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QUADRUPOLE FIELD INSTABILITY IN CYLINDRICAL DIELECTRIC WAKEFIELD ACCELERATORS

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Abstract

Dielectric Wakefield Acceleration is a technology under active research, providing the potential to accelerate charged particle bunches with gradients much greater than conventional RF-based metallic cavities. The stability of driving bunches needs to be solved before practical applications are seen. Strong transverse fields are known to be excited in DWAs, with previous research focusing on mitigating single-beam breakup instability (BBU) induced when a beam propagates off-centre due to orbit-jitter or misalignment. It is also known that quadrupole-like fields are excited in planar/slab DWA structures and research has been conducted on mitigating this effect. We present simulation results that demonstrate quadrupole-like fields are also excited in circular DLWs, induced by beam astigmatism. We have shown that this in an extra source of instability within circular DWA structures and calculate the size of the fields excited as a function of beam astigmatism.

INTRODUCTION

Dielectric wakefield acceleration (DWA) has been suggested as a method to produce accelerating fields much greater than conventional RF-based accelerating methods. DWA exploits the Cherenkov radiation generated by a charged particle bunch in a dielectric lined waveguide (DLW) to accelerate a trailing bunch [1]. It has been demonstrated that DLWs can sustain GV/m fields before breakdown effects are observed, and acceleration of bunches with ~300 MV/m gradients has been measured [2, 3].

Two DLW geometries are under active consideration, circular/cylindrical and planar/slab DLWs. Strong transverse fields are excited off-axis in both geometries, leading to beam breakup instability induced by small initial offsets [4]. A method for compensating this instability is required before applications of DWA can be realised. One proposed method is to line a circular DWA with a quadrupole wiggler, BNS damping, continuously compensating any offset and returning the beam to the DLW axis [4, 5]. This method can only be applied to a circular DWA structure. BNS damping also leads to an oscillating RMS transverse beam size through the circular DWA. The effect of a non-radially symmetric beam in a circular DWA has not been investigated. Evidence of transverse fields excited on-axis in circular DWA structures has been experimentally demonstrated, but the source of these fields has not been fully explained [6, 7]. In these proceedings, the field excited by non-radially symmetric beams have been calculated. Higher-order fields have been shown to be excited and a potential new source of beam instability demonstrated.

Table 1: Beam, Mesh, and Circular DLW Parameters for Field Calculations

Parameter	
Charge	250 pC
Longitudinal Momentum	250 MeV/c
RMS Bunch Length, σ_t	200 fs
Longitudinal Profile Shape	Gaussian
RMS Beam Width, $\sigma_{x,y}$	50 μm
Transverse Mesh Density, Cells per $\sigma_{x,y}$	5
Longitudinal Mesh Density, Cells per σ_t	3
DLW Vacuum Radius, a	500 μm
Dielectric Thickness, δ	200 μm
Dielectric Permittivity	3.75

METHODOLOGY

Fields have been calculated using DiWaCAT, a specialised planar/slab and circular DLW field solver and beam tracker simulation software [8]. For circular DLWs, the wake potential is calculated using the method outlined in [9]. The beam distribution is given by a large sample of uniform charge macroparticles which is sampled onto a regular 3D Cartesian grid, consequently each grid point (or cell) has an associated charge. The wake potential is calculated at each grid point and field given by the convolution of the wake potential and beam distribution at each grid point.

The fields in these proceedings are calculated for a single set of beam parameters, listed in Table 1. These parameters are chosen to match expected beam parameters at CLARA during Phase-2 of operation [10]. A large transverse mesh density was chosen to avoid adding a source of radial or azimuthal asymmetry when using a Cartesian grid in a circular DLW. Field strength is directly proportional to drive bunch charge, so beam deflection for a given set of transverse beam parameters is directly proportional to the ratio of bunch charge to longitudinal momentum [9]. The deflection for parameters in Table 1 would be equivalent to the deflection with nC-scale bunch charge and GeV-scale energy, comparable to a practical wakefield accelerator.

The transverse fields excited in DLWs are longitudinally varying, with no field excited at the head of the bunch

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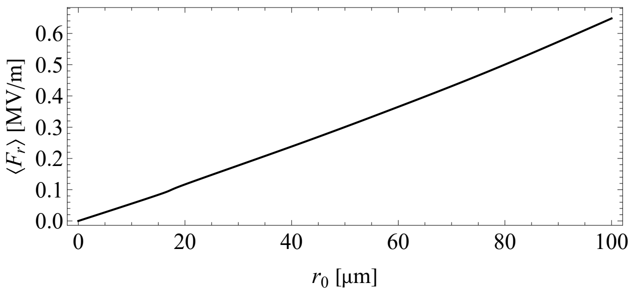


Figure 1: Average transverse field, at $(x, y) = (0, 0)$, as a function of beam offset from the centre of a circular DLW.

and field strength increasing towards the bunch tail. When evaluating the strength of fields, the average field strength weighted by the charge distribution will be used, defined by

$$\langle a \rangle = \frac{\int_{\infty}^{-\infty} a(t)\rho(t)dt}{\int_{\infty}^{-\infty} \rho(t)dt} ,$$

where $a(t)$ is the longitudinally varying field being evaluated and $\rho(t)$ is the longitudinal beam distribution. Using a multipole field expansion, the quadrupole-like component to the field is the transverse gradient to the transverse field excited.

Calculating the quadrupole field gradient numerically,

$$Q_i = \frac{F_i(x_i = 1\ \mu\text{m}) - F_i(x_i = 0)}{1\ \mu\text{m}} , \tag{1}$$

where x_i is the transverse axis in which the quadrupole strength is calculated. The quadrupole field strength can be defined in field units (MeV/m/m), equivalent quadrupole gradient (T/m), or the equivalent quadrupole gradient normalised to beam rigidity, k (m^{-2}).

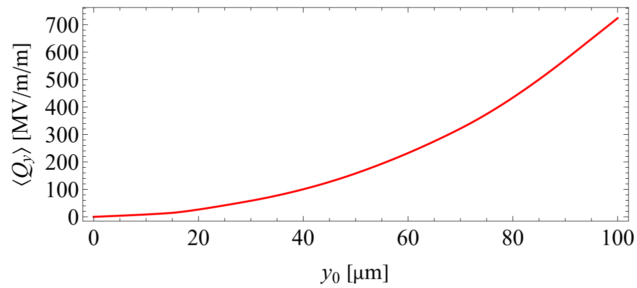
INSTABILITY FORMATION

Dipole-Like Field Excitation

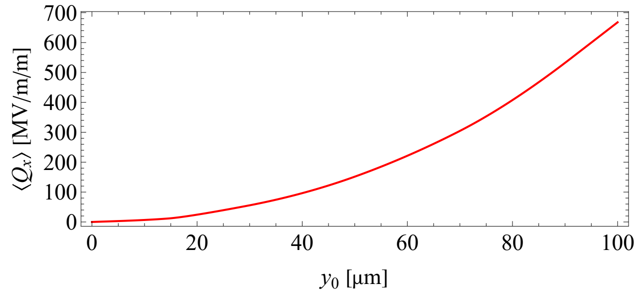
A beam off-axis in a circular DLW will excite transverse fields, a known cause of BBU in circular DWAs [4]. Using a multipole field expansion, the dipole-like component increases approximately linearly with small offsets with higher-order terms excited closer to the dielectric surface [11].

The average vertical field as a function of beam offset is shown in Fig. 1. Offsets close to the dielectric surface are not shown and — over the range shown — the field strength still increases approximately linearly with offset. The breaking of radial and azimuthal symmetry also leads to the excitation of higher-order fields. The strength of the quadrupole-like field, defined in Eq. (1), increases at a greater rate than the dipole-like field.

The strength of this field is approximately an order of magnitude lower than the dipole-like field. For example, with an offset of 100 μm the dipole-like field has an average strength of 0.6 MV/m, whilst the contribution of the quadrupole-like field 50 μm from the centre of bunch at the same offset is



(a) Average vertical defocusing quadrupole field strength, parallel to the direction of the offset.



(b) Average horizontal focusing quadrupole field strength, orthogonal to the direction of the offset.

Figure 2: Quadrupole-like field excited as a function of vertical beam offset from the centre of a circular DLW.

0.035 MV/m. This is the equivalent to a 2.2 T/m average quadrupole field gradient, or $k = 2.6$ m^{-2}. Therefore, the effect of exciting dipole-like fields is significantly greater than higher-order fields for beams off-axis in a circular DLW.

Quadrupole-Like Field Excitation

A finite beam propagating on-axis in a circular DLW can be considered the sum of point-like particles. The field excited by the beam is the sum of the fields excited by each point-like particle. These point-like particles are off-axis in the DLW so excite dipole-like and quadrupole-like fields as in Figs. 1 and 2. For a radially and azimuthally symmetric bunch, these individual contributions cancel; if this symmetry is broken a residual transverse field will be excited.

Fields have been calculated for a beam with parameters as in Table 1, with the exception of RMS horizontal beam size, σ_x, which was varied to change the beam astigmatism. The transverse field excited with $\sigma_x = 100$ μm is shown in Fig. 3. The shape of this field is asymmetric, i.e. $F_x(x, y)$ is not a mirror image of $F_y(x, y)$ as with a perfectly quadrupole-like field. The defocusing field, F_x, in the direction of the larger beam size is quadrupole-like, directly proportional to x and independent of y.

The defocusing field remains quadrupole-like with changing beam size (Fig. 4). The vertical field appears approximately sextupole-like. In the horizontal axis ($x = 0$), a proportional relationship between focusing strength and y is seen, however away from the axis a non-linear relationship is seen. Sextupole-like fields would lead to slice emittance growth in that transverse axis. Whilst slice emittance

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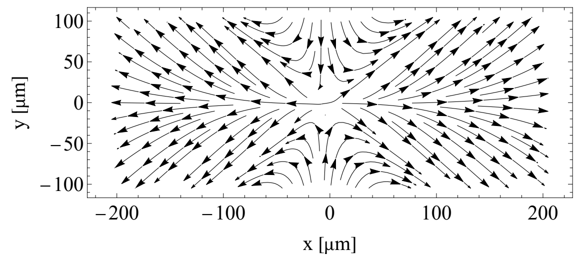
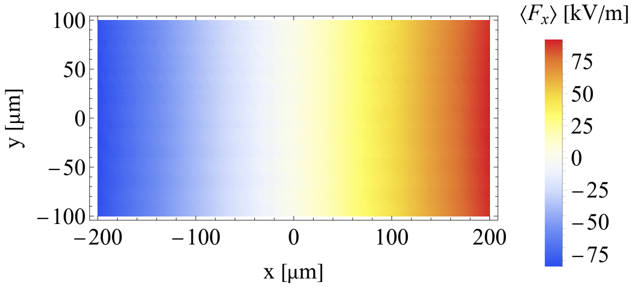
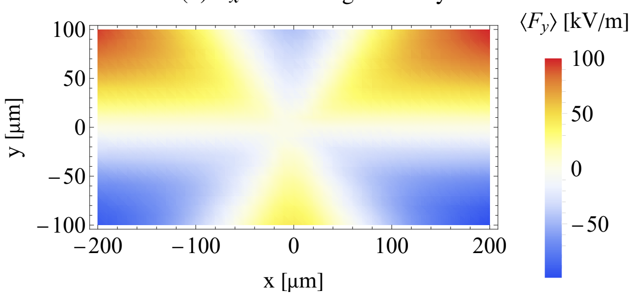


(a) Transverse field stream lines.



(b) F_x field strength density.



(c) F_y field strength density.

Figure 3: Transverse field excited on-axis by a beam with σ_x = 100 μm and σ_y = 50 μm.

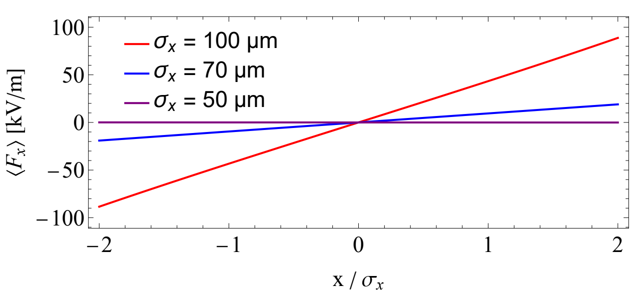
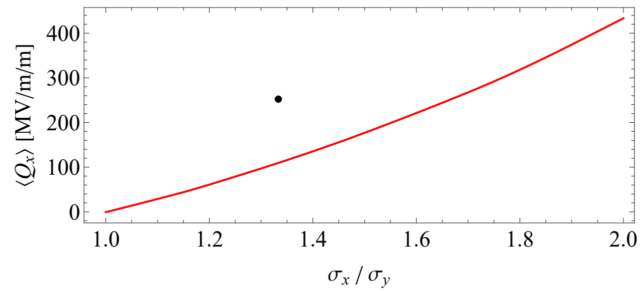


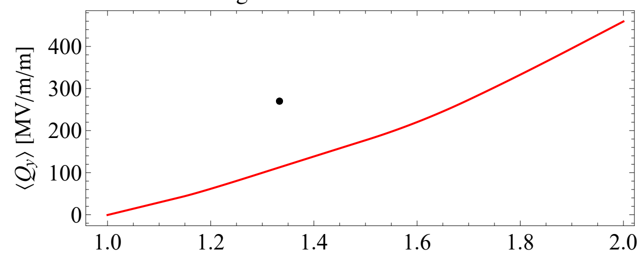
Figure 4: Horizontal variation, normalised to RMS beam width, in average horizontal field for varying beam astigmatism. For all cases σ_y = 50 μm.

would be preserved in the direction of quadrupole-like fields (larger transverse size), given that all transverse wakefields are longitudinally varying, projected emittance would not be preserved in either axis.

Defocusing in the direction of larger beam size suggests that beam astigmatism would grow as the beam propagates through a DWA stage. The strength of the defocusing grows exponentially with beam astigmatism (Fig. 5), leading to an extra source of instability in circular DWAs. The strength of these fields are significant, the average quadrupole-like field



(a) Average horizontal defocusing field strength, parallel to the direction of the axis of greatest beam width.



(b) Average vertical focusing field strength, orthogonal to the direction of the axis of greatest beam width.

Figure 5: Quadrupole-like field strength as a function of horizontal beam width with σ_y = 50 μm. The black point with σ_x = 100, σ_y = 75 μm is shown for reference.

for $\sigma_x/\sigma_y \approx 1.8$ is the equivalent to a defocusing quadrupole with a 1 T/m quadrupole strength, or $k = 1.2\,\mathrm{m}^{-2}$. Transverse field strength is a function of beam size in each axis. The black points in Fig. 5 demonstrates that a larger overall beam size, with the same aspect ratio, will excite stronger fields than a smaller total transverse beam size. This relationship leads to non-trivial beam dynamics, with the field strength a function of σ_x, σ_y, and the combination of the two.

CONCLUSIONS

In conclusion, simulations using benchmarked wakefield equations have been used to demonstrate the excitation of quadrupole-like and higher-order transverse fields when radial and azimuthal beam symmetry is broken. These fields can be explained as the non-symmetric summation of dipole-like fields excited off-axis in circular DLWs. The direction of defocusing, in the direction of larger beam size, would result in a growing instability with propagation distance.

This instability could be induced by a beam entering a circular DWA structure with transverse asymmetry, or by oscillating transverse beam sizes caused by a BNS damping scheme. Whilst the strength of the quadrupole-like field is considerably lower than deflecting fields excited by off-axis propagation, which can cause BBU over short distances, the effect of quadrupole-like fields on beam quality and stability over longer propagation distances needs further investigation.

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REFERENCES

[1] R. Keinigs and M. E. Jones, "The Cherenkov Wakefield Accelerator", *Part. Accel.*, vol. 24, pp. 223–229, 1989.

[2] M. C. Thompson *et al.*, "Breakdown Limits on Gigavolt-per-Meter Electron-Beam-Driven Wakefields in Dielectric Structures", *Phys. Rev. Let.*, vol. 100, p. 214801, 2008. doi:10.1103/PhysRevLett.100.214801

[3] B. D. O'Shea *et al.*, "Observation of Acceleration and Deceleration in gigaelectron-volt-per-metre gradient Dielectric Wakefield Accelerators", *Nature communications*, vol. 7, no. 1, pp. 1–7, 2016. doi:10.1038/ncomms12763

[4] C. Li *et al.*, "High Gradient Limits due to Single Bunch Beam Breakup in a Collinear Dielectric Wakefield Accelerator", *Phys. Rev. STAB.*, vol. 17, p. 091302, 2014. doi:10.1103/PhysRevSTAB.17.091302

[5] A. Zholents *et al.*, "A Preliminary Design of the Collinear Dielectric Wakefield Accelerator", *Nucl. Instrum. Methods - Section A*, vol. 829, pp. 190–193, 2016. doi:10.1016/j.nima.2016.02.003

[6] G. Andonian *et al.*, "Drive Bunch Train for the Dielectric Trojan Horse Experiment at the Argonne Wakefield Accelerator", *Instruments*, vol. 8, no. 2, p. 24, 2024. doi:10.3390/instruments8020028

[7] T. J. Overton *et al.*, "Experimental Study of Transverse Beam Dynamics in Dielectric Wakefield Structures", Submitted for publication, 2024.

[8] T. J. Overton, "DiWaCAT v1.0", 2023. doi:10.5281/zenodo.8321425

[9] K. Y. Ng, "Wake Fields in a Dielectric-lined Waveguide", *Phys. Rev. D.*, vol. 42, pp. 1819–1828, 1990. doi:10.1103/PhysRevD.42.1819

[10] E. S. Snedden *et al.*, "Specification and Design for Full Energy Beam Exploitation of the Compact Linear Accelerator for Research and Applications", *Phys. Rev. Acc. Beams*, vol. 27, p. 041602, 2024. doi:10.1103/PhysRevAccelBeams.27.041602

[11] S. Bettoni *et al.*, "Temporal Profile Measurements of Relativistic Electron Bunch based on Wakefield Generation", *Phys. Rev. Accel. Beams*, vol. 19, p. 021304, 2016. doi:10.1103/PhysRevAccelBeams.19.021304