11

Blowup Effect in Linear Accelerators

As has been mentioned occasionally, the stimulated emission could be treated as a radiative beam instability. As a matter of fact, the problem of electromagnetic waves generation and amplification can be reduced to the provocation of the controlled instability in a desirable frequency region.

The previous sections were devoted to numerous difficulties of the task. However, as it usually happens, instabilities are easily self-excited when and where they are not desirable or even harmful. According to well-known Murphy low, damping of these parasitic instabilities requires sometimes even more efforts than exciting the desirable ones.

The theory of collective instabilities in such complicated systems as high current particle accelerators deserves a special book and, in any case, is outside of our scope. Nevertheless, one example is worth to be mentioned here briefly.

We mean a so-called blowup effect experimentally found out in large linear accelerators at currents exceeding a rather low threshold value of order of several tens of milliamperes. The accelerated current pulse with typical duration of 2–3 ms was found out to shorten sharply. Increase in the injection current shortened the pulse even more so that the total accelerated charge remained the same or decreased. At the same time, hard x-ray radiation appeared indicating high-energy electrons bombarding the chamber walls. These effects were accompanied by electromagnetic radiation with frequency exceeding 1.5–2 times the frequency of the main accelerating mode.

The last obviously indicated a parasitic mode self-excitation, that is, the coherent radiation emission in a higher propagation band.[1] The electron bombardment proved that the excited mode had transverse components at the axis and was axially nonsymmetric.

Transverse focusing taken into account, one can consider each electron as an oscillator moving with a relativistic velocity in a system permitting prop-

[1] Remind that the dispersion characteristic of a linear accelerator's waveguide consists of bands of transparency. Certain spatial Fourier harmonics of propagating modes have phase velocity lesser than that of light.

agation of slow waves. Hence, in our conception we can talk about radiation under conditions of anomalous Doppler effect when growing of oscillations can be expected. In this short chapter, we pay attention to this effect because the negative energy waves had been considered above only as longitudinal space charge ones. In the present case, self-excitation and phasing of transverse displacement waves are of interest.

In a linear accelerator, the beam looks like a train of short bunches separated by the accelerating wave length which is not an integer number of the excited wave one. For this reason the microwave equilibrium structure of the beam is not of importance for self-excitation but gives a possibility to consider each bunch as an individual point-like particle.

A structure of a nonsymmetric wave in a periodic waveguide is rather complicated even if the waveguide itself is symmetric. Opposite to uniform systems, only axially symmetric modes belong to definite E or M types. In general, the proper waves have all six components and for this reason are called HEM-waves (Hybrid ElectroMagnetic). However, only quasi-synchronous harmonics with wavenumbers $k \approx \omega/\overline{\beta}c$ and phase velocities $\beta_{\mathrm{p}} \approx \overline{\beta}$ are of importance for interaction with a particle moving along z with a practically constant velocity $\overline{\beta}$. To avoid misunderstanding, note that the phase velocity $\beta_{\mathrm{p}} = \omega/kc$ should be considered in our case as a fixed parameter. Boundary conditions in a waveguide of period l can be provided only by cooperation of harmonics shifted in wavenumbers by multiples of $2\pi/l$ and not taking part in the synchronous interaction.

The field of the lowest synchronous harmonic with one variation over azimuth can be expressed in cylindrical coordinates via three components of the vector-potential. For a wave linearly polarized in a $x = r\cos\theta$ plane

$$
\begin{aligned}
A_{\mathrm{r}} &= I_2\left(kr\sqrt{1-\beta_{\mathrm{p}}^2}\right)\cos\theta\,;\\
A_{\theta} &= I_2\left(kr\sqrt{1-\beta_{\mathrm{p}}^2}\right)\sin\theta\,;\\
A_{\mathrm{z}} &= \mathrm{i}\sqrt{1-\beta_{\mathrm{p}}^2}I_1\left(kr\sqrt{1-\beta_{\mathrm{p}}^2}\right)\cos\theta\,,
\end{aligned}
\tag{11.1}
$$

where I_n is a Bessel function of an imaginary argument. Standard calculations yield for the field components:

$$
\begin{aligned}
E_{\mathrm{x}} &= -\mathrm{i}k\beta_{\mathrm{p}}\sqrt{1-\beta_{\mathrm{p}}^2}I_2\cos 2\theta\,;\\
E_{\mathrm{y}} &= -\mathrm{i}k\beta_{\mathrm{p}}I_2\sin 2\theta\,;\\
E_{\mathrm{z}} &= k\beta_{\mathrm{p}}\sqrt{1-\beta_{\mathrm{p}}^2}I_1\cos\theta\,;\\
\\
B_{\mathrm{x}} &= -\frac{\mathrm{i}}{2}k\sqrt{1-\beta_{\mathrm{p}}^2}\left(1+\beta_{\mathrm{p}}^2\right)I_2\sin 2\theta\,;\\
B_{\mathrm{y}} &= -\frac{\mathrm{i}}{2}k\sqrt{1-\beta_{\mathrm{p}}^2}\left[\left(1-\beta_{\mathrm{p}}^2\right)I_0-\left(1+\beta_{\mathrm{p}}^2\right)I_2\right]\cos 2\theta\,;
\end{aligned}
\tag{11.2}
$$

$$B_{\mathrm{z}} = k\sqrt{1-\beta_{\mathrm{p}}^2} I_1 \sin\theta \,.$$

In particular, considering the polarization plane, the components

$$E_{\mathrm{z}} = \frac{1}{2}k^2 x\beta_{\mathrm{p}}\left(1-\beta_{\mathrm{p}}^2\right) \,; \qquad B_{\mathrm{y}} = -\frac{\mathrm{i}}{2}k\left(1-\beta_{\mathrm{p}}^2\right)^{3/2} \,. \tag{11.3}$$

do not vanish at the axis. A schematic structure of the force lines in the paraxial region is presented in Fig. 11.1

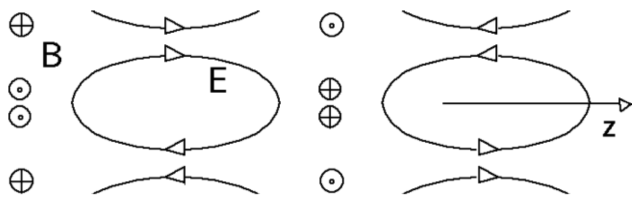

Fig. 11.1. Force lines of a HEM wave in the paraxial region

Two main deductions should be made. First of all, a particle travelling along the z-axis cannot radiate a HEM wave but experiences a deflecting Lorentz force. As a result, it is shifted in x-direction to the domain of possible emission and/or absorption. One can easily see that waves slightly faster than the particle accomplish negative work and, hence, are amplified. Correspondingly, slow waves are absorbed. This mechanism of the stimulated emission is in somewhat more complicated than the longitudinal phasing above. The latter effect also exists in our case but plays a secondary role.

In reality, the length of a single section is too small for developing of an absolute instability due to induced radiation.[2] But in a chain of many sections the instability occurs in spite of their electrodynamic independency. The necessary coupling takes place because information about a transverse displacement in a certain section is transported by the beam to all following ones.

To simplify the description, we neglect all transient effect at the ends of the sections constituting a waveguide of a large linear accelerator. Their independency can be imitated by putting zero the group velocity of the HEM wave. Radiative processes inside a single section cannot be considered in this model. We also suppose for simplicity the rigid structure of the beam bunched in the accelerating field. By the way this makes impossible the longitudinal bunching in the comparatively weak HEM wave.

Consider a sequence of particles (bunches) deflected by the synchronous HEM wave proportional to $\exp(\mathrm{i}kz - \mathrm{i}\omega t)$. When passing a point z the s-th particle is under action of the same force as the $(s-1)$-th one but shifted in phase by $-\omega T$ where T is a time interval between the particles. Besides,

[2] However, in industrial high-current accelerator, it can happen.

an additional force acts because of the $(s-1)$-th particle radiation. This is proportional to the particle deviation from the axis with a certain complex coefficient Z. For our purposes, an obvious fact is sufficient that Z value is proportional to the particle charge, that is, to the beam average current I. (Of course, calculations of the instability threshold would require the exact value of Z as well as the group velocity and damping constant of the HEM mode.) Now the equation for the transverse deviation of the s-th particle can be written in the form:

$$\left[\frac{\mathrm{d}^2 x_s}{\mathrm{d}z^2} + \nu^2 x_s\right] \exp\left(\mathrm{i}\omega T\right) = \frac{\mathrm{d}^2 x_{s-1}}{\mathrm{d}z^2} + \nu^2 x_{s-1} + Z x_{s-1} \,. \qquad (11.4)$$

Here ν^2 describes a possible external focusing and the phasor $\exp\left(\mathrm{i}\omega T\right)$ reflects the phase shift of the radiation field during the interval between particles.

The transverse deflections of two successive particles also have a phase shift of ωT. In any case, a formal substitution

$$x_s = X_s \exp\left(-\mathrm{i}\omega T s\right)$$

excludes the exponential factor from (11.4) and gives the following equation for slowly varying amplitudes:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}z^2} + \nu^2\right] \left(X_s - X_{s-1}\right) = Z X_{s-1} \,. \qquad (11.5)$$

Let us suppose now that particle-to-particle variations of the amplitude are small so that the index s can be considered as a continuous variable. As was mentioned above, it does not mean necessarily that the beam itself is continuous (the accepted model might fail only if the phase shift ωT is a multiple integer of 2π, i.e., if the HEM wave and the accelerating wave are coherent). In this approximation, Eq. (11.5) looks like

$$\frac{\partial}{\partial s}\left[\frac{\partial^2 X(z,s)}{\partial z^2} + \nu^2 X(z,s)\right] = Z X(z,s) \,, \qquad (11.6)$$

where the constant Z differs in somewhat from that of (11.4) and s can be treated now as time accounted from the moment when the head of the train passed the point z.

A solution of (11.6) depends on initial and boundary conditions, in particular on initial amplitude of the wave and on initial beam displacement. However, if time and distance are large enough, the amplitude asymptotic behavior is independent of initial conditions.

As far as the field vanishes ahead of the train Laplace transformation of (11.6) gives the second-order homogenous equation

$$\frac{\mathrm{d}^2 X(z,p)}{\mathrm{d}z^2} + \left(\nu^2 - \frac{Z}{p}\right) X(z,p) = 0 \qquad (11.7)$$

with a general solution

$$X(z,p) = L^{+}(p)\exp(\mathit{\Gamma} z) + L^{-}(p)\exp(-\mathit{\Gamma} z) \ , \tag{11.8}$$

where

$$\begin{aligned} \mathit{\Gamma}(p) &= \left(Z/p - \nu^2\right)^{1/2} \ ; \quad \operatorname{Re}\mathit{\Gamma} > 0\,; \\ L^{\pm} &= \left[X(z,p) \pm \mathit{\Gamma}^{-1}\frac{\partial X(z,p)}{\partial z}\right]_{z=0} \,. \end{aligned} \tag{11.9}$$

The asymptotic behavior of (11.8) for $z \to \infty$ is obviously determined by the first term in the right-hand side. The inverse Laplace transformation then yields

$$X(z,s) \asymp \frac{1}{2\pi \mathrm{i}}\int_{-\infty+\mathrm{i}0}^{+\infty+\mathrm{i}0} L^{+}(p)\exp\left[ps + \mathit{\Gamma}(p)z\right]\mathrm{d}p\,. \tag{11.10}$$

For large z and s, the integral value is determined by saddle points p_0 in the complex plane of p, which are the roots of the equation:

$$s + z\frac{\mathrm{d}\mathit{\Gamma}}{\mathrm{d}p} = 0 \quad \text{or} \quad p^2\sqrt{Z/p - \nu^2} = \frac{Zz}{2s}\,. \tag{11.11}$$

The root of interest corresponds to the maximal real part[3] of the exponent argument in (11.10). Passing the integration contour through the point and expanding the argument over powers of $p - p_0$:

$$ps + \mathit{\Gamma}(p)z \approx p_0 s + \mathit{\Gamma}(p_0)z + \frac{s^3 p_0^3}{Z^2 z^2}\left(3\frac{Z}{p} - 4\nu^2\right)(p - p_0)^2 \tag{11.12}$$

we obtain

$$X(z,s) \asymp \tag{11.13}$$
$$\frac{L^{+}(p_0)}{2\pi \mathrm{i}}\exp\left[p_0 s + \frac{Zz^2}{2sp_0^2}\right]\int\limits_{-\infty+\mathrm{i}0}^{+\infty+\mathrm{i}0}\exp\left[\frac{s^3 p_0^3}{Z^2 z^2}\left(3\frac{Z}{p_0} - 4\nu^2\right)(p - p_0)^2\right]\mathrm{d}p\,.$$

The substitution

$$u = (p - p_0)\left(4\nu^2 - 3\frac{Z}{p_0}\right)^{1/2}\frac{(sp_0)^{3/2}}{Zz}$$

gives for the integral (11.13):

$$X(z,s) \asymp -\frac{L^{+}(p_0)\,Zz}{2\sqrt{\pi}\,(sp_0)^{3/2}\,(3Z/p_0 - 4\nu^2)^{1/2}}\exp\left[p_0 s + \frac{Zz^2}{2sp_0^2}\right]\,. \tag{11.14}$$

---

[3] There can be two such roots but it does not make an essential difference.