Theoretical determination of $K_1(1270, 1400)$ mixing angle in QCD

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Abstract. In quark model, the strange axial vector mesons $K_1(1270)$ and $K_1(1400)$ are defined as the mixtures of orbital angular momentum states K_{1A} and K_{1B}. In this work, by using the orthogonality of the mass eigenstates, we have estimated the $K_1(1270, 1400)$ mixing angle θ_{K_1}, where we have found that $\theta_{K_1} \simeq -(39 \pm 4)^\circ$.

The Particle Data Group lists the axial vector, $I = 1/2$, P-wave strange mesons $K_1(1270)$ and $K_1(1400)$ with masses 1273 ± 7 MeV and 1402 ± 7 MeV, respectively [1]. The decays of these mesons are constrained by a selection rule such that $\Gamma(K_1(1270) \to K\rho) \gg \Gamma(K_1(1270) \to K^*\pi)$ and $\Gamma(K_1(1400) \to K^*\pi) \gg \Gamma(K_1(1400) \to K\rho)$. This rule suggests a large mixing angle close to $|45|^\circ$ between the pure orbital angular momentum and the G-parity states, $K_{1A}(1^3P_1)$ and $K_{1B}(1^1P_1)$, leading to the physical $K_1(1270)$ and $K_1(1400)$ states [2, 3]. This mixing is written as

$$\begin{aligned} |K_1(1270)\rangle &= \sin\theta_{K_1}|K_{1A}\rangle + \cos\theta_{K_1}|K_{1B}\rangle, \\ |K_1(1400)\rangle &= \cos\theta_{K_1}|K_{1A}\rangle - \sin\theta_{K_1}|K_{1B}\rangle, \end{aligned} \tag{1}$$

where θ_{K_1} is the mixing angle [4]. An accurate determination of θ_{K_1} is crucial to understand the physical properties of the axial-vector strange mesons. There are various approaches to this problem in the literature. In 1977, Carnegie *et al.* obtained $\theta_{K_1} = (41 \pm 4)^\circ$ by making a fit to the available data [3]. Brundell *et al.* estimated the mixing angle to be in the range $-30^\circ < \theta_{K_1} < 50^\circ$ by using the data on the ratio $BR(\tau \to K_1(1270)\nu_\tau)/BR(\tau \to K_1(1400)\nu_\tau)$. The favored value in this range from their analysis is $\theta_{K_1} \simeq 45^\circ$ [5]. It was found from experimental data that the mixing angle has a value $|\theta_{K_1}| \simeq 32^{+8^\circ}_{-2}$ or $|\theta_{K_1}| \simeq 57^{+2^\circ}_{-3}$ and among these $|\theta_{K_1}| \simeq 33^\circ$ is favored [6]. Burakovsky *et al* used a nonrelativistic constituent quark model and estimated $35^\circ < |\theta_{K_1}| < 55^\circ$[7]. In addition to these analyses the mixing angle has been extracted from the updated data on $\tau \to K_1(1270)\nu_\tau$ decays as $|\theta_{K_1}| \simeq 37^\circ$ or $|\theta_{K_1}| \simeq 58^\circ$[8].

It is apparent from above results in the literature that there exists a sign ambiguity in θ_{K_1} due to different definitions. In Ref. [9], the decay constants of K_{1A} and K_{1B} are defined positive and the sign ambiguity is removed which results in a negative value of θ_{K_1}. Hatanaka and Yang analyzed the latest data on $B \to K_1(1270, 1400)\gamma$ and $\tau \to K_1(1270, 1400)\nu_\tau$ and they have found $\theta_{K_1} = -(34 \pm 13)^\circ$.

In the present work we have used a theoretical approach, which has been introduced by T. M. Aliev, A. Ozpineci and V.S. Zamiralov [10], to determine the mixing angle θ_{K_1}. In this approach, the correlator function is constructed in terms of the interpolating currents of the pure K_{1A} and K_{1B} states. Then the mixing angle can be calculated using the orthogonality of the mass eigenstates, *i.e.*, $K_1(1270)$ and $K_1(1400)$. Using operator product expansion, the mixing is expressed in terms of quark and gluon degrees of freedom.

We start our analysis with the correlator function

$$\begin{aligned} \Pi_{\mu\nu}(p^2) &\equiv \Pi_{\mu\nu}^{K_1(1270)K_1(1400)}(p^2) \\ &= i\int d^4x e^{ipx}\langle 0|\mathcal{T}\{j_\mu^{K_1(1400)\dagger}(x) j_\nu^{K_1(1270)}(0)\}|0\rangle \;=\; 0+\ldots \end{aligned} \tag{2}$$

which should vanish since there is no transition possible between the mass eigenstates. To calculate the correlator in Eq. (2), the interpolating currents of $K_1(1270, 1400)$ mesons should be written in terms of pure states $K_{1(A,B)}$. Using the definition of mixing in Eq. (1), we obtain the relation

$$\begin{aligned} j^{K_1(1270)} &= \sin\theta_{K_1} j^{K_{1A}} + \cos j^{K_{1B}}, \\ j^{K_1(1400)} &= \cos\theta_{K_1} j^{K_{1A}} - \sin j^{K_{1B}}, \end{aligned} \tag{3}$$

where $j^{K_{1(A,B)}}$ are the currents for the pure states $K_{1(A,B)}$. In the SU(3) limit, K_{1A} and K_{1B} couple to axial-vector and tensor currents respectively. When SU(3) symmetry is broken, the contributions to pure currents are proportional to the Gagenbauer moments, and since they are either zero or comparable to zero, we can assume the pure axial-vector and tensor currents for K_{1A} and K_{1B}, respectively [4, 11]. Then we write

$$|K_{1A}(p,\epsilon)\rangle = \lambda_A \eta_\mu^A|0\rangle \quad \text{and} \qquad |K_{1B}(p,\epsilon)\rangle = \lambda_B \eta_\mu^B|0\rangle, \tag{4}$$

where $\lambda_A = (if_A m_A)^{-1}$, $\lambda_B = (f_B m_B^2)^{-1}$ are the coefficients, $\eta_\mu^A = \bar{s}\gamma_\mu\gamma_5 d$ and $\eta_\mu^B = \bar{s}\sigma_{\mu\alpha}p^\alpha\gamma_5 d$ are the pure axial-vector and tensor currents. Here $f_{A,B}$ and $m_{A,B}$ are the decay constants and masses of $K_{1(A,B)}$ states, respectively. After inserting the definitions of mixing in Eq. (3) and the pure currents in Eq. (4), the correlator in Eq. (2) can be written in terms of the mixing angle as follows:

$$\tan\theta_{K_1}(\lambda_A^2\Pi_{\mu\nu}^{AA} - \lambda_B^2\Pi_{\mu\nu}^{BB}) + (1+\tan^2\theta_{K_1})\lambda_A\lambda_B\Pi_{\mu\nu}^{AB} = 0 + \ldots\,, \tag{5}$$

where

$$\Pi_{\mu\nu}^{ij}(p^2) = i\int dx e^{ipx}\ \mathrm{Tr}[\Upsilon_\mu^{i\dagger}(x)(iS_d(-x))\Upsilon_\nu^j(0)(iS_s(x))] \tag{6}$$

is the two-point correlator of pure states with $i, j = A, B$, $\Upsilon_\mu^A = \gamma_\mu\gamma_5$ and $\Upsilon_\mu^B = \sigma_{\mu\alpha}p^\alpha\gamma_5$ are the Lorentz structures in pure currents and $iS_q(x)$ is the full quark propagator in position space. The correlator for pure states can be expressed in terms of Lorentz coefficients as $\Pi_{\mu\nu}^{ij} = \Gamma^{ij\prime}g_{\mu\nu} + \Gamma^{ij}p_\mu p_\nu$. We choose the $p_\mu p_\nu$ structure and apply a Borel transformation with respect to p^2. Then Eq. (5) results in an expression for the mixing angle as

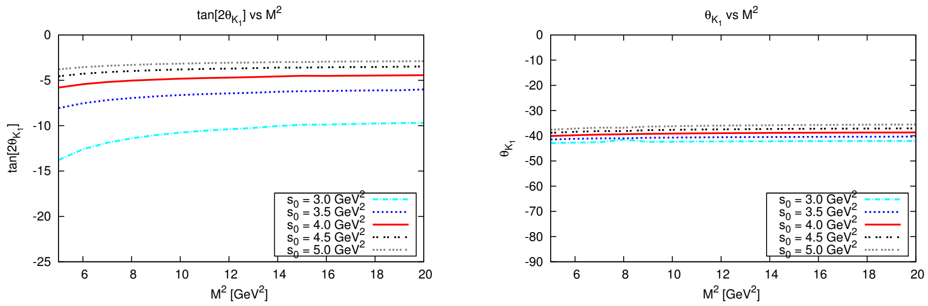
$$\tan(2\theta_{K_1}) = \frac{-2\lambda_A^\dagger\lambda_B\bar{\Gamma}^{AB}}{\lambda_A^2\bar{\Gamma}^{AA} - \lambda_B^2\bar{\Gamma}^{BB}}, \tag{7}$$

where $\bar{\Gamma}$ is the Borel transformed Γ with respect to p^2. Here M^2 is the Borel mass and s_0 is the continuum threshold.

In this work we have taken the following numerical inputs:

$$m_s = 100^{+30}_{-20}\ \text{MeV}, f_A = 250 \pm 13\ \text{MeV},\ f_B = 190 \pm 10\ \text{MeV},\ m_A = 1.31\text{GeV}, m_B = 1.34\ \text{GeV}. \quad (8)$$

Our numerical results for $\tan 2\theta_{K_1}$ and θ_{K_1} are shown in figures 1(a) and 1(b). We plot the dependence of $\tan 2\theta_{K_1}$ and θ_{K_1} on M^2 for different values of s_0. The plots suggest that $\tan 2\theta_{K_1}$ lies in the region $-13 \leq \tan 2\theta_{K_1} \leq -3$ and $\theta_{K_1} = -(39 \pm 4)^\circ$. This value is consistent with the previous results in the literature. We will give our detailed analytical expressions and improved analysis in a future work.



(a) (b)

Figure 1. (a)$\tan 2\theta_{K_1}$ vs. M^2 and (b) θ_{K_1} vs M^2, for different values of s_0.

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