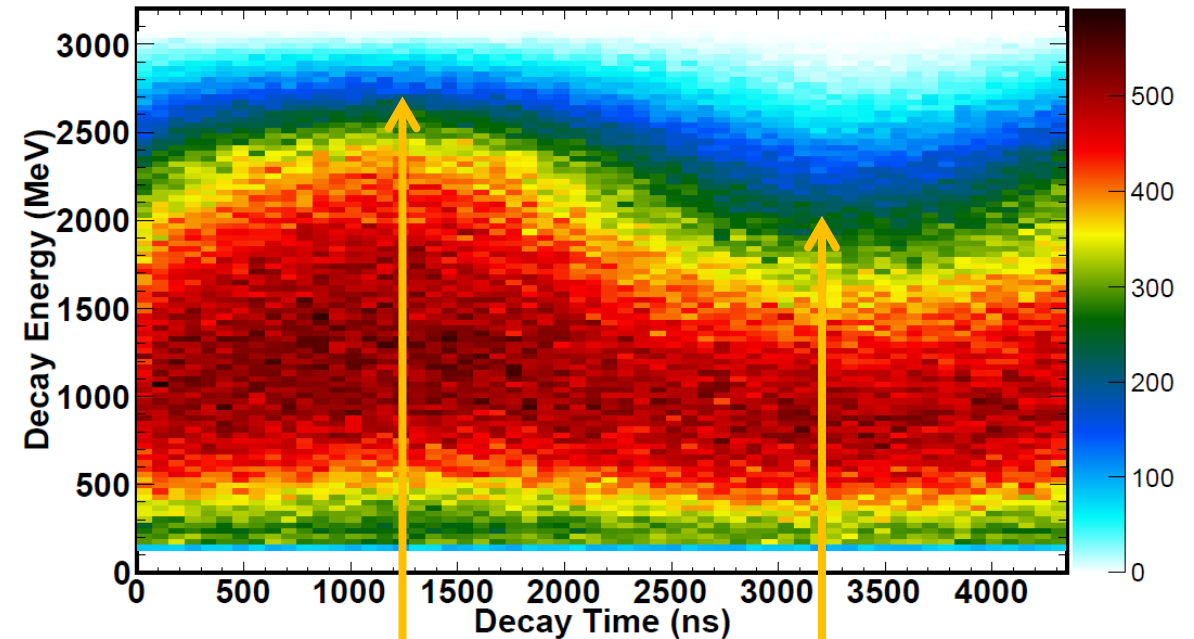
The Wobble Plot



$$f(t) = N_0 e^{-\lambda t} [1 + A \cos(\omega_a t + \phi)]$$

λ : exponential decay constant

ω_a : muon anomalous precession frequency



Muon spin and momentum
are aligned.

Muon spin and momentum
are anti-aligned.

Early-to-late phase change:

If, $\phi = \phi(t) = \phi_0 + \phi_1 t$, then

$$\begin{aligned} \cos(\omega_a t + \phi) &= \cos(\omega_a t + \phi_0 + \phi_1 t) = \\ &= \cos((\omega_a + \phi_1)t + \phi_0) \end{aligned}$$

Calculation of a_μ from Muon and Proton Spin Precession

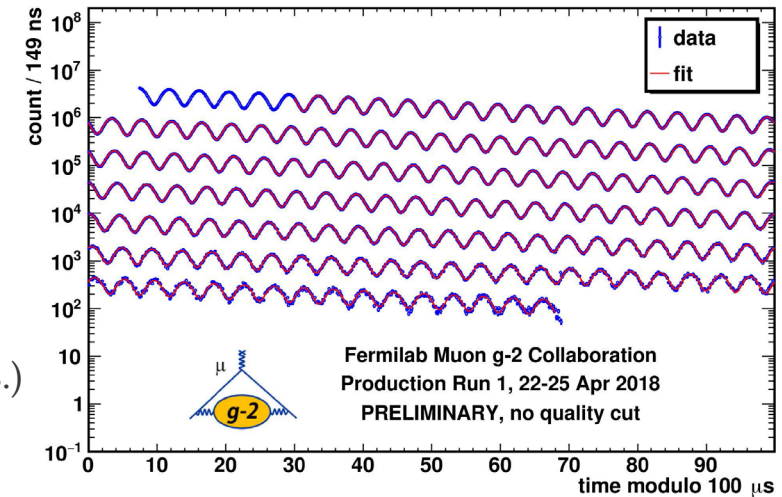
from decay e^+ time spectra

$$a_\mu = \frac{\frac{g_e}{2} \frac{m_\mu}{m_e} \omega_a}{\langle \omega_p \rangle \frac{\mu_e}{\mu_p}}$$

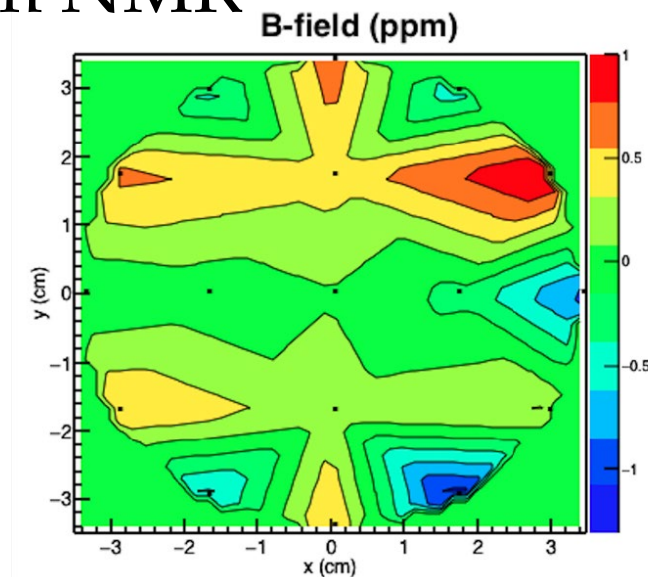
(140 ppb)

(70 ppb sys.
100 ppb stat.)

(70 ppb sys.)



from NMR



From CODATA [1]:

$$g_e = -2.002\,319\,304\,361\,82(52) \text{ (0.00026 ppb)}$$

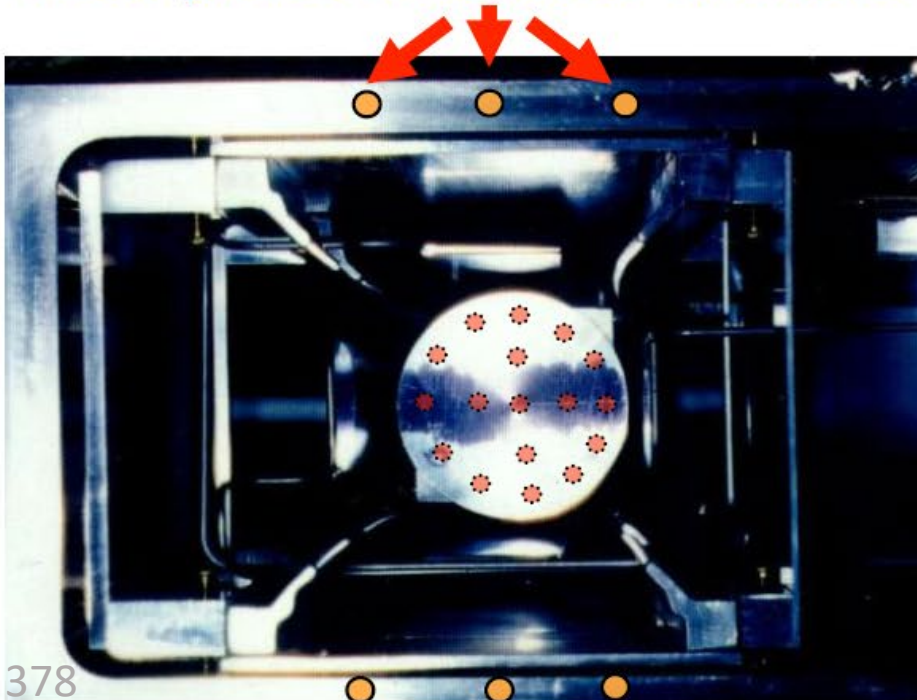
$$m_\mu/m_e = 206.768\,2826(46) \text{ (22 ppb)}$$

$$\mu_e/\mu_p = -658.210\,6866(20) \text{ (3.0 ppb)}$$

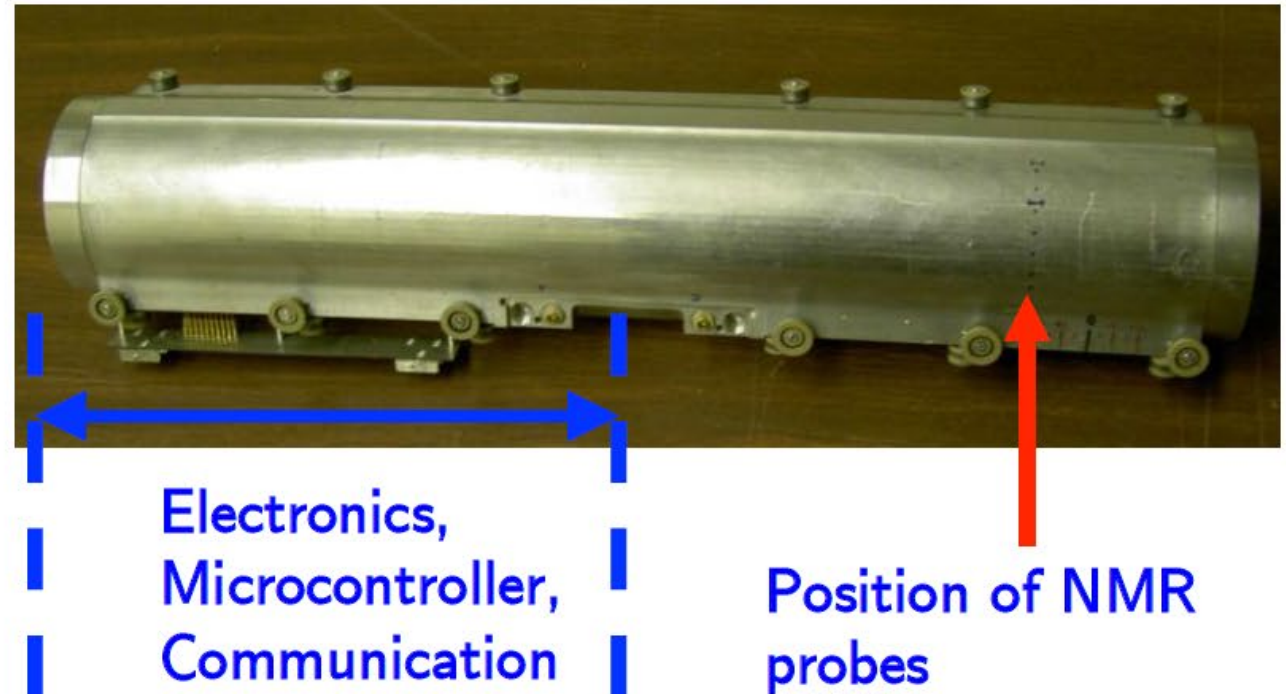
[1] P. J. Mohr, D. B. Newell and B. N. Taylor, Rev. Mod. Phys. 88, no. 3, 035009 (2016).

Fixed and Trolley-Mounted NMR Probes

Fixed probes on vacuum chambers



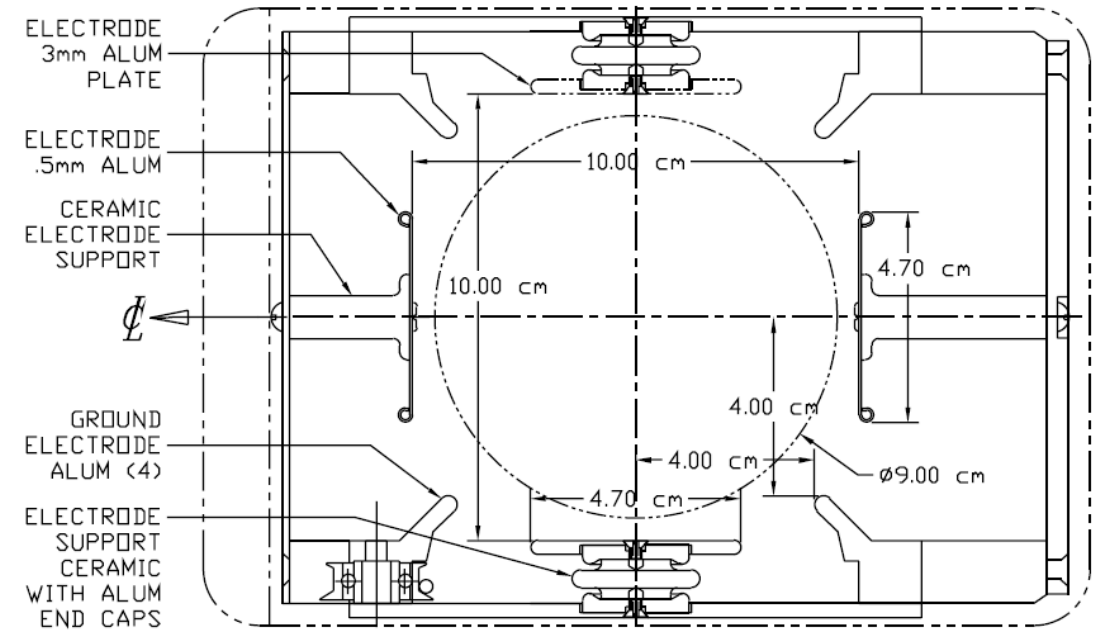
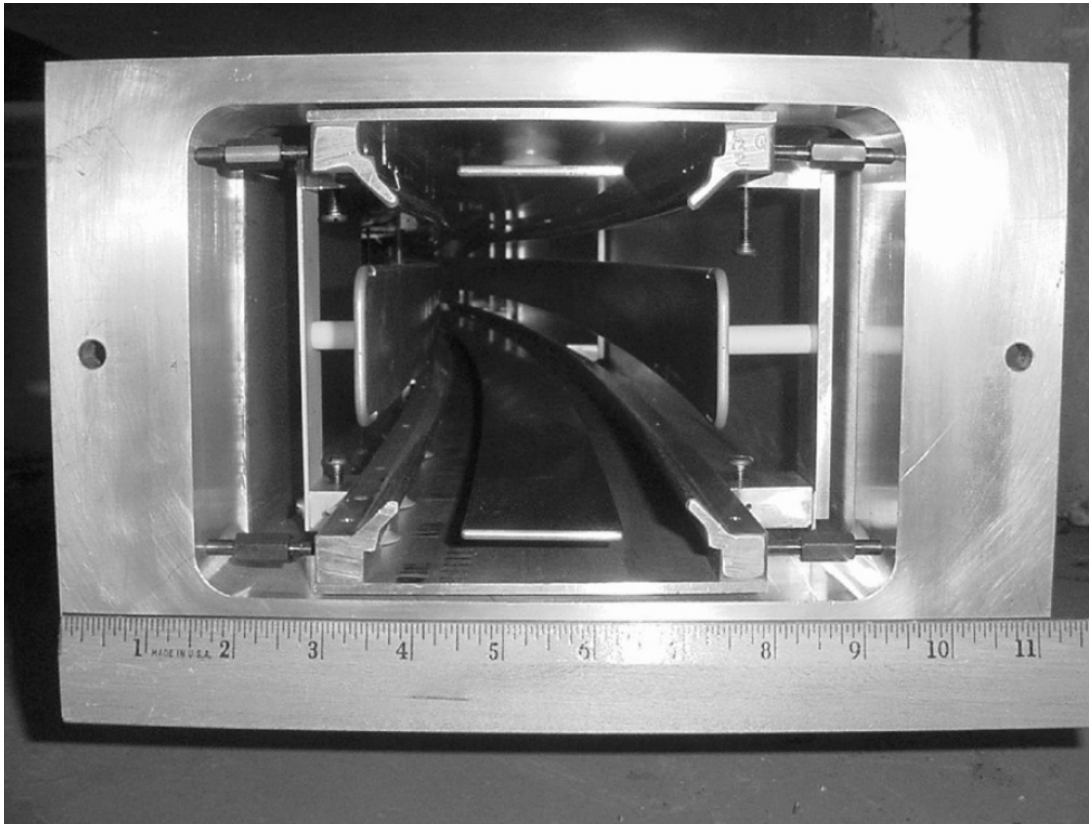
Trolley with matrix of 17 NMR probes



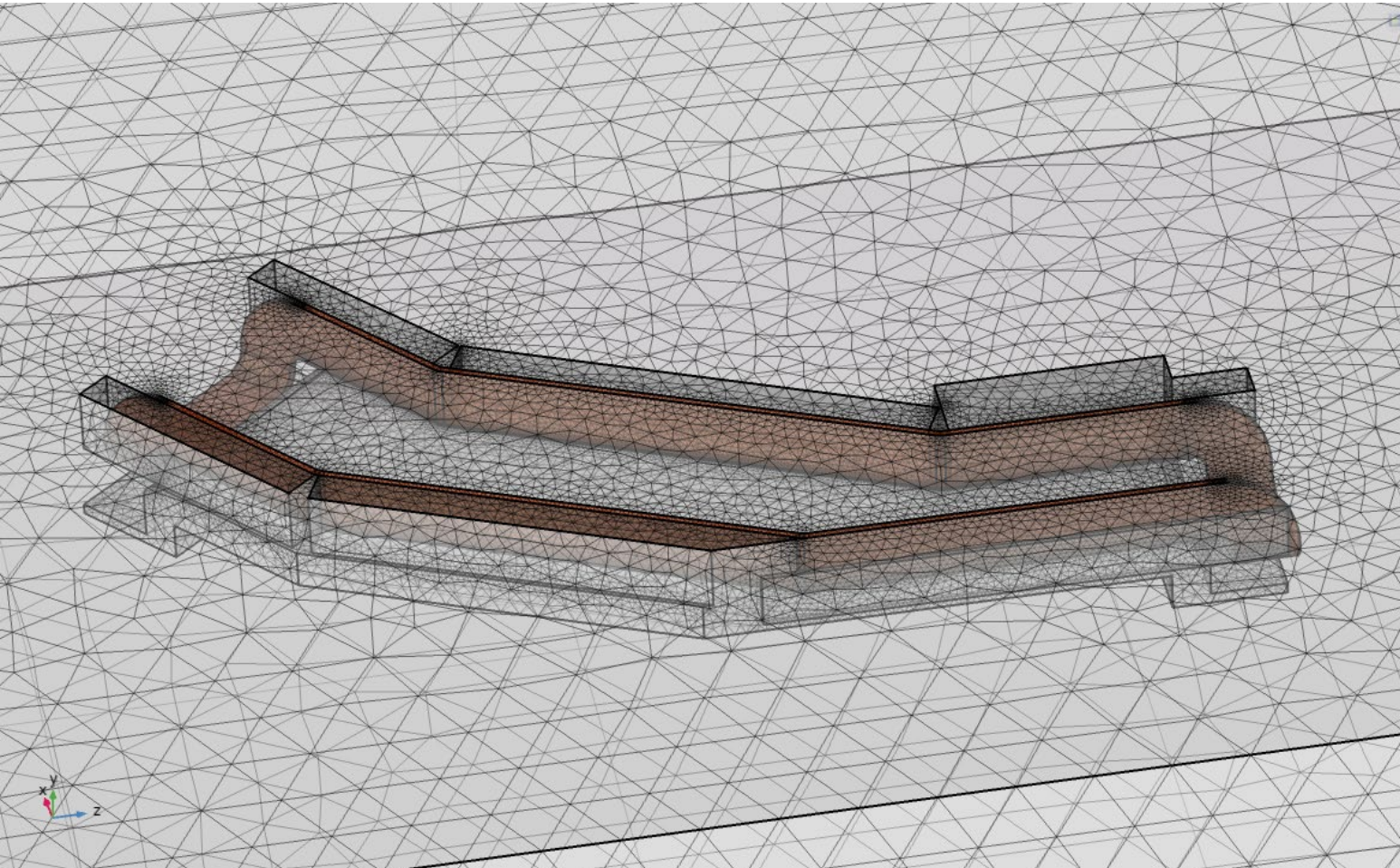
In the following slides, I will cover some beam dynamics and computational electromagnetics topics:

- *Field calculation for electrostatic quadrupoles*
- *Nonlinear transfer maps*
- *Calculation of aberrations, tunes, and chromaticity*

Muon g-2 Storage Ring High Voltage Quadrupoles

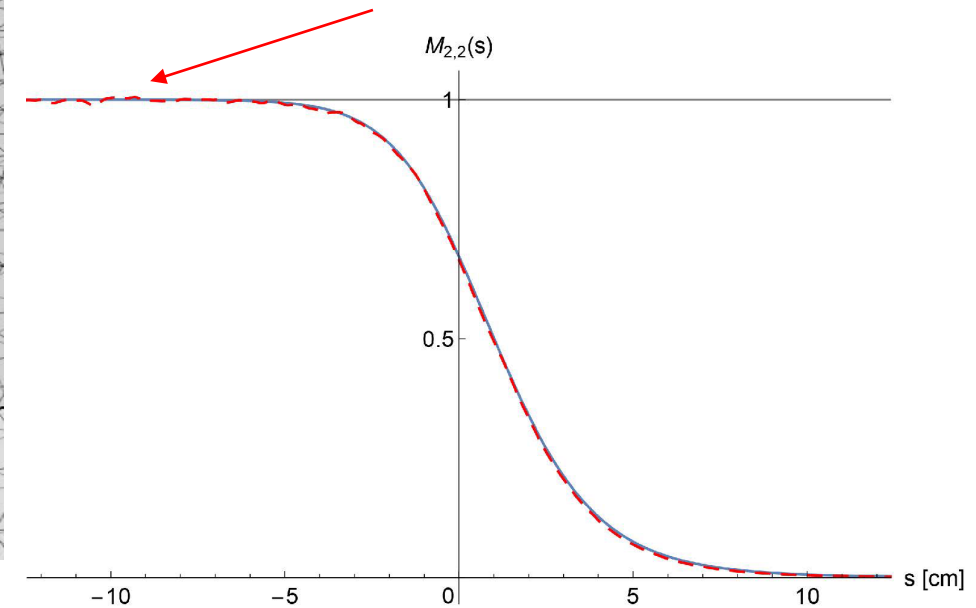


ELECTRODE AND SUPPORT FRAME - END VIEW

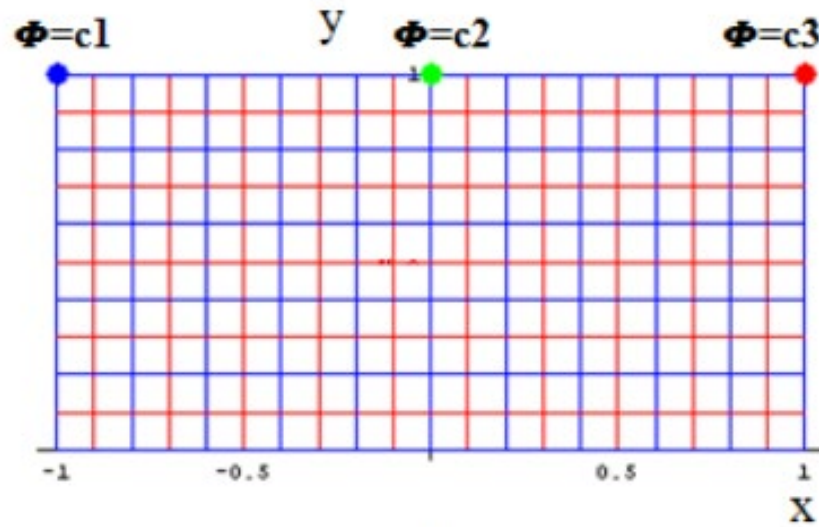


Conventional methods:

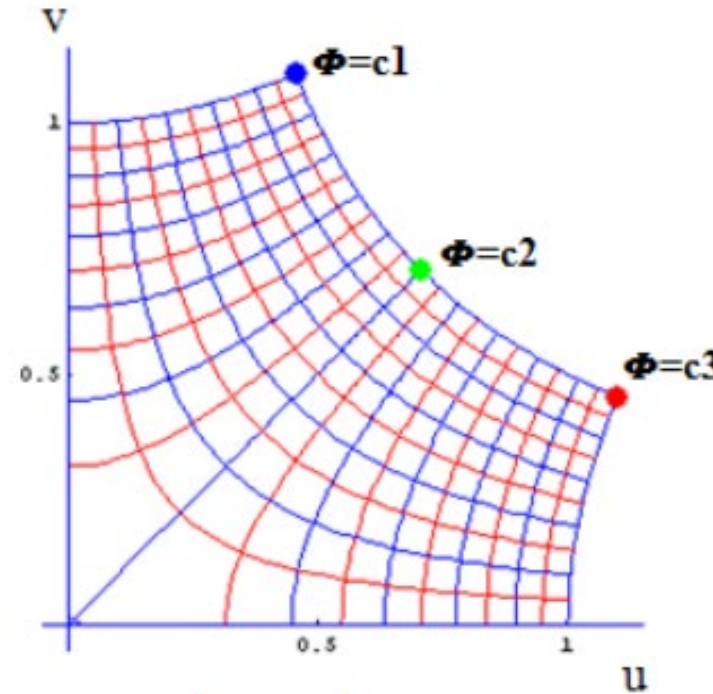
- Currently BEM, FEM, etc.
- Not automatically Maxwellian
- CPU intensive
- Computational noise



Conformal Mapping Methods



a. z plane



b. w plane

- Angle-preserving
- Solve $\Delta f = 0$ in a simpler geometry

$$f'(z) = c \operatorname{cn}(z|m) \operatorname{dn}(z|m) \prod_{j=1}^n (\operatorname{sn}(z|m) - \operatorname{sn}(x_j + iy_j|m))^{\alpha_j - 1}$$

Advantages of conformal mappings for field calculations:

- Fully Maxwellian
- Rapid recalculations with voltage asymmetries or plate offsets

Conformal Mapping Methods



Conformal Mapping Joukowski Transformation

Glenn
Research
Center

$$z = x + iy$$

$$A = z + \frac{1}{z}$$

$$A = B + iC$$

Rotating Cylinder

Airfoil

Use **complex variables**
to map from one geometry
to another

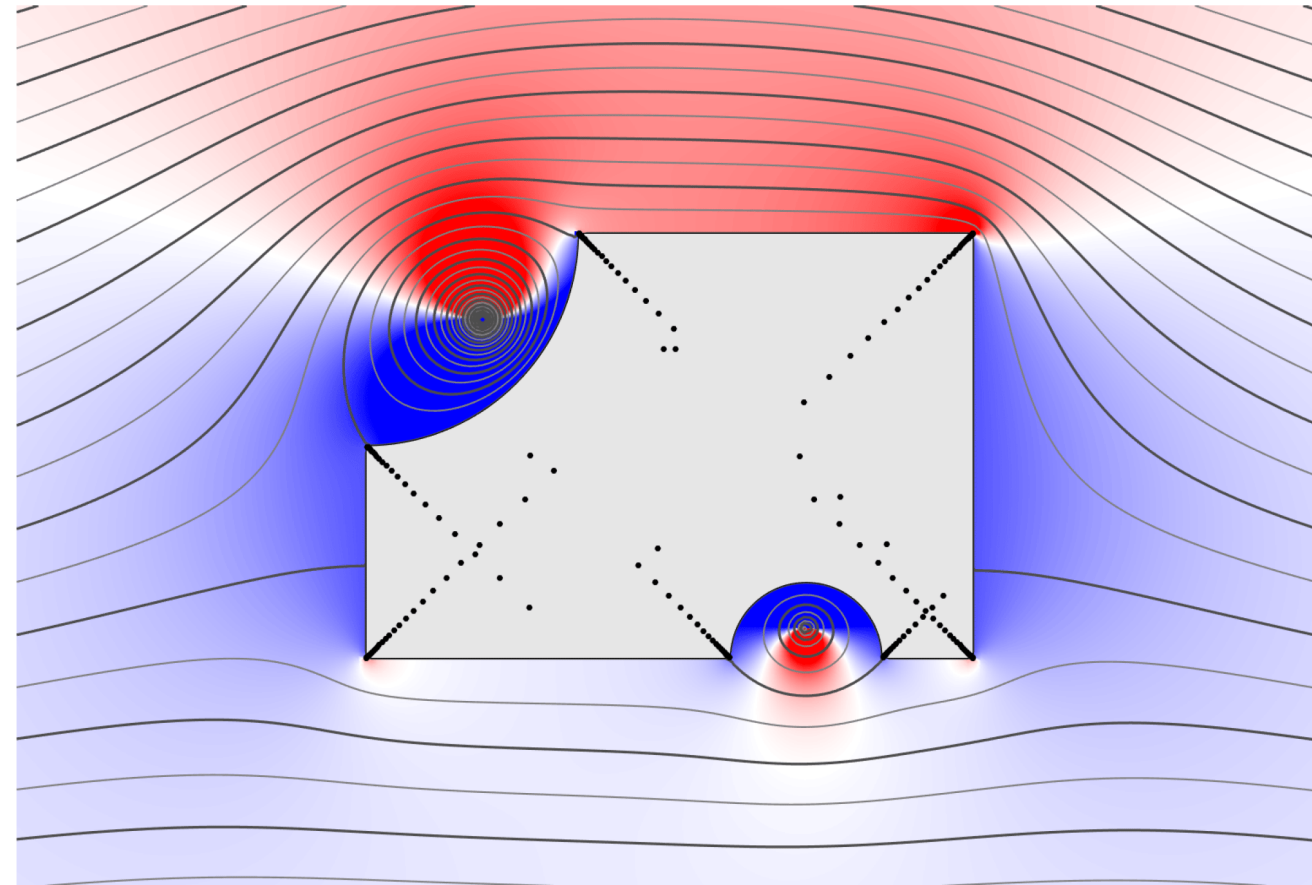
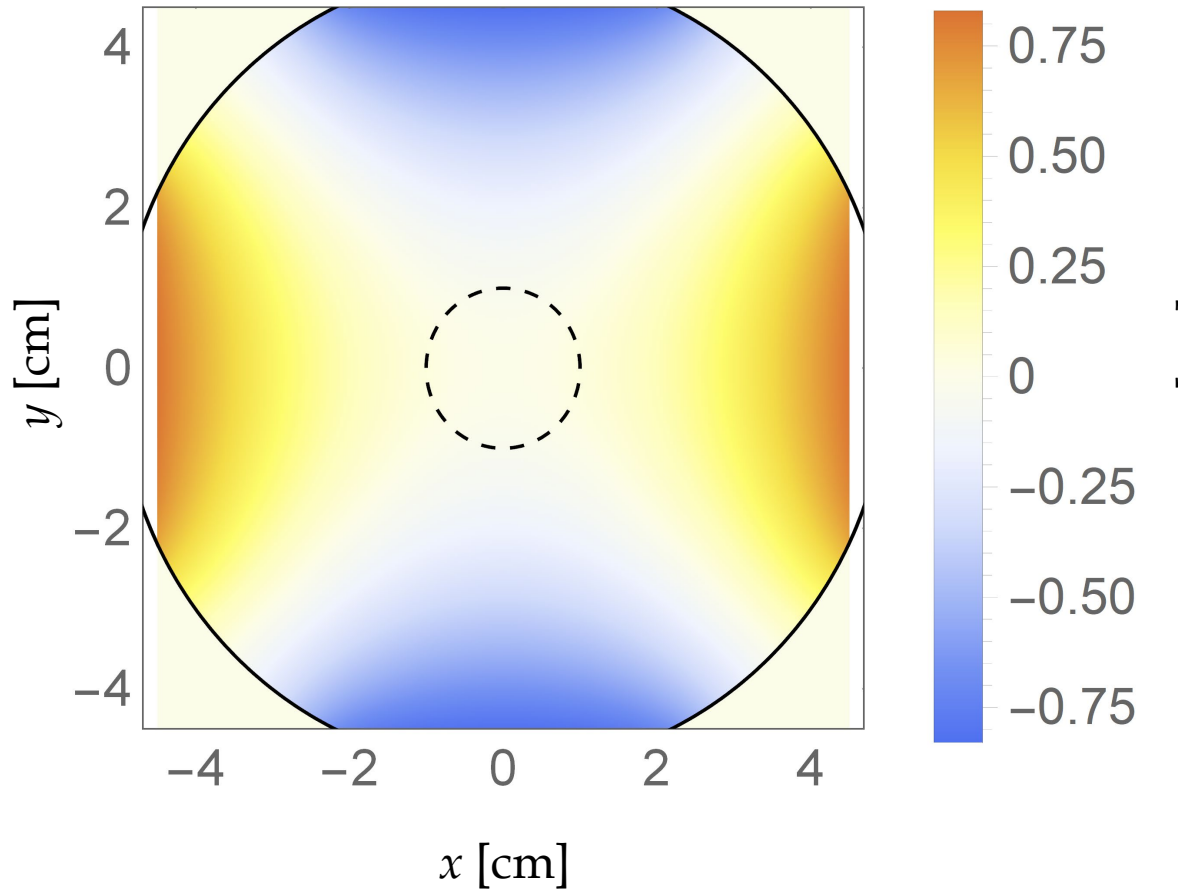


Image credit: NASA's Glenn Research Center.

Image credit: Peter Baddoo.

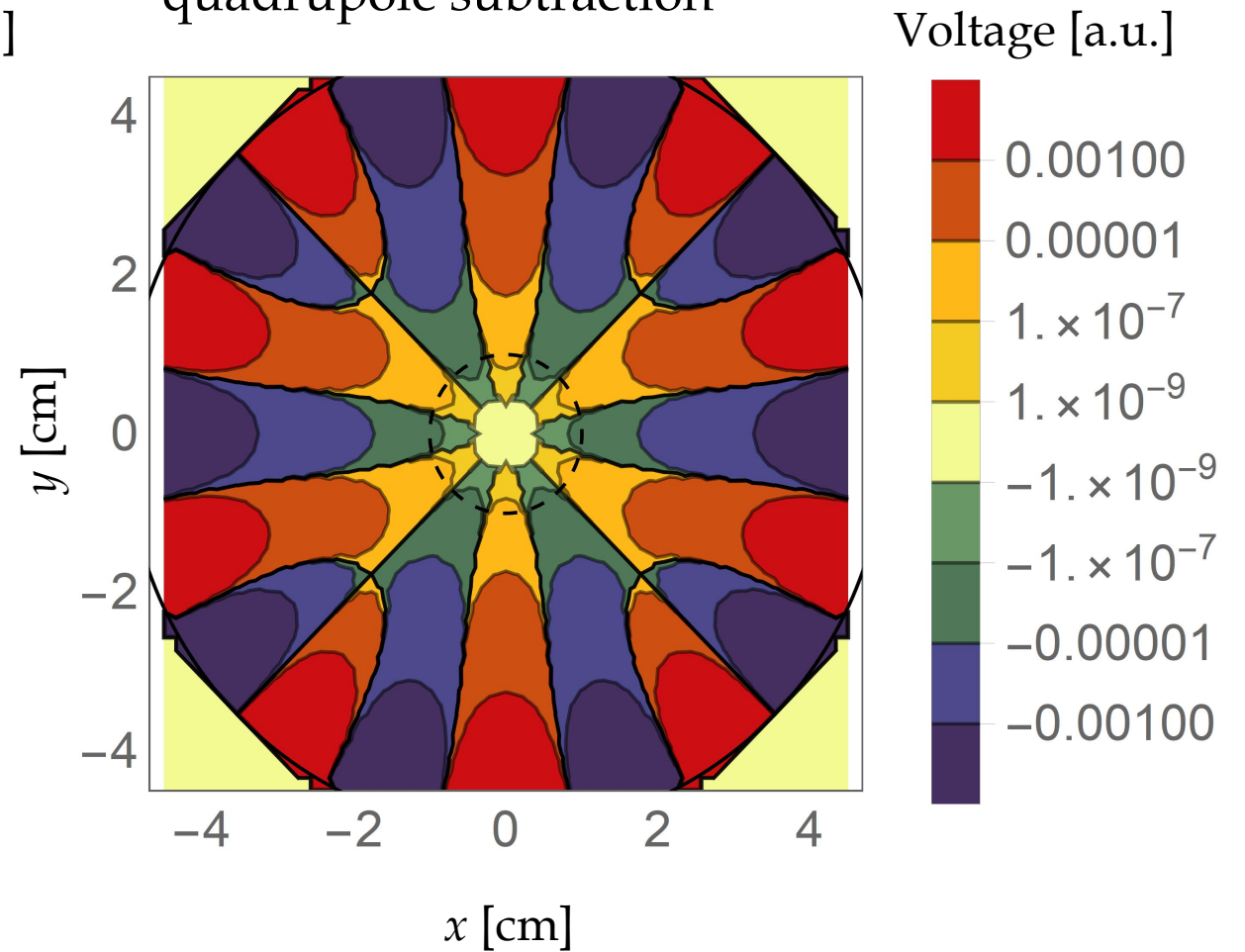
Multipole Expansion of the Quadrupole Potential

Field dominated by quadrupole moment



(Up to order 24)

Higher order multipoles after quadrupole subtraction



Nonlinear Transfer Map

Linear transfer matrix

$$\begin{pmatrix} x \\ a \\ y \\ b \\ l \\ \delta \end{pmatrix}_f =_1 \begin{pmatrix} M_{xx} & \cdots & M_{x\delta} \\ \vdots & \ddots & \vdots \\ M_{\delta x} & \cdots & M_{\delta\delta} \end{pmatrix} \begin{pmatrix} x \\ a \\ y \\ b \\ l \\ \delta \end{pmatrix}_i$$

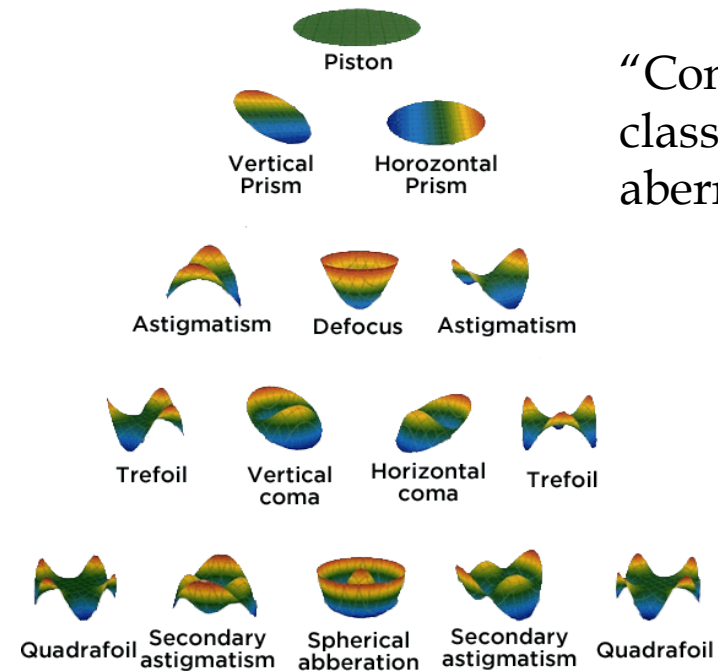
Nonlinear transfer map

$$z_i = \sum_{j \geq 1} \sum_{j_1 + \cdots + j_n = j} M_{j_1 \dots j_n}^i z_1^{j_1} \cdots z_n^{j_n}$$

$$z_i = \sum_{j \geq 1} \sum_{j_1 + \cdots + j_n = j} \frac{1}{j_1! \cdots j_n!} \left(z_i | z_1^{j_1} \cdots z_n^{j_n} \right) z_1^{j_1} \cdots z_n^{j_n}$$

Abserrations

“Conventional”
classification of
aberrations



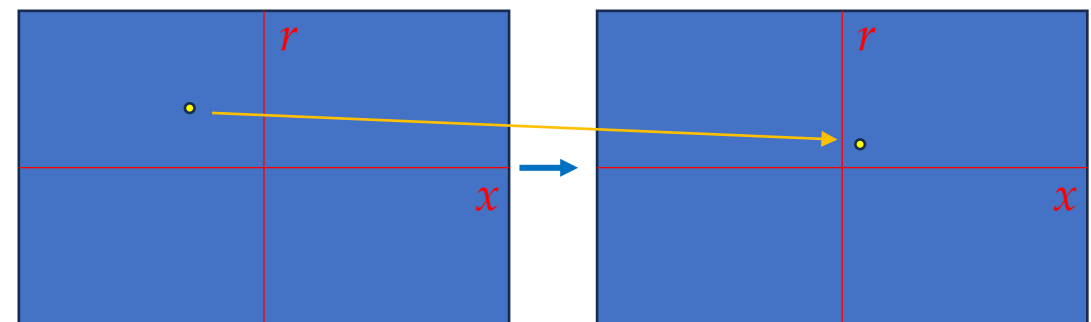
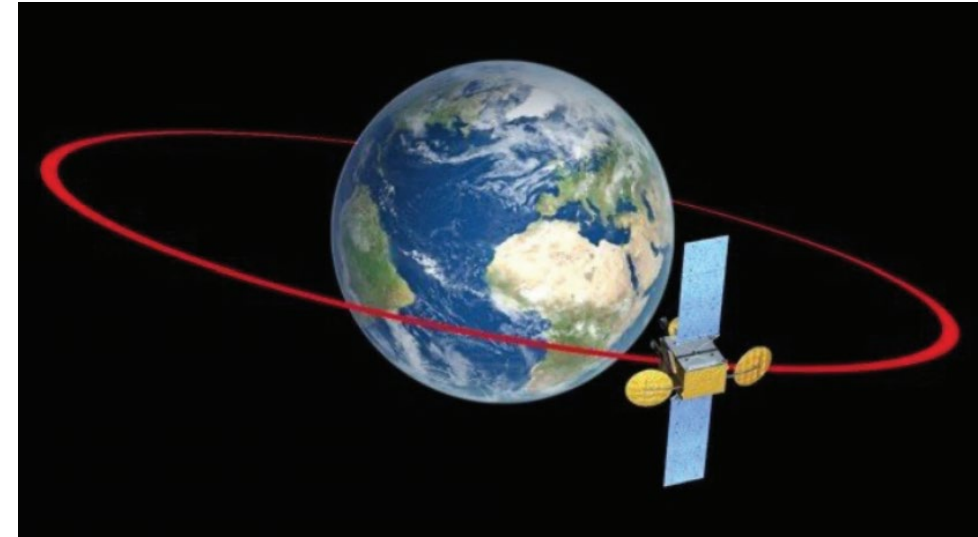


High order transfer maps for perturbed Keplerian motion

Alexander Wittig¹ · Roberto Armellin²

Received: 14 October 2014 / Revised: 21 March 2015 / Accepted: 29 April 2015
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Abstract The paper presents a new semi-analytical technique for the propagation of near-Earth satellite motion. The approach uses differential algebra techniques to compute the high order expansion of the solution of the system's ordinary differential equation for one orbital revolution, referred to as the transfer map. Once computed, a single high order transfer map (HOTM) can be reused to map an initial condition, or a set of initial conditions, forward in time for many revolutions. The only limiting factor is that the mapped objects must stay close to the reference orbit such that they remain within the region of validity of the HOTM. The performance of the method is assessed through a set of test cases in which both autonomous and non-autonomous perturbations are considered, including the case of continuously propelled trajectories.

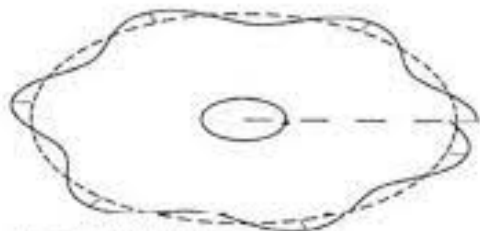


Turn n

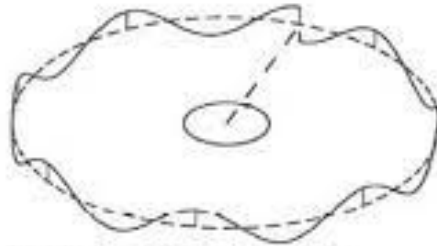
Turn $n + 1$

Tunes and Chromaticities

Tune: number of transversal oscillations per turn

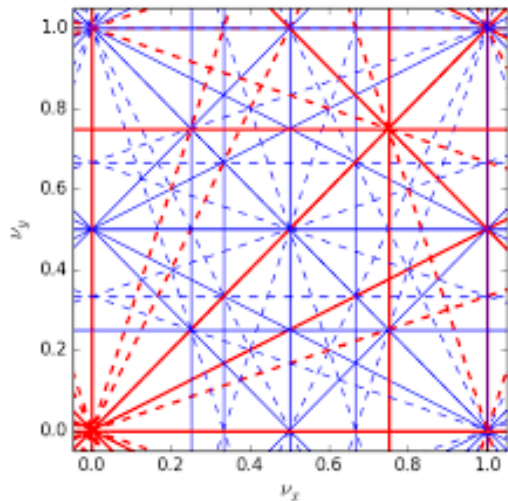


Horizontal Betatron Oscillation
with tune: $Q_H = 6.3$,
i.e., 6.3 oscillations per turn.



Vertical Betatron Oscillation
with tune: $Q_V = 7.5$,
i.e., 7.5 oscillations per turn.

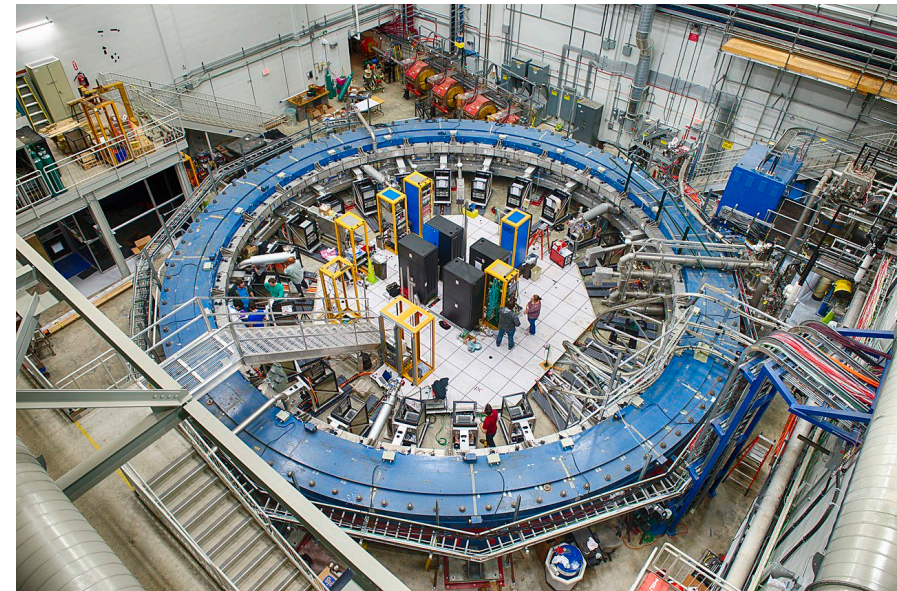
Tune / resonance diagram



Laser Hallway in Resident Evil

Chromaticity: dependence of the tune on relative energy offset

$$\xi_{x,y} = \frac{\partial Q_{x,y}}{\partial \delta_{KE}}$$



Accurate values of tunes and chromaticity critical in Muon g-2

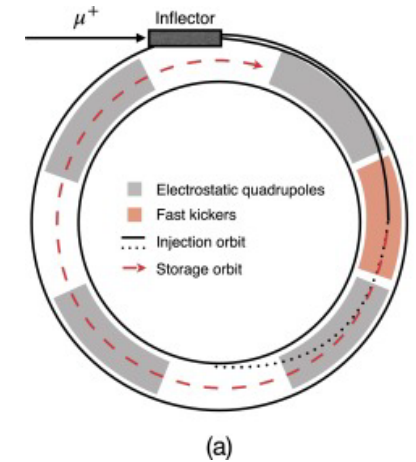
Calculation of Aberrations, Tunes, and Chromaticities

1. Analytic formulas aberrations:

- Order-by-order perturbation method
- Use transfer map methods:

$$\frac{d}{ds}\vec{r} = M(s)\vec{r} + \sum_{j=2}^{+\infty} N_j(\vec{r}, s)$$

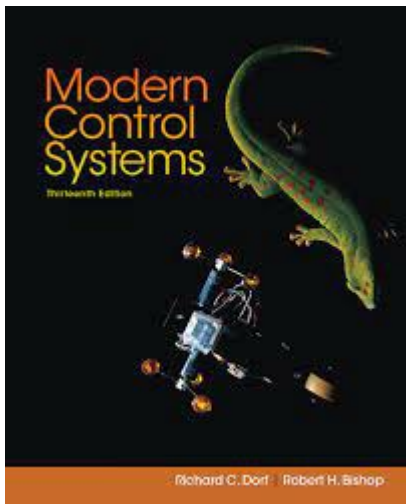
2. Obtain the transfer map for one turn around the storage ring by composition



3. Calculate the tunes:

$$Q_{x,y} = \frac{1}{2\pi} \arccos \left(\frac{\text{tr}(M_{x,y})}{2} \right)$$

4. Calculate the chromaticities: (Long formula in terms of first and second-order transfer map elements)



One beamline element:
partwise $M(s) = \text{const}$:
 $\vec{r}_f =_1 \exp(Ms)\vec{r}_i$

Muon g-2 Storage Ring Chromaticity

Notes on Chromaticity – Overview

G Minus 2 Experiment Document 29571

Eremey Valetov, Martin Berz, and Kyoko Makino

November 15, 2023

Horizontal Aberrations of the Muon $g-2$ High Voltage Electrostatic Quadrupole With Superimposed Dipole Field

G Minus 2 Experiment Document 29657

Eremey Valetov, Martin Berz, and Kyoko Makino

December 4, 2023

Department of Physics and Astronomy, Michigan State University

Chromaticity of the Muon $g-2$ Storage Ring

G Minus 2 Experiment Document 29717

Eremey Valetov, Martin Berz, and Kyoko Makino

December 10, 2023

Comments on E989 Note 167

G Minus 2 Experiment Document 2XXXX

Eremey Valetov, Martin Berz, and Kyoko Makino

December 11, 2023

Department of Physics and Astronomy, Michigan State University

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1.1 Transfer Maps and Aberrations	1
1.2 Coordinate Systems	2
2 Derivation of an Analytic Chromaticity Formula	2

For the second order horizontal DIQ aberrations, we obtained

$$(x|xx) = -2h^3\theta^{-2}(1-3n+(1-2n)\cos(\vartheta s))\sin^2(\vartheta s/2), \quad (13a)$$

$$(x|xa) = h^3\theta^{-3}(n+(1-2n)\cos(\vartheta s))\sin(\vartheta s), \quad (13b)$$

$$(x|aa) = 2h^3\theta^{-4}(n+(1-2n)\cos(\vartheta s))\sin^2(\vartheta s/2), \quad (13c)$$

$$(x|x\delta_K) = h^4\theta^{-4}\gamma_0^{-1}(1+\gamma_0)^{-1} \cdot \sin(\vartheta s/2s) \left(\vartheta ns(n-1+(2+n)\gamma_0^2)\cos(\vartheta s/2) - 2\gamma_0^2(3n-1+(2n-1)\cos(\vartheta s))\sin(\vartheta s/2) \right), \quad (13d)$$

$$(x|a\delta_K) = \frac{1}{2}h^4\theta^{-5}\gamma_0^{-1}(1+\gamma_0)^{-1} \cdot \left(n\sin(\vartheta s)(n-1+(2-n)\gamma_0^2) - \cos(\vartheta s)(\vartheta ns(n-1+(n+2)\gamma_0^2)+2(1-2n)\gamma_0^2\sin(\vartheta s)) \right), \quad (13e)$$

$$(x|\delta_K\delta_K) = h^5\theta^{-7}(1+\gamma_0)^{-2} \cdot \sin(\vartheta s/2) \left(2h^2(n-1)ns(n-1+(n+2)\gamma_0^2)\cos(\vartheta s/2) + 2\vartheta(n^2-1+(6n-1)\gamma_0^2+(2n-1)\gamma_0^2\cos(\vartheta s))\sin(\vartheta s/2) \right), \quad (13f)$$

$$(a|xx) = h^3\theta^{-1}n(\sin(\vartheta s)+\sin(2\vartheta s)), \quad (13g)$$

$$(a|xa) = h^3\theta^{-2}n(\cos(\vartheta s)-\cos(2\vartheta s)), \quad (13h)$$

$$(a|aa) = h^3\theta^{-3}(3n-1-2n\cos(\vartheta s))\sin(\vartheta s), \quad (13i)$$

$$(a|x\delta_K) = \frac{1}{2}h^4\theta^{-3}n\gamma_0^{-1}(1+\gamma_0)^{-1} \cdot \left(\vartheta s(\gamma_0^2(n+2)+n-1)\cos(\vartheta s) + \sin(\vartheta s)(n-1-4\gamma_0^2\cos(\vartheta s)-\gamma_0^2(n-2)) \right), \quad (13j)$$

$$(a|a\delta_K) = \frac{1}{2}h^4\theta^{-4}n\gamma_0^{-1}(1+\gamma_0)^{-1} \cdot \left(\vartheta s(\gamma_0^2(n+2)+n-1)\sin(\vartheta s)-2\gamma_0^2\cos(\vartheta s)+2\gamma_0^2\cos(2\vartheta s) \right), \quad (13k)$$

$$(a|\delta_K\delta_K) = h^5\theta^{-5}(1+\gamma_0)^{-2} \cdot \left(\sin(\vartheta s)(2\gamma_0^2n\cos(\vartheta s)+\gamma_0^2n^2+n-1) - \vartheta ns(\gamma_0^2(n+2)+n-1)\cos(\vartheta s) \right), \quad (13l)$$

and $(\delta_K|z_{j_1}z_{j_2})=0$ for all j_1 and j_2 .

Excellent agreement between analytic formulas and *COSY INFINITY*

Model DIEQ

Method	$\xi_x^{(p)}$
Valetov, analytic	-0.1423355243040865
<i>COSY INFINITY</i>	-0.1423355242929523

Model DIQ360

Method	$\xi_x^{(p)}$
Valetov, analytic	-0.14798273032639034
<i>COSY INFINITY</i>	-0.1479827303376410

Model DIEQ_ON

Method	$\xi_x^{(p)}$
Valetov, analytic	-0.16005036841645803
<i>COSY INFINITY</i>	-0.1600503684287856

Excellent agreement between analytic formulas
and the code *COSY INFINITY*

Model DIEQ

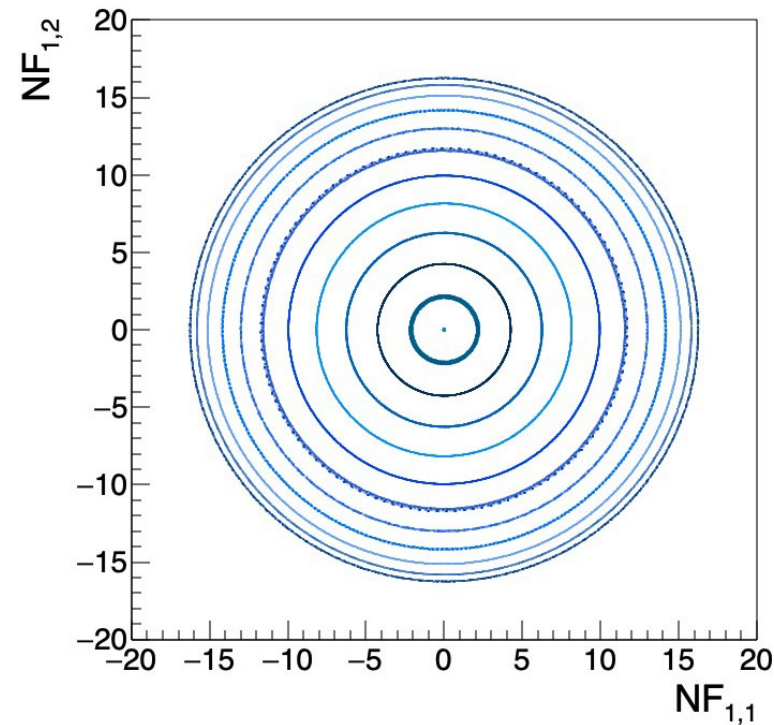
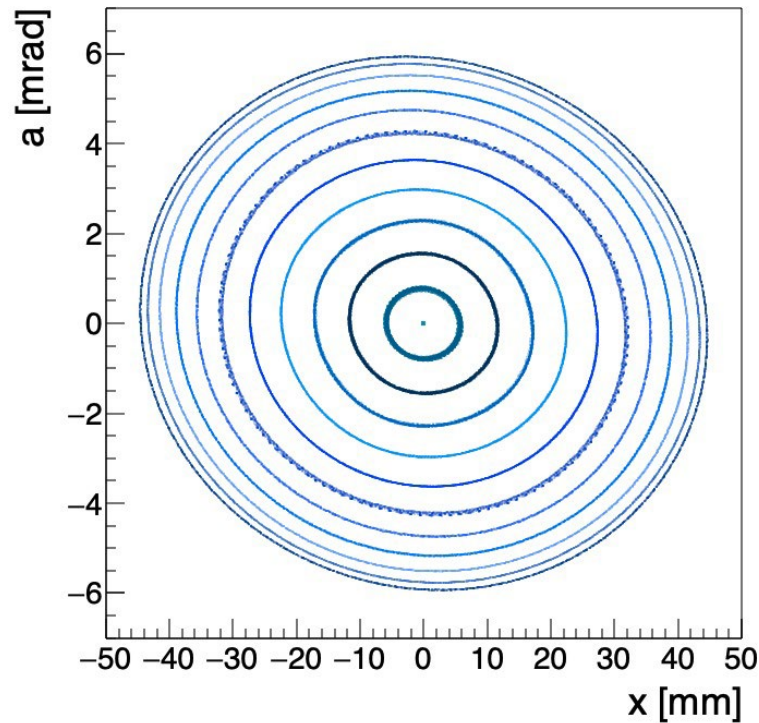
Method	$\xi_x^{(p)}$
Valetov, analytic	−0.1423355243040865
<i>COSY INFINITY</i>	−0.1423355242929523

Model DIQ360

Method	$\xi_x^{(p)}$
Valetov, analytic	−0.14798273032639034
<i>COSY INFINITY</i>	−0.1479827303376410

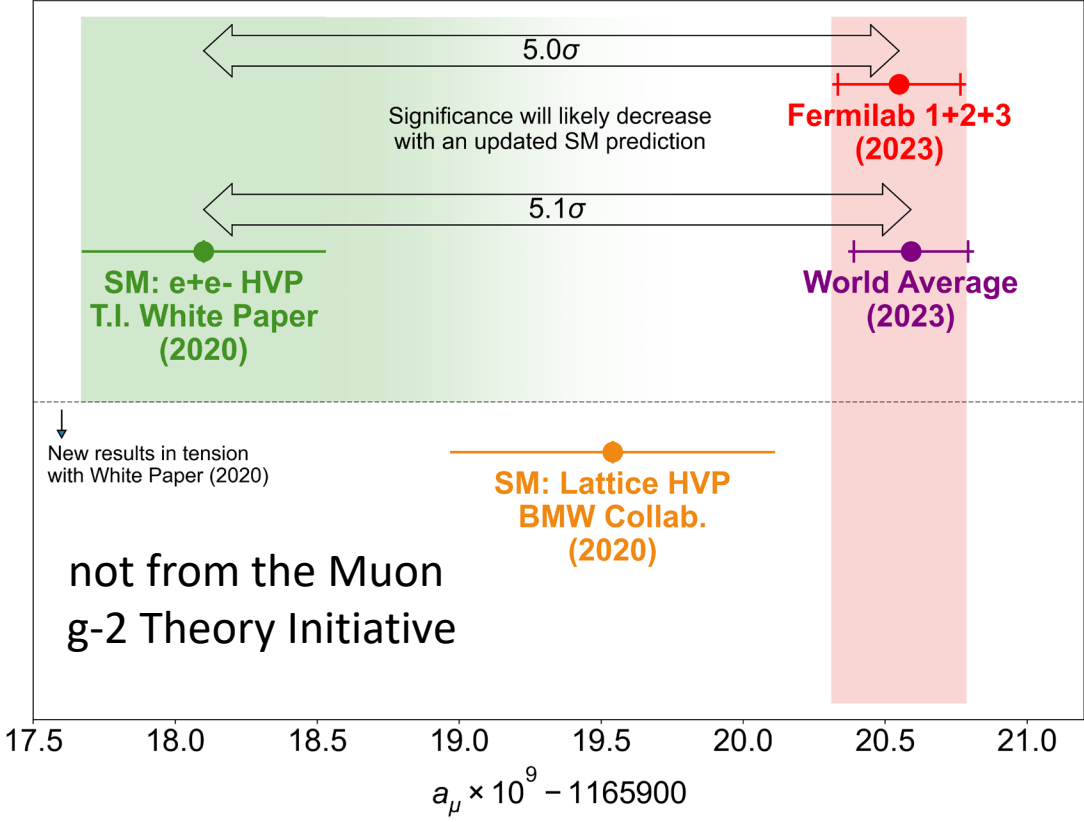
Model DIEQ_ON

Method	$\xi_x^{(p)}$
Valetov, analytic	−0.16005036841645803
<i>COSY INFINITY</i>	−0.1600503684287856



- Dimensionality reduction in a generic method of calculating beam dynamics corrections by two from the following properties
 - Particles have circular orbits in the Poincare sections in nonlinear normal form coordinates (with adjustments needed for resonances)
 - They sample points on the orbits uniformly

Result from Runs 1-3



Quantity	Correction [ppb]	Uncertainty [ppb]
ω_a^m (statistical)	–	201
ω_a^m (systematic)	–	25
C_e	451	32
C_p	170	10
C_{pa}	-27	13
C_{dd}	-15	17
C_{ml}	0	3
$f_{\text{calib}} \langle \omega'_p(\vec{r}) \times M(\vec{r}) \rangle$	–	46
B_k	-21	13
B_q	-21	20
$\mu'_p(34.7^\circ)/\mu_e$	–	11
m_μ/m_e	–	22
$g_e/2$	–	0
Total systematic	–	70
Total external parameters	–	25
Totals	622	215

a_μ (FNAL) = 0.00 116 592 055(24) [203 ppb]
 a_μ (Exp) = 0.00 116 592 059(22) [190 ppb]
 a_μ (Th) = 0.00 116 591 810(43) [370 ppb] (Review by Keshavarzi, 2022)

Runs 2-3 Release (August 10, 2023)

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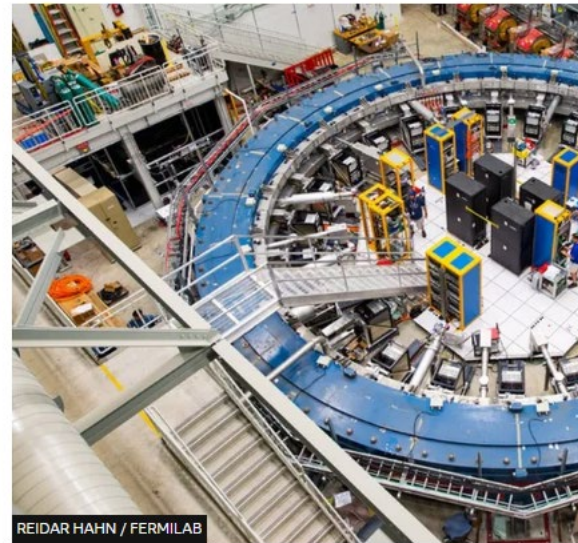
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Science

Scientists at Fermilab fifth force of nature

10 August 2023



REIDAR HAHN / FERMILAB

The findings come from the US muon g-2 experiment

The New York Times

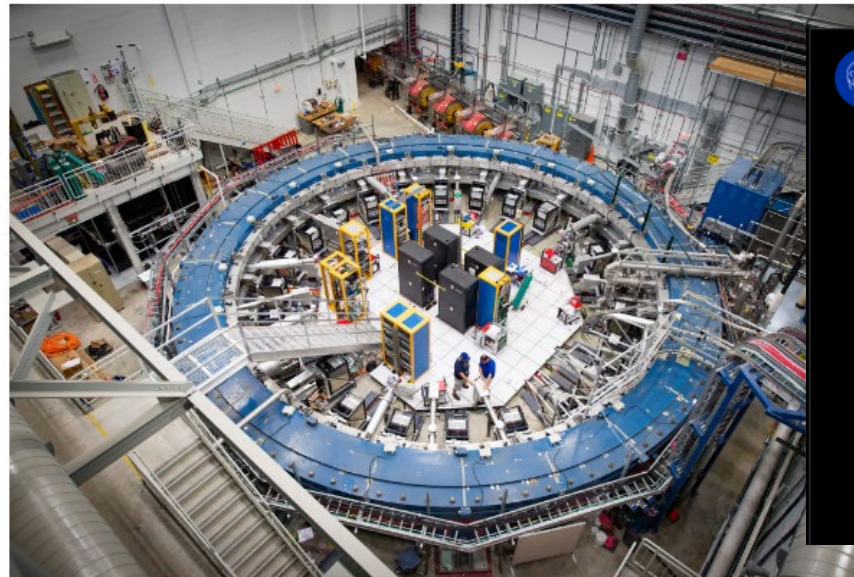
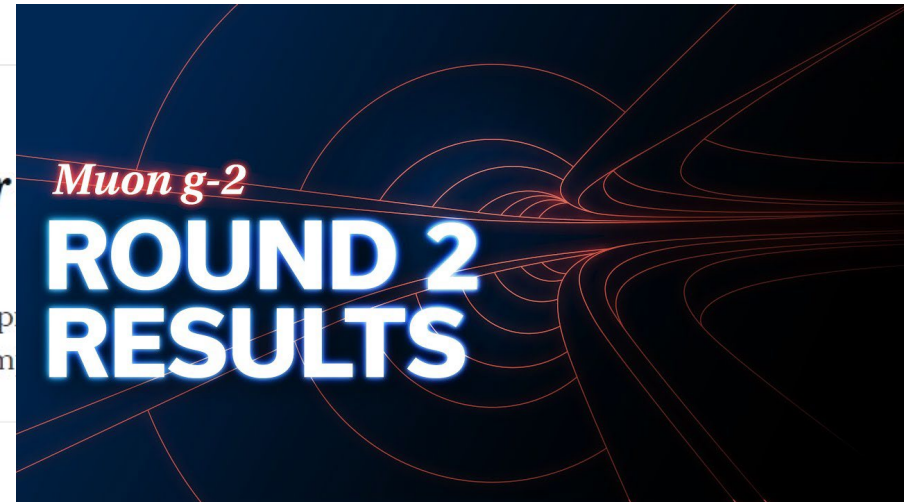
Physicists Move One Step Closer Theoretical Showdown

The deviance of a tiny particle called the muon might put one of the most well-tested theories in physics in jeopardy

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480



The Muon g-2 ring at the Fermilab particle accelerator complex in Batavia, Ill. Reidar Hahn/Fermilab, via US Department of Energy



CERN @CERN · Aug 11, 2023

“Congratulations to the **Muon g-2** collaboration and our @Fermilab colleagues for this result. It shows the power of experimental work alongside theoretical calculations to help us to understand the fundamental components of our Universe.” – Joachim Mnich, #CERN Director for...

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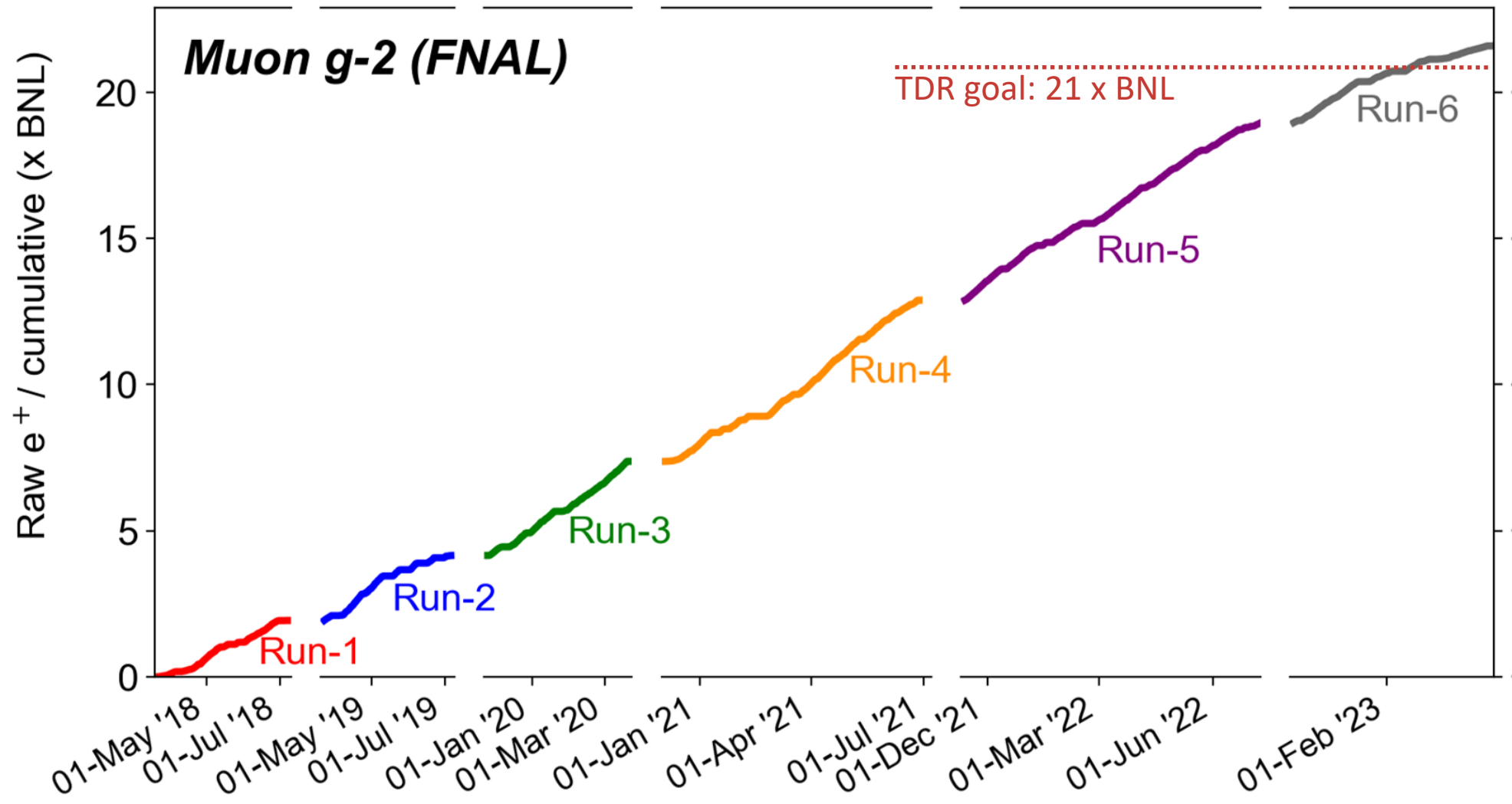
Fermilab @Fermilab · Aug 10, 2023

We're excited to announce that Fermilab's Muon g-2 experiment has achieved the world's most precise measurement of the anomalous magnetic moment of the muon!

Read more: news.fnal.gov/2023/08/muon-g-2/

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Data Acquisition as a Multiple of BNL Data



- Results based on Runs 2-3 were published in August 2023
- The experiment has completed its final Run 6 in July 2023
 - 21x more raw data than the Muon g-2 Experiment at BNL
 - Improving systematics:
 - Magnet temperature control
 - Magnetic field noise control
 - Analysis improvements (pileup reconstruction)
 - Achieved the TDR goal of 70 ppb systematic error
- Results from runs 4-6 planned to be released in 2025
- New theory results expected in 2025

Conclusion



USA

- Boston
- Cornell
- Illinois
- James Madison
- Kentucky
- Massachusetts
- Michigan
- Michigan State
- Mississippi
- North Central
- Northern Illinois
- Regis
- Virginia
- Washington

USA National Labs

- Argonne
- Brookhaven
- Fermilab

182 collaborators
33 Institutions
7 countries



China

- Shanghai Jiao Tong



Germany

- Dresden
- Mainz



Italy

- Frascati
- Molise
- Naples
- Pisa
- Roma Tor Vergata
- Trieste
- Udine



Korea

- CAPP/IBS
- KAIST



Russia

- Budker/Novosibirsk
- JINR Dubna



United Kingdom

- Lancaster/Cockcroft
- Liverpool
- Manchester
- University College London



Collaboration meeting at University of Liverpool, July 2023
Photo credit: McCoy Wynne

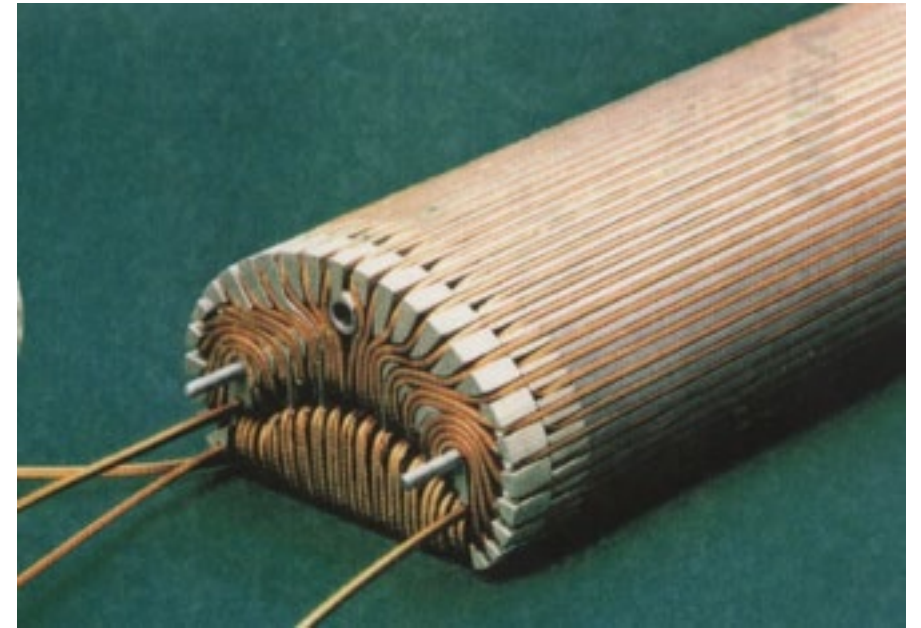
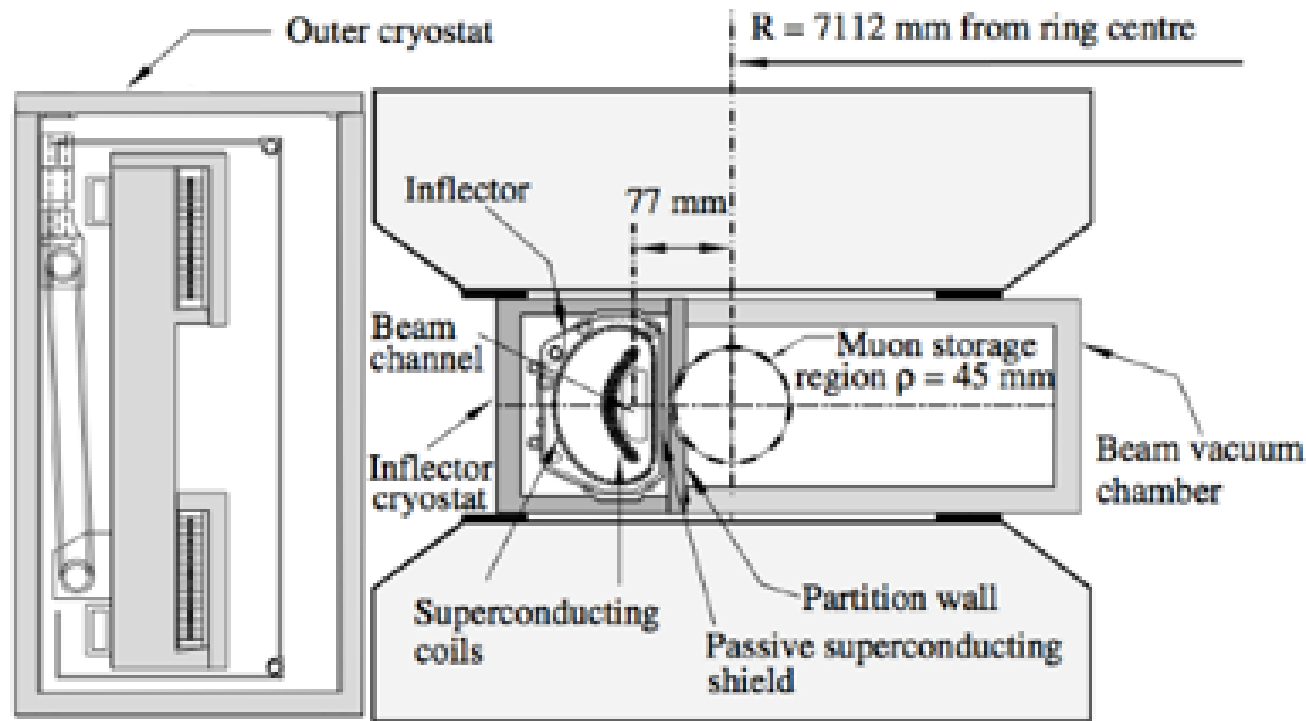


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Muon g-2 Inflector



- 1.45 T bucking field to cancel main field
- Can't perturb main field by more than ~ 1 ppm
- Interface optics of storage ring and the M5 beamline

Fitting Function Example: 20 Point

$$N = N_0 \Lambda N_{cbo} N_{2cbo} N_{vw} e^{-t/\tau} (1 - A A_{cbo} \cos(\omega_a t + \phi \phi_{cbo}))$$

$$N_{cbo} = 1 - A_{1cbo} e^{-\frac{t}{\tau_{cbo}}} \cos(\omega_{cbo} t + \phi_{1cbo})$$

$$N_{2cbo} = 1 - A_{2cbo} e^{-\frac{2t}{\tau_{cbo}}} \cos(2\omega_{cbo} t + \phi_{2cbo})$$

$$N_{vw} = 1 - A_{vw} e^{-\frac{t}{\tau_{vw}}} \cos(\omega_{vw} t + \phi_{vw})$$

$$A_{cbo} = 1 - A_{Acbo} e^{-\frac{t}{\tau_{cbo}}} \cos(\omega_{cbo} t + \phi_{Acbo})$$

$$\phi_{cbo} = 1 - A_{\phi_{cbo}} e^{-\frac{t}{\tau_{cbo}}} \cos(\omega_{cbo} t + \phi_{\phi_{cbo}})$$

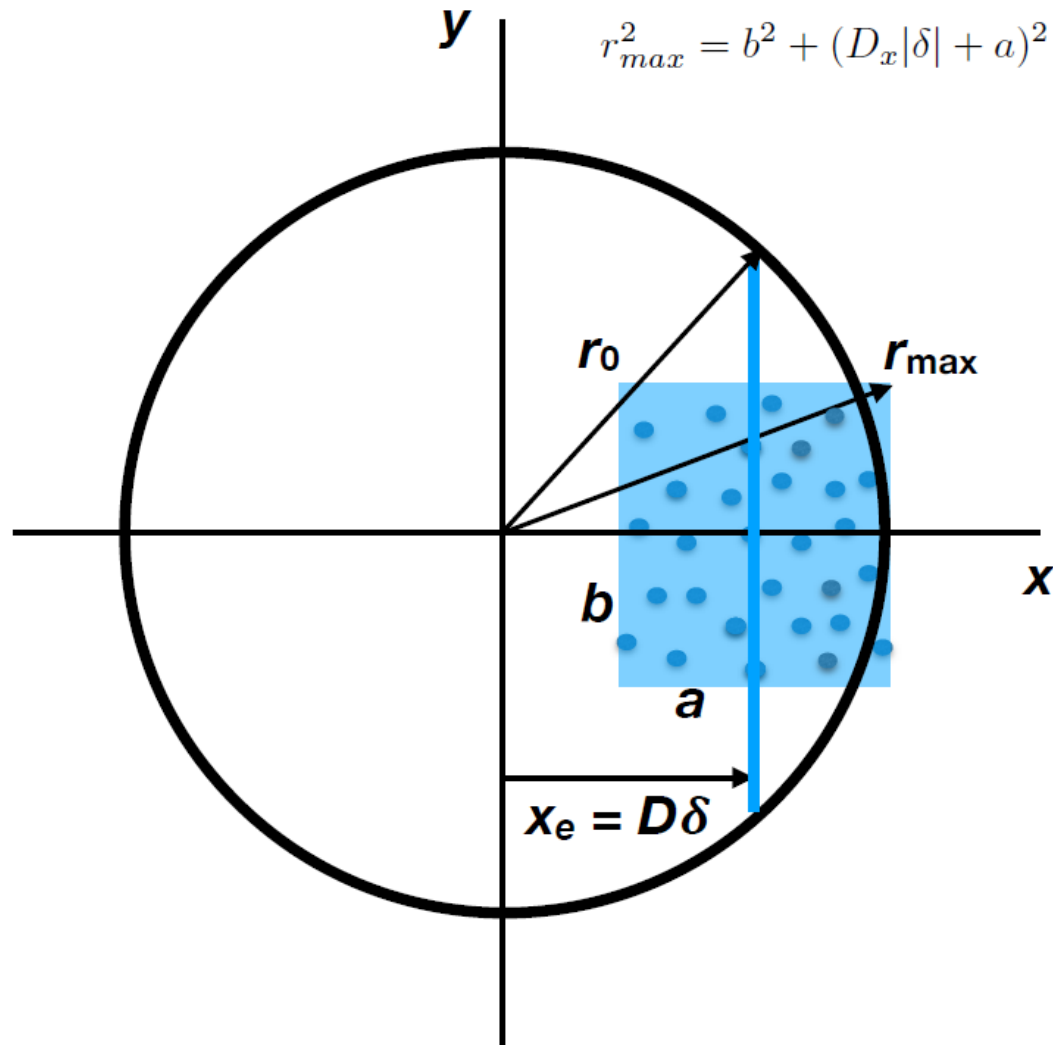
$$\omega_{cbo} = \omega_0 (1 + 2.875 e^{-\frac{t}{76}} / \omega_0 t + 5.47 e^{-\frac{t}{8.85}} / \omega_0 t)$$

$$\Lambda = 1 - K_{loss} \int L(t') e^{t'/64.4} dt$$

$$\chi^2 = \sum_{i=1}^{ndf} \left[\frac{N_{bin} - N_{fit}}{\sigma(N_{bin})} \right]^2$$

Momentum-Dependent Muon Losses

Mike Syphers



$$\frac{\Delta\omega_a}{\omega_a} = \frac{1}{\omega_a} \frac{d\langle\Phi\rangle}{d\langle dp/p_0\rangle} \frac{d\langle dp/p_0\rangle}{dt}$$

Straw Tracking Detectors

