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RECENT ASPECTS OF SUPERSYMMETRY BREAKING

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Introduction

One of the main goal of theoretical high energy physics consists in force unification. The Standard Model has provided a unified description of all non gravitational interactions as quantum field theories with gauge symmetries, while gravity is described by the General Relativity of Einstein. The fusion of these two theories in a unified framework is still lacking. Nowadays the most promising candidate is string theory, which describes the fundamental particles as vibrating strings and naturally includes gravity. However, a deep understanding of string theory is far to be achieved. Consistency of string theory demands a new symmetry in nature, which maps bosons into fermions and viceversa, the supersymmetry. Thus, if we consider the relative role of field theory and string theory, we can conclude that physics at distances well below the Planck scale is described by a local quantum field theory. Then the basic building blocks for any theory of elementary particle physics are supersymmetric field theories, and, most probably, supersymmetric Yang-Mills theories. Moreover ordinary quantum field theories still have many open questions such as the hierarchy problem, confinement, ultraviolet completion. Supersymmetric quantum field theories can then provide an important theoretical laboratory to learn about quantum properties of field theories. Indeed, their renormalization properties and their high level of symmetries enable a powerful non perturbative analisys. These are relevant reasons to investigate supersymmetric quantum field theories.

However, from experimental observations, we know that at low energy supersymmetry must be broken if we wish to build reasonable phenomenological models. This is the motivation to study the wide subject of supersymmetry breaking in quantum field theories. Any theoretical construction where there is supersymmetry, from string theory to supergravity and so on, should deal with the problem of how to break it.

One of the usual phenomenological pattern is that the supersymmetry breaking occurs in an hidden sector, which is not charged under the Standar Model gauge group, and that the breaking is then transmitted to a supersymmetric extension of the Standard Model (MSSM). The transmission can be mediated by another set of fields, the messengers, and can take place through gauge interaction and/or gravitational interaction. The transmission of supersymmetry breaking leads then to the generation of (soft) supersymmetry breaking terms in the lagrangian of the MSSM. Specifying the right hidden sector is a very hard task, even if some general properties to fit with the observations have to be required.

Metastable and dynamical supersymmetry breaking
In the last years there has been an effort to exploit the special simplifications of supersymmetric field theories to discover the behavior of these theories in the region of strong coupling. These investigations led to many wonderful realizations about these theories; in particular, it was discovered that many cases have remarkable nontrivial dual descriptions. The electric-magnetic duality of $\mathcal{N} = 1$ gauge theories has been discovered by Seiberg and has provided fundamental insights in the low energy dynamics of these theories.

A striking result was the understanding of the Affleck-Dine-Seiberg superpotential. In a specific range between the number of colours and the number of flavours of supersymmetric QCD there is a non perturbative generation of a superpotential. This superpotential is a pure quantum effect that modifies drastically the classical space of vacua of the theory. Classically there is a manifold of supersymmetric vacua, but at quantum level there is a runaway behaviour of the potential, and hence no vacua. This non perturbative superpotential has been deeply studied and exploited in model building, with the aim of breaking supersymmetry through a dynamical effect.

As already mentioned, Seiberg duality is one of the most relevant improvements in non perturbative properties of supersymmetric gauge theories. It represents the possibility of providing an alternative description for the original theory (the electric one) in terms of another theory, the magnetic one, which is weakly coupled when the first one is strongly coupled. The two theories describe at low energy the same physics. We can study the weakly coupled one with standard perturbative techniques to extract information about the strong dynamics of the other theory. This is why Seiberg duality is referred to as an electric/magnetic duality.

The recent breakthrough of Intriligator Seiberg and Shih has given a new and fundamental ingredient. They have shown that it is possible to find metastable vacua with supersymmetry breaking in the low energy description of massive supersymmetric QCD. Their analysis is based on the weakly coupled (magnetic) description and provides a mechanism of spontaneous breaking of supersymmetry. On the other hand, in the electric theory language, the existence of such vacua is a strong coupling phenomenon, and so the supersymmetry breaking is dynamical. Actually, these vacua are only metastable, i.e. they are local minima of the potential. They decay into the supersymmetric vacua of the theory but, if their lifetime is long, they can be phenomenologically acceptable.

Metastability is a crucial property of these vacua since there is a very strong constraint on the existence of supersymmetric vacua in supersymmetric theories: the non vanishing of the Witten index. As soon as it is different from zero, the theory admits supersymmetric vacua and hence supersymmetry cannot be spontaneously broken in the true vacuum of the theory. However, if we accept the possibility of false but long lived vacuum, we can search metastable vacua with supersymmetry breaking even in theories with Witten index different from zero.

These models can have direct phenomenological applications playing the role of an hidden sector. In this thesis we focus on schemes of gauge mediation, where the breaking transmission takes place through gauge interactions. This allows to make some predictions about the final spectrum, in particular about masses of the MSSM which are generated through the radiative corrections.

In this context a crucial role is played by R-symmetry. It is a $U(1)$ global symmetry which is usually manifest in the metastable vacua with broken supersymmetry. This is not satisfying since an R-symmetry forbids the perturbative generation of crucial supersymmetry breaking terms.

Explicit supersymmetry breaking
The supersymmetry breaking discussed so far is a spontaneous breaking of supersymmetry. However, as a general global symmetry, the breaking of supersymmetry can also be explicit, adding non supersymmetric terms to the Lagrangian. The explicit supersymmetry breaking should be soft, in order not to spoil the renormalization properties of the theory. The introduction of supersymmetry breaking terms can be achieved through the so called spurion superfields. The importance of this method of supersymmetry breaking is due to the fact that such soft terms naturally arise in the flat limit of supergravity theory with spontaneous breaking of supersymmetry. If we consider a supergravity theory where the vacuum breaks local superysmmetry and we perform the limit where we decouple gravity $M_{Planck} \to \infty$, keeping fixed the ratio between the gravitino mass and the Planck mass, then we obtain a theory with global supersymmetry plus soft terms.

Introduction to $\mathcal{N} = 1$ gauge theories
In the first part of the thesis we review some aspects of non perturbative $\mathcal{N} = 1$ gauge theories, the Intriligator Seiberg and Shih (ISS) model, and some elements of the gauge mediation mechanism. We focus on the basic analysis which are necessary in order to understand $\mathcal{N} = 1$ electric/magnetic duality, the breaking of supersymmetry and its consequences. This is a very wide subject, that connects old and recent results which are spread in the theoretical and also phenomenological literature of the last twenty years. Hence we think it can be useful to review some main aspects in order for the thesis to be more complete.

In particular in chapter 1 we review some elements of supersymmetry breaking, of non perturbative techniques and of electric/magnetic duality. We widely explain the extended electric/magnetic duality with adjoint fields, that will be necessary in the following.

In chapter 2 we review the ISS model, and we give a very brief outlook on the large amount of papers that have followed.

The gauge mediation of supersymmetry breaking, with particular attention to the gaugino mass generation, is reviewed in chapter 3.

Metastable vacua in $\mathcal{N} = 1$ gauge theories
From the fourth chapter we begin with the original part of the thesis. Chapters 4,5 and 6 are devoted to the analysis of supersymmetry breaking metastable vacua in different $\mathcal{N} = 1$ gauge theories, through a detailed study of their low energy dynamics. These chapters are based on papers written in collaboration with A. Amariti and L. Girardello [1, 2, 3]. Two main classes of theories have been studied.

The first one, analyzed in chapter 4 and 5, is characterized by the presence of fields in the adjoint representation of the gauge group, which brings to richer electric/magnetic dualities. This theory is also interesting since models with scalar fields in the adjoint representation naturally arise in string theory. In chapter 4 we investigate the possibility of non supersymmetric meta-stable vacua in the deep infrared of $SU(N)$ gauge theory with both fundamental and adjoint matter, and with non trivial superpotential for the adjoint. Such superpotential generates further mesons in the dual magnetic theory. This produce several flat directions which cannot be stabilized easily and which could lead to instability of the non supersymmetric vacua. We find that, adding proper deformations to the superpotential, there are non supersymmetric meta-stable vacua with parametrically large life-time. In our model the landscape of non supersymmetric vacua that appears at classical level is wiped out by quantum corrections and there is no $U(1)_R$ symmetry, differently from the most of the other models in literature.

Subsequentely, in chapter 5, we embed this model in a direct gauge mediation scenario, showing how it can be used as an hidden sector to comunicate the supersymmetry breaking to a visible sector (MSSM). The absence of an R-symmetry allows the generation of a non trivial gaugino mass.

In chapter 6 we discuss the second class of theories, the A_n quiver gauge theories, which can be derived directly from string theory. These models indeed arise in type IIB string theory as the world volume theory of $D5$-branes partially wrapped on Calabi-Yau singularities. Such theories are characterized by the presence of multiple gauge groups. We study the low energy dynamics by Seiberg dualizing alternate nodes in the quiver, and then we prove the existence of longliving metastable vacua. In order to rely on perturbative computations in the dual (magnetic) theory, we perform a carefull analysis of the RG flow of the different gauge groups which leads to non trivial constraints to be imposed on the strong coupling scales.

Also for this class of models we show how they can be embedded in a gauge mediation scenario. The R-symmetry problem is solved here by the mass term for the messenger fields, that breaks the $U(1)_R$ at tree level.

Dijkgraaf Vafa conjecture and explicit supersymmetry breaking

As already introduced, String Theory is the most promising candidate for force unification. In the last years the geometrical engineering of gauge theories has provided a method to embed gauge theories in string theory and the interplay between the two descriptions has given new important insights to the quantum field theories understanding. In chapter 7 we explore the topic of explicit supersymmetry breaking in this framework.

A very simple relation between the low energy effective action for a class of $\mathcal{N} = 1$ gauge theories and the free energy of an auxiliary matrix model has been found by Dijkgraaf and Vafa. The simplest case is $U(N)$ gauge theory with massive adjoint chiral matter multiplets with a polinomial tree-level superpotential. The proposal stems from a set of dualities in the framework of geometrical engineering of gauge theories, topological strings and matrix models and it has been tested and supported directly in field theory. In chapter 8, based on the work [4] in collaboration with L. Girardello and G. Tartaglino-Mazzucchelli, we study explicit soft supersymmetry breaking in the tree–level superpotential promoting the coupling constants to spurions. Holomorphy at large is lost, but holomorphic quantities such as the low–energy superpotential can be still analyzed. We succesfully compare the computation in the superfield formalism adapted to spurion fields with that one via algebraic curve underlying the effective gauge theory, where the holomorphic breaking terms are interpreted as Whitham deformations.

AdS/CFT correspondence

In the last years a lot of progresses have been made in the comprehension of field theories and string theory by means of dualities. The effort to understand the non perturbative regime of YM theories has brought to the search of dual descriptions of gauge theories which permit to learn about the strong coupling dynamics. The gauge/gravity correspondence provides a closed string description, based on classical supergravity, of the dynamics of gauge theories at large t Hooft coupling.

Generally, a possible way to uncover stringy effects in Yang-Mills theory consists of adding D-branes on the supergravity side and trying to find out the corresponding field theory dual. In order to add consistently probe D-branes, we have to control the supersymmetry preserved by the inclusion of these extra degrees of freedom. This also guarantees that the configuration found for the brane is stable. For this reason much effort has been spent to find supersymmetric configuration of D-branes in known supergravity solutions which are dual to gauge theories.

The AdS/CFT conjecture by Maldacena, originally formulated between the $\mathcal{N}=4$ SYM and type IIB string theory on $AdS_5 \times S^5$, has been generalized to theories with less supersymmetry. The class of models we are interested in are $\mathcal{N}=1$ marginal or massive deformations of $\mathcal{N}=4$ SYM, which have been study from the field theory perspective by Leigh and Strassler. The gravity dual of the exactly marginal beta deformation has been found by Lunin and Maldacena, while the Pilch Warner flow is the gravity solution dual to the single mass deformation of $\mathcal{N}=4$ SYM. The mass deformation is not marginal, but the renormalization group flow leads the theory to a conformal infrared fixed point. These theories are described in type IIB supergravity as a warped product of AdS_5 times an internal five dimensional manifold, which can support non trivial fluxes. This can be mapped to a warped product of a Minkowsky spacetime times a non compact six dimensional manifold with fluxes.

The description of supergravity solutions with fluxes has received a lot of attention in the framework of string compactification. There it was found that they can be better understood in the language of Generalized Complex Geometry (GCG). In this formalism the supergravity equations (dilatino and gravitino variations) can be recast as differential equations for pure spinors, formal objects (sums of even and odd forms) which encode the geometrical properties of the background. The GCG can be exploited also in the analysis of non compact supergravity solutions, and hence in the context of AdS/CFT correspondence. The description of the supergravity solutions dual to the marginal and to the mass deformation of $\mathcal{N} = 4$ SYM has been recently given in the language of GCG, as manifolds with $SU(2)$ structure. Moreover in the GCG formalism the supersymmetry condition for probe D-brane can be formulated as differential equation on the pullback of the pure spinors on the world volume of the brane.

In the last chapter of the thesis, based on [5], we study, with the GCG formalism, supersymmetric embeddings of D-branes in $SU(2)$ structure manifolds, covering the case of the gravity dual of massive and marginal deformation of $\mathcal{N} = 4$ SYM. We find supersymmetric configurations of different dimensionality and we propose their dual gauge theory interpretations. The topic of this last chapter is not directly related to the first part. Nevertheless, it is known that a method to break supersymmetry in the supergravity framework can be the insertion of objects, i.e. branes, that do not preserve supersymmetry. Thus, having as final purpose the breaking of supersymmetry, the analysis of supersymmetric configuration is anyhow the first step.

Chapter 1

Prologue on $\mathcal{N} = 1$ gauge theories

In this chapter we review some aspects of $\mathcal{N} = 1$ gauge theories which are relevant for non perturbative analysis. There are many textbooks [6, 7, 8, 9] and reviews of this subject [10, 11, 12, 13, 17, 18, 14, 15, 16, 19]. Some of them are introductions to supersymmetric gauge theories, others are more focused on electric magnetic duality or on supersymmetry breaking. We refer to these references for exhaustive introductions and explanations. As mentioned in the introduction, our purpose is to provide the basic elements to understand the wide subject of supersymmetry breaking in $\mathcal{N} = 1$ gauge theories, that often trace back to very old results and on the other hand can involve some very recent developments.

In the first part we give some basic elements of supersymmetry breaking. We explain the possible mechanisms to break supersymmetry, discussing their main properties, and we give some clarifying example. Then we move to quantum properties of supersymmetric gauge theories, setting the basis for a detailed non perturbative analysis. We analyze the supersymmetric version of QCD, exploring the classical moduli space of vacua and then investigating the quantum properties. This analysis brings to many unexpected and interesting results, such as dynamical supersymmetry breaking and electric magnetic duality. Finally, we present a quite detailed study of electric magnetic duality for theories with adjoint fields.

1.1 Order parameter for supersymmetry breaking

The order parameter for global supersymmetry breaking is the vacuum energy. This can be obtained from the supersymmetry algebra, which contains the translation operator P_μ

$$\{Q_\alpha \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu \tag{1.1}$$

where the Q's are the supersymmetry generators. The Hamiltonian can then be rewritten as

$$H = P^0 = \frac{1}{4}\left(\bar{Q}_1 Q_1 + \bar{Q}_2 Q_2 + \quad \text{h.c.}\right) \tag{1.2}$$

This is a non negative definite operator and its expectation values on the vacuum $|0\rangle$ is

$$\langle 0|H|0\rangle = \frac{1}{4}\left(|Q_1|0\rangle|^2 + |\bar{Q}_1|0\rangle|^2 + |Q_2|0\rangle|^2 + |\bar{Q}_2|0\rangle|^2\right) \geq 0 \tag{1.3}$$

A supersymmetric vacuum is annihilated by the supersymmetry generators, and hence has zero energy

$$E_{susy} = \langle 0|H|0\rangle = 0 \tag{1.4}$$

On the other hand, a supersymmetry breaking vacuum is not invariant under supersymmetry transformations, so $Q_\alpha|0\rangle \neq 0$ and $\bar{Q}_{\dot{\alpha}}|0\rangle \neq 0$, and hence its energy is positive

$$E_{nonsusy} = \langle 0|H|0\rangle > 0 \tag{1.5}$$

In order to know wheter global supersymmetry is spontaneously broken we therefore need to study the minima of the scalar potential, and see whether there is a minimum with zero energy.

1.2 $\mathcal{N} = 1$ global supersymmetry

Now, consider $\mathcal{N} = 1$ supersymmetric theories described by the following Lagrangian

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} K(\Phi_i^\dagger, \Phi^i) + \int d^2\theta W(\Phi_i) + \quad \text{h.c} \tag{1.6}$$

Here K is the Kahler potential and W is the superpotential for the chiral superfields Φ_i. The scalar potential is given by

$$V_F = K_{i\bar{j}}^{-1} \frac{\partial W}{\partial \Phi_i} \frac{\partial \bar{W}}{\partial \Phi_j^\dagger} = K_{i\bar{j}}^{-1} F_i \bar{F}_j \tag{1.7}$$

where

$$K_{i\bar{j}} = \frac{\partial^2 K}{\partial \Phi_i \partial \Phi_{\bar{j}}^\dagger} \tag{1.8}$$

The Kahler potential, where not specified, is taken as canonical, that is sum of the squared moduli of the fields.

If there are gauge interactions in the theory the scalar potential has an additional contributions

$$V = V_F + V_D \qquad V_D = \frac{1}{2}\sum_a (D^a)^2 \tag{1.9}$$

where $D^a = -g(\Phi_i^\dagger (T^a)^i_j \Phi^j)$, a runs on the generators of gauge group and the field involved are the charged ones. The Kahler potential for the charged field is modified as $K(\Phi_i^\dagger, e^{2q_i V}\Phi^i)$. We assume that the gauge group is simple so there is a single coupling constant g. The kinetic term for the gauge interaction is

$$\frac{1}{16\pi i}\int d^2\theta\tau\mathcal{W}_\alpha\mathcal{W}^\alpha + \quad \text{h.c} \tag{1.10}$$

where $\mathcal{W}_\alpha$ is the superfield strenght of the gauge superfield V and $\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}$ is the holomorphic gauge coupling.

As expected the scalar potential (1.9) is non negative. To look for zeros of the scalar potential in field space we have to find the subspace of fields for which $D^a = 0$. This is called the space of D-flat directions. If for a subspace of the D-flat directions we also have $F_i = 0$ for all $F_i's$ then the potential is zero. The space of field where this happens is the moduli space of (supersymmetric) vacua [21, 20].

Supersymmetry is broken by non zero F and/or D vacuum expectation value. Here we mainly discuss the case of F term supersymetry breaking.

1.3 Supersymmetry breaking

We are interested in mechanism of supersymmetry breaking. The breaking of a global symmetry can be spontaneous, if the symmetry of the action is not preserved by the vacuum. The symmetry can be instead explicitly broken, if we add terms in the Lagrangian that are not invariant under that symmetry. In supersymmetric gauge theories both method can be realized.

1.3.1 Explicit (soft) supersymmetry breaking

The explict supersymmetry breaking consist in adding terms in the Lagrangian that are not invariant under a supersymmetry trasformation. Tipycally they could be masses for some components in a supermultiplet, breaking the mass degeneracy. One of the main advantage of supersymmetric theories is their renormalizability properties, which follow from cancellations in the perturbative computation between fermionic and bosonic degrees of freedom. Hence we would like to keep this good UV behaviour when adding the explicit supersymetry breaking terms. The explicit terms of supersymmetry breaking with this property has been studied in [22]. They found all the possible terms which can be added to a supersymmetric Lagrangian without introducing quadratic divergencies, the so called soft terms. The soft breaking possibilities are given by the following recipe [22]: given any renormalizable superspace action, add to it terms which are product of ordinary superfields (and their derivatives) and of an x-independent but θ-dependent superfield (i.e. a spurion), restricted by the condition that if the spurion is set to 1, the resulting term leads to a renormalizable action or is a total derivative. The soft breaking possibilities are few, and given by the following terms in the Lagrangian

$$
\begin{aligned}
L_{break} &= \int d^2\theta\ \chi\ \Phi^2 + h.c. \qquad \chi = \mu^2\theta^2 \\
L_{break} &= \int d^2\theta\ \eta\ \Phi^3 + h.c. \qquad \eta = \mu\theta^2 \\
L_{break} &= \int d^2\theta\ \eta\ \mathcal{W}_\alpha\mathcal{W}^\alpha + h.c. \qquad \eta = \mu\theta^2 \\
L_{break} &= \int d^2\theta d^2\bar{\theta}\ U\ \Phi\Phi^\dagger \qquad U = \mu^2\theta^2\bar{\theta}^2 \\
L_{break} &= \int d^2\theta d^2\bar{\theta}\ U\ D^\alpha(\Phi\mathcal{W}_\alpha) + h.c \qquad U = \mu^2\theta^2\bar{\theta}^2
\end{aligned}
\tag{1.11}
$$

The first four of them can be easily obtained promoting the coupling constants in the supersymmetric Lagrangian to superfields which have aquired a non trivial expectation values for their auxiliary component. Observe that the third soft term is a mass for the gaugino field, since in component it leads to

$$
L_{break} = \frac{1}{2}\mu\lambda^\alpha\lambda_\alpha + h.c \tag{1.12}
$$

1.3.2 Spontaneous supersymmetry breaking

The spontaneous breaking of supersymmetry is interesting since is not introduced by hand in the Lagrangian. However, the supertrace theorem constraint too much the mass spectrum of a theory with spontaneous supersymmetry breaking. This theorem can be avoided via the gauge or gravitational mediation mechanism. In many phenomenological models supersymmetry is spontaneously broken in a hidden sector; the breakig is then communicated via gauge or gravitational interactions to the visible sector (a supersymmetric extension of the Standard model) where it give raise to soft supersymmetry breaking terms. This is the reason why spontaneous supersymmetry breaking is strongly studied for phenomenological applications. The spontaneous supersymmetry breaking can also be dynamical when it is not a tree level effect. Typically dynamical supersymmetry breaking is obtained with the addition of non perturbative contributions to the superpotential which drive the breaking.

Spontaneous supersymmetry breaking and the Goldstino

As already explained, spontaneous supersymmetry breaking occurs when the action is supersymmetric but the vacuum is not. A vacuum is a Lorentz invariant stable configuration. Lorenz invariance implies that all space time derivatives and all fields that are not scalars must vanish. Hence only scalar fields ϕ_i can have vacuum expectation value $\langle\phi_i\rangle$. Thus a vacuum is where

$$
\langle A_\mu^a\rangle = \langle\lambda^a\rangle = \langle\psi^i\rangle = \partial_\mu\langle\phi_i\rangle = 0 \tag{1.13}
$$

and the scalar potential $V(\langle\phi^i\rangle, \langle\phi_i^\dagger\rangle)$ is in a minimum. The minimum may be the global minimum of V, i.e. a true vacuum, or a local minimum of V. In this last case it is a false vacuum which decay via tunneling effects into the true vacuum. If the lifetime of this false vacuum is large enough, it could have phenomenological applications.

The minimum condition states [1]

$$\frac{\partial V}{\partial \phi^i} = \frac{\partial V}{\partial \phi_i^\dagger} = 0 \tag{1.14}$$

As already mentioned, in a supersymmetric theory the scalar potential is given by the sum of F and D terms

$$V = F_i^\dagger F^i + \frac{1}{2} D^a D^a \tag{1.15}$$

where

$$F_i^\dagger = \frac{\partial W}{\partial \phi^i} \qquad D^a = -g(\phi_i^\dagger (T^a)^i_j \phi^j + \xi^a) \tag{1.16}$$

Here W is the superpotential and ξ^a are the allowed Fayet Iliopoulos terms for $U(1)$ factors.

The scalar potential is non negative and it vanishes if $F^i = D^a = 0$. This is a global minimum of the scalar potential where supersymmetry is unbroken. There could be many of such true vacua. There could also be a manifold of such vacua (a moduli space), parametrized by the vacuum expectation values of the fields.

On the other hand, there can not be any solution to the equation $V = 0$. Hence the minimum of the scalar potential, where $V = V_{MIN} > 0$, is a vacuum with spontaneous breaking of supersymmetry.

The spontaneous breaking of a global symmetry always implies a massless Goldstone mode with the same quantum numbers as the broken symmetry generators. In the case of global supersymmetry, the broken generator is the fermionic charge Q_α. Hence the Goldstone particle is a massless Weyl fermion, called the Goldstino. The fermionic degrees of freedom in a supersymmetric $\mathcal{N} = 1$ gauge theory are the gaugino λ^a and the fermionic component of the chiral fields ψ^i. The mass matrix, in the (ψ_i, λ_a) basis, is

$$M = \begin{pmatrix} \langle \frac{\partial^2 W}{\partial \phi^i \phi^j} \rangle & -g\langle \phi_l^\dagger (T^a)^l_i \rangle \\ -g\langle \phi_l^\dagger \rangle (T^b)^l_j & 0 \end{pmatrix} \tag{1.17}$$

Now, the condition for the minimum of the scalar potential leads to

$$0 = \frac{\partial V}{\partial \phi^i} = F^j \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} - g^a D^a \phi_j^\dagger (T^a)^j_i \tag{1.18}$$

and the gauge invariance of the superpotential gives

$$0 = \delta^a_{gauge} W = \frac{\partial W}{\partial \phi^i} \delta^a_{gauge} \phi^i = F_i^\dagger (T^a)^i_j \phi^j \tag{1.19}$$

[1] The Kahler is taken to be canonical.

Combining these condition we obtain the following matrix equation

$$M\tilde{G} = 0 \qquad \tilde{G} = \begin{pmatrix} \langle F^j \rangle \\ \langle D^a \rangle \end{pmatrix} \tag{1.20}$$

which states that M annihilates the vector $\tilde{G}$. This means that the mass matrix M has a zero eigenvalue. The Goldstino is the fermionic fields corresponding to the eigenvector with zero mass, schematically

$$\psi_G \sim \sum_i \langle F_i \rangle \psi_i + \sum_a \langle D^a \rangle \lambda^a \tag{1.21}$$

It is non trivial if and only if at least one of the auxiliary fields has a vacuum expectation value, breaking supersymmetry. So we have prooved that if global supersymmetry is spontaneously broken, then there must be a massless Goldstino, and that its component among the various fermions in the theory are just proportional to the corresponding auxiliary field VEV's.

Supertrace theorem In theories with spontaneous symmetry breaking there is a useful sum rules that governs the tree level mass squared of particles. If supersymmetry is unbroken all particles within a supermultiplet have the same mass. If supersymmetry is broken this is no longer true, but the mass splitting in the multiplet can be computed as a function of the supersymmetry breaking parameters, i.e. the VEVs of the auxiliary fields. We introduce the short notations

$$D_i^a = \frac{\partial D^a}{\partial \phi^i} = -g(\phi^\dagger T^a)_i \qquad D^{ia} = \frac{\partial D^a}{\partial \phi_i^\dagger} = -g(T^a \phi)^i \qquad D_j^{ai} = -g T_j^{ai} \tag{1.22}$$

$$F^{ij} = \frac{\partial^2 \bar{W}}{\partial \phi_i^\dagger \partial \phi_j^\dagger} \qquad F_{ij} = \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \tag{1.23}$$

The mass matrix for the vectors is

$$(m_V^2)^{ab} = 2\langle D_i^a \rangle \langle D^{bi} \rangle \tag{1.24}$$

The vectors become massive by the standard Higgs mechanism when the charged scalars get VEVs.

The fermionic mass matrix is

$$m_F = \begin{pmatrix} \langle F_{ij} \rangle & \sqrt{2} i \langle D_i^b \rangle \\ \sqrt{2} i \langle D_j^a \rangle & 0 \end{pmatrix} \tag{1.25}$$

so the squared fermion mass matrix is

$$m_F m_F^\dagger = \begin{pmatrix} \langle F_{ij} \rangle \langle F^{ij} \rangle & -\sqrt{2} i \langle F_{ij} \rangle \langle D_i^b \rangle \\ \sqrt{2} i \langle D_j^a \rangle \langle F^{ij} \rangle & 2\langle D_j^a \rangle \langle D_i^b \rangle \end{pmatrix} \tag{1.26}$$

while the scalar mass matrix is

$$m_B^2 = \begin{pmatrix} \langle F_{il}\rangle\langle F^{kl}\rangle + \langle D^{ak}\rangle\langle D^a_i\rangle + \langle D^a\rangle D^{ak}_i & \langle F^l\rangle\langle F_{ilj}\rangle + \langle D^a_i\rangle\langle D^a_j\rangle \\ \langle F^\dagger_l\rangle\langle F^{pkl}\rangle + \langle D^{ap}\rangle\langle D^{ak}\rangle & \langle F_{jl}\rangle\langle F^{pl}\rangle + \langle D^{ap}\rangle\langle D^a_j\rangle + \langle D^a\rangle D^{ap}_j \end{pmatrix} \tag{1.27}$$

We can then compute the supertrace of the tree level squared mass matrices, defined as a weighted sum over particles with spin j

$$\mathrm{STr}(m^2) = \sum_j (-1)^j (2j+1)\mathrm{Tr}(m_j^2) \tag{1.28}$$

The supertrace results

$$\mathrm{STr}(m^2) = \mathrm{Tr}(m_B^2) - 2\mathrm{Tr}(m_F^\dagger m_F) + 3\mathrm{Tr}(m_V^2) = -2g^2\langle D^a\rangle \mathrm{Tr}T^a \tag{1.29}$$

We see that if $\langle D^a\rangle$ or $\mathrm{Tr}T^a = 0$ this supertrace vanishes. In particular the supertrace vanishes for F-term supersymmetry breaking or if all gauge group generators are traceless. In general the supertrace vanishes if the traces of the $U(1)$ charges over the chiral superfields are 0. This holds for any non anomalous gauge symmetry.

The supertrace theorem then states that the sum of the squared masses of all bosonic degrees of freedom equals the sum for all fermionic ones. In supersymmetric vacuum this is a trivial statement. In vacuum with spontaneous breaking of supersymmetry this is a very strong condition, which constraints the mass spectrum. This theorem have to be eluded in order to obtain sensible phenomenology from spontaneous supersymmetry breaking.

1.3.3 R-symmetry

The description of supersymmetric Lagrangian in superspace naturally suggests that the complex rotation of the fermionic coordinate θ_α should be a symmetry

$$\theta \to e^{i\alpha}\theta \tag{1.30}$$

This is the R symmetry. The component of a superfield are charged under this $U(1)_R$ symmetry. For a chiral superfield Φ with components ϕ the charges are as follows

$$\phi \to e^{in\alpha}\phi \tag{1.31}$$

$$\psi \to e^{i(n-1)\alpha}\psi \tag{1.32}$$

$$F \to e^{i(n-2)\alpha}F \tag{1.33}$$

By convenction, chiral supermultiplets are always labeled by the R-charge of their scalar component, in this case n.

R-symmetry may be broken if the superpotential does not transform correctly. Since the term in the Lagrangian following from the superpotential is

$$\mathcal{L} = \int d\theta^2 W = W_{|\theta^2} \tag{1.34}$$

the superpotential should have charge 2 under R symmetry

$$W \rightarrow e^{2i\alpha} W \,. \tag{1.35}$$

The Kahler potential should instead be neutral under R symmetry.

Clearly the charge assignement for the chiral fields Φ has to be done coherently with the requirement that the superpotential has R charge 2.

Finally, the kinetic term for the vector bosons should also have R charge 2 since

$$\mathcal{L}_{kin} = \int d\theta^2 \tau \mathcal{W}_\alpha \mathcal{W}^\alpha \tag{1.36}$$

Then the R charge is usually[2] normalize such that the charge of the gaugino λ_a is 1

$$\lambda_a \rightarrow e^{i\alpha} \lambda_a \tag{1.37}$$

and the charge of the gauge boson is zero. This also implies that the Kahler potential for the charged matter fields is neutral. Differently from the chiral superfields, the charge assignement for the gaugino is quite general (it does not depend on the form of the superpotential), and it is related to the requirement that tha kinetic term for the gauge fields has the right R charge.

It often happens that the canonical R symmetry is anomalous. In that case, it is sometimes possible to form a non anomalous R symmetry by combining the R symmetry with other global $U(1)$'s.

Gaugino mass and R symmetry It is very instructive to observe the R symmetry properties of the soft supersymmetry breaking terms that we have introduced previously. Such terms can be generated by quantum corrections in effective theories from interactions between an hidden sector with spontaneous breaking of supersymmetry and the visible sector. This procedure is crucial in order to avoid the supertrace theorem which constraints too much the tree level spectrum of a theory with spontanoues supersymmetry breaking, as will be explained in chapter 3.

As already mentioned, the R charges for the chiral fields can be model dependent, whereas the charge of the gauge superfields is fixed. Observing the soft supersymmetry breaking terms (1.11,1.12), and comparing with the expression for the R charges (1.37) we conclude that the term (1.12), i.e. the gaugino mass, is not R preserving. Hence if a supersymmetric theory have a vacuum which breaks supersymmetry but preserves R symmetry, the radiative corrections cannot

[2]Unless we give strange R charges to the coupling τ

generate a gaugino mass term. Then the low energy spectrum would have a massless fermion in the adjoint representation, which is not phenomenologically sensible.

Even if the R symmetry is anomalous and broken by quantum effects to a discrete subgroup, this problem is not solved unless this dicrete subgroup is reduced to Z_2, and in R symmetry preserving vacua a gaugino mass is forbidden.

1.3.4 F-term susy breaking: O'Raifeartaigh model

Models where spontaneous supersymmetry is achieved via non trivial F-term VEVs are known as O'Raifeartaigh models. The idea is to take a set of chiral superfields Φ_i and a superpotential such that the F-equations $F_i = 0$ cannot be all satisfied. Hence the scalar potential $V = \sum_i |F_i|^2$ is different from zero and supersymmetry is broken.

For semplicity we consider a theory of pure chiral fields, i.e. a theory without gauge interactions, and with canonical Kahler. If the superpotential has no linear term, the vacuum $\langle \phi_i = 0 \rangle$ will always be a supersymmetric solution.

We take a set of three chiral fields with canonical Kahler potential

$$K = \Phi_1 \bar{\Phi}_1 + \Phi_2 \bar{\Phi}_2 + X \bar{X} \tag{1.38}$$

and superpotential with a linear term

$$W = \mu X + \frac{1}{2} h X \Phi_1^2 + m \Phi_1 \Phi_2 \tag{1.39}$$

This is the basic O'Rafeirtaigh model. The R charges are $R[X] = R[\Phi_2] = 2$ and $R[\Phi_1] = 0$. The F-terms are

$$F_1^\dagger = X \phi_1 + m \phi_2 \qquad F_2^\dagger = m \phi_1 \qquad F_X^\dagger = \mu + \frac{1}{2} h \phi_1^2 \tag{1.40}$$

The two conditions $F_2 = 0$ and $F_X = 0$ are not compatible. Hence supersymmetry is broken. If $m^2 > h\mu$ the absolute minimum of the potential is at $\phi_1 = \phi_2 = 0$ with $\langle X \rangle$ undetermined. The X field is a flat direction in the potential

$$m_X = 0 \qquad m_{\psi_X} = 0 \tag{1.41}$$

The tree level mass squared for the other fields are

$$m_{1/2}^2 = \frac{1}{4} (|hX| \pm \sqrt{|hX|^2 + 4|m|^2})^2 \tag{1.42}$$

for the fermions, and

$$m_0^2 = \left(|m|^2 + \frac{1}{2} \eta |h\mu| + \frac{1}{2} |hX|^2 \pm \frac{1}{2} \sqrt{|h\mu|^2 + 2\eta |h\mu| |hX|^2 + 4|m|^2 |hX|^2 + |hX|^4} \right) \tag{1.43}$$

for the four real scalars ($\eta = \pm 1$). The non degeneracy of the masses of the scalars and of the fermions is a signal of supersymmetry breaking. The massless scalar field X parametrize the pseudomoduli space of vacua. The spectrum changes along this pseudomoduli space parametrized by X; these vacua are physically distinct. The massless fermion ψ_X is the Goldstino associated to the spontanoues breaking of supersymmetry, indeed from (1.21) we obtain that

$$\psi_G = \langle F_X \rangle \psi_X \tag{1.44}$$

The flat direction (the X direction) of the scalar potential is lifted by quantum corrections. Indeed computing the one loop Coleman Weimberg effective potential, we obtain that the scalar field X gets positive squared masses, and it is stabilized at $\langle X \rangle = 0$, which is the true global minimum at quantum level, and where R-symmetry is preserved. Of course the fermion ψ_X does not receive mass from quantum corrections, being associated to the broken supersymmetry. The parameter of supersymmetry breaking $\sqrt{F_X} \sim \mu$ is a tree level parameter of the superpotential. This problem is addressed in models of dynamical supersymmetry breaking, where the breaking scale is generated by the dynamics.

Deforming the O'Raifeartaigh model We now consider a small deformation to the prevoius model. It consist in adding the following term to the superpotential

$$W = \mu X + \frac{1}{2} h X \Phi_1^2 + m \Phi_1 \Phi_2 + \frac{1}{2} \epsilon m \Phi_2^2 \tag{1.45}$$

This term breaks explicitly R-symmetry, and could also lead to a supersymmetric vacuum. Indeed now the F-term equations

$$F_X^\dagger = \frac{1}{2} h \phi_1^2 + \mu \qquad F_1^\dagger = h X \phi_1 + m \phi_2 \qquad F_2^\dagger = m \phi_1 + \epsilon m \phi_2 \tag{1.46}$$

can have a solution

$$\langle \phi_1 \rangle = \pm \sqrt{-2\frac{\mu}{h}} \qquad \langle \phi_2 \rangle = \mp \frac{1}{\epsilon} \sqrt{-2\frac{\mu}{h}} \qquad \langle X \rangle = \frac{m}{h\epsilon} \tag{1.47}$$

However if we consider the parameter ϵ to be very small the supersymmetric vacuum is mooved very far in the field space. Moreover the potential near the origin of the field space is not modified a lot. This means that the supersymmetry breaking vacuum we have found in the previous model is now a local minimum of the scalar potential, not the global one, and hence is a false vacuum. However, if the parameter ϵ is sufficiently small, the supersymmetry breaking false vacuum can have a large lifetime, i.e. it can take a long time in decaying to the true (supersymmetric) vacuum. Hence the non supersymmetric vacuum is not stable, but it is metastable.

1-loop correction As we have seen in the previous examples, models of tree-level supersymmetry breaking often have a moduli space of vacua. The flat directions of the scalar potential can be associated to broken global symmetries, and hence are Goldstone bosons, and they remain flat even at quantum level. On the other hand the pseudoGoldstone bosons are classical flat directions non associated to any broken global symmetry, and hence not protected from quantum corrections. For example the scalar partner of the massless Goldstino is a massless field at classical level, but it could acquire positive or negative mass at quantum level.

In this sense the *pseudomoduli* space is a classical moduli space of vacua which could be lifted by quantum corrections.

The Coleman-Weimberg effective potential for the psudomoduli is obtained computing the 1-loop corrections to the vacuum energy

$$V_{eff}^{1loop} = \frac{1}{64\pi^2} S\mathrm{Tr}\left(m^4 \log \frac{m^2}{\Lambda_{UV}^2}\right) = \tag{1.48}$$

$$= \frac{1}{64\pi^2}\left[\mathrm{Tr}\left(m_B^4 \log \frac{m_B^2}{\Lambda_{UV}^2}\right) - 2\mathrm{Tr}\left(m_F^4 \log \frac{m_F^2}{\Lambda_{UV}^2}\right)\right] \tag{1.49}$$

where Λ_{UV} is a ultraviolet cutoff, and m_B^2 and m_F^2 are the tree-level bosons and fermion masses (1.26,1.27). They can be function of the pseudomoduli vevs; in this sense (1.48) is an effective potential for the pseudomoduli, which can fix their vevs after minimization.

The logarithmic divergent term ($\log \Lambda_{UV}\, S\mathrm{Tr}\ m^4$) in (1.48) is usually absorbed in the renormalization of the tree level coupling constants, since $S\mathrm{Tr}\ m^4$ is pseudomoduli independent. The effective potential for the pseudomoduli is hence given by the $S\mathrm{Tr}\ m^4 \log m^2$ term in (1.48).

Finally we remind that the effective potential also include two other contributes. The first one is quartic divergent, and is proportional to $\Lambda_{UV}^4\, S\mathrm{tr}\ \mathbf{1}$; the second one is quadratic divergent, proportional to $\Lambda_{UV}^2\, S\mathrm{Tr}\ m^2$. They both vanish in supersymmetric theories and also in theories with spontaneous breaking of supersymmetry due to the supertrace theorem, which guarantees that $S\mathrm{Tr}\ m^2 = 0$.

1.3.5 Dynamical supersymmetry breaking

In the previuos sections we have studied mechanism to spontaneous break supersymmetry. This is achieved when the Lagrangian is manifestly supersymmetric, whereas the vacuum is not. The superpotential and the Kahler potential give the scalar potential, whose minimization select the vacuum.

A supersymmetric vacuum is where the scalar potential vanishes $V = 0$. The non renormalization theorems state that the superpotential is not renormalized perturbatively. Hence if the theory admits a supersymmetric vacuum at tree-level, it couldn't be spoiled by perturbative corrections to the superpotential. The non renormalization theorem follows from the fact that the superpotential must be an holomorphic function of the fields and also of the couplings.

Then if supersymmetry is unbroken at tree level, it can only be broken by non perrturbative effects. In supersymmetric gauge theories, the holomorphic properties and the global symmetries of the theory can sometimes be enough to obtain the non perturbative corrections. This is why the study of non perturbative aspects of supersymmetric gauge theories is relevant also in connection with supersymmetry breaking.

Consider a theory with a tree-level supersymmetric vacuum. The energy of such vacuum is vanishing by definition. Only dynamical effects can modify this energy, and break supersymmetry. Then if the theory breaks supersymmetry spontaneously, via non perturbative effects, the supersymmetry breaking scale, i.e. the vacuum energy, is proportional to some strong coupling scale

$$E_{vac} \sim \Lambda \sim M_{cutoff} e^{-\frac{8\pi^2}{g^2(M_{cutoff})}} \tag{1.50}$$

where M_{cutoff} is a cutoff scale of the theory and $g(M_{cutoff})$ is the coupling constant of the interaction whose dynamics have driven the breaking, evaluated at the scale of the cutoff. Thus dynamical supersymmetry breaking can naturally generates large hierarchies [23], since this factor can be very small ($\sim 10^{-17}$).

The analysis of models with dynamical breaking of supersymmetry can be hard since the low energy of such models can be characterized by strongly coupled gauge theories. There are mainly three types of dynamical supersymmetry breaking models, depending on how much we can understand.

1. Using indirect arguments we can only conclude that supersymmetry is broken at the strong coupling scale, but we cannot obtain the potential and the spectrum.

2. We can derive the superpotential at low energy, in variables such that tha kahler potential is not singular, and conclude that supersymmetry is broken by non trivial F-terms. However we cannot compute the spectrum of the theory and the supersymmetry breaking scale.

3. We can calculate the superpotential, and we can also calculate the Kahler potential, since we find range of the parameters such that the theory is weakly coupled. For example this can be achieved by a dual and infrared free description of the theory. Then we can compute the supersymmetry breaking scale and the spectrum.

1.3.6 Witten Index

We have seen that the hamiltonian of a supersymmetric theory is the square of the supercharge (1.2). The supersymmetry generators Q also transform bosonic state into fermionic state and viceversa. Together, these properties imply that bosonic and fermionic state with non zero energy always come in pairs. On the other hand, states with vanishing energy are annihilated by Q, and so they are not necessarily paired. This property is crucial in understanding the Witten index

[24], which is defined as

$$\mathrm{Tr}(-1)^F = \sum_E n_B(E) - n_F(E) = n_B(0) - n_F(0) \tag{1.51}$$

where the sum is taken over the whole states and $n_B(E)$ and $n_F(E)$ are respectively the number of bosonic and fermionic states with energy E. As mentioned, $n_B(E) = n_F(E)$ as soon as $E \neq 0$. Thus only the supersymmetry preserving vacua $E = 0$ can cotribute to the index. We conclude that a sufficient condition for unbroken supersymmetry is $\mathrm{Tr}(-1)^F \neq 0$. Conversely, a necessary condition for broken supersymmetry is that $\mathrm{Tr}(-1)^F$ vanishes.

The important property of this index is that it is a discrete quantity, and it is not modified by continuos deformations of the parameter of the theory. With continuos deformations we mean those that do not modify the asymptotic behaviour of the action in the field space. For instance, a change in the large field behaviour of the superpotential can affect the index.

Witten has shown that the index for pure supersymmetric gauge theories is different from zero. The index does not change if we add massive vector matter, as it can be calculated in the large mass limit, where we integrate out the matter fields and obtain pure SYM. Thus there are supersymmetric vacua for any value of the mass, even in the massless limit. This constraint on the existence of supersymmetric vacua in $\mathcal{N} = 1$ supersymmetric gauge theories with vector matter has much restricted potentially phenomenological applications. However a possible way out is to consider theories which admit supersymmetric vacua, and where supersymmetry breaking happens in a false but long living vacuum.

1.3.7 Breaking by Rank condition

We now consider a more complicated model which share many properties with the O'Raifeartaigh one and that will be crucial in the following chapters [25]. Consider a theory of pure chiral fields Φ, $\tilde{\Phi}$ and M with the following global symmetries

	$SU(\tilde{N})$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_A$	$U(1)_R$
Φ	$\tilde{N}$	$\bar{N}_f$	1	1	1	0
$\tilde{\Phi}$	$\bar{\tilde{N}}$	1	N_f	-1	1	0
M	1	N_f	$\bar{N}_f$	0	-2	2

(1.52)

where we work in the regime

$$\tilde{N} < N_f \tag{1.53}$$

Tha Kahler potential is taken to be canonical

$$K = \mathrm{Tr}\Phi^\dagger\Phi + \mathrm{Tr}\tilde{\Phi}^\dagger\tilde{\Phi} + \mathrm{tr}\ M^\dagger M \tag{1.54}$$

and the non trivial superpotential is

$$W = h\mathrm{Tr}\Phi M\tilde{\Phi} - hm^2\mathrm{tr}\ M \tag{1.55}$$

The first term is the more general interaction compatible with the symmetries in 1.52. The second term breaks the global symmetries to $SU(\tilde{N}) \times SU(N_f)_D \times U(1)_B \times U(1)_R$ where $SU(N_f)_D$ is the diagonal subgroup of the two flavour symmetries $SU(N_f)$.

We compute the F term equation for the field M

$$-(F_M^\dagger)^{ij} = h\Phi_c^i \tilde{\Phi}^{jc} - hm^2\delta^{ij} \qquad i,j = 1\ldots N_f,\ c = 1\ldots\tilde{N} \tag{1.56}$$

This is an $N_f \times N_f$ matrix relation. However, the first term of (1.56) is a matrix of rank $\tilde{N}$, since (1.53). On the other hand, δ^{ij} has rank N_f and so the $F_M^\dagger$ cannot all vanish and supersymmetry is spontaneously broken.

The minimum of the scalar potential is

$$V_{min} = (N_f - \tilde{N})|h^2m^4| \tag{1.57}$$

and the moduli space of vacua is parametrized by

$$M = \begin{pmatrix} 0 & 0 \\ 0 & X_{N_f-\tilde{N}\times N_f-\tilde{N}} \end{pmatrix} \qquad \Phi = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix} \qquad \tilde{\Phi}^T = \begin{pmatrix} \tilde{\varphi}_0 \\ 0 \end{pmatrix} \tag{1.58}$$

with arbitrary X, and φ_0,$\tilde{\varphi}_0$ subjected to the constraint

$$\tilde{\varphi}_0\varphi_0 = m^2\mathbf{1}_{\tilde{N}} \tag{1.59}$$

In order to understand better the classical flat directions of this moduli space we select a point and look at the fluctuation [25]. We choose the vacua of maximal unbroken global symmetry

$$X = 0 \qquad \tilde{\varphi}_0 = \varphi_0 = m\mathbf{1}_{\tilde{N}} \tag{1.60}$$

and parametrize the fluctuation with fields ξ_i and ϕ_i and θ

$$M = \begin{pmatrix} \xi_1 & \phi_5 \\ \phi_6 & X \end{pmatrix} \qquad \Phi = \begin{pmatrix} \varphi_0 e^{\theta} + \xi_2 \\ \phi_1 \end{pmatrix} \qquad \tilde{\Phi}^T = \begin{pmatrix} \tilde{\varphi}_0 e^{-\theta} + \xi_3 \\ \phi_2 \end{pmatrix} \tag{1.61}$$

where in the quarks we have separeted the trace part of the fluctuation, θ, and the traceless part ξ_2, ξ_3. Here the global symmetries are broken to $SU(\tilde{N})_D \times SU(N_f - \tilde{N}) \times U(1)_{B'} \times U(1)_R$ where the first factor is the diagonal subgroup of the product $SU(\tilde{N}) \times SU(\tilde{N})_F$ (with $SU(\tilde{N})_F$ we denote one part of the diagonal flavour group $SU(N_f)_D \to SU(\tilde{N})_F \times SU(N_f - \tilde{N}) \times U(1)$).

The tree level scalar potential gives mass to most of the fluctuations. The massless Goldstino comes from the fermionic component of X. Some of the massless fluctuations are Goldstone bosons, corresponding to broken global symmetries, and they are exactly massless also at quantum level. The remaining massless fields, called pseudoGoldstone, are not associated to any global broken symmetry and they can acquire positive (stabilizing the vacuum) or negative masses (leading to runaway directions) by quantum corrections. They are

$$X \qquad \theta + \theta^\star \qquad \delta\chi = \frac{m^\star}{|m|}(\xi_2 - \xi_3) + h.c. \tag{1.62}$$

These pseudo moduli can acquire masses, starting at 1 loop, from their couplings to the other massive fields. The one loop contribution is the dominant one since the coupling h is marginally irrelevant in the infrared. The field X can be set in diagonal form using the residual flavour symmetry $SU(N_f - \tilde{N})$.

The behaviour of these directions can be understood better looking at the superpotential for the fluctuations. Inserting the expression for the fluctuations in the tree level superpotential and keeping up to trilinear order we obtain[3]

$$W_{fluct} = h\left(me^{\theta}\phi_5\phi_2 + me^{-\theta}\phi_4\phi_1 + \phi_1 X\phi_2 - m^2 X + me^{\theta}\xi_1\xi_3 + me^{-\theta}\xi_2\xi_1\right) + \dots \tag{1.63}$$

where the dots are higher order terms (typically third order in the fluctuations ϕ_i and ξ_i). We easily recognize a structure reminding the O'Raifeartaigh model, where the field X drives the spontaneous supersymmetry breaking. We are interested in computing the 1 loop corrections as a function of the pseudomoduli, and so we focus on the fluctuations which give non trivial contributions to the mass matrix (1.26,1.27). In particular, the fields ξ_i couple only with cubic interactions to the fields ϕ_i, which themselves do not appear at linear order[4]. Hence the ξ_i do not contribute to the bosonic mass matrix and constitute a decoupled supersymmetric sector, at this order. Neglecting them the fluctuation superpotential get a very simple expression

$$W_{fluct} = h\left(me^{\theta}\phi_5\phi_2 + me^{-\theta}\phi_4\phi_1 + \phi_1 X\phi_2 - m^2 X\right) \tag{1.64}$$

Here we recognize exactly $N_f - \tilde{N}$ copies of O'Raifeartaigh models. A calculation then yields the 1 loop effective potential as a function of the pseudomoduli

$$V_{eff}^{1-loop} = const + \frac{h^4 m^2(\log 4 - 1)\tilde{N}(N_f - \tilde{N})}{8\pi^2}\left(\frac{1}{2}m^2(\theta + \theta^\star) + |X|^2\right) \tag{1.65}$$

showing that the directions X and $\theta + \theta^\star$ are stabilized by quantum corrections. Hence the perturbative corrections remove the classical degeneracy and select the vacuum (1.60). The tracelessness part $\delta\chi$ is still a potentially dangerous flat directions which could destabilize this minimum. We will see that this direction can be stabilized with other mechanisms.

We have shown that this simple model exhibits spontaneous breaking of supersymmetry via the so called rank condition. The moduli space of vacua is lifted by quantum corrections which

[3] The trace is implicit.

[4] Only X appears at linear order.

select the vacuum (1.60) as the quantum minimum giving most of the pseudoGoldstone positive masses. The spectrum of the theory in this vacuum has a hierarchy of mass scales, dictated by the marginally irrelevant coupling h. Some fields have tree level masses proportional to $|hm|$, and the pseudomoduli acquire masses $\sim |h^2 m|$. The Goldstone bosons remain exactly massless also at quantum level and there is also a massless Goldstino associated to the breaking of supersymmetry. Observe that R symmetry is preserved in the supersymmetry breaking vacuum (1.60).

1.4 Non perturbative aspects of supersymmetric gauge theories

The analysis of the vacuum structure of four dimensional supersymmetric gauge theories often rely on the notion of an infrared effective action. There are two object which are usually called effective action: the 1PI effective action and the Wilsonian effective action. One is the standard generating functional of one particle irreducible Feynman diagrams. It is obtained by a Legendre transformation, and the momenum integrations in loop diagrams is from zero to the UV cut off. The Wilsonian effective action, instead, include momentum integration from the UV cut-off down to a scale μ. This is simply a local action describing the theory's degrees of freedom at energies below a given energy scale μ, which is the characteristic scale of the effective theory. When there are no interacting massless particles, these two effective actions are identical. However, when interacting massless particles are in the spectrum, the 1PI effective action suffers from IR ambiguities. Moreover in supersymmetric gauge theories the 1PI might suffer from holomorphic anomalies, which spoil the holomorphic dependence of the effective action from the scale μ.

The procedure to get the infrared effective action mainly consists in guessing an IR effective field content for the microscopic UV theory under investigation, and write down all the possible IR effective action built with these fields and which are consistent with supersymmetry and with the other global symmetries of the UV theory. This could be a powerful method to analyze the vacuum structure of the theory. Indeed, in supersymmetric gauge theories, the selection rules deduced from the global symmetries of the UV theory and the holomorphy of the superpotential sometimes constraint the IR effective action sufficiently to obtain exact results.

1.4.1 Holomorphy and symmetries

The basic approach is to consider the low effective action for the light fields, after having integrated out all the degrees of freedom above some scale μ. Assuming that at this scale supersymmetry is unbroken, the fields can be organized in supermultiplets. The light matter fields can be combined in chiral multiplets, whereas the gauge fields combine into vector multiplets. A particular contribution to the effective Lagrangian which is very constrained by symmetries is the quantum effective superpotential $W_e f f(\Phi_i, g_I, \Lambda)$. It is a function of the light chiral fields, of the tree level coupling constants and of the scale Λ associated to the gauge dynamics.

The key fact is that supersymmetry requires W_{eff} to be holomorphic in the chiral superfields Φ_i. We can think of all the couplings g_I in the tree level superpotential and the scale Λ as background fields[5], i.e. that they are chiral superfields whose scalar component has get an expectation values. This implies that the effective superpotential should also be an holomorphic function of the couplings g_I and of the scale Λ. This is a crucial property for the non renormalization theorems of the superpotential.

Then we can find a large global symmetry group of the theory assigning transformation laws to chiral fields and to background fields. The effective quantum superpotential should be invariant under this large symmetry group.

Finally W_{eff} can be analyzed approximately in various limits, and sometimes the singularities can be identified. Being W_{eff} an holomorphic function, it is determined by its asymptotic behaviour and by its singularities, giving non trivial results (such as the non renormalization theorem).

1.4.2 Renormalization of the gauge coupling

We here analyze the renormalization properties of supersymmetric gauge theories. The one loop renormalization group equations implies that the coupling runs with the beta function

$$\beta(g) = -\frac{b_0}{(4\pi)^2} g^3 + O(g^5) \tag{1.66}$$

where b_0 is the beta function coefficient which depends on the charged field content of the theory. For a supersymmetric Yang Mills theory with gauge group G_c and chiral superfields in the representation r_i the beta coefficient is

$$b_0 = \frac{3}{2} T(Adj) - \frac{1}{2} \sum_i T(r_i) \tag{1.67}$$

where $T(r)$ is the index of the representation r. For example, for $SU(N)$, $T(Adj) = 2N$ and $T(fund) = 1$.

The solution for the running of the coupling at one loop is then

$$\frac{1}{g^2(\mu)} = -\frac{b_0}{8\pi^2} \ln\left(\frac{|\Lambda|}{\mu}\right) \tag{1.68}$$

where Λ is the intrinsic scale of the non abelian gauge theory that enters through dimensional transmutation[6] $\Lambda = \mu_0 e^{\frac{-8\pi^2}{b_0 g^2(\mu_0)}}$. It is the energy scale at which the coupling diverges, and it is independent of μ_0.

The supersymmetric kinetic term for the vector superfield is

$$\frac{1}{16\pi i} \int d^2\theta \tau(\mu) \mathcal{W}_\alpha \mathcal{W}^\alpha \tag{1.69}$$

[5] As done for the spurions

[6] μ_0 is a fixed scale where we fix the value of $g(\mu_0)$.

where $\mathcal{W}_\alpha$ is the superfield strenght and $\tau(\mu)$ is the complexified gauge coupling evaluated at the scale μ. Taking into account the one loop gauge coupling running we have

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2(\mu)} = \frac{1}{2\pi i}\ln\left(\frac{|\Lambda|}{\mu}\right)^{b_0} e^{i\theta} \tag{1.70}$$

This coupling include the θ parameter which appear in the Lagrangian as the coupling of the CP non invariant term $\sim F\tilde{F}$.

It is then natural to define a complex scale in supersymmetric gauge theories which encodes the information about the θ angle and the intrinsic gauge theory scale as

$$\Lambda = |\Lambda| e^{i\frac{\theta}{b_0}} = \mu_0 e^{\frac{2\pi i\tau}{b_0}} \tag{1.71}$$

Such that the running of the holomorphic coupling is simply

$$\tau = \frac{b_0}{2\pi i}\ln\left(\frac{\Lambda}{\mu}\right) \tag{1.72}$$

but this renormalization effect can be further modified by non perturbative contributions.

Renormalization of the gauge coupling Let us assume that we are dealing with an asymptotically free theory, so if we take the scale $\mu \gg \Lambda$, then the theory is weakly coupled. We analyze the effective theory for energy lower than μ. If the theory remains weakly coupled, we can describe it with an effective theory with the same degrees of freedom. The effective kinetic term at a scale μ is then

$$W_{eff} = \tau(\Lambda,\mu)\mathcal{W}_\alpha \mathcal{W}^\alpha \tag{1.73}$$

where now $\tau(\Lambda,\mu)$ is the effective holomorphic coupling which include the perturbative and the non perturbative contributions. The non perturbative corrections to τ are constrained by the symmetry and by the fact that τ should be itself an holomorphic function. Hence the more general corrections have the expression

$$\tau(\Lambda,\mu) = \frac{b_0}{2\pi i}\ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda,\mu) \tag{1.74}$$

where $f(\Lambda,\mu)$ is an holomorphic function of Λ. Since in the weak couling limit $\Lambda \to 0$ we must recover the perturbative result (1.72), $f(\Lambda,\mu)$ must have positive series expansion in Λ. Moreover the physics must be periodic in the θ parameter, hence

$$\Lambda \to e^{\frac{2\pi i}{b_0}} \tag{1.75}$$

is a symmetry of the theory. This implies that the series expansion of $f(\Lambda, \mu)$ should be in powers of Λ^{b_0}, and we find that the corrections to the τ couplings are constrained to

$$\tau(\Lambda, \mu) = \frac{b_0}{2\pi i} \ln \left(\frac{\Lambda}{\mu} \right) + \sum_{n=1}^{\infty} a_n \left(\frac{\Lambda}{\mu} \right)^{b_0 n} \tag{1.76}$$

where the second term can be understood as the n instanton contributions, so there are no more perturbative corrections beyond the one loop.

1.4.3 Anomalies

Anomalies refer to classical symmetries which are broken by quantum effects. This means that in the full quantum theory there is no (gauge invariant or covariant) conserved current for an anomalous symmetry. This is important in the case of classical global symmetries, implying that the classical Ward identities are violated, but it does not affect the consistency of the theory. An example of an anomalous symmetry is scale invariance. Quantum effects in a Yang Mills theory, that classically is scale invariant, make the gauge coupling run with the energy.

The chiral anomaly is another kind of anomaly that occurs in the conservation of the currents for chiral rotations. If anomalous chiral rotations are gauged, then the resulting theory is inconsistent. This gives restrictions on the allowed gauge group representations of fermions in gauge theories. Chiral anomalies arise in four dimensional quantum field theories only when there are fermions with chiral symmetries charged under the gauge group. The anomalies can be computed in perturbation theory and arise at one loop. The existence of anomalies depends only on the field content and charges of the light fermions in the theory, and not on details of the interactions.

't Hooft anomaly matching There is one fundamental property of anomalies that has been pointed out by 't Hooft and can give important advances in understanding strongly coupled theories.

Consider a theory described by a Lagrangian at some scale, with a global symmetry group G. Now weakly gauge the symmetry G. The resulting theory may not be a consistent theory due to non vanishing anomaly A^{UV} for the newly gauged currents. We can however add a set of fermionic fields (called spectator) that have only G gauge coupling, such that their G anomaly A^S exactly cancel the other anomalies, i.e. $A^S = -A^{UV}$. In this way we have obtained a consistent (anomaly free) theory and we can investigate its infrared dynamics.

Since the spectator theory can be made arbitrarily weakly coupled $g_G \to 0$, the infrared dynamics of the enlarged theory are just the infrared dynamics of the original theory plus the arbitrarily weakly coupled spectator and gauge fields G. Thus the anomalies for the spectators are just the same as at high energy, and since the whole theory is anomaly free for the symmetry[7] G, we have

$$0 = A^{IR} + A^S \tag{1.77}$$

[7] We assume that the strong dynamics does not break the G symmetry

where A^{IR} is the anomaly of the symmetry G for the infrared description of the original theory (without spectators).

We then conclude that $A^{IR} = A^{UV}$, that is the anomalies for the global symmetry G must be the same in the ultraviolet and in the infrared region. The importance if this result is that the original theory might be strongly coupled in the infrared, so the effective action may be a priori be described by a completely different set of fermionic fields than in the miscroscopic description. 't Hooft arguments constrain the infrared fermion content, stating that their *global anomalies* are the same as those for the fermions in the ultraviolet description.

1.5 Supersymmetric SQCD

The supersymmetric QCD is the more natural generalization of QCD as a supersymmetric theory. Is is a $SU(N_c)$ super Yang Mills gauge theory coupled to N_f flavours of quark chiral superfields in the fundamental represantation of the gauge group. We denote with Q_i and $\tilde{Q}_i$ the quark and antiquark chiral superfields respectively, with $i = 1, \ldots, N_f$. The Lagrangian for the supersymmetric QCD is

$$\mathcal{L} = \int d^4\theta (Q_i^\dagger e^V Q_i) + (\tilde{Q}_i^\dagger e^V \tilde{Q}_i) - \frac{i}{16\pi} \int d^2\theta \tau \mathcal{W}^{\alpha a} \mathcal{W}_{\alpha a} + h.c. \tag{1.78}$$

The full global symmetry of the model is $SU(N_f) \times SU(N_f) \times U(1)_R \times U(1)_B$ where $U(1)_R$ is the R symmetry and $U(1)_B$ is the baryon number. Indeed there is an anomalous global R symmetry mixed with an anomalous global $U(1)$ flavour symmetry to give an anomaly free $U(1)_R$. There could also be a superpotential for the chiral matter fields that we have set to zero here, typically a mass term for the flavours

$$W = m^i_j Q^j \tilde{Q}_i \tag{1.79}$$

This is the more general renormalizable action for superQCD. We want to extract information about the IR physics of this theory, the vacuum structure, the massless particles, the effective superpotential, The main tools are holomorphy, the global symmetries and anomaly based arguments.

Symmetries and vacuum equation The RG running of the gauge coupling is governed by the one loop beta function (1.66). The coefficient for $SU(N_c)$ supersymmetric QCD with N_f flavors is

$$b_0 = 3N_c - N_f \tag{1.80}$$

The theory is then asymptotically free for $N_f < 3N_c$ and infrared free for $N_f > 3N_c$. With zero superpotential for the flavors, the theory has a non abelian $U(N_f)$ global symmetry. In addition there is some $U(1)$ symmetries, among which the R symmetry. The global symmetries are summarized in the following table

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_A$	$U(1)_R$	$U(1)_{R'}$
Q_a^i	N_c	N_f	1	1	1	1	$1-\frac{N_c}{N_f}$
$\tilde{Q}_i^a$	$\bar{N}_c$	1	$\bar{N}_f$	-1	1	1	$1-\frac{N_c}{N_f}$
W_α	adj	1	1	0	0	1	1

(1.81)

where $U(1)_B$ is the baryonic symmetry, $U(1)_A$ and $U(1)_R$ are the axial and R symmetry respectively, which are anomalous. The last column $U(1)_{R'}$ is an anomaly free R symmetry obtained combining $U(1)_R$ and $U(1)_A$. We can also assign charges to the coupling constant, regarding them as background chiral superfields. The anomalous global symmetries can be compensated by appropriate transformation laws of the τ parameter. This select the charges we have to assign to the strong coupling scale Λ.

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_A$	$U(1)_R$	$U(1)_{R'}$
m	1	$\bar{N}_f$	N_f	0	-2	0	$2\frac{N_c}{N_f}$
$\Lambda^{3N_c-N_f}$	1	1	1	0	$2N_f$	$2N_c$	0

(1.82)

When $N_f > 0$, and without a superpotential, the tree level scalar potential for the flavors is given by the D term, $V = \text{tr } D^2$, where

$$D = \sum_A \left(Q_i^\dagger T_A Q^i + \tilde{Q}^{\dagger i} T_A \tilde{Q}_i \right) = \sum_A T_A \left(Q_i^\dagger Q^i - \tilde{Q}^{\dagger i} \tilde{Q}_i \right) \tag{1.83}$$

We have used the fact that the generators for the fundamental and the antifundamental representation are equal except for a sign.

If there is a massive superpotential (1.79) for the flavors there are also F term equations

$$m_j^i Q^j = m_j^i \tilde{Q}_i = 0 \tag{1.84}$$

By sending the mass of one flavor to infinity we can decouple that flavor from the theory, leaving SQCD with one less flavor. The scale of this theory is related to the one with all the flavor with the matching condition

$$\Lambda_{N_f-1}^{3N_c-N_f+1} = m\, \Lambda_{N_f}^{3N_c-N_f} \tag{1.85}$$

1.5.1 Classical Vacua

The classical moduli space of vacua for supersymmetric QCD is defined by solving the D term equation of motion. Using appropriate flavor and gauge rotations we can put the Q fields in diagonal form obtaining the following vacuum structure.

$N_f < N_c$

$$Q = \tilde{Q}^T = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \dots & \\ & & & a_{N_f} \end{pmatrix} \tag{1.86}$$

with a_i arbitrary. The gauge invariant description of the classical moduli space is in term of the invariant combination $M^i_j = Q^i \tilde{Q}_j$. This is a massless chiral superfields with N_f^2 components, which is exactly the number of components of the flavors not eaten by the superHiggs mechanism. Indeed the gauge group is broken to $SU(N_c - N_f)$. Of course, for non generic values of the squarks vacuum expectation values, the unbroken gauge symmetry can be enhanced, corresponding to point were $\det M = 0$.

The Kahler potential for the flavors is

$$K = Q^\dagger_i Q^i + \tilde{Q}^{\dagger i} \tilde{Q}_i \tag{1.87}$$

We can write it in term of the invariant monomial M

$$K = 2\mathrm{tr}\,\sqrt{\bar{M}M} \tag{1.88}$$

This is obtained squaring the D term equation which implies that $\tilde{Q}^\dagger \tilde{Q} = \sqrt{(M\bar{M})}$. The Kahler metric (1.88) is singular whenever M is not invertible, i.e. where there are enhanced gauge symmetry.

$N_f \geq N_c$

$$Q = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \dots & \\ & & & a_{N_c} \\ & & & \end{pmatrix} \qquad \tilde{Q} = \begin{pmatrix} \tilde{a}_1 & & & \\ & \tilde{a}_2 & & \\ & & \dots & \\ & & & \tilde{a}_{N_c} \\ & & & \end{pmatrix} \tag{1.89}$$

with

$$|\tilde{a}_i|^2 = |a_i|^2 + \rho \tag{1.90}$$

where ρ is an arbitrary real constant. Thus, generically, the gauge symmetry is completetly broken on the moduli space by the vacuum expectation values of the flavours. Hence there are $2N_f N_c - (N_c^2 - 1)$ massless chiral supermultiplets left over by the superHiggs mechanism. These light degrees of freedom are described in terms of the gauge invariant mesons and baryons

$$M^i_j = Q^i \tilde{Q}_j \tag{1.91}$$

$$B_{i_{N_c+1} \dots i_{N_f}} = \epsilon_{i_1 \dots i_{N_f}} Q^{i_1}_{a_1} \dots Q^{i_{N_c}}_{a_{N_c}} \epsilon^{a_1 \dots a_{N_c}} \tag{1.92}$$

This basis is overcomplete. Indeed there are classical relations between mesons and baryons. For example in the case $N_f = N_c$ the following contraint holds classically

$$\det M - B\tilde{B} = 0 \tag{1.93}$$

At the origin of the moduli space, where $M = B = \tilde{B} = 0$, there is again a singularity associated with the enhanced gauge symmetry. Among the other classical constraints, the rank of the meson is bounded

$$\text{rank} M \leq N_c \tag{1.94}$$

Adding mass terms We consider now to add masses to all the flavours in the theory

$$W_{tree} = \text{Tr} m Q\tilde{Q} \tag{1.95}$$

For masses larger than the strong coupling scale Λ we can integrate out the quarks and studying the theory at lower energy as a theory of pure gauge, i.e. pure SYM. The scale matching condition before and after having integrated out the quarks is

$$\Lambda_{SYM}^{3N_c} = \det m \; \Lambda^{3N_c - N_f} \tag{1.96}$$

In the low energy theory the gaugino condensation leads to the following effective superpotential

$$W_{eff} = N_c \Lambda_{SYM}^3 = N_c \left(\det m \; \Lambda^{3N_c - N_f}\right)^{\frac{1}{N_c}} \tag{1.97}$$

which we have rewritten in terms of the original scale Λ. This superpotential can be considered as a generating functional for the operator M and $S \sim \text{Tr} W_\alpha W^\alpha$ with sources m and $\log \Lambda^{3N_c - N_f}$ respectively. We can then find the vacuum expectation values for these operators in the supersymmetric vacua

$$\begin{aligned} \langle M \rangle &= \partial_m W_{eff} = \left(\det m \; \Lambda^{3N_c - N_f}\right)^{\frac{1}{N_c}} \frac{1}{m} \\ \langle S \rangle &= \partial_{\log \Lambda^{3N_c - N_f}} W_{eff} = \left(\det m \; \Lambda^{3N_c - N_f}\right)^{\frac{1}{N_c}} \end{aligned} \tag{1.98}$$

Indeed there are N_c different vacua, in agreement with the Witten index for pure $SU(N_c)$ SYM theories. Observe that the results (1.98) is valid for all N_f. Note in particular that for $N_f > N_c$ the matrix $\langle M \rangle$ does no satisfy the classical constraints (1.94) of the theory with massless flavours. However, taking $m \to 0$ in (1.98) does bring $\langle M \rangle$ back to the classical moduli space.

Performing a Legendre transformation between m and M, we can use (1.97) to derive the effective action

$$W_{eff} = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}} + \text{Tr} m M \tag{1.99}$$

We remind that this is not a Wilsonian effective action for the light field, it is not an effective superpotential on the moduli space. It can only be used to derive vacuum expectation values and is meaningfull only with $m \neq 0$.

1.6 Quantum vacua

1.6.1 $N_f = 0$, Gaugino condensation

We consider the case where there are no matter, i.e. pure $\mathcal{N} = 1$ gauge theory. Witten showed that in this theory the index is

$$\mathrm{Tr}(-1)^F = \frac{1}{2}T(\mathrm{adj}) \tag{1.100}$$

This implies that supersymmetry is not broken and there are at least $\frac{1}{2}T(\mathrm{adj})$ discrete, degenerate vacua. The theory content is the gauge field and the gaugino, a spinor in the adjoint representation of the gauge group. The coefficient of the beta function is $b_0 = 3N_c$. The $U(1)_R$ symmetry is anomalous. However, it is possible to build an anomaly free $U(1)$ combinig the R symmetry with a shift in the θ parameter. Because of the anomaly, an R symmetry transformation on the gaugino

$$\lambda \to e^{i\alpha}\lambda \tag{1.101}$$

is a symmetry of the theory if it is combined by the following shift of the θ parameter

$$\theta \to \theta + 2N_c\alpha \qquad \tau \to \tau + \frac{N_c\alpha}{\pi} \tag{1.102}$$

Since the physics is periodic in θ with period 2π, it is understood that when the phase is a multiple of $\frac{2\pi}{2N_c}$ no compensation is needed on the θ angle. This means that a Z_{2N_c} discrete subgroup of the original $U(1)_R$ symmetry survives as a symmetry at quantum level.

On the other hand, treating the holomorphic gauge coupling τ as a background chiral superfield we can define the following global symmetry of the theory

$$\lambda \to e^{i\alpha}\lambda \tag{1.103}$$

$$\tau \to \tau + \frac{N_c\alpha}{\pi} \tag{1.104}$$

which is a combination of the continuos R symmetry with a shift of the background superfield τ.

We then assume that supersymmetric Yang Mills has no massless particles, just massive color-singlet composites. Thus the low energy superpotential could contain only the background field τ and the requirement that W_{eff} has R charge 2 specifies its form uniquely as

$$W_{eff} = a\mu^3 e^{\frac{2\pi i\tau}{N_c}} = a\Lambda^3 \tag{1.105}$$

where a is a constant.

Now, in the supersymmetric Yang Mills action the F component of the background chiral superfield τ act as a source for the gaugino condensate. With the assumption that there are no massless particle at low energy, the effective action is given only by the effective superpotential (1.105). Hence the gaugino condensate can be computed

$$\langle\lambda\lambda\rangle = 16\pi i \frac{\partial}{\partial F_\tau} \int d^2\theta W_{eff} = 16\pi i \frac{\partial}{\partial \tau} W_{eff} = -\frac{32\pi^2}{N_c} a\mu^3 e^{\frac{2\pi i \tau}{N_c}} = -\frac{32\pi^2}{N_c} a\Lambda^3 \tag{1.106}$$

This vacuum expectation value for the gaugino bilinear breaks the discrete Z_{2N_c} symmetry to Z_2, since (1.106) is invariant under the R symmetry

$$\langle\lambda^a\lambda^a\rangle \to e^{2i\alpha}\langle\lambda^a\lambda^a\rangle \tag{1.107}$$

only if α is $\alpha = 0, \pi$. The general Z_{2N_c} transformation with $\alpha = \frac{k\pi}{N_c}$ sweeps out N_c different values for $\langle\lambda^a\lambda^a\rangle$, i.e. N_c distinct vacua.

The effective superpotential (1.105) can be used to find the tension of domain walls interpolating between these vacua. Moreover, we can consider the kinetic term for the gauge field (1.69) as a superpotential where the term ($\tau \sim 3N_c \ln\Lambda$) act as a source for the operator $S \sim \mathrm{Tr}\mathcal{W}_\alpha\mathcal{W}^\alpha$, leading to a vacuum expectation value for S as in (1.106). We can then perform a Legendre transformation to integrate in S and obtain the Veneziano Yankielowicz superpotential

$$W_{eff}(S) = N_c S\left(1 - \ln\frac{S}{\Lambda^3}\right) \tag{1.108}$$

The superfield S is usually called the glueball superfield. However it is important to stress that it is not associated to any light field of the spectrum. The superpotential (1.108) can be used only to find tensions of domain walls and vacuum expectation values.

1.6.2 $N_f < N_c$

Consider now $SU(N_c)$ SQCD with N_f flavours and $N_f < N_c$. We have already analyzed the classical vacuum solutions. We now wonder if there could be non perturbative contributions to the effective superpotential which are compatible with the symmetries of the theory. From the charges in the tables 1.81 and 1.82 there is a unique superpotential which can be generated, the Affleck Dine Seiberg [26] one

$$W_{eff} = C\left(\frac{\Lambda^{3N_c-N_f}}{\det Q\tilde{Q}}\right)^{\frac{1}{N_c-N_f}} \tag{1.109}$$

with C a constant which can depend on the numbers N_f and N_c. This superpotential has been shown to be generated by direct instanton computation for the case $N_f = N_c - 1$, where the gauge group is completely broken and the instanton calculation is reliable (there is no infra-red divergence), leading to $C = N_c - N_f$. For $N_f < N_c - 1$ it can be associated to gaugino condensation in the unbroken $SU(N_c - N_f)$ factor of the gauge group.

The dynamically generated superpotential (1.109) leads to a squark potential which slopes to zero for $\det M \to \infty$. Therefore, the quantum theory does not have a ground state. We started with a moduli space of vacua in the classical theory and ended up in the quantum theory without a vacuum.

Observe that the ADS superpotential does not make sense for $N_f \geq N_c$. Indeed for $N_f = N_c$ the exponent diverges whereas for $N_f > N_c$ the constraint (1.94) implies $\det M = 0$.

Mass perturbation Now we add masses to the quarks and we analyze the effects on the ADS effective superpotential. Suppose giving mass to just the N_fth flavour

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\det Q\tilde{Q}}\right)^{\frac{1}{N_c-N_f}} + mQ_{N_f}\tilde{Q}_{N_f} \tag{1.110}$$

We can parametrize the meson $M = Q\tilde{Q}$ as

$$M = \begin{pmatrix} \tilde{M}_{N_f-1} & y \\ y & t \end{pmatrix} \tag{1.111}$$

The equations of motion fix $y = 0$ and

$$t = \left(m\left(\frac{\Lambda^{3N_c-N_f}}{\det Q\tilde{Q}}\right)^{\frac{1}{N_c-N_f}}\right)^{\frac{N_c-N_f}{N_c-N_f+1}} \tag{1.112}$$

Plugging this into the superpotential, we obtain

$$W = (N_c - N_f + 1)\left(m\frac{\Lambda^{3N_c-N_f}}{\det \tilde{M}}\right)^{\frac{1}{N_c-N_f+1}} \tag{1.113}$$

Relating the strong coupling scale with (1.85) we recognize in (1.113) the ADS superpotential for a SQCD with $N_f - 1$ flavour.

If we now consider adding masses to all flavours, the exact superpotential is

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\det Q\tilde{Q}}\right)^{\frac{1}{N_c-N_f}} + mQ\tilde{Q} \tag{1.114}$$

Of course it has the same form of the effective action we derive integrating in the meson field M (1.99), which is valid for all N_f. However, we observe that in this interval ($N_f < N_c$) this expression is sensible even in the limit of zero mass and it is the true effective superpotential of the theory.

We can solve the equations for the vacuum expectation values for the meson

$$\langle M^i_j\rangle = (\det m\ \Lambda^{3N_c-N_f})^{\frac{1}{N_c}}\left(\frac{1}{m}\right)^i_j \tag{1.115}$$

For large m, the matter fields are very massive and decouple, leaving a low energy $SU(N_c)$ pure Yang Mills theory. Evaluating the superpotential using (1.115) yields

$$W = N_c(\Lambda^{3N_c-N_f} \det m)^{\frac{1}{N_c}} \tag{1.116}$$

Using the scale matching condition between the scale before and after having integrated out the massive quarks

$$\Lambda_{SYM}^{3N_c} = \det m \ \Lambda_{N_f}^{3N_c-N_f} \tag{1.117}$$

we obtain as the low energy superpotential

$$W = N_c\Lambda_{SYM}^3 \tag{1.118}$$

which is exactly the superpotential we have derived for SYM via gaugino condensation (1.105).

1.6.3 $N_f = N_c$

Where $N_f = N_c$ the moduli space is described by mesons and baryons constrined by the classical relation (1.93). However Seiberg argued that this relation is modified at quantum level as

$$\det M - B\tilde{B} = \Lambda^{2N_c} \tag{1.119}$$

This lifting of the moduli space is indeed consistent with the ADS superpotential. To check this we consider giving mass to the N_fth flavour. We can again parametrize the meson as in (1.111). Now the equations of motion set $y = B = \tilde{B} = 0$ and the constraint (1.119) gives $\det \tilde{M}\ t = \Lambda^{2N_c}$. Pluggin into the superpotential we obtain

$$W = \frac{m\ \Lambda^{2N_c}}{\det \tilde{T}} \tag{1.120}$$

Once again using the scale matching $\Lambda_{N_f-1}^{3N_c-N_f+1} = m\ \Lambda_{N_f}^{3N_c-N_f}$, and reminding in this case $N_f = N_c$, this superpotential can be recognized as the ADS superpotential for $N_f = N_c - 1$.

Hence on the quantum moduli space (1.119) there are no singularities. The difference between the classical and the quantum moduli space is negligible for large expectation values of M, B and $\tilde{B}$, which correspond to the weak coupling region.

1.6.4 $N_f = N_c + 1$

The classical moduli space is parametrized by mesons and baryons subject to the classical constraint

$$\det M \left(\frac{1}{M}\right)_j^i = B^i\tilde{B}_j \tag{1.121}$$

$$M_j^iB^i = M_i^j\tilde{B}_j = 0 \tag{1.122}$$

Unlike the previous case, here the classical and the quantum moduli space coincide. However, the singularities at strong coupling are not associated to massless gluons, but to additional massless mesons and baryons. Indeed at the origin of the field space all the mesons and baryons are massless and physical. This can be checked by using the 't Hooft anomaly matching condition between the description in terms of elementary fields and the one with composite fields. At weak coupling the moduli space is as in the classical theory. The superotential which encode these results is

$$W = \frac{1}{\Lambda^{2N_c-1}}\left(M_i^j B_j \tilde{B}^i - \det\ M\right) \tag{1.123}$$

Indeed at the origin all the fields are massless, whereas at a generic point in the moduli space the equations of motion for this superpotential reproduce the classical constraints.

Finally the superpotential (1.123) is consistent with the superpotential for the $N_f = N_c$ case if we add the mass for one flavour and integrate it out.

1.6.5 $N_f > N_c$

For SQCD with $N_f > N_c$ the moduli space is still parametrized by the gauge singlet fields mesons and baryons. However there is no deformation of the classical moduli space which is compatible with holomorphy and the symmetries in Tables 1.81 1.82. So there is a quantum moduli space of vacua which coincide with the classical one. In order to understand what happens in this window between the number of colors and the number of flavours we distinguish among different cases.

$N_f \geq 3N_c$ In this range the theory is not asymptotically free. The coupling constant become smaller in the flow towards low energy. Therefore the theory becomes more and more weakly coupled at lower energy (it is IR free) and the elementary degrees od freedom are still the quarks and the gluons. The theory is in a non Abelian free electric phase. It has a Landau pole at high energy where it is strongly coupled and then it can be considered as a consistent quantum theory only as an effective theory at low energy.

$\frac{3}{2}N_c < N_f < 3N_c$ The theory is here asymptotically free. However there is a fixed point in the renormalization group flow, and so this range between the number of color and the number of flavors is called conformal window. This can be understood analyzing the exact beta functions

$$\beta(g) = -\frac{g^3}{16\pi^2}\frac{3N_c - N_f(1-\gamma)}{1 - N_c\frac{g^2}{8\pi^2}} \tag{1.124}$$

where γ is the anomalous dimension of the mass of the quarks

$$\gamma = -\frac{g^2}{8\pi^2}\frac{N_c^2-1}{N_c} + O(g^4) \tag{1.125}$$

Since there are values for N_f and N_c where the one loop is negative whereas the two loop is positive, there might be a non trivial fixed point. Now take the number of flavours to be infinitesimally close to the point where asymptotic freedom is lost, i.e. $N_f = 3N_c - \epsilon N_c$, we have

$$16\pi^2\beta(g) = -g^3\epsilon N_c - \frac{g^5}{8\pi^2}(3(N_c^2-1) + O(\epsilon)) + O(g^7) \tag{1.126}$$

So there is an approximate solution of the condition $\beta = 0$ where the two terms cancel. This corresponds to a perturbative infrared fixed point

$$g_{sc}^2 = \frac{8\pi^2}{3}\frac{N_c}{N_c^2-1}\epsilon \tag{1.127}$$

The theory is then a non trivial superconformal field theory. The elementary quarks and gluons are not confined but appear as interacting massless particles. Given that such a fixed point exists, we can use the superconformal algebra to extract information about this theory. It follows from the algebra of the superconformal group that the dimension of an operator is related to its R charge, precisely

$$D \geq \frac{3}{2}|R| \tag{1.128}$$

which is saturated for chiral ($D = \frac{3}{2}R$) and antichiral operators ($D = -\frac{3}{2}R$). The chiral operators form a ring where the dimensions simply add. The R symmetry of the superconformal fixed point is not anomalous and commutes with the flavour symmetry. Therefore, it must be the non anomalous R charge of table 1.81. We can compute the scaling dimension of the gauge invariant operator $M = Q\tilde{Q}$

$$D[M] = D[Q\tilde{Q}] = 2 + \gamma_{sc} = \frac{3}{2}2(1 - \frac{N_c}{N_f}) \tag{1.129}$$

where γ_{sc} is the anomalous dimension at the infrared fixed point, which then results

$$\gamma_{sc} = 1 - 3\frac{N_c}{N_f} \tag{1.130}$$

Now, the superconformal algebra gives constraint on the possible unitary representation. For example, a scalar field ϕ should satisfy the unitary bound

$$D[\phi] \geq 1 \tag{1.131}$$

Requiring $D[M] \geq 1$ we obtain the constraint

$$N_f > \frac{3}{2}N_f \tag{1.132}$$

Thus the theory flows to an infrared interacting fixed point for $\frac{3}{2}N_c < N_f < 3N_c$. In order to understand what happens for smaller N_f we observe that the dimension of the meson M approaches 1 for $N_f = \frac{3}{2}N_c$, which shows that in this limit M is a free field. This suggest that in the correct description the field M, and perhaps the whole IR theory, is free.

1.7 Seiberg duality

We have seen that for $N_f > N_c+1$ we cannot generate deformation of the moduli space, i.e. a non perturbative superpotential, compatible with the global symmetries. The natural generalization of the superpotential for the $N_f = N_c + 1$ case does not have R charge equal to 2.

To understand the right description we observe that the baryon superfield have $\tilde{N} = N_f - N_c$ indices. Thus they can be viewed as bound states of $\tilde{N}$ components. We can associate these components with new superfields q and $\tilde{q}$. To bind these consituents into the gauge invariant baryon superfield, we need a Yang Mills theory with gauge group $SU(\tilde{N})$, for which q and $\tilde{q}$ transform in the fundamental and antifundamental representation. Then the baryons would have the dual description as $B_{ij\ldots k} = \epsilon_{a_1\ldots a_{\tilde{N}}} q_i^{a_1} \ldots q_k^{a_{\tilde{N}}}$.

The precise duality proposed by Seiberg [27] is that the original electric theory can be described by an equivalent magnetic theory. It is based on the gauge group $SU(\tilde{N} = N_f - N_c)$, with N_f flavours q and $\tilde{q}$, a gauge invariant field $(M_m)^i_j$ and a non trivial superpotential

$$W_{magn} = (M_m)^i_j q_i \tilde{q}^j = \frac{1}{\mu} M^i_j q_i \tilde{q}^j \tag{1.133}$$

The exact symmetries of the theories and the charges of the fields and of the strong scale intended as a background field are summarized in the following table:

	$SU(N_f-N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$	$U(1)_A$
q	$N_f - N_c$	$\bar{N}_f$	1	$\frac{N_c}{N_f-N_c}$	$1-\frac{N_f-N_c}{N_f}$	1
$\tilde{q}$	$\overline{N_f - N_c}$	1	N_f	$-\frac{N_c}{N_f-N_c}$	$1-\frac{N_f-N_c}{N_f}$	1
M_m	1	$\bar{N}_f$	N_f	0	$2\frac{N_f-N_c}{N_f}$	-2
$\tilde{\Lambda}^{3(N_f-N_c)-N_f}$	1	1	1	0	0	$2N_f$

Note that the two theories have different gauge group and different numbers of interacting particles. Nevertheless, they describe the same physics, they are quantum equivalent. In particular the global non anomalous symmetries of the theories are the same and all the 't Hooft anomaly matching conditions for these symmetries are satisfied.

The presence of the scale μ in (1.133) can be understood analyzing the theory in the conformal window. In the electric description $M = Q\tilde{Q}$ has dimension 2 at the UV fixed point and acquires anomalous dimension (1.129) at the IR fixed point. In the magnetic description, M_m is an elemenatry field of dimension 1 at the UV fixed point and dimension (1.129) at the IR fixed point. In order to relate M_m and M we must introduce a scale μ such that $M = \mu M_m$. This intermediate scale also enters the matching condition between the strong coupling scale of the electric Λ and of the magnetic $\tilde{\Lambda}$ theory

$$\Lambda^{3N_c-N_f}\tilde{\Lambda}^{3\tilde{N}-N_f} = (-1)^{N_f-N_c}\mu^{N_f} \tag{1.134}$$

This shows explicitly that Seiberg duality is a strong/weak duality.

The Seiberg duality is valid in the whole region $N_f > N_c + 1$ but remember that it is a low energy duality, it is not a electric magnetic duality valid at all energy scales as the Olive-Montonen duality on $\mathcal{N} = 4$ supersymmetric gauge theories. We now comment on the different intervals

- For $N_f > 3N_c$ the electric theory is IR free and so it is the right description for the low energy.
- In the conformal window the electric and magnetic theories are both UV free, and they differ in the UV. The two different UV starting points flow under the renormalization group (RG) to the same interacting RG fixed point in the IR.
- For $N_c + 1 < N_f \leq \frac{3}{2}N_c$ the magnetic theory is IR free, with irrelevant interactions. The UV electric theory flows at long distance to the IR free magnetic theory. The magnetic theory provides the right degrees of freedom which describe the low energy physics.

The Seiberg duality has passed many consistency check. We already mentioned the 't Hooft anomaly matching conditions. The moduli space of the electric and the magnetic theory coincide, and the gauge invariants operators are precisely matched. We can give mass to one flavour in the electric description and integrate it out, obtaining gauge $SU(N_c)$ with $N_f - 1$ flavours. The electric mass corresponds in the dual superpotential as a linear term for the meson which forces the dual quarks to have a vacuum expectation values which Higgses the theory down to a $SU(N_f - N_c - 1)$ with $N_f - 1$ flavours. Finally, it is easy to check that performing the duality transformation twice, we come back to the original theory.

1.8 Kutasov-Seiberg-Schwimmer duality

We have seen in the previous section how Seiberg duality provides a very elegant description of the strong coupling dynamics of supersymmetric gauge theories, in particular of SQCD. This electric-magnetic duality gives a description of strongly coupled gauge theories in term of a dual theory, weakly coupled in the infrared, which have the same long distance behavior. The main test which suppurt this duality are the following:

- The two dual theory have the same global symmetries and the 't Hooft anomaly matching conditions are satisfied.
- The moduli space of vacua of the two theories coincide.
- The equivalence of the moduli space of vacua and of the chiral ring is preserved under deformation of the theories by F-components of chiral operators (e.g. adding masses).

The equivalence of the two theories is valid at quantum level, where non perturbative phenomena are taken into account.

We consider now supersymmetric Yang-Mills theory with gauge group $SU(N_c)$ coupled to a chiral adjoint field X_α^β and to N_f flavours in the fundamental and antifundamental representation of the gauge group Q_α^i, $\tilde{Q}_j^\beta$, with $(\alpha, \beta = 1 \dots N_c)$, $(i, j = 1, \dots, N_f)$. This theory is asymptotically free for $N_f < 2N_c$. We denote with Λ its dynamically generated strong scale. Without a superpotential for the adjoint field this model is still pourly understood. The theory simplifies if we add the following superpotential

$$W = \frac{s_0}{k+1} \mathrm{Tr} X^{k+1} \tag{1.135}$$

This superpotential is, for generic k, a dangerously irrelevant operator and thus cannot be ignored in the IR (for a discussion about dangerously irrelevant operators see [30]). For $k = 1$ it is a mass term for the adjoint that can be integrated out, obtaining the supersymmetric QCD. The superpotential (1.135) removes many flat direction of the undeformed theory and truncates the chiral ring of the theory, imposing the relation

$$(X^k)_\alpha^\beta - \frac{1}{N_c} (\mathrm{Tr} X^k) \delta_\alpha^\beta = \text{D-term} \tag{1.136}$$

Hence the chiral operators involving X in the presence of the superpotential (1.135) are $\mathrm{Tr} X^l$, $l = 2 \dots k$ and operators involving X^l with $l < k$. There are two kinds of gauge invariant operators. Meson operators

$$(M_m)_j^i = \tilde{Q}_j X^{m-1} Q^i \qquad m = 1, \dots, k \tag{1.137}$$

and baryons operator that are built using the dressed quarks

$$Q_{(l)} = X^{l-1} Q \qquad l = 1 \dots k \tag{1.138}$$

and constructed as follows

$$B^{(n_1 \dots n_k)} = Q_{(1)}^{n_1} \dots Q_{(k)}^{n_k} \qquad \sum_{l=1}^{k} n_l = N_c \tag{1.139}$$

where the colour indices are contracted with an ϵ tensor.

The anomaly free global symmetries of this model and the corresponding charges are summarized in this table

	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
Q	N_f	1	1	$1 - \frac{2}{k+1} \frac{N_c}{N_f}$
Q	1	$\bar{N}_f$	-1	$1 - \frac{2}{k+1} \frac{N_c}{N_f}$
X	1	1	0	$\frac{2}{k+1}$

(1.140)

This theory has been deeply studied in this references [30, 28, 29].

1.8.1 Stability

We can further deform the theory and consider the more general superpotential, which do not change the properties of the chiral ring described above,

$$W = \sum_{i=0}^{k-1} \frac{s_i}{k+1-i} \mathrm{Tr} X^{k+1-i} + \lambda \mathrm{Tr} X \tag{1.141}$$

Here λ is a Lagrange multiplier enforcing the tracelessness condition $\mathrm{Tr}X = 0$. For non zero $\{s_i\}$, the R symmetry is broken. Using transformation in the complexified gauge group $SU(N_c)_{\mathbf{C}}$ we can diagonalized the matrix X, where the eigenvalues are the roots of W'

$$W'(x) = \sum_{i=0}^{k-1} s_i x^{k-i} + \lambda = s_0 \prod_{l=1}^{k} (x - c_l) \tag{1.142}$$

If all the eigenvalues c_l are different from each other, the theory splits in the infrared into a set of decouples SQCD theories. Ground state are labelled by the integers $r_1 \leq r_2 \leq r_k$ counting the number of eigenvalues of X which are the l-th root of W'. These integers are constrained such that

$$\sum_{l=1}^{k} r_l = N_c \tag{1.143}$$

and the values of λ is obtained imposing that the sum of the eigenvalues vanishes

$$\sum_{l=1}^{k} c_l r_l = 0 \tag{1.144}$$

In each vacuum the field X is massive and can be integrated out. The expectation value of X break the gauge group to

$$SU(N_c) \to SU(r_i) \times SU(r_2) \times \cdots \times SU(r_l) \times U(1)^{k-1} \tag{1.145}$$

Some of the r_l may vanish or some of the c_l may coincide, in which case the picture is modified in an obvious way. For generic $\{s_i\}$ the classical infrared behaviour of the theory, a set of decoupled SQCD, is insensitive to the values of the $\{s_i\}$. Quantum mechanically each of the SQCD has a strong couplling scale, Λ_i, which depends non trivially (through a scale matching condition) on the couplings $\{s_i\}$ and on the strong coupling scale of the theory with the adjoint field Λ.

Each of the $SU(r_l)$ factors describe a supersymmetric QCD model, and it is well known that it has no stable vacuum when the numbers of flavours is smaller than the number of colors. Then the system has stable vacuum if and only if

$$r_l \leq N_f \qquad \forall \quad l = 1 \dots k \tag{1.146}$$

which implies that the theory has a vacuum if and only if

$$N_f \geq \frac{N_c}{k} \tag{1.147}$$

1.8.2 Duality 1

We now discuss the dual description found in [29] for the theory with the superpotential (1.135) and next we will turn to the more general case (1.141).

We have already introduce the gauge group, the field content and the global symmetries of the theory (1.140). The dual theory proposed in [29] has dual gauge group $SU(kN_f - N_c)$ and the following matter content: N_f flavours of magnetic quarks q_i^α, $\tilde{q}^i_\beta$, an adjoint field Y_α^β and gauge singlets $(M_m)_i^j$, $m = 1 \dots k$ representing in the dual description the generalized mesons (1.137) of the original, electric theory. The global symmetries are the same of the electric description with charges

	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
q	$\bar{N}_f$	1	$\frac{N_c}{kN_f-N_c}$	$1-\frac{2}{k+1}\frac{kN_f-N_c}{N_f}$
$\tilde{q}$	1	N_f	$-\frac{N_c}{kN_f-N_c}$	$1-\frac{2}{k+1}\frac{kN_f-N_c}{N_f}$
Y	1	1	0	$\frac{2}{k+1}$
M_m	N_f	$\bar{N}_f$	0	$2-\frac{4}{k+1}\frac{N_c}{N_f}+\frac{2}{k+1}(m-1)$

(1.148)

The mesons M_m have in the magnetic theory standard kinetic terms, rescaled by powers of s_0 and μ. Note again that $j \leq k$ since X^l is not an independent chiral operator for $l \geq k$. The superpotential in the dual magnetic description is schematically

$$W_{dual} = \frac{\bar{s}_0}{k+1}\mathrm{Tr}Y^{k+1} + \frac{s_0}{\mu^2}\sum_{m=1}^{k} M_m \tilde{q} Y^{k-m} q \tag{1.149}$$

where the auxiliary scale μ is needed for dimensional reason. This superpotential preserve the R symmetry, and in the case $k = 1$ we come back to Seiberg duality. The 't Hooft anomaly matching conditions for the electric and the magnetic theory are satisfied. The auxiliary scale μ is actually not an independent parameter. The scale of the electric theory Λ, that of the magnetic theory $\tilde{\Lambda}$, and μ satisfy the following scale matching relation

$$\Lambda^{2N_c-N_f}\tilde{\Lambda}^{2\bar{N}_c-N_f} = s_0^{-2N_f}\mu^{2N_f} \tag{1.150}$$

It is easy to check that (1.150) is invariant under all global symmetries. This scale matching condition implies that when the electric theory is weakly coupled the magnetic one is strongly coupled and viceversa.

1.8.3 Duality 2.

Now we present the complete duality map when the electric theory is deformed with the full superpotential (1.141). One expects the magnetic superpotential (1.149) to be deformed as well. We will start with a discussion of the deformation of the first term on the r.h.s. of (1.149), the superpotential for Y, and then turn to the second term, proportional to M_j. We report the main results, the detailed derivation can be found in [30].

The electric superpotential (1.141) describes a space of theories parametrized by the couplings s_i. The general form of the dual superpotential is

$$\bar{W} = \sum_{i=0}^{k-1} \frac{\bar{s}_i}{k+1-i} \mathrm{Tr} Y^{k+1-i} + \bar{\lambda} \mathrm{Tr} Y + \alpha(s) \tag{1.151}$$

where $\bar{s}_i = \bar{s}_i(s)$ are the magnetic coupling constants and $\alpha(s)$ is a constant, that are non trivial functions of the electric couplings $\{s_i\}$.

In complete analogy with the discussion of the electric theory, the magnetic theory has, for generic $\{\bar{s}_i\}$s a large number of vacua, parametrized by integers $\bar{r}_l$ corresponding to the number of eigenvalues of the matrix Y with the value $\bar{c}_l$, where $\bar{c}_l$ is the l-th root of $\bar{W}'$, analogously than (1.142). Clearly $\sum_l \bar{r}_l = \tilde{N}_c = kN_f - N_c$. The low energy magnetic theory is a direct product of decoupled copies of SQCD with N_f flavors of quarks, with the gauge group broken as

$$SU(\tilde{N}_c) \to SU(\bar{r}_1) \times SU(\bar{r}_2) \times \cdots \times SU(\bar{r}_k) \times U(1)^{k-1} \tag{1.152}$$

The proposal of [30] states that the duality between the deformed theories, (1.141) and (1.151) reduces to a direct product of the Seiberg SQCD dualities for the separate factors in (1.145), (1.152). That means that the magnetic multiplicities $(\bar{r}_1, \cdots, \bar{r}_k)$ are related to the electric ones via the SQCD duality relation:

$$\bar{r}_i = N_f - r_i \tag{1.153}$$

The fact that this is a one to one map of the sets of vacua of the electric and magnetic theories follows from non perturbative effects, that can lift the moduli space and even remove some vacua.

For completeness, we remind that when two or more of the critical points of W coincide, the same number of critical points of $\bar{W}$ should coincide. If the order of a critical point c_i (and therefore that of $\bar{c}_i$ as well) is n_i, the degeneracies r_i and $\bar{r}_i$ of this critical point in the electric and magnetic theories are related by

$$\bar{r}_i = n_i N_f - r_i. \tag{1.154}$$

The duality is in this case between an electric theory with gauge group $SU(r_i)$ and a superpotential $\mathrm{Tr} X^{n_i+1}$, and a magnetic one with gauge group $SU(\bar{r}_i) = SU(n_i N_f - r_i)$ with a similar superpotential.

For the above scenario to be realized, the electric and magnetic superpotentials must be closely related. In particular, the fact that whenever any number of critical points c_i coincide, the same number of dual critical points $\bar{c}_i$ must coincide as well is a very strong constraint on the dual couplings $\bar{s}_i(s)$.

Another important constraint is obtained observing that the duality map relating $\mathrm{Tr}X^j$ to $\mathrm{Tr}Y^l$ is closely related to the mapping $\bar{s}(s)$. Define the free energy of the model as

$$e^{-\int d^4xd^2\theta F(s_i)+c.c} = \langle e^{-\int d^4xd^2\theta W(X,s_i)+c.c.}\rangle \tag{1.155}$$

where s_i are background chiral superfields. Then, correlation functions of the operators $\mathrm{Tr}X^j$ are given by derivatives of the free energy F with respect to the superfields s_i:

$$\frac{1}{k+1-i}\langle \mathrm{Tr}X^{k+1-i}\rangle = \frac{\partial F}{\partial s_i} \tag{1.156}$$

and similarly in the magnetic theory, in terms of the dual free energy $\bar{F}$:

$$\frac{1}{k+1-i}\langle \mathrm{Tr}Y^{k+1-i}\rangle = \frac{\partial \bar{F}}{\partial \bar{s}_i} \tag{1.157}$$

where duality implies

$$\bar{F}(\bar{s}_i(s)) = F(s_i). \tag{1.158}$$

Since $\mathrm{Tr}X^j$, $\mathrm{Tr}Y^j$ are tangent vectors to the space of theories, we find that the electric and magnetic operators are related by:

$$\frac{1}{k+1-i}\mathrm{Tr}X^{k+1-i} = \sum_j \frac{\partial \bar{s}_j}{\partial s_i}\frac{1}{k+1-j}\mathrm{Tr}Y^{k+1-j} + \frac{\partial \alpha}{\partial s_i} \tag{1.159}$$

Taking the expectation values of both sides of this equation we find that the mapping $\bar{s}_i(s)$ must satisfy a constraint in addition to the previously described one on the degeneration of eigenvalues. The expectation values of the left and right hand sides of 1.159 which depend in a highly non-trivial way on the particular vacuum chosen (the set of r_i) must satisfy a relation that is independent of the particular vacuum chosen.

1.8.4 The precise mapping

It is convenient to think of X, Y as general $U(N)$ matrices, with a dynamical Lagrange multiplier λ ($\bar{\lambda}$) imposing the tracelessness of X (Y). Consider the electric theory. We define a shifted X, denoted by X_s as:

$$X_s \equiv X + b\mathbf{1} \qquad b = \frac{s_1}{s_0 k}. \tag{1.160}$$

This shift cancels the first subleading term in W, leading to the superpotential:

$$W_s(X_s) = \sum_{i=0}^{k-1} \frac{t_i}{k+1-i} \mathrm{Tr} X_s^{k+1-i} + \lambda_s \left(\mathrm{Tr} X_s - b N_c \right) + \beta N_c \tag{1.161}$$

where $W_s(X_s) \equiv W(X)$ and

$$t_i = \sum_{j=0}^{i} \binom{k-j}{i-j} (-b)^{i-j} s_j \tag{1.162}$$

$$\lambda_s = \lambda + \sum_{j=0}^{k-1} (-b)^{k-j} s_j \tag{1.163}$$

$$\beta = -\sum_{j=0}^{k-1} \frac{k-j}{k+1-j} (-b)^{k+1-j} s_j. \tag{1.164}$$

Note that $t_0 = s_0$ and $t_1 = 0$. The transformation (1.160) corresponds to an analytic coordinate transformation on the space of theories. A similar transformation can be performed on the magnetic side, replacing Y by Y_s and $\bar{\lambda}$ with $\bar{\lambda}_s$. The $k-1$ independent couplings s_i, $i = 1, \cdots, k-1$ are replaced by the $k-2$ couplings t_i, $i = 2, \cdots, k-1$, and b. In the X_s variables the coefficient of the first subleading term $\mathrm{Tr} X_s^k$ in the superpotential (1.161) always vanishes (i.e. $t_1 = 0$). The information about that coefficient in the original description is in b. In a sense, the transformation (1.160) allowed us to trade the operator $\mathrm{Tr} X^k$ for the operator λ.

The duality suggested in [30] for the eigenvalues of X_s (a_i) and Y_s ($\bar{a}_i$), defined analogously to (1.142), is simply $\bar{a}_i = a_i$, $i = 1, \ldots k$ which means

$$\begin{aligned} \bar{t}_i &= -t_i \\ \bar{\lambda}_s &= -\lambda_s. \end{aligned} \tag{1.165}$$

The second equation here is an operator identity; it can be thought of as arising from the coupling relations:

$$\bar{b} \bar{N}_c = -b N_c; \quad \alpha_s = \text{independent of } b \tag{1.166}$$

using (1.159). Equations (1.165,1.166) specify the mapping $\bar{s}_i(s)$ completely. The explicit expression for α_s can be found in [30].

Summarizing, the mapping of the couplings in the electric superpotential t_i, b, defined by (1.162), to their magnetic counterparts is given by equations (1.165, 1.166). Of course the simple transformation laws described here become more complicated when we translate them back to the original coordinates s_i [30].

1.8.5 The M_j terms in the magnetic superpotential

So far our discussion focused on the way the first term in the magnetic superpotential (1.149) is deformed as we deform the electric superpotential as (1.141). In this subsection we will use these results to determine the deformation of the second term in W_{magn}. We will work in the parametrization of the space of theories described previously, the $\{t_i\}$.

When one turns on non-vanishing couplings t_i in (1.141), the magnetic superpotential (1.149) can in principle receive contributions proportional to t_j, $t_j t_l$, etc, consistently with the global symmetries. The way to fix all these terms is to require that duality act in the way described before. Namely, for generic t_i we expect the magnetic theory to split into an approximately decoupled set of SQCD theories that are dual to the different decoupled factors in (1.145).

This requirement of decoupling is rather non-trivial since the second term on the r.h.s. of (1.149) tends to couple the different $SU(\bar{r}_i)$ theories. Indeed, denote the first r_1 components (in color) of the electric quarks Q by Q_1, the next r_2 by Q_2 and so on. Similarly, the first $\bar{r}_1$ components of q are denoted by q_1, the next $\bar{r}_2$ by q_2, etc.. Then expanding around $\langle X_s \rangle$ we find

$$M_m = \tilde{Q}_1 Q_1 a_1^{m-1} + \tilde{Q}_2 Q_2 a_2^{m-1} + \cdots + \tilde{Q}_k Q_k a_k^{m-1} \qquad (1.167)$$

where $\tilde{Q}_l Q_l$ are the mesons of the l'th electric SQCD theory with gauge group $SU(r_l)$. Similarly we write:

$$\tilde{q} Y_s^j q = \tilde{q}_1 q_1 a_1^j + \tilde{q}_2 q_2 a_2^j + \cdots + \tilde{q}_k q_k a_k^j \qquad (1.168)$$

where we denote the shifted Y field appropriate for the $\bar{t}_i$ coordinate system on theory space by Y_s. In the above formula we used the fact that in the coordinates t_i the duality map is trivial, $\bar{a}_i = a_i$.

The second term in W_{magn} (1.149) has to be corrected in such a way that the different SQCD theories do not couple – there should not be any cross terms coupling $\tilde{q}_j q_j \ \tilde{Q}_i Q_i$ with $i \neq j$. The unique solution to this requirement is

$$W_{magn} = \sum_l \frac{\bar{t}_l}{k+1-l} \mathrm{Tr} Y_s^{k+1-l} + \frac{1}{\mu^2} \sum_{l=0}^{k-1} t_l \sum_{j=1}^{k-l} M_j \tilde{q} Y_s^{k-j-l} q \qquad (1.169)$$

All the numerical coefficients in the second term on the r.h.s. of (1.169) are fixed by the requirement that when we substitute (1.167), (1.168) into it, cross terms such as $\tilde{Q}_1 Q_1 \tilde{q}_2 q_2$ vanish.

The form (1.169) for the dual superpotential must satisfy additional consistency conditions. The simplest of these involves getting the right behavior when some of the roots a_i coincide [30]. Others consistency conditions involve also the matching of scale (1.150). This relation and the dual magnetic superpotential are consistent with various deformation of the model, including adding mesonic deformations, i.e. terms proportional to M_m.

Chapter 2

Dynamical supersymmetry breaking in metastable vacua

2.1 Introduction and overview of recent developments

At a first attemp, dynamical supersymmetry breaking can appear to be a rather non generic phenomenon in supersymmetric gauge theories. The non zero Witten index of $\mathcal{N} = 1$ Yang Mills theory implies that any supersymmetric gauge theory with vector like massive matter has supersymmetric vacua. So theories with no supersymmetric vacua must be either chiral [31, 32] or they should have massless matter [33, 34].

The way out of this constraint resides in abandoning the idea that models of dynamical supersymmetry breaking must have no supersymmetric vacua. Indeed it is phenomenologically acceptable considering a false vacuum, i.e. a local minimum of the scalar potential, where supersymmetry is spontaneously broken, and that the supersymmetric vacua stay elsewhere in the field space. The necessary requirement is of course that the decay rate of this false vacuum to the true global minimum of the theory is parametrically small, or, in other words, that the lifetime of the false vacuum is parametrically large. This brings to the idea of meta-stable supersymmetry breaking vacua, already explored in the literature [35, 37, 36]. The novelty of the approach of [25] is in considering dynamical supersymmetry breaking in meta-stable vacua. A crucial role in their model is played by Seiberg duality, which allows to find a weakly coupled version of a very simple theory, massive SQCD, in order to describe the low energy. In this description they found metastable supersymmetry breaking vacua, which are long lived. These vacua appear at a semiclassical level in the magnetic description, but they are purely strongly quantum mechanically in the electric description, and in this sense we denote this model as dynamical breaking of supersymmetry.

The existence of metastable vacua with dynamical breaking of supersymmetry seems by now a generic phenomenon in $\mathcal{N}=1$ gauge theories. It has been proved successfully in many classes of models [38, 39, 1, 40, 14, 41]. Much effort has been spent in order to find the right D-brane configurations in string theory and in M theory in order to engineer these gauge theories with metastable vacua [38, 42]. Another relevant aspect is the description in the AdS/CFT correspondence language. The proposal of [43] relate these vacua to metastable state of anti D-branes in throat geometries. Also the realization of this kind of dynamical supersymmetry breaking in string theory has recently attracted a lot of attention, in particular in many quiver gauge theories emerging from D-branes at singularities [53, 56, 3, 44]. Related results has been obtained studying configurations of D-branes and anti D-branes [45]. On the other hand, these models with supersymmetry breaking can also be inserted in supergravity theories [46], for instance with the aim of F-terms uplifting. Finally, the cosmological properties of such vacua and their role in the heating process of the universe has been studied [47]. For references about metastable vacua and gauge mediation see chapter 3.

2.2 ISS model

We consider a very simple model, $SU(N_c)$ SQCD with N_f massive quarks Q and $\tilde{Q}$. We have already introduced this theory in section (1.5). The mass m_0 of the quarks are taken to be much smaller than the strong coupling scale of the theory $m_0 < \Lambda$, and hence has to be taken in the low energy description. The superpotential for the electric theory is, in the case of equal masses,

$$W = m_0 \text{tr } Q\tilde{Q} \tag{2.1}$$

We work in the window

$$N_c + 1 \leq N_f < \frac{3}{2} N_c \tag{2.2}$$

such that the electric theory is asymptotically free and the dual description is infrared free. Indeed, as explained in section 1.7, the low energy of this theory can be better described in terms of a Seiberg dual theory, which is a $SU(\tilde{N} = N_f - N_c)$ gauge theory with N_f magnetic flavours q, $\tilde{q}$ and a gauge singlet M, coupled with a non trivial superpotential. In the free magnetic range (2.2) the metric on the moduli space is smooth around the origin of the field space. Therefore the Kahler potential is regular and can be expanded as

$$K = \frac{1}{\beta} \text{Tr}(q^\dagger q + \tilde{q}^\dagger \tilde{q}) + \frac{1}{\alpha |\Lambda|^2} \text{Tr} M^\dagger M \tag{2.3}$$

where the scale Λ appears because the field M is identified with the microscopic combination $Q\tilde{Q}$ which has dimension two. The dimensionless coefficient α and β are positive real numbers that do not affect our reasoning. The superpotential for the dual theory is

$$W = \frac{1}{\mu} \text{tr } M q \tilde{q} + m_0 \text{tr } M \tag{2.4}$$

where the intermediate scale μ is related to the strong scale of the two theories by the usual matching condition

$$\Lambda^{3N_c-N_f}\tilde{\Lambda}^{3(N_f-N_c)-N_f} = (-1)^{N_f-N_c}\mu^{N_f} \tag{2.5}$$

The theory just presented is the same as the model studied in section 1.3.7 with the dictionary

$$\frac{M}{\sqrt{\alpha}\Lambda} \to M \qquad \Phi = q \qquad \tilde{\Phi} = \tilde{q} \qquad h = \frac{\sqrt{\alpha}\Lambda}{\mu} \qquad m^2 = -m_0\mu \qquad \tilde{N} = N_f - N_c \tag{2.6}$$

This theory hence has supersymmetry breaking vacua, stabilized by quantum corrections, where

$$M = \left(\begin{array}{cc} 0 & 0 \\ 0 & X_{N_f-\tilde{N}\times N_f-\tilde{N}} \end{array}\right) \qquad q = \left(\begin{array}{c} m\mathbf{1}_{\tilde{N}} \\ 0 \end{array}\right) \qquad \tilde{q}^T = \left(\begin{array}{c} m\mathbf{1}_{\tilde{N}} \\ 0 \end{array}\right) \tag{2.7}$$

and the scalar potential is

$$V_{min} = (N_f - \tilde{N})|h^2m^4| \tag{2.8}$$

The main difference between the theory presented here and the one of section 1.3.7 is that here the symmetry group $SU(\tilde{N})$ is gauged. This fact could have many consequences on the stability and even on the existence of such vacua. Moreover we observe that we expect to recover, elsewhere in the field space, the N_c supersymmetric vacua of massive $SU(N_c)$ SQCD labelled by the Witten index.

2.3 Gauge symmetry

So we consider now the effect of gauging the symmetry $SU(\tilde{N})$ on this theory, that is the dual description of massive SQCD. We are in the IR free window, hence the magnetic theory is weakly coupled. There is a scale (a Landau Pole) $\tilde{\Lambda}$, above which the theory is strongly coupled. For such high energies the right description is the electric one.

Now, having gauged the $SU(\tilde{N})$ symmetry, the scalar potential has a new contribute coming from the D terms

$$V_D = \frac{1}{2}g^2\sum_A \left(\text{Tr}q^\dagger T_A q + \text{Tr}\tilde{q}T_A\tilde{q}^\dagger\right) \tag{2.9}$$

This potential has several important consequences. First of all it vanishes in the vacua (2.7), so (2.7) remains a minimum of the tree level potential. The $SU(\tilde{N})$ symmetry is completely Higgsed in this vacuum, and the Goldstone bosons associated to the breaking of this symmetry are eaten through the superHiggs mechanism. On the other hand, the scalar potential (2.9) give tree level masses $\sim gm$ to the fields $\delta\chi$. Thus, only the fields X and $\theta + \theta^\star$ remain as classical pseudo moduli. We have seen in the section 1.3.7 that these pseudomoduli are stabilized by quantum corrections. We wonder if the gauging modify the quantum corrections. We know from section 1.3.2 that the mass matrices get non trivial contributions from the D terms. However, observe that the $SU(\tilde{N})$ gauge fields do not directly couple to the supersymmetry breaking and the D terms vanish on the pseudo flat space. Moreover, the non zero expectation values for the charged quarks q and $\tilde{q}$ do not couple directly to any non zero F term and hence do not contribute to the supertrace.

We then conclude that the leading order effective potential is the one computed in section 1.3.7 and it is unaffected by the gauging of the $SU(\tilde{N})$ gauge group. Hence the vacuum (2.7) is a local minimum of the scalar potential with spontaneous breaking of supersymmetry.

2.4 Supersymmetric vacua

We know, from the Witten index of the electric theory, that also the magnetic theory must admit N_c supersymmetric vacua, and we search for them in another region of field space. Consider giving large expectation values to the meson M. The superpotential (2.4) gives then the flavours a mass $\langle hM \rangle$, and they can be integrated out. The low energy theory is then $SU(\tilde{N})$ Yang Mills with a dynamical scale given by

$$\Lambda_{low}^{3\tilde{N}} = \frac{h^{N_f} \det M}{\Lambda^{N_f - 3\tilde{N}}} \tag{2.10}$$

where $\tilde{\Lambda}$ is the Landau Pole of the magnetic theory. The low energy theory get contribution from gaugino condensation and so the effective superpotential is

$$W_{low} = \tilde{N}\Lambda_{low}^3 - hm^2 \text{tr } M = \tilde{N}\left(h^{n_f}\tilde{\Lambda}^{-(N_f-3\tilde{N})}\det M\right)^{\frac{1}{\tilde{N}}} - hm^2\text{tr } M \tag{2.11}$$

This superpotential leads to the $N_f - \tilde{N} = N_c$ supersymmetric vacua[1]

$$\langle hM \rangle = \tilde{\Lambda}\epsilon^{\frac{2\tilde{N}}{N_f-\tilde{N}}}\mathbf{1}_{N_f} = m\epsilon^{\frac{3\tilde{N}-N_f}{N_f-\tilde{N}}}\mathbf{1}_{N_f} \qquad \epsilon = \frac{m}{\tilde{\Lambda}} \tag{2.12}$$

where ϵ is a parameter that we can take arbitrary small, such that

$$|m| \ll |\langle hM \rangle| \ll |\tilde{\Lambda}| \tag{2.13}$$

This vacuum expectation values is then far below the Landau Pole, making reliable the analysis we have done. On the other hand, the supersymmetric vacuum is far from the non supersymmetric one whose scale is given by the parameter m.

Hence we recover in another region of the field space, far from the origin but below the Landau Pole, the supersymmetric vacua. The restoration of supersymmetry is driven by non perturbative effects (gaugino condensation) associated to the dynamics of the magnetic gauge group $SU(\tilde{N})$. In the supersymmetric vacua R symmetry is broken by the vacuum expectation values for the

[1] We take the meson proportional to the identity matrix.

meson field. We have found dynamical supersymmetry restoration in a theory which breaks supersymmetry at tree level.

The existence of these supersymmetric vacua elsewhere in field space implies that the non supersymmetric vacua of the previous section become only metastable upon gauging $SU(\tilde{N})$, and we have to check if their lifetime is large.

2.5 Effects from underlying microscopic theories

The theory we have studied (the magnetic one) is infrared free and therefore we cannot trust it as a complete theory. Its description breaks down at the Landau Pole $\tilde{\Lambda}$ where its gauge interactions become large. We should verify that any effects from the microscopic theory does not infer our results. We have already introduced the dimensionless parameter

$$|\epsilon| = \left|\frac{m}{\tilde{\Lambda}}\right| \ll 1 \tag{2.14}$$

that we have assumed very small. This implies that the procedure to obtain (2.12) and the result itself, were consistent. We will find that this parameter controls also other possible corrections coming from the microscopic theories.

We first consider as a potentially dangerous contribution the effects of loops of modes from the high energy theory. These can be summarized in corrections to the Kahler potential, that up to now we have considered to be canonical, which have the form

$$\delta K = \frac{c}{|\tilde{\Lambda}|^2}\mathrm{Tr}(M^\dagger M)^2 \tag{2.15}$$

with c a dimensionless constant. The right dimension is obtained using the natural scale of the theory. Then such high dimension operator are suppressed by inverse power of the large scale $\tilde{\Lambda}$ and do not affect the low energy dynamics. More precisely, the contributions to the effective potential coming from these corrections goes like $\sim |m^2\epsilon^2|$, whereas the one loop corrections like $\sim |m|^2$. Higher order corrections are even more soppressed by powers of $\tilde{\Lambda}$. Hence the contributions of 2.15 are negligeable with respect to the one loop ones.

The other main question regards the fact that in the derivation of (2.12) we have considered a typically non perturbative effects of the superpotential, i.e. gaugino condensation, without taking into account possible Kahler corrections like (2.15). The leading effects of the Kahler corrections (2.15) to the effective scalar potential are schematically

$$\delta_{Kahler}V_{eff} \sim \left|\frac{m^2 M}{\tilde{\Lambda}}\right|^2 \sim |m^2\epsilon^2||M|^2 \tag{2.16}$$

which, as already said, are negligeable with respect to the one loop contribute (). The correction to the scalar potential coming from the gaugino condensation contribution in the superpotential

(2.11) are of the form

$$\delta_{superpot} V_{eff} \sim \left| \frac{m^2 M^{\frac{N_f - \tilde{N}}{\tilde{N}}}}{\tilde{\Lambda}^{\frac{N_f - \tilde{N}}{\tilde{N}}}} \right| \tag{2.17}$$

Then, for small M, precisely for $|M| < |\tilde{\Lambda}\epsilon^{\frac{2\tilde{N}}{N_f - 3\tilde{N}}}|$ both corrections are negligeable, confirming the results about the non supersymmetric vacuum and its stability. For $|M| \gg |\tilde{\Lambda}\epsilon^{\frac{2\tilde{N}}{N_f - 3\tilde{N}}}|$ the correction from the superpotential (2.17) is more important than the correction from the Kahler potential (2.16).

Concluding, the possible corrections coming from the high energy theory do not invalidate our analysis of the low energy dynamics, where we evidenciate the existence of local quantum minimum of the scalar potential with spontaneous breaking of supersymmetry, and also the existence of supersymmetric vacua, elsewhere in the field space, where the restoration of supersymmetry is driven by non perturbative effects. Moreover observe that the global picture we have found is very similar to the deformed O'Rafeartaigh model (section 1.3.4). Indeed the non perturbative contribution in the superpotential due to gaugino condensation is similar to the deformation added to the O'Rafeartaigh model, which restore supersymmetry in the large field region, whereas is negligeable in the small field region where there are metastable supersymmetry breaking vacua. One of the main novelty in the ISS model is that the hierarchy between the different contribution to the superpotential is naturally generated dynamically.

2.6 Lifetime: Bounce action

We have found that the non supersymmetric vacuum is a metastable state of the theory, that have to decay into the true vacuum, which is supersymmetric. We want to estimate the lifetime of this false vacuum, computing the decay rate, which is proportional to the semi classical decay probability. The semi classical decay probability is given by the exponential of the bounce action [48]. The bounce action is the difference in the Euclidean action between the tunneling configuration and the metastable vacuum. In order to estimate the bounce, we need to find a trajectory in field space which costs the minimal amount of energy to connect the non supersymmetric vacuum to the supersymmetric one. This means finding the direction in the field space where the potential barrier is minimum. The classical potential is approximately $V \sim |hqM|^2 + |hM\tilde{q}|^2$, hence a large potential energy is necessary in order to have both q and M different from zero. Starting from the non supersymmetric vacuum (2.7) the most efficient path is then to climb up to a point where

$$M = 0 \qquad q = \tilde{q} = 0 \qquad V = N_f |h^2 m^4| \tag{2.18}$$

From there, we move in the M direction towards the supersymmetric vacuum (2.12). The thin wall approximation is not appropriate in this case, since the two vacuum have not the same energy

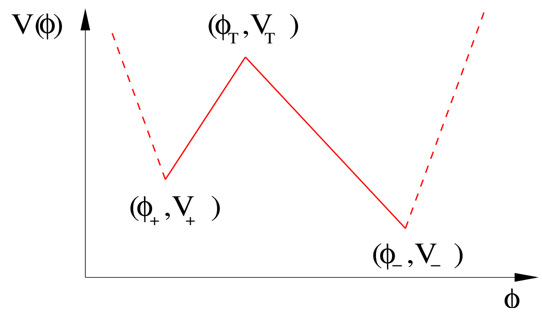


Figure 2.1: A triangular barrier potential for a generic field ϕ

and are very far from each other in the field space. We use the triangular approximation [49], which is reliable in cases where the gradient of the potential is approximately constant at both sides of the peak. In this case the bounce action is

$$S \sim \frac{|(\Delta\phi_+)^2 + (\Delta\phi_-)^2|^2}{\Delta V_+} \tag{2.19}$$

Where we have defined

$$\Delta\phi_\pm = \pm(\phi_T - \phi_\pm) \qquad \Delta V_\pm = (V_T - V_\pm) \tag{2.20}$$

and the fields ϕ_+, ϕ_-, ϕ_T and the values V_T, V_+ and V_- are reported in the figure 3.1. We have showed the triangular barrier for a generic field ϕ. However, in the case at hand, the first displacement $\Delta\phi_+$ is associated to the quarks field, whereas the second one $\Delta\phi_-$ is associated to the meson field. Now, since $\Delta\phi_+ \ll \Delta\phi_-$, and using that $\Delta V_+ = V_{min}$ (2.8), we estimate the bounce action as

$$S \sim \frac{(\Delta\phi_+)^4}{V_{min}} = \frac{1}{\epsilon^{\frac{4(N_f - 3\tilde{N})}{N_f - \tilde{N}}}} \tag{2.21}$$

Since the exponent is positive in our window (2.2), in the limit $\epsilon \to 0$, we can make the bounce action parametrically large and hence the lifetime of the non supersymmetric vacuum parametrically long. Of course any other trajectory will cost more than this in energy, and then will take more time to be done.

2.7 Conclusions and comments

We conclude that SQCD with light massive flavours (with masses smaller than the strong coupling scale Λ) has metastable supersymmetry breaking vacua in the window $N_c + 1 < N_f < \frac{3}{2}N_c$

We have not commented on the interesting special case $N_f = N_c + 1$, where the magnetic gauge group is trivial. In the superpotential there is an extra term which encodes the quantum constraint on the moduli space (see sec. 1.6.4). For $N_c > 2$ the determinant piece in this new term is negligeable near the origin of the field space and hence we remain with the same theory analized in section, which has metastable vacua. Hence metastable vacua are present in the low energy description of massive SQCD in the window $N_c + 1 \leq N_f < \frac{3}{2}N_c$. In [25] it was also conjectured that the same is true for the $N_f = N_c$ case.

In the case of equal masses, the non supersymmetric vacua have a global flavour symmetry $U(N_f)$ which is spontaneously broken to $U(N_f - N_c) \times U(N_c)$. Thus the metastable dynamical supersymmetry breaking vacua form a compact moduli space of vacua

$$\mathcal{M} = \frac{U(N_f)}{U(N_f - N_c) \times U(N_c)} \tag{2.22}$$

There is a bigger configuration space of vacua with broken supersymmetry versus the isolated supersymmetric vacua. This suggest a cosmological advantage in populating the vacua with broken supersymmetry with respect to the supersymmetric ones.

Chapter 3

Gauge mediation

A relevant aspect of theory showing metastable vacua consists in the possibility of implementing gauge mediation mechanism [51, 37, 52]. This topic has been studied in many recent works [53, 54, 55, 2, 57]. An important aspect for phenomenology is naturalness of any scale appearing in the theory, which can be achieved with the so called retrofitting [58]. We briefly explain here some aspects of the extensive subject of gauge mediated supersymmetry breaking. For a complete review see [19] (see also [15, 59, 6, 7, 16]).

3.1 Introduction

Supersymmetry is an elegant solution to the naturalness problem. However, in the real world supersymmetry must be broken. Hence it must be an approximate symmetry of the theory above the TeV scale. This is possible when supersymmetry is broken softly, i.e. without introducing quadratic divergencies. The mass spectrum of the thoery is determinated by the mechanism of supersymmetry breaking. A very stringent constraint on the mass spectrum is given by the Supertrace theorem. It states that the sum of the particle tree level masses weighted by the number of degrees of freedom, is equal in the bosonic and fermionic sector

$$\mathrm{STr}(m^2) = \sum_j (-1)^j (2j+1)\mathrm{Tr}(m_j^2) = 0 \tag{3.1}$$

This theorem applyies to models with tree level supersymmetry breaking and without gravitational anomalies. This rules out the possibility of constructing simple models with tree level supersymmetry breaking. However, the supertrace theorem follows from the properties of renormalizability that constraint the kinetic terms to the canonical form.

The way out of this constraint consist in assuming that the sector of supersymmetry breaking is coupled to the observable sector via non renormalizable tree level couplings. This can be achieved by taking the supersymmetry breaking fields heavy, and then consider the effective theory obtained integrating them out. This effective theory can have non canonical, and non renormalizable, kinetic terms for matter and guage fields which couple to the supersymmetry breaking sector. This leads to soft supersymmetry breaking terms such as scalar and gaugino masses avoiding the supertrace theorem. Therefore, the supersymmetry breaking mechanism have to be understood studying the supersymmetry breaking effective action and its interaction with the observable sector.

Gravitational mediation

One possibility consists in considering a theory which is not renormalizable, where supertrace theorem does not hold. The natural candidate is gravity. Spontaneous supersymmetry breaking in supergravity leads to soft terms in the effective theories with rigid (non local) supersymmetry. This is a widely considered scenario in phenomenological models, where gravity plays a fundamental role.

Flavour changing neutral current One of the main problem in theories with gravity mediated supersymmetry breaking is that there is no obvious reason why the supersymmetry breaking masses for squarks and leptons should be flavour invariant. Gravity has no reason to arrange its interactions so that they are diagonal in the same basis in which the Higgs couples to the fermions. Even if at tree level, for some accidental reason, they are flavour symmetric, loop corrections will distort their structure. This leads to mass non universalities and eventually to flavour changing neutral current.

In order to respect the stringent bound given by the experiment, it would be preferable for the mass degeneracies among the squarks and sleptons, rather than be accidental, to be guaranteed by the nature of the mediation mechanism. If the scalar soft masses were functions only of the gauge quantum numbers of the individual sparticles, universality would be automatic. This can be achieved in model with gauge mediated supersymmetry breaking, where the ordinary gauge interactions are responsible for the appareance of soft supersymmetry breaking in the MSSM.

Gauge mediation

In this case the dynamics at microscopic level is described by a renormalizable lagrangian, which satisfy the supertrace theorem. However the low energy description is governed by an effective lagrangian where non renormalizable terms have been induced by quantum effects, through gauge interactions. This effective theory can communicate supersymmetry breaking to an observable sector, and the supertrace constraint is avoided.

In gauge mediated theories, it is possible to describe the dynamics without gravity. This is not appreciate in view of unification of forces. On the other hand it allows to study models with field theoretical tools only, without have to deal with quantum gravity. This is particularly interesting because lot of progress have been made in understanding non perturbative aspects of supersymmetric gauge theories, where supersymmetry breaking mechanisms, and their comunication to the standard model sector, can be investigated.

3.2 Gauge mediation

As already introduced, the basic idea of gauge mediation is that the ordinary gauge interactions give raise through loop corrections to the soft supersymmetry breaking terms in the MSSM. The gravitational interaction is a subleading effect.

The standard construction consists in the following three sectors.

1. The *visible* sector: this is a supersymmetric extension of the Standard Model, typically the MSSM.

2. The *hidden* sector[1]: this is the sector where supersymmetry breaking occurs. It should be a singlet under MSSM gauge transformation. Details of this sector are model dependent. We can summarize the effect of this sector considering a set of chiral superfields S_i, which are SM gauge singlets, that acquire non zero vacuum expectation values for both their scalar and auxiliary components
$$\langle S_i \rangle = s_i + \theta^2 F_i \tag{3.2}$$
where the F_i components set the supersymmetry breaking scale. In the simplest case, where there is only one S, this coincides with the Goldstino superfield.

3. The *messenger* sector: this sector is formed by some new superfields Φ $\tilde{\Phi}$ that transform under the gauge group as a real non trivial representation (such as they can have gauge invariant masses and be very heavy) and couple at tree level with the superfields S_i
$$W \sim \sum_i \lambda_i S_i \Phi_i \tilde{\Phi}_i \tag{3.3}$$
This coupling generates a supersymmetric mass of order s_i for the messengers and mass squared splitting of order F_i. In the minimal model of gauge mediated supersymmetry breaking the messengers are chiral superfields transforming as a $5 + \bar{5}$ of $SU(5) \subset SU(3) \times SU(2) \times U(1)$. This choice is sufficient to give masses to all of the MSSM scalars and gauginos.

   We have assumed in (3.3) that the interaction superpotential is diagonal in the messenger fields Φ_i. This can be obtained with a rotation in field space. In this case there can be non trivial relation between the neutral chiral fields S_n, leading to cancellation. We will see an example in the appendix A.

   Also the messenger sector is quite model dependent, and we expect it has a common origin with the hidden sector. In model of direct gauge mediation the two sectors are unified.

[1] Sometimes it is called secluded sector in order to distinguish it from the hidden sector which is the usual name in gravity mediated scenarios.

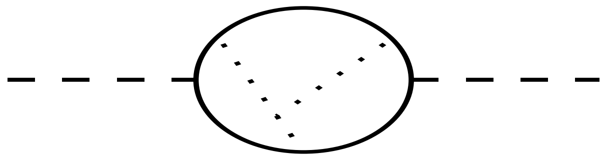


Figure 3.1: Corrections to gauge superfield propagator

The mass splitting of the messengers fields due to the interaction superpotential (3.3) can be found as follows. We consider the superpotential (3.3) and compute the corresponding mass matrices. The messenger fermions have masses $|\lambda_i s_i|$. On the other hand, the squared mass matrix for the scalars is

$$M_0 = \begin{pmatrix} |\lambda_i s_i|^2 & \lambda_i F_i \\ \lambda_i F_i^* & |\lambda_i s_i|^2 \end{pmatrix} \tag{3.4}$$

with squared mass eigenvalues $|\lambda_i s_i|^2 \pm |\lambda_i F_i|$. The requirement that the mass eigenvalues are positive yields the constraint

$$|F_i| < |\lambda_i||s_i|^2 \tag{3.5}$$

The masses of the messenger superfield component still satisfy the supertrace sum rule. Nevertheless, the splitting between the masses of the scalar and fermionic components of the messengers superfields indicates supersymmetry breaking.

The supersymmetry violation, apparent in this messenger spectrum for $F_i \neq 0$, is communicated to the MSSM through radiative corrections.

The interaction of the messengers superfields both with the other superfields of the hidden sector and with the SM gauge superfields can be expected to produce a breakdown of supersymmetry in the propagators of the component fields of the SM gauge superfields. To lowest order in the SM gauge coupling, the leading contribution to the propagators comes from diagram as the one shown in figure 4.1, where dashed lines are any component fields of the gauge superfields, solid lines are component fields of the messengers, and dotted lines are component fields of the hidden sector superfields.

The breakdown of supersymmetry in the propagators of gauge superfields is comunicated to the scalars sleptons and squarks of the supersymmetric Standard Model through diagram like in figure 3.2, where a component field of the gauge superfield of the standard model is emitted and reabsorbed by the squark or by the slepton.

3.3 Dynamical supersymmetry breaking and gauge mediation

Dynamical supersymmetry breaking provides an attractive way to explain the hierarchy between the weak scale and the Plank scale. For this reason there are many attempts to build realistic

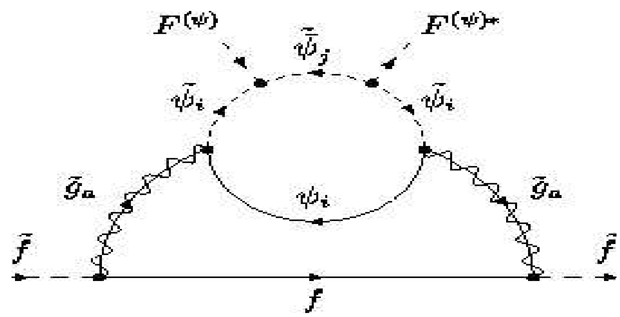


Figure 3.2: Corrections to the scalar propagators

models of dynamical supersymmetry breaking with gauge mediation. Many of the gauge theories that are known to break dynamically supersymmetry, with gauge group G_{dyn} have flavour symmetries G_f, which can remain unbroken in the vacuum, and which can be anomaly free.

The strategy consists then in embedding the SM group in this non anomaly flavour group $G_f \supset SU(3) \times SU(2) \times U(1)$ and in gauging it. The resulting model still break supersymmetry as long as the SM interaction is weak compared to the gauge interaction G_{dyn} whose dynamics controls the supersymmetry breaking. Now, the supersymmetry breaking is obviously comunicated to the whole SM sector just by the SM gauge interactions.

Landau Pole problem One of the main difficulties in this context is the so called Landau pole problem. Typically, models that break dynamically supersymmetry and have a flavour group G_f, have a large gauge group G_{dyn}. The different colours G_{dyn} behave as distinct flavours for the SM gauge interactions. Hence there are in general many flavours of SM matter in the messenger sector. This can imply that the SM gauge factors lose asymptotic freedom, and that the gauge couplings diverge above the messenger mass scale but below the GUT scale.

3.4 Direct gauge mediation

Another interesting approach is based on the attempt to remove the messenger sector. The strategy is to unify the hidden and the messenger sectors. The sector with supersymmetry breaking, the one that is hidden in the usual gauge mediation scenario, should then now play also the role of the messenger sector to comunicate the breaking[2].

Once again the requirement is that the supersymmetry breaking sector possesses an anomaly-free global symmetry large enough to embed the SM gauge group and which is not broken in the vacuum with supersymmetry breaking.

[2]Hence here the messengers are charged under the gauge group G_{dyn}.

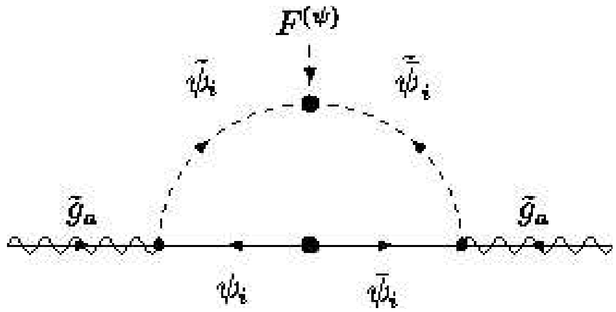


Figure 3.3: Corrections to the gaugino propagator

Also in this case the large size of the dynamical group feeds back in the SM sector potentially leading to Landau pole problems.

3.5 Gaugino mass

Because the Φ_i fields are charges under the SM gauge groups, the gauginos of the MSSM can receive masses through loops of these fields. In particular, gaugino propagators get corrrections at one loop from the diagram shown in figure 4.3. The particles in the loop are messenger fields. Recall that the interaction vertices are of gauge coupling strength even though they do not involve gauge bosons.

The resulting gaugino mass which is induced by the radiative corrections is

$$M_a \simeq \frac{\alpha_a}{4\pi} \sum_i T_a(i) \frac{|F_i|}{|s_i|} \, g\left(\frac{|F_i|}{|\lambda_i||s_i|^2}\right) \qquad a = 1,2,3 \tag{3.6}$$

where

$$g(x) = \frac{1}{x^2}\left[(1+x)\ln(1+x) + (1-x)\ln(1-x)\right] = 1 + \frac{x^2}{6} + \frac{x^4}{15} + \frac{x^6}{28} + \dots \tag{3.7}$$

and the $T_a(i)$ are the Dynkin index for the pair Φ, $\tilde{\Phi}$, associated to the representation by which the pair Φ, $\tilde{\Phi}$ transforms under the gauge group G_a which is a factor of the SM gauge group [3]. For example $n_a = 1$ for the $N + \bar{N}$ of $SU(N)$.

The expansion in (3.7) is valid when $x < 1$. In the small x limit, all terms in x are neglected. When $F_i \sim s_i$ we cannot neglect the higher terms.

A detailed example of computation of gaugino mass in cases where the interaction superpotential is not diagonal in the messengers is reported in appendix A.

[3] The index a runs over the $SU(3) \times SU(2) \times U(1)$ factors of the SM gauge group $SU(5)$.

Chapter 4

Metastable vacua in SQCD with adjoint fields

4.1 Introduction

Long living meta-stable vacua breaking supersymmetry exist in classes of $\mathcal{N} = 1$ gauge theories of the SQCD type with massive fundamental matter.

The novelty of the approach of Intriligator Seiberg and Shih, that we have reviewed in the previous chapters, relies on theories for which Seiberg-like duality exists i.e. (electric) theories which are asymptotically free in the ultraviolet and strongly coupled in the infrared, where the physics can be described in terms of weakly coupled dual (magnetic) theories. In the region of small fields this dual description can be studied as a model of pure chiral fields. Supersymmetry is broken by the rank condition, i.e. not all the F-term conditions can be satisfied. Roughly speaking the next step is to recover, in this magnetic infrared, a generalized chiral O'Raifeartaigh model with supersymmetry breaking vacua.

These non supersymmetric vacua have typically classical flat directions which can be lifted by quantum corrections. Intriligator Seiberg and Shih have proved that such corrections generate positive mass terms for the pseudo-moduli leading to long lived metastable vacua. These facts should be tested in different supersymmetric theories. Some generalization have already appeared upholding the notion that such phenomenon is rather generic.The relative stability of the vacua is a rather delicate issue. Remarks about the corresponding string configurations corroborating the stability analysis have also appeared.

In this chapter we study theories with adjoint chiral fields with cubic superpotential à la KSS [28, 29, 30]. Such superpotentials generate a further meson in the dual magnetic theory: this might produce several pseudo-Goldstone excitations and jeopardize the 1-loop stability of the non supersymmetric vacua. There must be enough F and/or D equations to give tree level masses. A viable model, of string origin, with two gauge groups has been presented in [39].

We consider a theory with one gauge group $SU(N_c)$ and two massive electric adjoint fields, where the most massive one gets integrated out. This amounts to add a massive mesonic deformation in the dual theory. This avoids dangerous extra flat directions which cannot be stabilized at 1-loop. A discussion of the possible interpretation via D-brane configurations can be found in [60, 61].

In the study of the magnetic dual theory we find a tree-level non supersymmetric vacuum which is stabilized by quantum corrections; we show that this is a metastable state that decays to a supersymmetric one after a parametrically long time. A landscape of non supersymmetric metastable vacua, present at classical level, disappears at quantum level. Differently from [25, 38, 39] in our model there is no $U(1)_R$ symmetry and our minimum will not be at the origin of the field space, making our computation much involved. We present most of our results graphically, giving analytic expressions in some sensible limits. We follow the computational strategy of [25].

In section 5.2 we recall some basic elements of the KSS duality and introduce the model that we consider through the chapter. In section 5.3 we solve the D and F equations finding an energy local minimum where supersymmetry is broken by a rank condition. In section 5.4 we compute the 1-loop effective potential around this vacuum and find that it is stabilized by the quantum corrections. In section 5.5 we restore supersymmetry by non perturbative gauge dynamics and recover supersymmetric vacua. Using this result we estimate the lifetime of our metastable vacuum in section 5.6.

4.2 $\mathcal{N} = 1$ SQCD with adjoint matter

Here we introduce some useful elements about electric/magnetic duality for supersymmetric gauge theories with an adjoint field [28, 29, 30]. We consider $\mathcal{N} = 1$ supersymmetric $SU(N_c)$ Yang Mills theory coupled to N_f massive flavours $(Q^i_\alpha, \tilde{Q}^{j\beta})$ in the fundamental and antifundamental representations of the gauge group $(\alpha, \beta = 1, \dots N_c)$ and in the antifundamental and fundamental representations of the flavour group $(i, j = 1, \dots N_f)$, respectively. We also consider a charged chiral massive adjoint superfield X^α_β with superpotential[1]

$$W_{el} = \frac{g_X}{3}\mathrm{Tr}X^3 + \frac{m_X}{2}\mathrm{Tr}X^2 + \lambda_X \mathrm{Tr}X \tag{4.1}$$

where λ_X is a Lagrange multiplier enforcing the tracelessness condition $\mathrm{Tr}X = 0$. The Kahler potential for all the fields is taken to be canonical. This theory is asymptotically free in the range $N_f < 2N_c$ and it admits stable vacua for $N_f > \frac{N_c}{2}$ [29].

The dual theory [28, 29, 30] is $SU(2N_f - N_c \equiv \tilde{N})$ with N_f magnetic flavours $(q, \tilde{q})$, a magnetic adjoint field Y and two gauge singlets build from electric mesons ($M_1 = Q\tilde{Q}$, $M_2 = QX\tilde{Q}$), with

[1](Tr) means tracing on the color indices, while (tr) on the flavour ones.

magnetic superpotential

$$W_{magn} = \frac{\tilde{g}_Y}{3}\text{Tr}Y^3 + \frac{\tilde{m}_Y}{2}\text{Tr}Y^2 + \tilde{\lambda}_Y \text{Tr}Y - \frac{1}{\mu^2}\text{tr}\ \left(\frac{\tilde{m}_Y}{2}M_1 q\tilde{q} + \tilde{g}_Y M_2 q\tilde{q} + \tilde{g}_Y M_1 qY\tilde{q}\right) \tag{4.2}$$

where the relations between the magnetic couplings and the electric ones are

$$\tilde{g}_Y = -g_X, \qquad \tilde{N}\tilde{m}_Y = N_c m_X. \tag{4.3}$$

The intermediate scale μ takes into account the mass dimension of the mesons in the dual description. The matching between the microscopic scale (Λ) and the macroscopic scale ($\tilde{\Lambda}$) is

$$\Lambda^{2N_c - N_f}\tilde{\Lambda}^{2\tilde{N}-N_f} = \left(\frac{\mu}{g_X}\right)^{2N_f}. \tag{4.4}$$

We look for a magnetic infrared free regime in order to rely on perturbative computations at low energy. The b coefficient of the beta function is $b = (3\tilde{N} - N_f) - \tilde{N}$, negative for $N_f < \frac{2}{3}N_c$ and so we will consider the window for the number of flavours

$$\frac{N_c}{2} < N_f < \frac{2}{3}N_c \qquad \Rightarrow \qquad 0 < 2\tilde{N} < N_f \tag{4.5}$$

where the magnetic theory is IR free and it admits stable vacua.

4.2.1 Adding mesonic deformations

We now add to the electric potential (4.1) the gauge singlet deformations

$$W_{el} \to W_{el} + \Delta W_{el} \qquad \Delta W_{el} = \lambda_Q\,\text{tr}\ QX\tilde{Q} + m_Q\,\text{tr}\ Q\tilde{Q} + h\,\text{tr}\ (Q\tilde{Q})^2 \tag{4.6}$$

The first two terms are standard deformations of the electric superpotential that don't spoil the duality relations (e.g. the scale matching condition (4.4)) [30]. The last term of (4.6) can be thought as originating from a second largely massive adjoint field Z in the electric theory with superpotential

$$W_Z = m_Z \text{Tr}Z^2 + \text{Tr}\ ZQ\tilde{Q} \tag{4.7}$$

and which has been integrated out [60, 61, 62]. The mass m_Z has to be considered larger than Λ_{2A}, the strong scale of the electric theory with two adjoint fields. This procedure leads to the scale matching relation

$$\Lambda_{2A}^{N_c - N_f} = \Lambda_{1A}^{2N_c - N_f} m_Z^{-N_c} \tag{4.8}$$

where Λ_{2A} and Λ_{1A} are the strong coupling scales before and after having integrated out the adjoint field Z, i.e. with two or one adjoint fields respectively.

The other masses in this theory have to be considered much smaller than the strong scale $\Lambda_{2A} \gg m_Q, m_X$. This forces, via (4.8), the scale Λ_{1A} and the masses to satisfy the relations

$$\frac{m_Q m_Z}{\Lambda_{1A}^2} \ll 1 \qquad\qquad \frac{m_X m_Z}{\Lambda_{1A}^2} \ll 1 \tag{4.9}$$

We will work in this range of parameters, translating these inequalities in the dual (magnetic) context.

We also observe that in (4.8) the coefficient b of the beta function for the starting electric theory with two adjoint fields is $b = N_c - N_f$ and the theory is asymptotically free for $N_f < N_c$. This range is still consistent with our magnetic IR free window (4.5). The dimensional coupling h in our effective theory (4.6) results $h = \frac{1}{m_Z}$ so it must be thought as a small deformation. In analogy with [62][2] we can suppose that when h is small the duality relations are still valid and obtain the full magnetic superpotential

$$\begin{aligned} W_{magn} &= \frac{\tilde{g}_Y}{3}\text{Tr}Y^3 + \frac{\tilde{m}_Y}{2}\text{Tr}Y^2 + \tilde{\lambda}_Y \text{Tr}Y - \frac{1}{\mu^2}\,\text{tr}\,\left(\frac{\tilde{m}_Y}{2}M_1 q\tilde{q} + \tilde{g}_Y M_2 q\tilde{q} + \tilde{g}_Y M_1 qY\tilde{q}\right) \\ &\quad + \lambda_Q\ \text{tr}\ M_2 + m_Q\ \text{tr}\ M_1 + h\ \text{tr}\ (M_1)^2 \end{aligned} \tag{4.10}$$

For this dual theory the scale matching relation is the same as (4.4) with $\Lambda \equiv \Lambda_{1A}$ defined in (4.8).
We consider the free magnetic range (4.5), where the metric on the moduli space is smooth around the origin [25]. The Kahler potential is thus regular and has the canonical form

$$K = \frac{1}{\alpha_1^2\Lambda^2}\text{tr}\ M_1^\dagger M_1 + \frac{1}{\alpha_2^2\Lambda^4}\text{tr}\ M_2^\dagger M_2 + \frac{1}{\beta^2}\text{Tr}Y^\dagger Y + \frac{1}{\gamma^2}(\text{tr}\ q^\dagger q + \text{tr}\ \tilde{q}^\dagger\tilde{q}) \tag{4.11}$$

where $(\alpha_i, \beta, \gamma)$ are unknown positive numerical coefficients.

4.3 Non supersymmetric meta-stable vacua

We solve the equations of motion for the chiral fields of the macroscopic description (4.10). We will find a non supersymmetric vacuum in the region of small fields where the $SU(\tilde{N})$ gauge dynamics is decoupled. The gauge dynamics becomes relevant in the large field region where it restores supersymmetry via non perturbative effects (see sec.5).

We rescale the magnetic fields appearing in (4.10) in order to work with elementary fields with mass dimension one. We then have a $\mathcal{N} = 1$ supersymmetric $SU(\tilde{N})$ gauge theory with N_f magnetic flavours $(q, \tilde{q})$, an adjoint field Y, and two gauge singlet mesons M_1,M_2, with canonical

[2] Where it was done in the context of Seiberg duality.

Kahler potential. The superpotential, with rescaled couplings, reads

$$\begin{aligned} W_{magn} &= \frac{g_Y}{3}\mathrm{Tr}Y^3 + \frac{m_Y}{2}\mathrm{Tr}Y^2 + \lambda_Y \mathrm{Tr}Y + \mathrm{tr}\,(h_1 M_1 q\tilde{q} + h_2 M_2 q\tilde{q} + h_3 M_1 q Y \tilde{q}) \\ & -h_1 m_1^2 \,\mathrm{tr}\, M_1 - h_2 m_2^2 \,\mathrm{tr}\, M_2 + m_3 \,\mathrm{tr}\, M_1^2 \end{aligned} \tag{4.12}$$

where the rescaled couplings in (4.12) are mapped to the original ones in (4.10) via

$$h_1 = -\frac{\tilde{m}_Y}{2\mu^2}(\alpha_1\Lambda)\gamma^2 \qquad h_2 = -\frac{\tilde{g}_Y}{\mu^2}\left(\alpha_2\Lambda^2\right)\gamma^2 \qquad h_3 = -\frac{\tilde{g}_Y}{\mu^2}(\alpha_1\Lambda)\gamma^2\beta$$

$$h_1 m_1^2 = -m_Q\,\alpha_1\,\Lambda \qquad h_2 m_2^2 = -\lambda_Q\,\alpha_2\,\Lambda^2 \qquad m_3 = h(\alpha_1\Lambda)^2 \tag{4.13}$$

We can choose the magnetic quarks $q, \tilde{q}^T$ (which are $N_f \times \tilde{N}$ matrices) to solve the D equations as

$$q = \left(\begin{array}{c} k \\ 0 \end{array}\right) \qquad \tilde{q}^T = \left(\begin{array}{c} \tilde{k} \\ 0 \end{array}\right) \tag{4.14}$$

where $k, \tilde{k}$ are $\tilde{N} \times \tilde{N}$ diagonal matrices.

We impose the F equations of motion for the superpotential (4.12)

$$\begin{aligned} F_{\lambda_Y} &= \mathrm{Tr}Y = 0 \\ F_Y &= g_Y Y^2 + m_Y Y + \lambda_Y + h_3 M_1 q\tilde{q} = 0 \\ F_q &= h_2 M_2 \tilde{q} + h_1 M_1 \tilde{q} + h_3 M_1 Y \tilde{q} = 0 \\ F_{\tilde{q}} &= h_2 M_2 q + h_1 M_1 q + h_3 M_1 q Y = 0 & (4.15) \\ F_{M_1} &= h_1 q\tilde{q} + h_3 q Y\tilde{q} - h_1 m_1^2 \delta_{ij} + 2 m_3 M_1 = 0 \qquad & i,j = 1,\dots N_f \\ F_{M_2} &= h_2 q\tilde{q} - h_2 m_2^2 \delta_{ij} = 0 & i,j = 1,\dots N_f \quad (4.16) \end{aligned}$$

Since we are in the range (4.5) where $N_f > \tilde{N}$ the equation (4.16) is the rank condition of [25]: supersymmetry is spontaneously broken at tree-level by these non trivial F-terms.

We can solve the first $\tilde{N}$ equations of (4.16) by fixing the product $k\tilde{k}$ to be $k\tilde{k} = m_2^2 \mathbf{1}_{\tilde{N}}$. We then parametrize the quarks vevs in the vacuum (4.14) with complex θ

$$q = \left(\begin{array}{c} m_2 e^{\theta}\,\mathbf{1}_{\tilde{N}} \\ 0 \end{array}\right) \qquad \tilde{q}^T = \left(\begin{array}{c} m_2 e^{-\theta}\,\mathbf{1}_{\tilde{N}} \\ 0 \end{array}\right) . \tag{4.17}$$

The other $N_f - \tilde{N}$ equations of (4.16) cannot be solved and so the corresponding F-terms don't vanish ($F_{M_2} \neq 0$). However we can find a vacuum configuration which satisfies all the other F-equations (4.15) and the D-ones. We solve the equations (4.15) for M_1, Y and λ_Y and we choose Y to be diagonal, finding

$$\lambda_Y = \frac{h_3 h_1 m_2^2}{2m_3}\left(m_2^2 - m_1^2\right) - \frac{m_Y^2}{g_Y}\left(1 - \frac{h_3^2 m_2^4}{2 m_3 m_Y}\right)^2 \frac{n_1 n_2}{(n_1 - n_2)^2} \tag{4.18}$$

where the integers (n_1, n_2) count the eigenvalues degeneracy along the Y diagonal, with $(n_1+n_2 = \tilde{N})$

$$\langle Y \rangle = \begin{pmatrix} y_1 \mathbf{1}_{n_1} & 0 \\ 0 & y_2 \mathbf{1}_{n_2} \end{pmatrix} \qquad y_1 = -\frac{m_Y - \frac{h_3^2 m_2^4}{2m_3}}{g_Y} \frac{n_2}{n_1 - n_2} \qquad y_2 = \frac{m_Y - \frac{h_3^2 m_2^4}{2m_3}}{g_Y} \frac{n_1}{n_1 - n_2}$$

We choose the vacuum in which the magnetic gauge group is not broken by the adjoint field choosing $n_1 = 0$, so y_2 vanishes and $\langle Y \rangle = 0$. We observe that other choices for $\langle Y \rangle$ with $n_1 \neq 0 \neq n_2$ wouldn't change the tree-level potential energy of the vacua which is given only by the non vanishing F_{M_2}. This classical landscape of vacua will be wiped out by 1-loop quantum corrections[3]. In our case $(n_1 = 0)$ we have

$$\langle M_1 \rangle = \begin{pmatrix} \frac{h_1}{2m_3}(m_1^2 - m_2^2)\, \mathbf{1}_{\tilde{N}} & 0 \\ 0 & \frac{h_1 m_1^2}{2m_3}\, \mathbf{1}_{N_f - \tilde{N}} \end{pmatrix} = \begin{pmatrix} p_1^A & 0 \\ 0 & p_1^B \end{pmatrix} \tag{4.19}$$

The two non trivial blocks are respectively $\tilde{N}$ and $N_f - \tilde{N}$ diagonal squared matrices.

The $(q, \tilde{q})$ F equations fix the vev of the M_2 meson to be

$$\langle M_2 \rangle = \begin{pmatrix} -\frac{h_1^2}{2h_2 m_3}(m_1^2 - m_2^2)\, \mathbf{1}_{\tilde{N}} & 0 \\ 0 & \mathcal{X} \end{pmatrix} = \begin{pmatrix} p_2^A & 0 \\ 0 & \mathcal{X} \end{pmatrix} \tag{4.20}$$

where the blocks have the same dimensions of M_1, with $\mathcal{X}$ undetermined at the classical level.

Since supersymmetry is broken at tree level by the rank condition (4.16) the minimum of the scalar potential is

$$V_{MIN} = |F_{M_2}|^2 = (N_f - \tilde{N})|h_2 m_2^2|^2 = (N_f - \tilde{N})\, \alpha_2^2\, |\lambda_Q \Lambda^2|^2 \tag{4.21}$$

It depends on parameters that we can't compute from the electric theory (e.g. α_2); in any case we are only interested in the qualitative behaviour of the non supersymmetric state. The potential energy of the vacuum (4.21) doesn't depend on θ and $\mathcal{X}$; they are massless fields at tree level, not protected by any symmetry and hence are pseudo-moduli. Their fate will be decided by the quantum corrections.

Since there isn't any $U(1)_R$ symmetry we don't expect the value of $\mathcal{X}$ in the quantum minimum to vanish. Indeed, computing the 1-loop corrections, we will find that in the quantum minimum the value of θ is zero while $\mathcal{X}$ will get a nonzero vev. This makes our metastable minimum different from the one discovered in [25, 38, 39] where the quantum corrections didn't give the pseudo-moduli a nonzero vev. Notice also that although we have many vevs different from zero in the non supersymmetric vacua they are all smaller than the natural breaking mass scale $|F_{M_2}|^{\frac{1}{2}} = |h_2 m_2^2|^{\frac{1}{2}}$.

[3] This agrees with an observation in [39].

4.4 1-Loop effective potential

In this section we study the 1-loop quantum corrections to the effective potential for the fluctuations around the non supersymmetric vacuum selected in the previous section with $\langle Y \rangle = 0$. The aim is to estabilish the sign of the mass corrections for the pseudo-moduli $\mathcal{X}, \theta$. The 1-loop corrections to the tree level potential energy depend on the choice of the adjoint vev $\langle Y \rangle$: as a matter of fact they are minimized by the choice $\langle Y \rangle = 0$.

The 1-loop contributions of the vector multiplet to the effective potential vanish since the D equations are satisfied by our non supersymmetric vacuum configuration.

The 1-loop corrections will be computed using the supertrace of the bosonic and fermionic squared mass matrices built up from the superpotential for the fluctuations of the fields around the vacuum. The standard expression of the 1-loop effective potential is

$$V_{1-loop} = \frac{1}{64\pi^2} S\mathrm{Tr}\mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} = \frac{1}{64\pi^2} \sum \left(m_B^4 \log \frac{m_B^2}{\Lambda^2} - m_F^4 \log \frac{m_F^2}{\Lambda^2} \right) \tag{4.22}$$

where the F contributions to the mass matrices are read from the superpotential W

$$m_B^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & W^{\dagger abc} W_c \\ W_{abc} W^{\dagger c} & W_{ac} W^{\dagger cb} \end{pmatrix} \qquad m_f^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & 0 \\ 0 & W_{ac} W^{\dagger cb} \end{pmatrix} \tag{4.23}$$

We parametrize the fluctuations around the tree level vacuum as

$$q = \begin{pmatrix} k e^{\theta} + \xi_1 \\ \phi_1 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} k e^{-\theta} + \xi_2 \\ \phi_2 \end{pmatrix} \qquad Y = \delta Y \tag{4.24}$$

$$M_1 = \begin{pmatrix} p_1^A + \xi_3 & \phi_3 \\ \phi_4 & p_1^B + \xi_4 \end{pmatrix} \qquad M_2 = \begin{pmatrix} p_2^A + \xi_5 & \phi_5 \\ \phi_6 & \mathcal{X} \end{pmatrix} \tag{4.25}$$

We expand the classical superpotential (4.12) up to trilinear order in the fluctuations $\phi_i, \xi_i, \delta Y$. Most of these fields acquire tree level masses, but there are also massless fields. Some of them are Goldstone bosons of the global symmetries, considering $SU(\tilde{N})$ global, the others are pseudo-Goldstone bosons.

In this set up, ξ_1 and ξ_2 combine to give the same Goldstone and pseudo-Goldstone bosons as in [25]. Gauging the $SU(\tilde{N})$ symmetry these Goldstone bosons are eaten by the vector fields, and the other massless fields, except $\theta + \theta^\star$, acquire positive masses from D-term potential as in [25]. Combinations of the ϕ_i fields give the Goldstone bosons related to the breaking of the flavour symmetry $SU(N_f) \to SU(\tilde{N}) \times SU(N_f - \tilde{N}) \times U(1)$. The off diagonal elements of the classically massless field $\mathcal{X}$ are Goldstone bosons of the $SU(N_f - \tilde{N})$ flavour symmetry as in [38]. We then end up with the pseudo-moduli $\theta + \theta^\star$ and the diagonal $\mathcal{X}$.

We now look for the fluctuations which give contributions to the mass matrices (4.23). They are only the ϕ_i fields, while the ξ_i and δY represent a decoupled supersymmetric sector. Indeed

ξ_i and δY do not appear in bilinear terms coupled to the ϕ_i sector, so they do not contribute to the fermionic mass matrix (4.23). Even if they appear in trilinear terms coupled to the ϕ_i, they do not have the corresponding linear term[4]: they do not contribute to the bosonic mass matrix (4.23). Since $(\xi_1, \xi_2, \delta Y)$ do not couple to the breaking sector at this order, also their D-term contributions to the mass matrices vanish and all of them can be neglected. We can then restrict ourselves to the chiral ϕ_i fields for computing the 1-loop quantum corrections to the effective scalar potential using (4.23). Without loss of generality we can set the pseudo-moduli $\mathcal{X}$ proportional to the identity matrix.

The resulting superpotential for the sector affected by the supersymmetry breaking (the ϕ_i fields) is a sum of $\tilde{N} \times (N_f - \tilde{N})$ decoupled copies of a model of chiral fields which breaks supersymmetry at tree-level

$$\begin{aligned} W &= h_2 \left(\mathcal{X}\phi_1\phi_2 - m_2^2 \mathcal{X}\right) + h_2 m_2 \left(e^{\theta}\phi_2\phi_5 + e^{-\theta}\phi_1\phi_6\right) + \\ &\quad + h_1 m_2 \left(e^{\theta}\phi_2\phi_3 + e^{-\theta}\phi_1\phi_4\right) + 2m_3\phi_3\phi_4 + \frac{h_1^2 m_1^2}{2m_3}\,\phi_1\phi_2 \end{aligned} \tag{4.26}$$

This superpotential doesn't have any $U(1)_R$ symmetry, differently from the ones studied in [25, 38, 39]. This may be read as an example of a non generic superpotential which breaks supersymmetry [50], without exact R symmetry.

The expressions for the eigenvalues, and then for the 1-loop scalar potential, are too complicated to be written here. We can plot our results numerically to give a pictorial rapresentation.

The computation is carried out in this way: we first compute the eigenvalues of the bosonic and fermionic mass matrices (4.23) using the superpotential (4.26); we evaluate them where all the fluctuations ϕ_i are set to zero; finally we compute the 1-loop scalar potential using (4.22) as a function of the pseudo-moduli $\mathcal{X}, \theta + \theta^\star$. The corrections will always be powers of $\theta + \theta^\star \equiv \tilde{\theta}$ so from now on we will treat only the $\tilde{\theta}$ dependence. We give graphical plots of the 1-loop effective potential treating fields and couplings as real. We have checked that our qualitative conclusions about the stability of the vacuum are not affected by using complex variables.

We redefine the couplings in order to have the mass matrices as functions of three dimensionless parameters (ρ, η, ζ)

$$\rho = \frac{h_1}{h_2} \qquad \eta = \frac{2m_3}{h_2 m_2} \qquad \zeta = \frac{h_1^2 m_1^2}{2h_2 m_2 m_3} \quad , \qquad\qquad \zeta < \rho < \eta \tag{4.27}$$

and we rescale the superpotential with an overall scale $h_2 m_2$ which becomes the fundamental unit of our plots. The inequality in (4.27) is a consequence of the range (4.9) and the redefinitions (4.13). We notice also that ρ, η, ζ have absolute values smaller than one.

In figure 4.1 we plot the 1-loop scalar potential as a function of $\mathcal{X}, \tilde{\theta}$ and for fixed values of the parameters ρ, η, ζ. We can see that there is a minimum, so the moduli space is lifted

[4]The possible linear terms in ξ_i and δY factorize the F-equations (4.15) and so they all vanish.

by the quantum corrections, the pseudo-moduli get positive masses, and there is a stable non supersymmetric vacuum. Making a careful analysis we find that the quantum minimum in the 1-loop scalar potential is reached when $\langle\tilde{\theta}\rangle = 0$ but $\langle\mathcal{X}\rangle \neq 0$ and its vev in the minimum depends on the parameters (ρ, η, ζ). This agrees with what we observed in the previous section. It can be better seen in the second picture of figure 4.1 where we take a section of the first plot for $\tilde{\theta} = 0$.

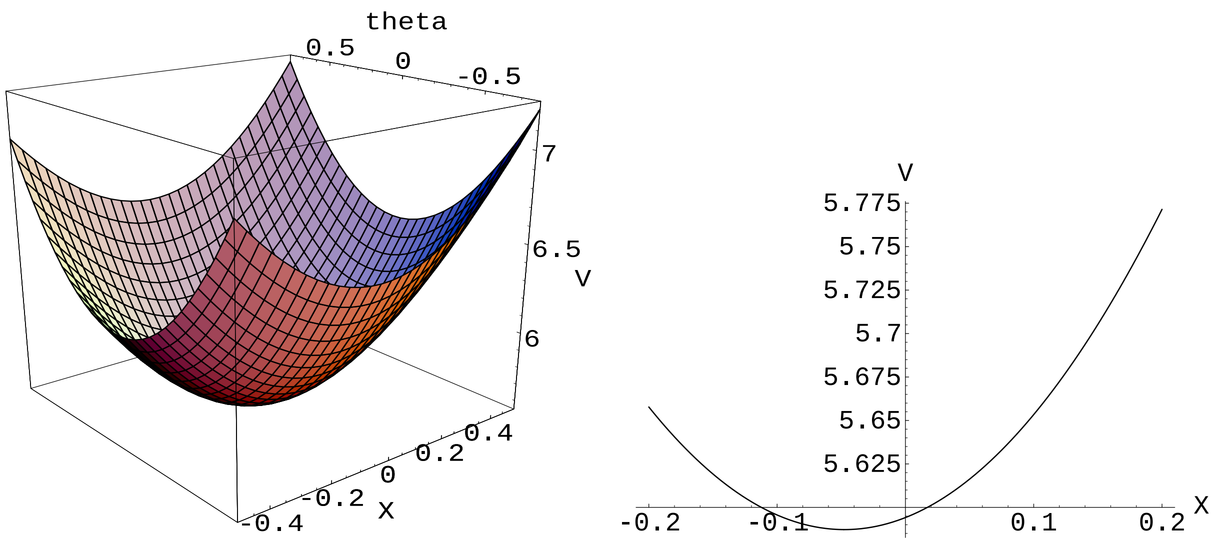


Figure 4.1: Scalar potential V^{1-loop} for $(\eta = 0.5, \rho = 0.1, \zeta = 0.05, \mathcal{X} = -0.5\ldots0.5, \tilde{\theta} = -0.8\ldots0.8)$, and its section for $\tilde{\theta} = 0$; $\mathcal{X}$ is in unit of m_2, while V is in unit of $|h_2^2 m_2^2|^2$.

In figure 4.2 we plot the 1-loop scalar potential for $\tilde{\theta} = 0$ as a function of $\mathcal{X}$ and of the parameter ρ, fixing η and ζ. For each value of ρ the curvature around the minimum gives a qualitative estimation of the generated mass for the pseudo-moduli $\mathcal{X}$. We note that for large ρ the scalar potential becomes asymptotically flat, and so the 1-loop generated mass goes to zero, but this is outside our allowed range.

As already observed, there is a minimum for $\langle\mathcal{X}\rangle$ slightly different from zero due to quantum corrections, and we have found that it goes to zero in the limit $(\zeta \to 0, \rho \to 0)$. We can give analytic results in this limit[5]. We found at zero order in ρ and ζ, with arbitrary η, that the 1-loop generated masses for the pseudo-moduli are

$$\begin{aligned} m_{\mathcal{X}}^2 &= \frac{\tilde{N}(N_f - \tilde{N})}{8\pi^2}|h_2^2 m_2|^2(\log[4] - 1) + o(\rho) + o(\zeta) \\ m_{\tilde{\theta}}^2 &= \frac{\tilde{N}(N_f - \tilde{N})}{16\pi^2}|h_2^2 m_2^2|^2(\log[4] - 1) + o(\rho) + o(\zeta) \end{aligned} \tag{4.28}$$

[5]Considering η, ρ, ζ real.

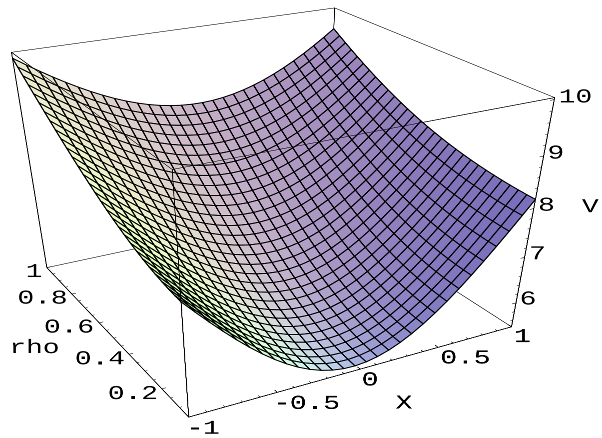


Figure 4.2: Scalar potential V^{1-loop} for ($\mathcal{X} = -1\ldots 1, \rho = 0.05\ldots 1, \eta = 0.5, \zeta = 0.05, \tilde{\theta} = 0$); $\mathcal{X}$ is in unit of m_2, while V is in unit of $|h_2^2 m_2^2|^2$.

so in the limit of small ρ (and small ζ) quantum corrections don't depend on η. We can then write the 1-loop scalar potential in the limit of small ζ and ρ obtaining

$$
\begin{aligned}
V^{(1)} \;=\; & \frac{\tilde{N}(N_f - \tilde{N})}{64\pi^2}|h_2^2 m_2|^2 \Bigg\{ |m_2|^2 \Bigg(\log\big(\frac{|h_2 m_2|^2}{\Lambda^2}\big) + 2\rho^4 \log[\rho^2] - 4(1+\rho^2)^2 \log[1+\rho^2] + \\
& +2(2+\rho^2)^2 \log[2+\rho^2] \Bigg) + \Bigg(4(2+\rho^2)^2 \log[2+\rho^2] - 4\rho^4 \log[\rho^2] + \\
& -8(1+\rho^2)(1+2\log[1+\rho^2]) \Bigg) |\mathcal{X} + m_2\zeta|^2 + |m_2|^2 \Bigg(2(1+\rho^2) \Big[(2+\rho^2)^2 \log[2+\rho^2] + \\
& -\rho^4 \log[\rho^2] - 2(1+\rho^2)(1+2\log[1+\rho^2]) \Big] + 4(\log[4] - \frac{5}{3})\zeta^2 \Bigg) (\theta + \theta^\star)^2 \Bigg\} (1 + o(\zeta))
\end{aligned}
\tag{4.29}
$$

In these approximations the vev for $\langle \mathcal{X} \rangle$ in the minimum is shifted linearly with ζ; however, in general, the complete behaviour for $\langle \mathcal{X} \rangle$ is more complicated and depends non trivially on η. We observe that, being ζ a simple shift for the vev of $\mathcal{X}$, it doesn't affect its mass, while it modifies $\tilde{\theta}$ mass.

From (4.29) we can read directly the masses expanding for small ρ

$$m^2_{\mathcal{X}} = \frac{\tilde{N}(N_f-\tilde{N})}{8\pi^2}|h_2 m_2|^2\Big(|h_2|^2(\log[4]-1)+|h_1|^2(\log[4]-2)\Big) \tag{4.30}$$

$$m^2_{\tilde{\theta}} = \frac{\tilde{N}(N_f-\tilde{N})}{16\pi^2}|h_2 m_2^2|^2\Bigg(|h_2|^2(\log[4]-1)+\left|\frac{h_1^2 m_1^2}{2m_2 m_3}\right|^2(\log[4]-\frac{5}{3})+ + |h_1|^2(2\log[4]-3)\Bigg). \tag{4.31}$$

These expressions are valid up to cubic order in ρ,ζ. The first term in (4.30,4.31), being independent of the deformations (ρ,η,ζ), agrees with [25]. The second term in (4.30) is the same as in [39].

4.5 Supersymmetric vacuum

Supersymmetry is restored via non perturbative effects [26], away from the metastable vacuum in the field space, when the $SU(\tilde{N})$ symmetry is gauged [25]. The non supersymmetric vacuum discovered in the sections 3 and 4 is a metastable state of the theory which decays to a supersymmetric one. We are interested in evaluating the lifetime of the metastable vacuum. We need an estimation of the vevs of the elementary magnetic fields in the supersymmetric state.

We first integrate out the massive fields in the superpotential (4.12) using their equations of motion. In (4.12) there are two massive fields (M_1, Y). We integrate out the meson M_1 and the adjoint field Y tuning λ_Y in such a way that the gauge group $SU(\tilde{N})$ is not broken by the adjoint[6], as in the metastable state, so $\langle Y\rangle = 0$. Using this last condition the equation of motion for the meson M_1 gives the simple relation $M_1 = \frac{h_1}{2m_3}\left(m_1^2 - q\tilde{q}\right)$. Integrating out the charged field Y the scale matching condition reads

$$\tilde{\Lambda}^{2\tilde{N}-N_f} = \tilde{\Lambda}_{int}^{3\tilde{N}-N_f}\hat{m}_Y^{-\tilde{N}} \tag{4.32}$$

where we have indicated with $\hat{m}_Y$ the resulting mass for Y which is a combination of its tree-level mass m_Y and a term proportional to $\frac{h_3^2}{m_3}(q\tilde{q})^2$ which will be shown to be zero in the supersymmetric vacuum.

We obtain a superpotential for the meson M_2 and the flavours $(q,\tilde{q})$

$$W_{int} = \mathrm{tr}\,\left(\frac{h_1^2}{4m_3}\left(2m_1^2 q\tilde{q} - (q\tilde{q})^2\right) + h_2 M_2 q\tilde{q} - h_2 m_2^2 M_2\right) \tag{4.33}$$

We expect that the supersymmetric vacua lie in the large field region, where the $SU(\tilde{N})$ gauge dynamics becomes relevant [25]. We then consider large expectation value for the meson M_2. We

[6]We are not interested in finding all the supersymmetric vacua.

can take as mass term for the flavours $(q, \tilde{q})$ only the vev $\langle h_2 M_2 \rangle$ neglecting the other contribution in (4.33) coming from the couplings of the magnetic theory.

We then integrate out the flavours $(q, \tilde{q})$ using their equations of motion $(q = 0, \tilde{q} = 0)$. The corresponding scale matching condition is

$$\Lambda_L^{3\tilde{N}} = \tilde{\Lambda}_{int}^{3\tilde{N}-N_f} \det(h_2 M_2) = \tilde{\Lambda}^{2\tilde{N}-N_f} \det(h_2 M_2)\, m_Y^{\tilde{N}}\,. \tag{4.34}$$

The low energy effective $SU(\tilde{N})$ superpotential gets a non-perturbative contribution from the gauge dynamics related to the gaugino condensation proportional to the low energy scale Λ_L

$$W = \tilde{N}\Lambda_L^3 \tag{4.35}$$

that can be written in terms of the macroscopical scale $\tilde{\Lambda}$ using (4.34). This contribution should be added to the M_2 linear term that survives in (4.33) after having integrated out the magnetic flavours $(q, \tilde{q})$. Via the scale matching relation (4.34) we can then express the low energy effective superpotential as a function of only the M_2 meson

$$W_{Low} = \tilde{N}\left(\tilde{\Lambda}^{2\tilde{N}-N_f} \det(h_2 M_2)\right)^{\frac{1}{\tilde{N}}} m_Y - m_2^2 h_2 \,\mathrm{tr}\, M_2 \tag{4.36}$$

Using this dynamically generated superpotential we can obtain the vev of the meson M_2 in the supersymmetric vacuum. Considering M_2 proportional to the identity $\mathbf{1}_{N_F}$ we minimize (4.36) and obtain

$$\langle h_2 M_2 \rangle = \tilde{\Lambda}\epsilon^{\frac{\tilde{N}}{N_f-\tilde{N}}}\xi^{\frac{\tilde{N}}{N_f-\tilde{N}}}\mathbf{1}_{N_f} = m_2\left(\frac{1}{\epsilon}\right)^{\frac{N_f-2\tilde{N}}{N_f-\tilde{N}}}\xi^{\frac{\tilde{N}}{N_f-\tilde{N}}}\mathbf{1}_{N_f} \tag{4.37}$$

where

$$\epsilon = \frac{m_2}{\tilde{\Lambda}} \qquad \xi = \frac{m_2}{m_Y}\,. \tag{4.38}$$

ϵ is a dimensionless parameter which can be made parametrically small sending the Landau pole $\tilde{\Lambda}$ to infinity. ξ is a dimensionless finite parameter which doesn't spoil our estimation of the supersymmetric vacuum in the sensible range $\epsilon < \frac{1}{\xi}$. All the exponents appearing in (4.37) are positive in our window (4.5).

We observe that in the small ϵ limit the vev $\langle h_2 M_2 \rangle$ is larger than the typically mass scale m_2 of the magnetic theory but much smaller than the scale $\tilde{\Lambda}$

$$m_2 \ll \langle h_2 M_2 \rangle \ll \tilde{\Lambda}. \tag{4.39}$$

This fact justifies our approximation in integrating out the massive flavours $(q, \tilde{q})$ neglecting the mass term in (4.33) except $\langle h_2 M_2 \rangle$. It also shows that the evaluation of the supersymmetric vacuum is reliable because the scale of $\langle h_2 M_2 \rangle$ is well below the Landau pole.

4.6 Lifetime of the metastable vacuum

We make a qualitative evaluation of the decay rate of the metastable vacuum. At semi classical level the decay probability is proportional to e^{-S_B} where S_B is the bounce action from the non supersymmetric vacuum to a supersymmetric one. We have to find a trajectory in the field space such that the potential energy barrier is minimized. We remind the non supersymmetric vacuum configuration (4.17,4.19,4.20) and the supersymmetric one

$$q = 0 \qquad \tilde{q} = 0 \qquad Y = 0 \qquad \langle h_1 M_1 \rangle = \frac{h_1^2 m_1^2}{2m_3} \, \mathbf{1}_{N_f} \qquad \langle h_2 M_2 \rangle \neq 0 \tag{4.40}$$

where $\langle h_2 M_2 \rangle$ can be read from (4.37).

By inspection of the F-term contributions (4.15) to the potential energy it turns out that the most efficient path is to climb from the local non supersymmetric minimum to the local maximum where all the fields are set to zero but for M_1 which has the value $M_1 = \frac{h_1 m_1^2}{2m_3}\mathbf{1}_{N_f}$ as in the supersymmetric vacuum, and M_2, which is as in (4.20). This local maximum has potential energy

$$V_{MAX} = N_f |h_2 m_2^2|^2 \tag{4.41}$$

We can move from the local maximum to the supersymmetric minimum (4.40) along the M_2 meson direction. The two minima are not of the same order and so the thin wall approximation of [48] can't be used. We can approximate the potential barrier with a triangular one using the formula of [49]

$$S \simeq \frac{(\Delta\Phi)^4}{V_{MAX} - V_{MIN}} \tag{4.42}$$

We neglect the difference in the field space between all the vevs at the non supersymmetric vacuum and at the local maximum. We take as $\Delta\Phi$ the difference between the vevs of M_2 at the local maximum and at the supersymmetric vacuum. Disregarding the M_2 vev at the local maximum we can approximate $\Delta\Phi$ as (4.37). We then obtain as the decay rate

$$S \sim \left(\left(\frac{1}{\epsilon}\right)^{\frac{N_f - 2\tilde{N}}{N_f - \tilde{N}}} \xi^{\frac{\tilde{N}}{N_f - \tilde{N}}} \right)^4 \sim \left(\frac{1}{\epsilon}\right)^{4\frac{N_f - 2\tilde{N}}{N_f - \tilde{N}}} \tag{4.43}$$

This rate can be made parametrically large sending to zero the dimensionless ratio ϵ (i.e. sending $\tilde{\Lambda} \to \infty$) since the exponent $\left(4\frac{2\tilde{N} - N_f}{\tilde{N} - N_f}\right)$ is always positive in our window (4.5).

In conclusion we have found that the $SU(N_c)$ SQCD with two adjoint chiral fields and mesonic deformations admits a metastable non supersymmetric vacuum with parametrically long life. It seems that particular care is needed in building models with adjoint matter exhibiting such vacua. The same can be said about the string geometrical construction realizing the gauge model we have studied [60, 61].

Chapter 5

Gauge mediation in SQCD with adjoint fields

5.1 Introduction

As mentioned, following the strategy of the ISS model many examples of dynamical supersymmetry breaking in metastable vacua have been studied in supersymmetric $\mathcal{N} = 1$ gauge theories and in string theory. The approach of ISS relies on theories for which Seiberg-like dualities exist, i.e. the IR strong dynamics of the electric theory can be studied perturbatively in the dual magnetic theory in the range where it is weakly coupled.

In the previous chapter we studied a SQCD-like model with $SU(N)$ gauge group and adjoint fields with non trivial superpotential [1]. Models with adjoint fields exhibit richer structure. In [39, 1] it has been necessary to add gauge singlet deformations in order to stabilize the non supersymmetric vacuum. These models show classical landscape of vacua parametrized by the adjoint vevs that can [1] or cannot [39] be wiped out at quantum level.

In the ISS model the fundamental fields are massive with mass lower than the natural scale, while in [38, 39] the fundamental fields are massless. In [1] we considered massive fundamental matter but we will show that our previuos results are valid in the limit of vanishing quark masses.

The ISS model and its generalizations can have phenomenological applications in connection with gauge mediation of dynamical supersymmetry breaking to the standard model sector. R-symmetry plays here a relevant role since a $U(1)$ R-symmetry, even broken to Z_n, forbids a gaugino mass generation. To obtain a gaugino mass, deformations can be added to the superpotential making the R-symmetry trivial, and this might require a further careful analysis of its stability. Quite recently, meta-stable models have been analysed in this direction. In most cases extra terms, breaking R-symmetry, have been added to known models of dynamical supersymmetry breaking, leading to gaugino mass at 1 loop at the first or at the third order in the breaking scale.

The model of the previous chapter, [1], which has meta-stable vacua, is rather non generic (in the sense of [50]), it has no R-symmetry and it is suitable for direct gauge mediation. We show, indeed, that a gaugino mass gets generated at 1 loop at third order in the breaking parameter.

5.2 $\mathcal{N} = 1$ SQCD with adjoint matter

We consider $\mathcal{N} = 1$ supersymmetric $SU(N_c)$ Yang Mills theory coupled to N_f massive flavours $(Q^i_\alpha, \tilde{Q}^{j\beta})$ in the fundamental and antifundamental representations of the gauge group ($\alpha, \beta = 1, \ldots N_c$) and in the antifundamental and fundamental representations of the flavour group ($i, j = 1, \ldots N_f$), respectively. We also consider a charged chiral massive adjoint superfield X^α_β with superpotential[1]

$$W_{el} = \frac{g_X}{3}\text{Tr}X^3 + \frac{m_X}{2}\text{Tr}X^2 + \lambda_X \text{Tr}X \tag{5.1}$$

where λ_X is a Lagrange multiplier enforcing the tracelessness condition $\text{Tr}X = 0$. This theory is asymptotically free in the range $N_f < 2N_c$ and it admits stable vacua for $N_f > \frac{N_c}{2}$ [29]. The matching between the microscopic scale (Λ) and the macroscopic scale ($\tilde{\Lambda}$) is

$$\Lambda^{2N_c - N_f}\tilde{\Lambda}^{2\tilde{N} - N_f} = \left(\frac{\mu}{g_X}\right)^{2N_f} . \tag{5.2}$$

where the intermediate scale μ takes into account the mass dimension of the mesons in the dual description.

We add to the electric potential (5.1) the gauge singlet deformations

$$\Delta W_{el} = \lambda_Q \,\text{tr}\, QX\tilde{Q} + m_Q \,\text{tr}\, Q\tilde{Q} + h\,\text{tr}\,(Q\tilde{Q})^2 \tag{5.3}$$

The first two terms are standard deformations of the electric superpotential that do not spoil the duality relations (e.g. the scale matching condition (5.2)) [30]. The last term of (5.3) can be thought as originating from a second largely massive adjoint field Z in the electric theory with superpotential

$$W_Z = m_Z \text{Tr}Z^2 + \text{Tr}\, ZQ\tilde{Q} \tag{5.4}$$

and which has been integrated out. The mass m_Z has to be considered larger than Λ_{2A}, the strong scale of the electric theory with two adjoint fields. This procedure leads to the scale matching relation

$$\Lambda_{2A}^{N_c - N_f} = \Lambda_{1A}^{2N_c - N_f} m_Z^{-N_c} \tag{5.5}$$

where Λ_{2A} and Λ_{1A} are the strong coupling scales before and after the integration of the adjoint field Z. The other masses in this theory have to be considered much smaller than the strong scale: $\Lambda_{2A} \gg m_Q, m_X$. We can suppose that when $h = \frac{1}{m_z}$ is small the duality relations are still valid.

[1](Tr) means tracing on the color indices, while (tr) on the flavour ones.

The dual theory [29, 30, 28] is $SU(2N_f - N_c \equiv \tilde{N})$ with N_f magnetic flavours $(q, \tilde{q})$, a magnetic adjoint field Y and two gauge singlets build from electric mesons ($M_1 = Q\tilde{Q}$, $M_2 = QX\tilde{Q}$), with magnetic superpotential

$$\begin{aligned} W_{magn} &= \frac{\tilde{g}_Y}{3}\mathrm{Tr}Y^3 + \frac{\tilde{m}_Y}{2}\mathrm{Tr}Y^2 + \tilde{\lambda}_Y \mathrm{Tr}Y - \frac{1}{\mu^2}\,\mathrm{tr}\,\left(\frac{\tilde{m}_Y}{2}M_1 q\tilde{q} + \tilde{g}_Y M_2 q\tilde{q} + \tilde{g}_Y M_1 qY\tilde{q}\right) \\ &\quad + \lambda_Q\ \mathrm{tr}\ M_2 + m_Q\ \mathrm{tr}\ M_1 + h\ \mathrm{tr}\ (M_1)^2 \end{aligned} \tag{5.6}$$

For this dual theory the scale matching relation is the same as (5.2) with $\Lambda \equiv \Lambda_{1A}$ defined in (5.5).
We consider the range where the magnetic theory is IR free and it admits stable vacua

$$\frac{N_c}{2} < N_f < \frac{2}{3}N_c \qquad \Rightarrow \qquad 0 < 2\tilde{N} < N_f \tag{5.7}$$

In this range the metric on the moduli space is smooth around the origin [27], and the Kahler potential is regular and can be considered canonical.

5.3 Non supersymmetric meta-stable vacua

Rescaling the fields and the coupling the superpotential (5.6) is

$$\begin{aligned} W_{magn} &= \frac{g_Y}{3}\mathrm{Tr}Y^3 + \frac{m_Y}{2}\mathrm{Tr}Y^2 + \lambda_Y \mathrm{Tr}Y + \mathrm{tr}\ (h_1 M_1 q\tilde{q} + h_2 M_2 q\tilde{q} + h_3 M_1 qY\tilde{q}) \\ &\quad - h_1 m_1^2\ \mathrm{tr}\ M_1 - h_2 m_2^2\ \mathrm{tr}\ M_2 + m_3\ \mathrm{tr}\ M_1^2 \end{aligned} \tag{5.8}$$

Solving the equations of motion we find the supersymmetry breaking tree level vacua:

$$q = \begin{pmatrix} m_2 e^{\theta}\,\mathbf{1}_{\tilde{N}} \\ 0 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} m_2 e^{-\theta}\,\mathbf{1}_{\tilde{N}} \\ 0 \end{pmatrix} \qquad \langle Y \rangle = \begin{pmatrix} y_1 \mathbf{1}_{n_1} & 0 \\ 0 & y_2 \mathbf{1}_{n_2} \end{pmatrix} \tag{5.9}$$

Where y_i are functions of n_1 and $n_2 = \tilde{N} - n_1$ as in [1]. We choose the vacuum in which the magnetic gauge group is not broken by the adjoint field $n_1 = 0$, which implies $y_2 = 0$, so $\langle Y \rangle = 0$. In this case we have

$$\langle M_1 \rangle = \begin{pmatrix} p_1^A & 0 \\ 0 & p_1^B \end{pmatrix} \qquad \langle M_2 \rangle = \begin{pmatrix} p_2^A & 0 \\ 0 & \mathcal{X} \end{pmatrix} \tag{5.10}$$

where the explicit expressions can be found in [1]. The two non trivial blocks of the mesons are respectively $\tilde{N}$ and $N_f - \tilde{N}$ diagonal squared matrices.

Supersymmetry is broken at tree level by the rank condition, i.e. the F equations of motion of M_2 field cannot be all satisfied

$$0 \neq F_{M_2} = F_{\mathcal{X}} = h_2 m_2^2 \tag{5.11}$$

The minimum of the scalar potential in this tree level vacuum is then different from zero, and results proportional to $|F_{M_2}|^2$. The potential energy of the vacuum does not depend on θ and $\mathcal{X}$; they are massless fields at tree level, not protected by any symmetry and hence are pseudo-moduli.

In [1] the detailed study of the 1-loop quantum corrections to the effective potential has been performed. These corrections depend on the choice of the adjoint vev $\langle Y \rangle$: they are minimized by the choice $\langle Y \rangle = 0$. This is a true quantum minimum of the scalar potential where the pseudomoduli get positive mass squared from the Coleman-Weinberg potential. The field $\mathcal{X}$ gets a non trivial vacuum expectation value from the quantum corrections, $\langle \mathcal{X} \rangle \neq 0$, moving slightly the minimum away from the origin.

We report the masses for the pseudomoduli in the regime of small $\rho = \frac{h_1}{h_2}$ and small $\zeta = \frac{h_1^2 m_1^2}{2h_2 m_2 m_3}$ ($\eta = \frac{2m_3}{h_2 m_2}$ has been neglected as in [1])

$$
\begin{aligned}
m_{\mathcal{X}}^2 &= \frac{\tilde{N}(N_f - \tilde{N})}{8\pi^2}|h_2 m_2|^2 \Big(|h_2|^2(\log[4] - 1) + |h_1|^2(\log[4] - 2) \Big) \\
m_{\tilde{\theta}}^2 &= \frac{\tilde{N}(N_f - \tilde{N})}{16\pi^2}|h_2 m_2^2|^2 \left(|h_2|^2(\log[4] - 1) + \left| \frac{h_1^2 m_1^2}{2m_2 m_3} \right|^2 (\log[4] - \frac{5}{3}) + |h_1|^2(2\log[4] - 3) \right)
\end{aligned}
$$

Other choices for $\langle Y \rangle$ with $n_1 \neq 0 \neq n_2$ would not change the tree level potential energy of the vacua, so there is a landscape of vacua at classical level. This is wiped out by 1-loop corrections. We can give more details about the computation in the case where $n_1 \neq 0$ and how we excluded the possibility of a landscape at quantum level. We parametrize the fluctuations around the non supersymmetric vacua in the case of non trivial vev for the adjoint field

$$
q = \begin{pmatrix} ke^{\theta} + \xi_1 \\ \phi_1 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} ke^{-\theta} + \xi_2 \\ \phi_2 \end{pmatrix} \qquad \langle Y \rangle = \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix} + \delta Y \tag{5.12}
$$

$$
M_1 = \begin{pmatrix} p_1^A + \xi_3 & \phi_3 \\ \phi_4 & p_1^B + \xi_4 \end{pmatrix} \qquad M_2 = \begin{pmatrix} p_2^A + \xi_5 & \phi_5 \\ \phi_6 & \mathcal{X} \end{pmatrix} \tag{5.13}
$$

The resulting superpotential for the sector affected by the supersymmetry breaking (the ϕ_i chiral fields) is

$$
\begin{aligned}
W &= h_2 \left(\mathcal{X}\phi_1\phi_2 - m_2^2 \mathcal{X} \right) + h_2 m_2 \left(e^{\theta}\phi_2\phi_5 + e^{-\theta}\phi_1\phi_6 \right) + \\
&\quad + (h_1 + h_3 y_i) m_2 \left(e^{\theta}\phi_2\phi_3 + e^{-\theta}\phi_1\phi_4 \right) + 2m_3\phi_3\phi_4 + \frac{h_1 m_1^2}{2m_3}(h_1 + h_3 y_i)\, \phi_1\phi_2
\end{aligned} \tag{5.14}
$$

where $i = 1, 2$. Exactly we have n_1 copies of (5.14) with $i = 1$ and $\tilde{N} - n_1$ copies with $i = 2$. The fields appearing in (5.14) are the only ones which contribute to the one loop potential.

Comparing with the one in [1], we observe that having $n_1 \neq 0$ contributes only in a shift in the ζ and ρ parameters. We can then compute the 1-loop quantum corrections to the scalar potential

$V^{1-loop}(n_1)$, which depends non trivially on n_1 through $y_i(n_1)$. This contribution is minimized when $n_1 = 0$, i.e. $\langle Y \rangle = 0$, implying that this is the lowest energy vacuum [1].

5.4 Decay to the supersymmetric vacuum

Supersymmetry is restored when the $SU(\tilde{N})$ symmetry is gauged via non perturbative effects [26], away from the metastable vacuum in the field space. The non supersymmetric vacuum is then a metastable state of the theory which decays into a supersymmetric one. In [1] we find a supersymmetric vacuum in the large field region for the meson M_2 where

$$\langle h_2 M_2 \rangle = \tilde{\Lambda} \epsilon^{\frac{\tilde{N}}{N_f-\tilde{N}}} \xi^{\frac{\tilde{N}}{N_f-\tilde{N}}} \mathbf{1}_{N_f} = m_2 \left(\frac{1}{\epsilon}\right)^{\frac{N_f-2\tilde{N}}{N_f-\tilde{N}}} \xi^{\frac{\tilde{N}}{N_f-\tilde{N}}} \mathbf{1}_{N_f}\,, \qquad \epsilon = \frac{m_2}{\tilde{\Lambda}} \quad \xi = \frac{m_2}{m_Y}\,. \tag{5.15}$$

ϵ is a dimensionless parameter which can be made parametrically small sending the Landau pole $\tilde{\Lambda}$ to infinity. ξ is a dimensionless finite parameter which does not spoil the estimation of the supersymmetric vacuum in the sensible range $\epsilon < \frac{1}{\xi}$. All the exponents appearing in (5.15) are positive in the window (5.7).

We observe that in the small ϵ limit the vev $\langle h_2 M_2 \rangle$ is larger than the mass scale m_2 of the magnetic theory but much smaller than the scale $\tilde{\Lambda}$

$$m_2 \ll \langle h_2 M_2 \rangle \ll \tilde{\Lambda}. \tag{5.16}$$

making the evaluation of the supersymmetric vacuum reliable.

We now make a qualitative evaluation of the decay rate of the metastable vacuum. At semi classical level the decay probability is proportional to e^{-S_B} where S_B is the bounce action from the non supersymmetric vacuum to a supersymmetric one. We obtain as the decay rate [1]

$$S \sim \left(\left(\frac{1}{\epsilon}\right)^{\frac{N_f-2\tilde{N}}{N_f-\tilde{N}}} \xi^{\frac{\tilde{N}}{N_f-\tilde{N}}} \right)^4 \sim \left(\frac{1}{\epsilon}\right)^{4\frac{N_f-2\tilde{N}}{N_f-\tilde{N}}} \tag{5.17}$$

This rate can be made parametrically large sending to zero the dimensionless ratio ϵ (i.e. sending $\tilde{\Lambda} \to \infty$) since the exponent $\left(4\frac{2\tilde{N}-N_f}{\tilde{N}-N_f}\right)$ is always positive in the window (5.7).

5.5 Massless quarks

The non supersymmetric meta-stable vacua survive the limit $m_Q \to 0$. Indeed this limit corresponds in the magnetic description to send to zero the linear term for M_1, i.e. $m_1 \to 0$, and all the results are smooth in this limit.

At classical level there are only small differences in the field vacuum expectation values. At quantum level this limit set the parameter ζ to zero, but the qualitative behaviour of the 1-loop corrections is the same: the classical flat directions are still lifted.

Finally the computation of the supersymmetric vacuum for $m_1 = 0$ is even more straightforward, giving a vanishing vacuum expectation values for the meson M_1. The lifetime estimation of the non-supersymmetric vacuum is not affected by this limit, and it is still parametrically large. Setting $m_1 = 0$ the model become more similar to the one studied in [39].

5.6 R-symmetry and gauge mediation

We are interested in direct gauge mediated supersymmetry breaking. In this framework the gauge group of the SM has to be embedded into a flavour group of the dynamical sector. The gauge sector of the SM directly couples to the supersymmetry breaking dynamics and a natural question for model building is whether the gauginos of the MSSM acquire masses.

We can embed the SM gauge group into the subgroups of the flavour symmetry $SU(2N_f - N_c)$ or $SU(N_c - N_f)$ provide $(2N_f - N_c > 5)$ or $(N_c - N_f > 5)$, respectively. As in [53] we can compute the beta function coefficient $b_{SU(3)}$ at different renormalization scales and we conclude that in order to avoid Landau pole problems the embedding should be done in $SU(2N_f - N_c)$.

The full model has no R-symmetry, and, unlike [27, 39], no accidental R-symmetry arises at the non-supersymmetric meta-stable vacuum, and hence a gaugino mass generation is not forbidden [27]. Moreover the absence of R-symmetry implies that the non supersymmetric minimum is not at the origin of the moduli space, i.e. $\langle \mathcal{X} \rangle \neq 0$.

The R-breaking terms are the quadratic massive terms $\phi_1\phi_2$ and $\phi_3\phi_4$ in (5.14). The first one can be eliminated shifting the field $\mathcal{X}$. The second one cannot be eliminated rearranging the fields. If the mass m_3 is larger than the supersymmetry breaking scale, ϕ_3 and ϕ_4 could be integrated out, supersymmetrically, recovering an accidental R-symmetry: this, however, is not our range of parameters.

We analyze the dynamics at the meta-stable vacuum where the breaking of supersymmetry generates a gaugino mass proportional to the breaking scale $F_{\mathcal{X}}$. Contribution to this mass comes from the superpotential[2] of the messengers ϕ_i

$$W \supset h_2 \left(\mathcal{X}\phi_1\phi_2\right) + h_2 m_2 \left(\phi_2\phi_5 + \phi_1\phi_6\right) + h_1 m_2 \left(\phi_2\phi_3 + \phi_1\phi_4\right) + 2m_3\phi_3\phi_4 + \frac{h_1^2 m_1^2}{2m_3}\,\phi_1\phi_2 \quad (5.18)$$

which, in a matrix notation, reads

$$\left(\begin{array}{ccc} \phi_1 & \phi_3 & \phi_5 \end{array} \right) \mathcal{M} \left(\begin{array}{c} \phi_2 \\ \phi_4 \\ \phi_6 \end{array} \right) \quad (5.19)$$

[2]For simplicity we consider only one copy of the chiral superpotentials.

where $\mathcal{M}$ is a mass matrix for the messenger fields

$$\mathcal{M} = \begin{pmatrix} h_2\langle\mathcal{X}\rangle + \frac{h_1^2 m_1^2}{2m_3} & h_1 m_2 & h_2 m_2 \\ h_1 m_2 & 2m_3 & 0 \\ h_2 m_2 & 0 & 0 \end{pmatrix} \equiv h_2 m_2 \begin{pmatrix} \frac{\langle\mathcal{X}\rangle}{m_2} + \zeta & \rho & 1 \\ \rho & \eta & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{5.20}$$

This matrix does not generate a gaugino mass at one loop at first order in $F_{\mathcal{X}}$ as in [63]. However at the third order in $F_{\mathcal{X}}$, the gaugino mass arises as in [55, 63]. This contribution is not negligible when $\frac{F_{\mathcal{X}}}{h_2^2 m_2^2} \sim 1$, which is admitted in our range of parameters.

Diagonalization of (5.20) and use of the general formula in [64] for the computation of the 1 loop diagrams contributing to the gaugino mass m_λ lead to

$$m_\lambda \sim \frac{F_{\mathcal{X}}^3}{(h_2 m_2)^5} \left[\frac{1}{4} \left(\frac{\langle\mathcal{X}\rangle}{m_2} + \zeta \right) + \rho^2 \eta \right] \tag{5.21}$$

The coefficient of $F_{\mathcal{X}}^3$ in (5.21) is evaluated at the third order in the adimensional small parameters (ρ, η, ζ): indeed by direct inspection we find that also the term $\left(\frac{\langle\mathcal{X}\rangle}{m_2} + \zeta\right)$ gives at least third order contributions in (η, ρ, ζ).

Chapter 6

Metastable A_n quivers

6.1 Introduction

The existence of long living metastable vacua seems by now a rather generic phenomenon in large classes of supersymmetric gauge theories. It provides an attractive way for dynamical breaking of supersymmetry and the interest in these theories has been enhanced by the possibilities of their embedding in supergravity and string theory and of their use in gauge mediation mechanisms.

Metastability is a low energy phenomenon for UV free theories and in general the key ingredient which makes a perturbative analysis possible is Seiberg duality to IR free theories described in terms of macroscopic fields.

An interesting set of theories in which to study metastability à la ISS is the ADE class of quiver gauge theories [65, 66, 67].

These theories can be derived in type IIB string theory from $D5$-branes partially wrapping 2-cycles of non compact Calabi-Yau threefolds. These manifolds are ADE-fold geometries fibered over a plane, and the 2-cycles are blown up S_i^2 in one to one correspondence with the simple roots of ADE.

In this chapter we investigate metastability in A_n $\mathcal{N} = 2$ (non affine) quiver gauge theories deformed to $\mathcal{N} = 1$ by superpotential terms in the adjoint fields. In the presence of many gauge groups we have, in principle, a large number of dualization choices.

In [39, 53, 56] A_2, A_3, A_4 quivers have been studied dualizing only one node in the quiver, where dynamical supersymmetry breaking occurs.

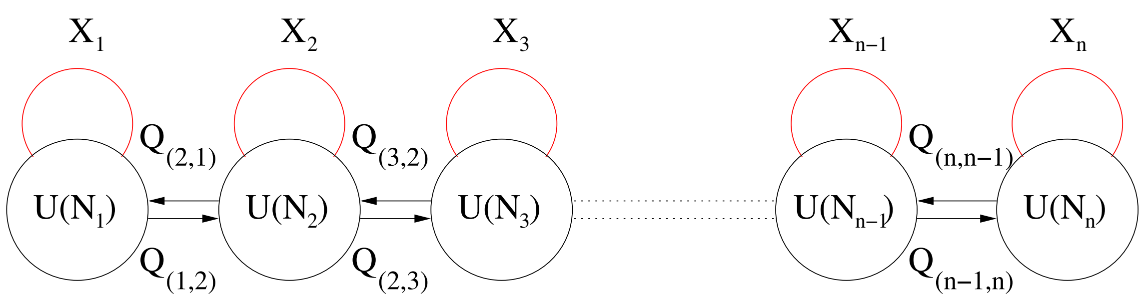
Here we consider A_n theories with arbitrary n, where several Seiberg dualities take place. In particular we will explore theories obtained by dualizing alternate nodes. This leads to a low energy description in terms of only magnetic fields.

In the duality process the dualized groups are treated as genuine gauge groups whereas the other ones have to be weakly coupled at low energy, so that they act as flavour groups i.e. global symmetries. The procedure depends on the interplay of the RG flows of the dualized and of the non dualized gauge groups and is governed by the associated beta-functions. This translates into inequalities among the ranks of the gauge groups and in hierarchies among the strong coupling scales.

The chapter is organized as follows. In section 7.2 we describe the $\mathcal{N}=2$ quiver gauge theories, explicitly broken to $\mathcal{N}=1$ by superpotential terms. After the integration of the massive adjoint fields, we give the general form of the superpotential. In section 7.3 we investigate Seiberg duality on the alternate nodes of the quiver. The general theory obtained with this procedure on an A_n is expressed in terms of only magnetic fields. In section 7.4 we consider the simplest case, i.e. A_3 quiver, showing that it possesses long living metastable vacua à la ISS. The analysis is done neglecting the gauge contributions of the odd nodes, which are treated as flavour symmetries. This last approximation is justified in section 7.5, where an analysis of the running of the couplings has been performed. The general result, metastability in an A_n quiver theory, is explained in section 7.6, giving an explicit example. In section 7.7 we comment on the possible ways of enforcing gauge mediation of supersymmetry breaking. Appendix B.1 explains how to find the metastable vacua upon changing the masses of the quarks in the electric description. Appendix B.2 provides details in the analysis on the running of the gauge couplings of section 5. Appendix B.3 adds to section 6, giving all the possible choices of A_5 which show metastable vacua.

6.2 A_n quiver gauge theories with massive adjoint fields

We consider a $\mathcal{N}=2$ (non affine) A_n quiver gauge theory, deformed to $\mathcal{N}=1$ by superpotential terms in the adjoint fields. The theory is associated with a Dynkin diagram where each node is a $U(N_i)$ gauge group.



The arrows connecting two nodes represent fields $Q_{i,i+1}, Q_{i+1,i}$ in the fundamental of the incoming node and anti fundamental of the out-coming node. The adjoint fields X_i refer to the i-th gauge group.

The gauge group of the whole theory is the product $\prod_{i=1}^{n} U(N_i)$. We call Λ_i the strong coupling scale of each gauge group.

The $\mathcal{N}=1$ superpotential is

$$W=\sum_{i=1}^{n} W_i(X_i)+\sum_{i,j} s_{i,j}(Q_{i,j})_{\alpha}^{\beta}(X_j)_{\beta}^{\gamma}(Q_{j,i})_{\gamma}^{\alpha} \tag{6.1}$$

where $s_{i,j}$ is an antisymmetric matrix, with $|s_{i,j}|=1$. The Latin labels run on the different nodes of the A_n quivers, the Greek labels runs on the ranks of the groups of each site. In the case of A_n theories the only non zero terms are $s_{i,i+1}$ and $s_{i,i-1}$. The superpotentials for the adjoint fields $W_i(X_i)$ break supersymmetry to $\mathcal{N}=1$.

We choose these superpotentials to be

$$W_i(X_i)=\lambda_i TrX_i+\frac{m_i}{2}TrX_i^2 \tag{6.2}$$

As a consequence the adjoint fields are all massive. We consider the limit where the adjoint fields are so heavy that they can be integrated out, and we study the theory below the scale of their masses.

Integrating out these fields we obtain the effective superpotential describing the A_n theory (traces on the gauge groups are always implied).

$$\begin{aligned} W &= \sum_{i=1}^{n-1}\left(\left(\frac{\lambda_{i+1}}{m_{i+1}}-\frac{\lambda_i}{m_i}\right)Q_{i,i+1}Q_{i+1,i}-\frac{1}{2}\left(\frac{1}{m_i}+\frac{1}{m_{i+1}}\right)(Q_{i,i+1}Q_{i+1,i})^2\right)\\ &+\sum_{i=2}^{n-1}\frac{1}{m_i}Q_{i-1,i}Q_{i,i+1}Q_{i+1,i}Q_{i,i-1} \end{aligned} \tag{6.3}$$

A final important remark is that for the A_n theories the D-term equations of motion can be decoupled and simultaneously diagonalized [68].

6.3 Seiberg duality on the even nodes

We investigate the low energy dynamics of the gauge groups of the Dynkin diagram, governed by the ranks and by the hierarchy between the strong coupling scales of each node. We work in the regime where the even nodes develop strong dynamics and have to be Seiberg dualized.

We set all the strong coupling scales of the even nodes to be equal $\Lambda_{2i}\equiv\Lambda_G$ and we require the odd nodes to be less coupled at this scale. We impose the following window for the ranks of the nodes

$$N_{2i}+1\le N_{2i-1}+N_{2i+1}<\frac{3}{2}N_{2i}\qquad i=1,\ldots,\frac{n-1}{2} \tag{6.4}$$

We take n odd, the even case can be included setting to zero one of the ranks of the extremal nodes.

Along the flow toward the IR, we have to change the description at the scale Λ_G performing Seiberg duality on the even nodes. The even nodes are treated as gauge groups, whereas the odd nodes are treated as flavours. We will discuss the consistency of this description in section 6.5.

It is convenient to list the elementary fields of the dualized theory, i.e. the electric gauge singlets and the new magnetic quarks.

	$U(N_{2i-1})$	$U(\widetilde{N}_{2i})$	$U(N_{2i+1})$
$M_{2i+1,2i-1}$	N_{2i-1}	1	$\bar{N}_{2i+1}$
$M_{2i+1,2i+1}$	1	1	Bifund.
$M_{2i-1,2i-1}$	Bifund.	1	1
$M_{2i-1,2i+1}$	$\bar{N}_{2i-1}$	1	N_{2i+1}
$q_{2i-1,2i}$	N_{2i-1}	$\bar{\widetilde{N}}_{2i}$	1
$q_{2i,2i-1}$	$\bar{N}_{2i-1}$	$\widetilde{N}_{2i}$	1
$q_{2i,2i+1}$	1	$\widetilde{N}_{2i}$	$\bar{N}_{2i+1}$
$q_{2i+1,2i}$	1	$\bar{\widetilde{N}}_{2i}$	N_{2i+1}

The mesons are proportional to the original electric variables: $M_{2i+k,2i+j} \sim Q_{2i+k,2i}Q_{2i,2i+j}$. The even magnetic groups have ranks $\widetilde{N}_{2i} = N_{2i+1} + N_{2i-1} - N_{2i}$. The superpotential in the new magnetic variables results

$$\begin{aligned} W &= hM^{(2i)}_{2i+k,2i+j}q_{2i+j,2i}q_{2i,2i+k} + h\mu^2_{2i+k,(2i)}M^{(2i)}_{2i+k,2i+k} + \\ &+ hmM^{(2i)}_{2i+1,2i+1}M^{(2i+2)}_{2i+1,2i+1} + hm\left(M^{(2i)}_{2i+k,2i+k}\right)^2 + hmM^{(2i)}_{2i-1,2i+1}M^{(2i)}_{2i+1,2i-1} \end{aligned} \tag{6.5}$$

where the index i runs from 1 to $\frac{n-1}{2}$, and k and j are $+1$ or -1. The upper index $(2i)$ of the mesons indicates which site the meson refers to: it is necessary because some mesons have the same flavor indexes, but they are summed on different gauge groups, so they have to be labeled differently. We denote with hm_i the meson masses, related to the quartic terms in the electric superpotential, and with $h\mu_i^2$ the coefficients of the linear deformations, corresponding to the masses of the quarks in the electric description. In (6.5) we wrote a single coupling hm, for all the different mesons, considering all their masses of the same order.

The b coefficients of the beta functions before dualization are

$$b_i = 3N_i - N_{i-1} - N_{i+1} \qquad i = 1, \ldots, n \tag{6.6}$$

where $N_0 = N_{r+1} = 0$. After the dualization the coefficients $\widetilde{b}$ for the beta functions in the internal nodes result

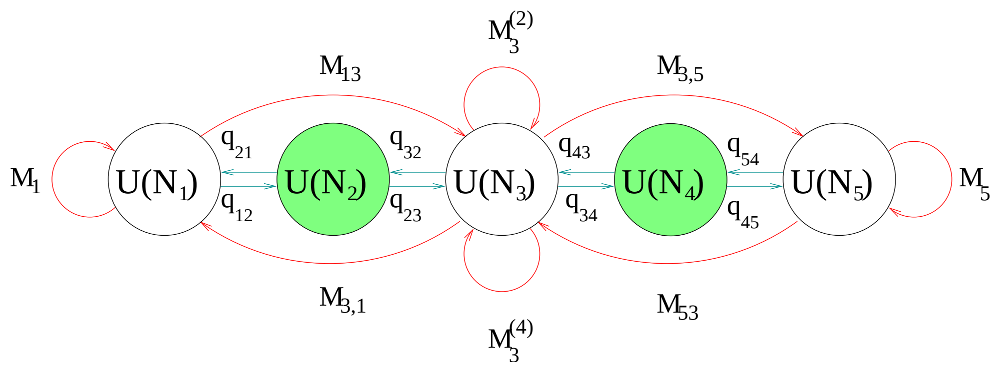
$$\widetilde{b}_{2k} = 2N_{2k+1} + 2N_{2k-1} - 3N_{2k} \tag{6.7}$$

$$\widetilde{b}_{2k+1} = N_{2k} + N_{2k+2} - N_{2k+1} - 2N_{2k-1} - 2N_{2k+3} \tag{6.8}$$

where k runs from 1 to $\frac{n-1}{2}$, and $N_{n+1} = N_{n+2} = 0$. For the external nodes we have

$$\widetilde{b}_1 = N_1 + N_2 - 2N_3 \qquad \widetilde{b}_n = N_n + N_{n-1} - 2N_{n-2} \tag{6.9}$$

To visualize the resulting magnetic theory (6.5) we exhibit below the content of the magnetic dual theory for an A_5 quiver, which encodes the relevant features.



The superpotential is

$$
\begin{aligned}
W &= h\left(M_{11}q_{12}q_{21} + M_{13}q_{32}q_{21} + M_{31}q_{12}q_{23} + M_{33}^{(2)}q_{32}q_{23}\right) + \\
&+ h\left(M_{33}^{(4)}q_{34}q_{43} + M_{35}q_{54}q_{43} + M_{53}q_{34}q_{45} + M_{55}q_{54}q_{45}\right) + \\
&+ hm\left(M_{11}^2 + M_{13}M_{31} + {M_{33}^{(2)}}^2 + M_{33}^{(2)}M_{33}^{(4)} + {M_{33}^{(4)}}^2 + M_{35}M_{53} + M_{55}^2\right) + \\
&+ h\left(\mu_1^2 M_{11} + \mu_{3,(2)}^2 M_{33}^{(2)} + \mu_{3,(4)}^2 M_{33}^{(4)} + \mu_5^2 M_{55}\right)
\end{aligned}
\qquad (6.10)
$$

6.4 Metastable vacua in A_3 quivers

We start studying the existence and the slow decay of non supersymmetric meta-stable vacua in A_3 quiver gauge theory, the simplest example of an A_n theory. The A_3 gauge group is $U(N_1) \times U(N_2) \times U(N_3)$. As already mentioned in section 6.2 for a A_n theory, we integrate out the adjoint fields and we perform Seiberg duality on the central node under the constraint

$$
N_2 + 1 \le N_1 + N_3 < \frac{3}{2}N_2 \qquad (6.11)
$$

The superpotential reads

$$
\begin{aligned}
W &= h\left(M_{1,1}q_{1,2}q_{2,1} + M_{1,3}q_{3,2}q_{2,1} + M_{3,1}q_{1,2}q_{2,3} + M_{3,3}q_{3,2}q_{2,3}\right) + \\
&+ h\mu_1^2 M_{1,1} + h\mu_3^2 M_{3,3}
\end{aligned}
\qquad (6.12)
$$

where all the mass terms for the mesons have been neglected. Turning on these terms does not ruin the metastability analysis at least for very small masses compared to the supersymmetry breaking scale. Such deformations slightly shift the value of the pseudomoduli in the non supersymmetric minimum, breaking R-symmetry [54]. We neglect them in the following.

The central node yields the magnetic gauge group $U(N_1+N_3-N_2)$ whereas the groups at the two external nodes are considered as flavour groups, much less coupled. We discuss in section 6.5 the consistency of this assumption. Since the gauge group is IR free in the low energy description, and the flavours are less coupled, we are allowed to neglect Kahler corrections and take it as canonical [25]. Moreover the D-term corrections to the one loop effective potential due to the flavour nodes are negligible with respect to the F-term corrections.

Now, there are two different choices of ranks for the A_3 theories, which can give meta-stable vacua: the first possibility is that $N_1 < N_2 \leq N_3$, the second one is $N_1 < N_2 > N_3$. We study separately the two cases which show meta-stable vacua in a similar manner.

$N_1 < N_2 \leq N_3$

We analyze here the case $N_1 < N_2 < N_3$; the equal ranks limit can be easily included. After the dualization the ranks obey the following inequalities $N_1 < \widetilde{N}_2 = N_1 + N_3 - N_2 < N_3$.

We work in the regime where $|\mu_1| > |\mu_3|$, and we comment on what happens in the opposite limit in the appendix B.1, where we shall discuss dangerous tachyonic directions in the quark fields.

We find that the following vacuum is a non supersymmetric tree level minimum

$$\begin{aligned} q_{1,2} = q_{2,1} &= \mu_1 \left(\mathbf{1}_{N_1} \quad 0\right) \qquad q_{2,3} = q_{3,2} = \begin{pmatrix} 0 & \mu_3 \mathbf{1}_{\widetilde{N}_2 - N_1} \\ 0 & 0 \end{pmatrix} \\ M_{1,1} &= 0 \qquad M_{1,3} = M_{3,1} = 0 \qquad M_{3,3} = \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix} \end{aligned} \tag{6.13}$$

where the field X is the pseudomodulus, which is a massless field not associated with any broken global symmetries. This flat direction has to be stabilized by the one loop corrections. Westart the one loop analysis by rearranging the fields and expanding around the vevs

$$\begin{aligned} q = \left(\frac{q_{1,2}}{q_{3,2}}\right) &= \left(\begin{array}{cc} \mu_1 + \Sigma_1 & \Sigma 2 \\ \hline \Sigma_3 & \mu_3 + \Sigma_4 \\ \Phi_1 & \Phi_2 \end{array}\right) \qquad \widetilde{q} = \left(\begin{array}{c|c} q_{2,1} & q_{2,3} \end{array}\right) = \left(\begin{array}{c|cc} \mu_1 + \Sigma_5 & \Sigma_6 & \Phi_3 \\ \Sigma_7 & \mu_3 + \Sigma_8 & \Phi_4 \end{array}\right) \\ M &= \left(\begin{array}{c|c} M_{1,1} & M_{1,3} \\ \hline M_{3,1} & M_{3,3} \end{array}\right) = \left(\begin{array}{c|cc} \Sigma_9 & \Sigma_{10} & \Phi_5 \\ \hline \Sigma_{11} & \Sigma_{13} & \Phi_6 \\ \Phi_7 & \Phi_8 & X + \Sigma \end{array}\right) \end{aligned} \tag{6.14}$$

We now compute the superpotential at the second order in the fluctuations. We find that the non supersymmetric sector is a set of decoupled O'Raifeartaigh like models with superpotential

$$W = h\mu_3^2 X + hX(\Phi_1\Phi_3 + \Phi_2\Phi_4) + h\mu_3(\Phi_1\Phi_5 + \Phi_2\Phi_6) + h\mu_1(\Phi_3\Phi_7 + \Phi_4\Phi_8) \tag{6.15}$$

In this way all the pseudomoduli can get a mass. The quantum corrections behave exactly as in [25], which means that the pseudomoduli get positive squared mass around the origin of the field space.

The choice (6.13) guarantees that there are no tachyonic directions and have to be made coherently with the hierarchy of the couplings μ_i; see the Appendix B.1 for details.

The lifetime of the non supersymmetric vacuum is related to the value of the scalar potential in the minimum, and to the displacement of the vevs of the fields between the false and the true vacuum. The scalar potential in the non supersymmetric minimum is

$$V_{min} = (N_3 + N_1 - \widetilde{N}_2)|h\mu_3^2|^2 = N_2|h\mu_3^2|^2 \tag{6.16}$$

The vevs of the fields in the supersymmetric vacuum have to be studied considering the non perturbative contributions arising from gaugino condensation. When we take into account these non perturbative effects, we expect that the mesons get large vevs and this allows us to integrate out the quarks using their equation of motion, $q_{i,j} = 0$. In the supersymmetric vacua also $M_{1,3} = 0$ and $M_{3,1} = 0$. If we define

$$M = \begin{pmatrix} M_{1,1} & 0 \\ 0 & M_{3,3} \end{pmatrix} \tag{6.17}$$

the effective superpotential is

$$W = (N_1 + N_3 - N_2)\left(\det(hM)\Lambda_{2i}^{2N_1+2N_3-3N_2}\right)^{\frac{1}{N_1+N_3-N_2}} - h\left(\mu_1^2 trM_{1,1} + \mu_3^2 trM_{3,3}\right) \tag{6.18}$$

We have now to solve the equation of motion for M_1 and M_3. The equations to be solved are

$$\begin{aligned} \left(h^M M_{1,1}^{(N_2-N_3)} M_{3,3}^{N_3} \Lambda_{2i}^{(2N_1+2N_3-3N_2)}\right)^{\frac{1}{N_1+N_3-N_2}} - \mu_1^2 &= 0 \\ \left(h^{N_2} M_{1,1}^{N_1} M_{3,3}^{(N_2-N_1)} \Lambda_{2i}^{(2N_1+2N_3-3N_2)}\right)^{\frac{1}{N_1+N_3-N_2}} - \mu_3^2 &= 0 \end{aligned} \tag{6.19}$$

The vevs of the mesons follow solving (6.19)

$$\langle hM_{1,1}\rangle = \mu_1^{2\frac{N_1-N_2}{N_2}} \mu_3^{2\frac{N_3}{N_2}} \Lambda_{2i}^{\frac{3N_2-2N_3-2N_1}{N_2}} \mathbf{1}_{N_1} \qquad \langle hM_{3,3}\rangle = \mu_1^{2\frac{N_1}{N_2}} \mu_3^{2\frac{N_3-N_2}{N_2}} \Lambda_{2i}^{\frac{3N_2-2N_3-2N_1}{N_2}} \mathbf{1}_{N_3} \tag{6.20}$$

Since $|\mu_1| > |\mu_3|$, it follows that $\langle hM_{3,3}\rangle > \langle hM_{1,1}\rangle$. This implies that in the evaluation of the bounce action, with the triangular barrier [49], we can consider only the displacement of M_3 in the field space. We obtain for the bounce action

$$S \sim \frac{(\Delta\Phi)^4}{\Delta V} = \left(\frac{\mu_1}{\mu_3}\right)^{\frac{3N_2-2N_3}{N_2}} \left(\frac{\Lambda_{2i}}{\mu_1}\right)^{4\frac{3N_2-2N_3-2N_1}{N_2}} \tag{6.21}$$

Both exponents are positive in the range (6.11). This implies that $S_B \gg 1$, and the vacuum is long living.

$N_1 < N_2 > N_3$

The ranks of the groups after the duality obey the relation $N_1 > \widetilde{N}_2 = N_1 + N_3 - N_2 < N_3$. We choose now $|\mu_1| > |\mu_3|$, but we show in the appendix B.1 that also the other choice is possible, leading to other vacua. In the meta-stable vacuum all the vevs of the fields have to be chosen to be zero except a block of the quarks $q_{1,2}$ and $q_{2,1}$ and the pseudomoduli. The vevs are

$$q_{1,2} = \mu_1 \begin{pmatrix} \mathbf{1}_{N_1} \\ \mathbf{0} \end{pmatrix} \qquad q_{2,1}^T = \mu_1 \begin{pmatrix} \mathbf{1}_{N_1} \\ \mathbf{0} \end{pmatrix} \tag{6.22}$$

The pseudomoduli come out from the meson $M_{3,3}$ and a $(\widetilde{N}_2 - N_1) \times (\widetilde{N}_2 - N_1)$ diagonal block of the other meson, $M_{1,1}$. The one loop analysis is the same as before and lifts all the flat directions.

In order to estimate the lifetime we need the vevs of the fields in the supersymmetric vacuum, which are again (6.20), and the value of the scalar potential in the non supersymmetric vacuum (6.22)

$$V_{min} = (N_2 - N_3)|h\mu_1|^2 + N_3|h\mu_3|^2 \tag{6.23}$$

Since $|\mu_1| > |\mu_3|$ we approximate the scalar potential by the term $\sim |\mu_1|^2$ and the field displacement by $\langle hM_3 \rangle$, obtaining as bounce action

$$S \sim \left(\frac{\mu_1}{\mu_3}\right)^{2\frac{N_2-N_3}{N_2}} \left(\frac{\Lambda_{2i}}{\mu_1}\right)^{4\frac{3N_2-2N_1-2N_3}{N_2}} \gg 1 \tag{6.24}$$

6.5 Renormalization group flow

The analysis of sections 6.3 and 6.4 relies on the fact that we neglect the contributions to the dynamics due to the odd nodes. It means that these groups have to be treated as flavours groups, i.e. global symmetries. However, in the A_n quiver theory each node represents a gauge group factor and we have to analyze how its coupling runs with the energy.

The magnetic window (6.4) constraints the even nodes to be UV free in the high energy description, i.e. $b_{2i} > 0$. The odd groups are not uniquely determined by (6.4) and can be both UV free or IR free in the electric description. In the first case we will choose their scale Λ_{2i+1} to be much lower than the even one

$$\Lambda_{2i+1} \ll \Lambda_{2i}. \tag{6.25}$$

In the second case, when $b_{2i+1} < 0$, Λ_{2i+1} is a Landau pole and we take

$$\Lambda_{2i+1} \gg \Lambda_{2i}. \tag{6.26}$$

In these regimes the even nodes become strongly coupled before the odd ones in the flow toward the infrared. This means that we need a new description provided by Seiberg dualities on the even nodes.

In order to trust the perturbative description at low energy, we have to impose that at the supersymmetry breaking scale (typically μ_i) the odd nodes (flavour), are less coupled than the even ones (gauge), which are always IR free. This requirement will give other constraints on the scales.

As already said there are two possible behaviors of the flavour groups above the scale Λ_{2i}: they can be IR free or UV free. For both cases there are three different possibilities about the beta coefficients in the low energy description.

We start discussing the case when the flavours group are UV free in the electric description. The following three possibilities arise for each flavour group $U(N_{2k+1})$ in the dual theory (Plots 1,2,3 in Figure 1).

1. The first one is characterized by

$$b_{2k+1} > 0 \qquad\qquad \widetilde{b}_{2k+1} < \widetilde{b}_{2i} < 0 \tag{6.27}$$

   In this case the flavour groups $U(N_{2k+1})$ are more IR free than the even nodes after Seiberg duality. The couplings of the flavour groups become more and more smaller than the couplings of the gauge groups along the flow toward low energy. Hence we do not need other constraints on the scales except (6.25).

2. The second possibility is reported in Plot 2 in Figure 1

$$b_{2k+1} > 0 \qquad\qquad \widetilde{b}_{2i} < \widetilde{b}_{2k+1} < 0 \tag{6.28}$$

   The flavour groups $U(N_{2k+1})$ are IR free in the dual theory, but less than the $U(\widetilde{N}_{2i})$ gauge groups (6.28). Below a certain energy scale the flavours become more coupled than the gauge groups. If this happens before the supersymmetry breaking scale we cannot trust our description anymore. To solve this problem we have to choose the correct hierarchy between the electric scales of the flavour and the gauge groups, and the supersymmetry breaking scale. We impose that the couplings of the flavours are smaller than the couplings of the gauge groups at the breaking scale, in the magnetic description. This condition can be rewritten in terms of electric scales only using the matching between the magnetic and the electric scales of the flavours. This procedure is explained in the Appendix B and gives the following condition on Λ_{2k+1}

$$\Lambda_{2k+1} \ll \left(\frac{\mu}{\Lambda_{2i}}\right)^{\frac{\widetilde{b}_{2k+1}-\widetilde{b}_{2i}}{b_{2k+1}}} \Lambda_{2i} \ll \Lambda_{2i} \tag{6.29}$$

   This imposes a constraint stronger than (6.25) on the strong coupling scale of the flavours.

3. The third possibility (Plot 3 Figure 1) is

$$b_{2k+1} > 0 \qquad\qquad \widetilde{b}_{2k+1} > 0 \tag{6.30}$$

In this case the flavour group $U(N_{2k+1})$ is asymptotically free in the low energy description. Once again we have to impose that at the breaking scale the flavours are less coupled than the gauge groups. The procedure is the same outlined above, and the condition is the same as (6.29). This case may become problematic in the far infrared. Indeed, since the flavour group is UV free, it develops strong dynamics at low energy. If we take into account the non perturbative contributions they could restore supersymmetry. Another interesting feature is the appearance of cascading gauge theories, flowing in the IR. We do not discuss these issues here.

If the flavour groups $U(N_{2k+1})$ are IR free in the electric description the same three possibilities discussed above arise (see Plots 4, 5, and 6 of Figure 1).

4. The plot 4 of Figure 1 is characterized by

$$b_{2k+1} < 0 \qquad\qquad \widetilde{b}_{2k+1} < \widetilde{b}_{2i} < 0 \tag{6.31}$$

Here we do not need any other constraint except (6.26).

5. The plot 5 in Figure 1 is

$$b_{2k+1} < 0 \qquad\qquad \widetilde{b}_{2i} < \widetilde{b}_{2k+1} < 0 \tag{6.32}$$

The requirement that the odd nodes are less coupled than the even ones at the supersymmetry breaking scale give once again non trivial constraints, with the same procedure outlined previously

$$\Lambda_{2k+1} \gg \left(\frac{\Lambda_{2i}}{\mu}\right)^{\frac{\widetilde{b}_{2i}-\widetilde{b}_{2k+1}}{b_{2k+1}}} \Lambda_{2i} \gg \Lambda_{2i} \tag{6.33}$$

where now the strong coupling scale of the flavour groups in the electric description is a Landau pole.

6. The last possibility (Plot 6 of Figure 1)

$$b_{2k+1} < 0 \qquad\qquad \widetilde{b}_{2k+1} > 0 \tag{6.34}$$

lead to the same constraint (6.33). In the far infrared the strong dynamics of the flavours node can lead to non perturbative phenomena, as in the case 3.

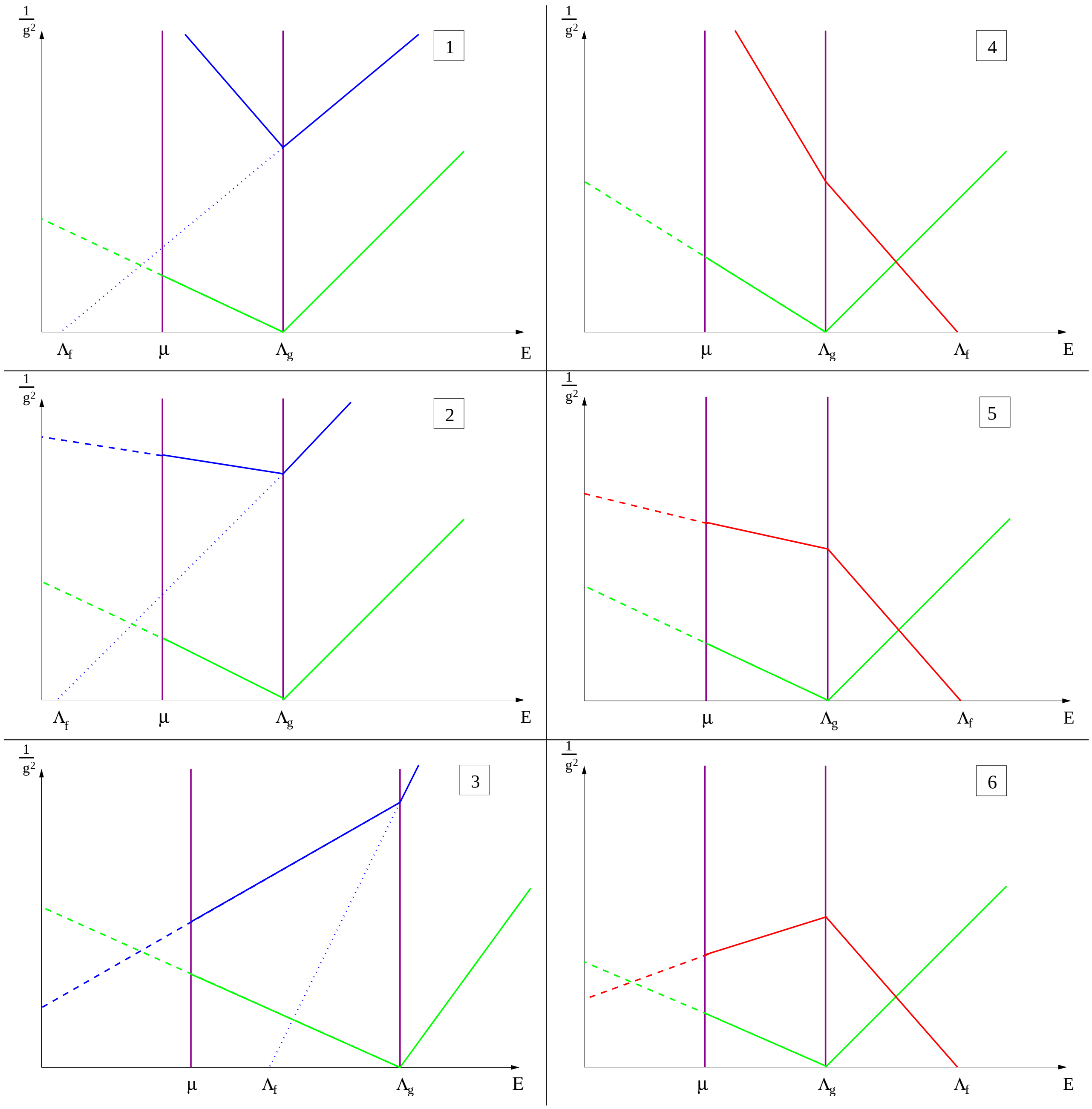


Figure 1: *The blue lines refer to flavour/odd groups which are UV free in the electric description, while the red ones are IR free. The green lines refer to the gauge/even group couplings. We denote with μ the supersymmetry breaking scale, and Λ_G and Λ_F are the strong coupling scales of the*

gauge and the flavour groups, respectively.

6.6 Meta-stable A_n

We work in the regime where the ratio $\frac{\mu_i^2}{m}$ is larger than the strong scale of the even nodes Λ_{2i}. This requirement is satisfied if $\lambda_i \gg \Lambda_{2i}^2$ in the electric theory. This allows us to ignore in the dual superpotential (6.5) the presence of quadratic deformations in the mesonic fields.

In this approximation the superpotential of the A_n quiver (6.5) reduces to $\frac{n-1}{2}$ copies of A_3 superpotentials. Hence a generic A_n diagram results decomposable in copies of A_3 quivers, where every adjacent pair shares an odd node.

For each A_3 the even nodes provide the magnetic gauge groups, and each A_3 has long living metastable vacua, if the perturbative window is correct. It follows that the A_n quiver theory, which is a set of metastable A_3 quivers, possesses metastable vacua.

We still have to be sure of the perturbative regime. This means that we have to control the gauge contributions from the odd nodes of the A_n diagram. We have to proceed as in section 6.5, and study the beta coefficients of the groups. From (6.8) we can see that the magnetic beta coefficients of the internal odd nodes involve the ranks of the next to next neighbor groups, i.e. they depend on five integer numbers. This means that in order to know these beta coefficients it is enough to study the A_5 consistent with (6.4). In the appendix B.3 we classify all the possible metastable A_5 diagrams and we give the corresponding electric and magnetic beta coefficients of the central flavour node. This classification describes the RG behaviour of all the internal odd nodes of the A_n.

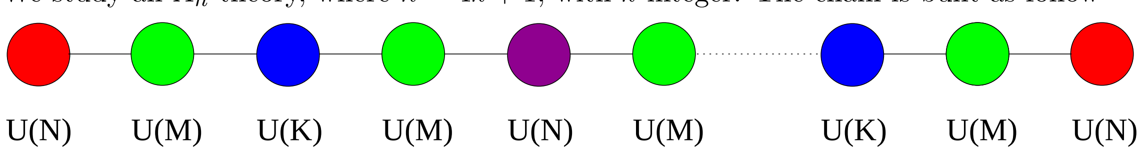
The running of the first and of the n-th node of the A_n quiver is still undefined and it is discussed in the appendix B.3.

This provides a classification of metastable A_n quiver gauge theories with alternate Seiberg dualities.

6.6.1 Example

We show now a simple example of metastable A_n diagram. We choose the even nodes in the electric description to become strongly coupled at the same scale Λ_{2i}. We require that at such scale the flavours (odd nodes) are less coupled than the gauge ones. Moreover we will show that we can also require that in the low energy description all the nodes are IR free and also that the flavour groups (odd nodes) are less coupled than the gauge groups (even nodes) at any scale below the Λ_{2i}.

We study an A_n theory, where $n = 4k + 1$, with k integer. The chain is built as follow



with $N < M < K$. This range allows for metastable vacuum in each A_3 piece as showed previously. We perform alternate Seiberg dualities, working in the in the window

$$M + 1 < N + K < \frac{3}{2}M$$

Thanks to the simple choice for the ranks we have four values for the b coefficients of the beta functions in the electric description, and four values for the coefficients $\widetilde{b}$. They are summarized in the following table

node	b	$\widetilde{b}$	
$1, n$ (red)	$3N - M$	$N - 2K + M$	
$2i$ (green)	$3M - N - K$	$2K + 2N - 3M$	
$4i - 1$ (blue)	$3K - 2M$	$2M - 4N - K$	$i = 1, \ldots, \frac{n-1}{4}$
$4j + 1$ (violet)	$3N - 2M$	$2M - 4K - N$	$j = 1, \ldots, \frac{n-5}{4}$

We require that in the magnetic description all the nodes are IR free. Moreover we require the beta coefficients of the odd groups to be lower than the even group ones, i.e. $\widetilde{b}_{\text{odd}} < \widetilde{b}_{2i}$. This restricts the window to

$$K > 2N \qquad\qquad 3N < 2M < 4N + K \tag{6.35}$$

In this regime all the nodes in the electric description are UV free except the $4j + 1$-th ones. Seiberg duality is allowed on the even nodes, if we impose the following hierarchy of scales

$$\Lambda_1, \Lambda_n, \Lambda_{4i-1} \ll \Lambda_{2i} \ll \Lambda_{4j+1} \tag{6.36}$$

The running of the gauge couplings of the different nodes are depicted in Figure 2.

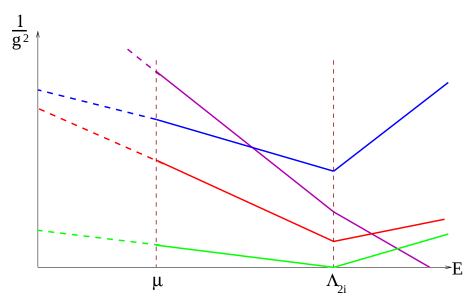


Figure 2: *The green line represents the running of the coupling of the even sites. The violet line is related to the $4j + 1$-th sites, the blue one to the $4i - 1$-th sites and the red to the first and the last nodes.*

At high energy the $4j + 1$-th nodes are strongly coupled, while the other nodes are all UV free. At the scale Λ_{2i} the even nodes become strongly coupled and Seiberg dualities take place. All the runnings of the couplings are changed by these dualities, and all the coefficients of the beta functions $\widetilde{b}_i$ become negative. Hence at energy scale lower than Λ_{2i} the theory is weakly coupled. Furthermore the beta coefficients of the odd nodes are more negative than the even node ones. This guarantees that we can rely on perturbative computations, treating the odd nodes as flavours.

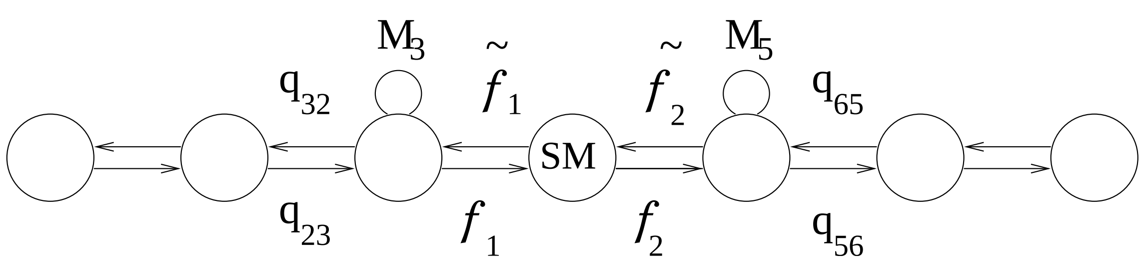
6.7 Gauge mediation

The models analyzed in this work can admit mechanisms of gauge mediation. This means that the breaking of supersymmetry can be transmitted to the Standard Model sector via a gauge interaction. This idea has already appeared in the literature of metastable vacua in A_n theories [53, 56].

Different realizations are possible here. A first one, of direct gauge mediation, identifies the SM gauge group with a subgroup of a flavour group in the quiver [53] and leads to a gaugino mass consistently with the bound of [54].

A second possibility [56] is to connect one of the extremal nodes of the A_n quiver with a new gauge group, which represents the Standard Model gauge group. The arrows connecting these nodes are associated with the messengers f and $\widetilde{f}$, which communicate the breaking of supersymmetry to the standard model. Neglecting all the quartic terms, except the term which couples the messengers $f, \widetilde{f}$ with the last meson, it is possible to show that also in this case gaugino masses arise at one loop.

In our models of metastable A_n quivers another possibility arises for gauge mediation. It consists in substituting an even node with the Standard Model gauge group.



The low energy description is constituted by two metastable A_n (A_3 in this case) which are connected through the SM sector. Both communicate the supersymmetry breaking to the standard model. The superpotential leads to two copies of messengers fields related to the two different hidden sectors

$$W = \left(m_1 + \theta^2 h_1 F_{M_3}\right) f_1 \widetilde{f}_1 + \left(m_2 + \theta^2 h_2 F_{M_5}\right) f_2 \widetilde{f}_2 \tag{6.37}$$

A gaugino mass arises at one loop proportional to $\left(h_1 \frac{F_{M_3}}{m_1} + h_2 \frac{F_{M_5}}{m_2}\right)$.

Conclusions

We have studied metastability in models of A_n quiver gauge theories. The low energy description in terms of macroscopic fields can be achieved via Seiberg dualities at chosen nodes in the A_n diagram. This choice defines, to a certain extent, the models.

A strategy for building acceptable models unfolds from the request for a reliable perturbative analysis. This constrains the ranks of the gauge groups associated with the nodes and their strong coupling scales. We chose to dualize alternate nodes and we fixed two scales: a unique breaking scale μ and a common strong coupling scale Λ_G for each dualized node. The RG flows of the dualized and non dualized gauge groups must be such that at energy scale higher than μ the gauge groups of the dualized nodes are more coupled than the other ones.

The RG properties of the different nodes of an A_n quiver can be studied decomposing it in A_5 quivers and the decomposition of the A_n in A_3 patches gives the structure of the metastable vacuum. In this way we classify all the possible A_n quiver gauge theories which show metastable vacua with the technique of alternating Seiberg dualities.

Finally we have discussed different patterns of gauge mediation.

Chapter 7

Supersymmetry breaking and the Dijkgraaf Vafa conjecture

7.1 Introduction

In this chapter we study explicit supersymmetry breaking in the Dijkgraaf Vafa conjecture. For a review about this conjecture see [69].

Dijkgraaf and Vafa have proposed that the low-energy glueball effective superpotential of $\mathcal{N} = 1$ supersymmetric gauge theories in four dimensions can be computed via an auxiliary matrix model [70]. The simplest case is a $U(N)$ gauge theory coupled to a massive adjoint chiral matter multiplet Φ with a tree–level superpotential $W(\Phi)$. The proposal stems from a set of string dualities in the framework of geometrically engineered gauge theories, topological strings and matrix models [65, 71, 70]. The large-N matrix model analysis brings in an algebraic curve which may correspond to a Calabi–Yau dual geometry [65]. We shall consider gauge theories that can be obtained from string theories that lead to such geometries. The DV proposal has been tested and supported directly on the field theoretical side by perturbative computation via superfields formalism [72] and then by using arguments based on anomaly equations [73].

We study here the case where susy is broken explicitly (soft and/or non soft) by the introduction of spurionic fields [22]. Holomorphy at large is lost, but holomorphic quantities such as the glueball superpotential can be still analyzed and one can compare the computation in the superfields formalism adapted to spurion fields with that one using the algebraic curve underlying the effective gauge theory. In order to discuss such breaking we utilize two notions: a closed string realization of the method of the spurions [74] and Whitham deformations [75, 76].

It is worth to recall the geometrical origin of such gauge theories for type IIB string theory in order to insert the notion of spurion in a natural way in this language. We have in mind D-branes partially wrapped over non trivial 2-cycles of non compact CY and the dual description where D-branes have been replaced by fluxes [65]. In the UV, adjoint chiral multiplets Φ arise from holomorphic deformations of the supersymmetric cycles and of open string gauge bundles on these cycles. A four-dimensional superpotential for these fields can arise and can be written as $W = W(\Phi, g_k)$ where g_k depend only on the complex structure. From the perspective of the $D3$–brane action in the low–energy limit, where supergravity decouples, the g_k can be interpreted as couplings. As already suggested in [74], the susy breaking parameters are described by auxiliary components of the closed string fields, typically magnetic fluxes along CY directions, depending on the complex structure moduli. Such fluxes are introduced by hand without back reaction of the string or of the supergravity backgrounds. In the four dimensional supergravity language they are F–components of chiral multiplets which depend only on the complex structure moduli. Vev of such F–terms cause spontaneous breaking of local susy and, in the appropriate flat limit with decoupling of supergravity, they appear as explicit breaking terms which can be written in the spurionic fashion in the rigid susy action.

A non–perturbative analysis of susy broken effective dynamics has been done in [77] for $\mathcal{N} = 2$ supersymmetric gauge theories. In that context the connection between the Seiberg–Witten solution [78] and integrable systems (Whitham hierarchy) [75] was used. The authors of [77] break susy promoting the Whitham parameters of the hierarchy to spurions and then compute the broken effective potential using the $\mathcal{N} = 2$ integrable structure.
As in the $\mathcal{N} = 2$ case, a relation between the Whitham systems and the $\mathcal{N} = 1$ effective geometry was established in [76]. This suggests to break supersymmetry promoting the Whitham parameters to spurions as in the $\mathcal{N} = 2$ case. In the $\mathcal{N} = 1$ geometry the Whitham parameters are precisely the tree–level coupling costants of the matter superpotential [76]. We will break the $\mathcal{N} = 1$ supersymmetry promoting them to spurions, and the Whitham hierarchy can then be interpreted as a family of supersymmetry breaking deformations of the original theory. Using this interpretation, we will compute directly from the geometrical data the holomorphic supersymmetry breaking contributions in the low–energy effective glueball superpotential.

We have also analyzed with perturbative supergraph techniques the effective glueball superpotential when susy is broken with spurions. Arguments for the computability of the effective superpotential have been presented in [74]. If supersymmetry is broken, holomorphicity in the coupling constants is no longer guaranteed, the computation is much harder than in the $\mathcal{N} = 1$ case and the simplifications of [72] do not work in general. Anyway, we can restrict ourselves to a particular subclass of contributions for which a spurionic superfields generalization of the techniques in [72] can be done. Within such strong approximation and with unbroken $U(N)$ gauge group, we find that to all order in the glueball superfield the effective superpotential has the same functional form of the $\mathcal{N} = 1$ case where the coupling constants are replaced by spurions and so it results still holomorphic.

The chapter is organized as follows: In section 8.2 we review the geometry underlying the Dijkgraaf–Vafa proposal. In section 8.3 we introduce supersymmetry breaking by spurions and discuss the low–energy glueball superpotential. In section 8.4 we discuss the geometry as a Whitham system and use it in the susy broken case. In section 8.5 we treat the explicit example of a deformed susy broken cubic tree–level superpotential. In section 8.6 we use perturbative superspace techniques along the line mentioned above. Section 8.7 is devoted to conclusions. In two appendices C.1 and C.2, we describe the computational details of section 8.5 and section 8.6.

7.2 The geometrical picture

We consider the particular case of a $\mathcal{N}=1$, $U(N)$ gauge theory with a degree $n+1$ polynomial tree–level superpotential $W(\Phi)$ for the chiral matter superfields in the adjoint representation of the gauge group

$$W(\Phi)=\sum_{k=1}^{n+1}\frac{g_k}{k}\mathrm{Tr}\,\Phi^k\ . \tag{7.1}$$

In a generic vacuum the gauge group $U(N)$ is broken to $U(N_1)\times\cdots\times U(N_n)$. In the IR limit the effective low–energy degrees of freedom are described by the glueball superfields $S_i=\frac{1}{32\pi^2}\mathrm{Tr}\,W_i^{\alpha}W_{\alpha i}$ where W_i^{α} is the fermionic chiral superfield, field strength of the vector multiplet of the unbroken gauge group $U(N_i)$.

The expression for the non perturbative glueball superpotential reads

$$W_{eff}(S_i)=-\sum_{i=1}^{n}\left[N_i\frac{\partial\mathcal{F}}{\partial S_i}+2\pi i\tau_i S_i\right]\ , \tag{7.2}$$

where $\mathcal{F}$ is the prepotential which can be computed from the geometrical data [65, 71]. In [70] it has been proposed to reinterpret and compute this prepotential as the free energy of an associated matrix model. In [73, 79] it was also deduced directly on the field theoretical ground using generalized Konishi anomaly equations.

The geometry associated with the low–energy theory is described by a family of genus $g=n-1$ Riemann surfaces and by a meromorphic differential dS

$$y^2 = [W(x)']^2+f^{(n-1)}(x)\ , \tag{7.3}$$

$$dS = y\,dx=\sqrt{[W(x)']^2+f^{(n-1)}(x)}\,dx\ . \tag{7.4}$$

The degree $n-1$ polynomial $f^{(n-1)}(x)=\sum_{l=0}^{n-1}f_l x^l$, is associated with the quantum contributions and the coefficients f_l $(l=0,\cdots,n-2)$ are the moduli of the complex curve; the derivatives of the meromorphic differential (7.4) with respect to the moduli gives holomorphic differentials.

A basis of canonical cycles [76, 79] is $\{\alpha^i,\beta_i,\alpha^0,\beta_0\}$, where $i=2,\ldots,n$, with intersection numbers $(\beta_b\cap\alpha^a=\delta_b^a)$. The cycles are all compact except β_0. We label the cuts starting from the larger real root of the algebraic curve (7.3), so from right to left. The α^i–cycle surrounds counterclockwise the i–th cut while the α^0–cycle encircles all the cuts and then gives the residue at infinity. The dual β_i–cycle $(i=2,\ldots,n)$ passes clockwise through the i-th and the first cut, while β_0 goes from the second sheet infinity to the first passing through the first cut. The periods s_i, the parameter t_0 and the conjugated periods are defined as

$$s_i = \oint_{\alpha^i} dS \ , \qquad t_0 = \oint_{\alpha^0} dS = -Res_\infty(dS) = \frac{f_{n-1}}{2g_{n+1}} \ , \tag{7.5}$$

$$\Pi_i = \frac{1}{2}\oint_{\beta_i} dS \ , \qquad \Pi_0 = \frac{1}{2}\int_{\beta_0} dS \ . \tag{7.6}$$

In these variables the effective superpotential computed by the geometry is

$$-W_{eff} = N\Pi_0 + \sum_{i=2}^{n} N_i \Pi_i = N\frac{\partial \mathcal{F}}{\partial t_0} + \sum_{i=2}^{n} N_i \frac{\partial \mathcal{F}}{\partial s_i} \ , \tag{7.7}$$

where $\sum_{j=1}^{n} N_j = N$. In the previous formula we have introduced the prepotential[1] $\mathcal{F}$ such that its derivatives w.r.t. the $\{s_i, t_0\}$ periods give the dual ones $\{\Pi_i, \Pi_0\}$.

Upon getting the superpotential as a function of the variables s_i and t_0, we return to the variables of [70, 65] using[2]

$$\begin{aligned} s_i &= -2S_i \ , \qquad i = 2, \ldots, n \ , \\ t_0 &= -2\sum_{j=1}^{n} S_j \ , \end{aligned} \tag{7.8}$$

in fact the S_i are the physical variables which are interpreted as the glueball superfields.

7.3 Supersymmetry breaking

The introduction of spurionic fields provides the standard mechanism for the explicit (soft and/or non soft) breaking of global supersymmetry. In the $\mathcal{N} = 1$ case the tree–level superpotential W_{tree}, and the effective glueball prepotential $\mathcal{F}$, depend on the coupling constants g_m associated with the operators $\mathrm{Tr}\,\Phi^m$ in the ultraviolet action. In order to break $\mathcal{N} = 1$ supersymmetry down to $\mathcal{N} = 0$ we promote the coupling constants g_m to $\mathcal{N} = 1$ chiral superfields G_m and then we freeze the scalar and the auxiliary F–components to constant values. In this way the chiral spurions $G_m = g_m + \theta^2\Gamma_m$ produce non supersymmetric terms in the superpotential W_{tree}. We want to study their effects on the low energy glueball effective superpotential under the assumption that the low energy degrees of freedom are still the glueballs. The breaking parameters Γ_m must be

[1] The prepotential differs from the usual one [70, 65] for a multiplicative factor due to the change of variables. Anyway, this difference is not felt by the effective superpotential W_{eff} which is a function of the dual periods, the quantities which really enter in the computation, as in (7.7).

[2] The variables of [70, 65] are $S_j = -\frac{1}{2}\oint_{A^j} dS$ and $\Pi_j = \frac{1}{2}\oint_{B_j} dS$, with $\{A_j, B^j;\ j = 1, \ldots, n\}$ a different set of cycles with all B_j non compact.

considered the smallest scales in the theory. They are thought as small perturbations of the $\mathcal{N}=1$ theory by keeping fixed the $\mathcal{N}=1$ vacuum structure and the gauge symmetry breaking patterns $U(N) \to U(N_1) \times \cdots \times U(N_n)$.

We set the scalar components of G_m equal to the coupling constants g_m for $m \leq n+1$, zero for $m > n+1$, and the F–components Γ_m will be considered as small susy breaking parameters for all G_m. Explicitly

$$
\begin{aligned}
G_k &= g_k + \theta^2 \Gamma_k \quad , \quad k \leq n+1 \ , && (7.9)\\
G_j &= \theta^2 \Gamma_j \qquad\quad , \quad j > n+1 \ , && (7.10)
\end{aligned}
$$

and hence we will consider tree–level superpotential (7.1) perturbed as

$$
W_{tree}(\Phi) = \sum_{k=1}^{n+1} \frac{G_k}{k} \mathrm{Tr}\, \Phi^k + \theta^2 \sum_{j>n+1} \frac{\Gamma_j}{j} \mathrm{Tr}\, \Phi^j \quad . \tag{7.11}
$$

Notice that besides having promoted to spurion the coupling constants already appearing in the tree–level superpotential, we have also added pure auxiliary F–terms. For $k > 3$ these spurionic terms are not soft and quadratic divergences can appear in the wave function renormalization; in any case they have to be considered as dangerously irrelevant operators with the usual warning [30, 74]. The Γ_m for $m \leq n+1$ can be interpreted as vacuum expectations values of fluxes [74], whereas it is not obvious that this is the case for $m > n+1$. In any case, we will see that the generalization to all the Γ_m terms is of some interest in the application of the Witham approach.

Let now analize what happens in the effective theory when the Γ_m are turned on. We will restrict ourselves to a discussion of some formal aspects which can be extracted from the geometry of the $\mathcal{N}=1$ case. We assume that in the effective dynamics the emergence of the spurions G_m are controlled by the holomorphic dependence of the $\mathcal{N}=1$ prepotential $\mathcal{F}(S_i, g_m)$ on the coupling constants. If we restrict ourselves to holomorphic terms in the low–energy glueball superpotential, the prepotential in the susy broken phase has the same functional form as the $\mathcal{N}=1$ case where now the coupling constants g_m are replaced by the spurions as G_m. This is essentially a naturalness assumption on the effective superpotential [80]. In section 6 we will discuss these assumptions using superfields perturbative techniques extending [72] to the susy broken case.
We make some comments about the interpretation of the couplings Γ_j ($j > n+1$). They must be understood as coming from tree–level superpotential W_{tree} of degree greater than $n+1$ where also the scalar coupling constants g_j above the $(n+1)$–degree are turned on. The low energy glueball prepotential will also depend on all these couplings. We then consider the effective theory of (7.11) as obtained from that one of higher degree in the limit where $g_j \to 0$ ($j > n+1$) and in the same vacuum of the theory of $(n+1)$–degree[3]. In conclusion the prepotential depends on the n glueball superfields S_i (in our conventions t_0 and s_i) and it is evaluated where $g_j = 0$.

[3]We can choose $\mathcal{N}=1$ massive theories with classical vacua configuration which are nonsingular for $g_j \to 0$ such that there is analitycity of the glueball superpotential around $g_j = 0$.

We expand now the prepotential $\mathcal{F}(S_i, G_m)$ around the supersymmetric vacuum. If we consider the case of broken supersymmetry with G_m having the form (7.9, 7.10) the terms with more than one power of Γ_m will not give any contribution and we have

$$\begin{aligned}\mathcal{F}(s_i, g_k, \Gamma_k, \Gamma_j) &= \mathcal{F}(s_i, g_m)|_{g_j=0} + \\ &+ \theta^2 \sum_{k=1}^{n+1} \Gamma_k \frac{\partial \mathcal{F}(s_i, g_m)}{\partial g_k}\Big|_{g_j=0} + \theta^2 \sum_{j>n+1} \Gamma_j \frac{\partial \mathcal{F}(s_i, g_m)}{\partial g_j}\Big|_{g_j=0} . \end{aligned} \tag{7.12}$$

The first term in this expression is the prepotential of the supersymmetric case for a theory with tree–level superpotential of $(n+1)$–degree. As just discussed the last term is interpreted as coming from an higher degree theory in the appropriate limit.
We now insert this expression in (7.7) and we obtain the holomorphic glueball superpotential associated with a tree–level susy breaking superpotential as (7.11)

$$\begin{aligned}-W_{eff} &= N\left[\frac{\partial \mathcal{F}}{\partial t_0} + \theta^2 \sum_{k=1}^{n+1} \Gamma_k \frac{\partial^2 \mathcal{F}}{\partial t_0 \partial g_k} + \theta^2 \sum_{j>n+1} \Gamma_j \frac{\partial^2 \mathcal{F}}{\partial t_0 \partial g_j}\right] + \\ &+ \sum_{i=2}^{n} N_i \left[\frac{\partial \mathcal{F}}{\partial s_i} + \theta^2 \sum_{k=1}^{n+1} \Gamma_k \frac{\partial^2 \mathcal{F}}{\partial s_i \partial g_k} + \theta^2 \sum_{j>n+1} \Gamma_j \frac{\partial^2 \mathcal{F}}{\partial s_i \partial g_j}\right] . \end{aligned} \tag{7.13}$$

In this expression the first terms within the square bracket are supersymmetric, whereas the others break susy explicitily: they involve second derivatives of the prepotential evaluated where the $g_j \equiv 0$ $(j > n+1)$. We will show in the next section how to obtain directly and efficiently from the geometrical data of the $\mathcal{N} = 1$ theory the mixed second derivatives of $\mathcal{F}$ appearing in (7.13) in order to extract the effective supersymmetry breaking contributions.

7.4 The $\mathcal{N} = 1$ geometry and Whitham systems

The geometry of the $\mathcal{N} = 1$ low–energy effective theory is associated with the generating meromorphic differential dS (7.4) and it can be thought as coming from a Seiberg–Witten geometry of a $\mathcal{N} = 2$ theory [71]. The addition of a superpotential together with a geometric transition and a desingularization leads to such geometry with parameters g_k and complex moduli f_l [65]. The couplings g_k can be viewed as Whitham deformations of the previous SW geometry. Performing a Whitham deformation mean extending the parameter space of the curve with extra variables [75]. As a consequence of this deformation the moduli of the curve and also the generating differential become functions of these new parameters.

As shown in [76] the $\mathcal{N} = 1$ geometry can be embedded into the Whitham framework. The moduli f_l of the curve (7.3) are functions $f_l = f_l(g_k, t_0, s_i)$ of the Whitham parameters g_k and of $(t_0,\, s_i)$, the periods of the generating differential dS along the α–cycles. We review some results

of [76] and set up our conventions.
One of the advantages we gain using Whitham description is that it provides an efficient way to compute the mixed second derivatives appearing in (7.13) directly in terms of geometrical data since the coupling constants are considered as independent parameters.

Using the whole set of variables (g_k, t_0, s_i) characterizing the curve (7.3) and the generating differential (7.4), the Whitham system can be defined by the following set of equations [76]

$$\frac{\partial dS}{\partial s_i} = d\omega_i \quad , \qquad \frac{\partial dS}{\partial t_0} = d\Omega_0 \quad , \qquad \frac{\partial dS}{\partial g_k} = d\Omega_k \quad , \tag{7.14}$$

where $d\omega_i$ are normalized holomorphic differentials

$$\oint_{\alpha_i} \frac{\partial dS}{\partial s_j} = \oint_{\alpha_i} d\omega_j = \delta_{ij} \quad . \tag{7.15}$$

The differentials $d\Omega_k$ are meromorphic of the second kind with poles only at the infinity points $\pm\infty$; $d\Omega_0$ is a differential of the third kind with residue at $\pm\infty$. They have vanishing α–periods and behave at infinity as ($\xi = \frac{1}{x}$)

$$\oint_{\alpha^i} d\Omega_0 = \frac{\partial s_i}{\partial t_0} = 0 \quad , \quad \oint_{\alpha^i} d\Omega_k = \frac{\partial s_i}{\partial g_k} = 0 \quad ; \qquad d\Omega_l = -(\xi^{-l-1} + O(1))d\xi \quad . \tag{7.16}$$

These normalization conditions characterize s_i, t_0 and g_k as independent variables.
The generating differential dS is then a linear combination of the differentials (7.14)

$$dS = \sum_{i=2}^{n} s_i \, d\omega_i + t_0 \, d\Omega_0 + \sum_{k=1}^{n+1} g_k \, d\Omega_k \;\; = \sqrt{[W(x)']^2 + \sum_{k=0}^{n-2} f_k x^k + 2g_{n+1} t_0 x^{n-1}} \, dx \quad . \tag{7.17}$$

Consinstency of the equality in (7.17) requires that

$$g_k = -Res_{\infty+}(x^{-k} dS) \quad , \tag{7.18}$$

which can be verified [76]. Using (7.17) the meromorphic differentials $d\Omega_l$ can be written as

$$\begin{aligned}
d\Omega_0 \quad &= \quad \frac{\partial dS}{\partial t_0} = \frac{g_{n+1} x^{n-1}}{y} dx + \frac{1}{2} \sum_{l=0}^{n-2} \frac{\partial f_l}{\partial t_0} \frac{x^l}{y} dx \quad , \\
d\Omega_k \quad &= \quad \frac{\partial dS}{\partial g_k} = \frac{W'(x) x^{k-1}}{y} dx + \frac{1}{2} \sum_{l=0}^{n-2} \frac{\partial f_l}{\partial g_k} \frac{x^l}{y} dx \; , \quad k = 1, \ldots, n \quad , \\
d\Omega_{n+1} \quad &= \quad \frac{\partial dS}{\partial g_{n+1}} = \frac{[W'(x) x^n + t_0 x^{n-1}]}{y} dx + \frac{1}{2} \sum_{l=0}^{n-2} \frac{\partial f_l}{\partial g_{n+1}} \frac{x^l}{y} dx \quad .
\end{aligned} \tag{7.19}$$

In this framework, the prepotential $\mathcal{F}$ and so the special geometry can be introduced thanks to the Riemann bilinear relations which guarantee the integrability condition of the prepotential [76]. We must define correctly the first derivatives of $\mathcal{F}$ with respect to both the periods and the coupling constants

$$\frac{\partial \mathcal{F}}{\partial s_i} = \Pi_i = \frac{1}{2}\oint_{\beta_i} dS \quad , \quad \frac{\partial \mathcal{F}}{\partial t_0} = \Pi_0 = \frac{1}{2}\int_{\infty-}^{\infty+} dS \quad , \quad \frac{\partial \mathcal{F}}{\partial g_k} = Res_{\infty+}\left(\frac{x^k}{k}dS\right) \quad . \tag{7.20}$$

As we have seen in the previous section, the supersymmetry breaking contributions appearing in the effective glueball superpotential (7.13) are mixed second derivatives of the prepotential with respect to the Whitham parameters g_k. Starting from the expressions (7.20) it results

$$\frac{\partial^2 \mathcal{F}}{\partial s_i \partial g_k} = Res_{\infty+}\left(\frac{x^k}{k}d\omega_i\right) \quad , \quad \frac{\partial^2 \mathcal{F}}{\partial t_0 \partial g_k} = Res_{\infty+}\left(\frac{x^k}{k}d\Omega_0\right) \quad . \tag{7.21}$$

The right hand side of these formulae express the susy breaking contributions in (7.13) as geometrical quantities which can then be read directly as residues. Nevertheless we have to remind the interpretation of the mixed second derivatives appearing in (7.13). As already mentioned, they should be thought to come from an appropriate higher degree system taking $g_j \to 0$ ($j > n+1$), with the genus of the curve and (t_0, s_i) kept fixed. Using (7.21), the residues can be computed directly with $g_j = 0$; therefore, $\frac{\partial^2 \mathcal{F}}{\partial t_0 \partial g_j}|_{g_j=0}$ and $\frac{\partial^2 \mathcal{F}}{\partial s_i \partial g_j}|_{g_j=0}$ can be properly obtained from the curve of $(n+1)$–degree which depend only on the couplings g_k, $k = 1, \cdots, n+1$. We can then extract all the mixed second derivatives, included those with respect to g_j, using the $(n+1)$–degree geometry.

This simplification is one of the advantages of the embedding of the geometry in the Whitham framework. With this approach we compute the holomorphic supersymmetry breaking terms in the effective glueball superpotential corresponding to a non–supersymmetric perturbation of the $(n+1)$–degree tree–level superpotential (7.11), without the explicit knowledge of the prepotential $\mathcal{F}$.

7.5 Tree–level cubic superpotential

We consider the simple case of a supersymmetric $U(N)$ gauge theory with tree–level superpotential

$$W_{tree}(\Phi) = \frac{m}{2}\mathrm{Tr}\Phi^2 + \frac{g}{3}\mathrm{Tr}\Phi^3 \ . \tag{7.22}$$

As suggested before we break supersymmetry promoting the coupling constants of the tree–level superpotential to spurions (7.9,7.10) deforming (7.22) as

$$W_{tree}(\Phi) = \frac{m+\theta^2\Gamma_2}{2}\mathrm{Tr}\,\Phi^2 + \frac{g+\theta^2\Gamma_3}{3}\mathrm{Tr}\,\Phi^3 + \theta^2\sum_{j>3}\frac{\Gamma_j}{j}\mathrm{Tr}\,\Phi^j \ . \tag{7.23}$$

The geometry of the $\mathcal{N}=1$ solution is described by the following complex curve of genus one with meromorphic differential

$$y^2 = g^2(x-a_1)^2(x-a_2)^2 + f_0 + f_1 x \ , \tag{7.24}$$

$$dS = y\,dx = \sqrt{g^2(x-a_1)^2(x-a_2)^2 + f_0 + 2gt_0 x}\,dx \ . \tag{7.25}$$

with $a_1 = 0$ and $a_2 = -\frac{m}{g}$ the classical roots.

We do the computation of the supersymmetry breaking parts as a series with small width of the cuts and then small values of s_i and t_0. The approach is the same as in [65]. In particular, we have considered the case of classical susy vacua with unbroken gauge group and also the case with $U(N) \to U(N_1) \times U(N_2)$ gauge symmetry breaking pattern. Using (7.21), we compute directly the second mixed derivatives of the prepotential, i.e. the susy breaking contributions. The details of the computations are in appendix A. We express our results directly in terms of the physical glueball superfields S_i $(i = 1, \dots, n)$ using the change of variables (7.8) at the end of the computation. We will write explicitly only the novel supersymmetry breaking contributions to the low–energy glueball superpotential referring the reader to the literature [65, 81] for the well known $\mathcal{N}=1$ part.

In the case $U(N) \to U(N)$, using (7.13) with superpotential of the form (7.23) we find

$$\begin{aligned}
-\frac{1}{N} W_{eff} &= -\frac{1}{N} W_{eff}^{\mathcal{N}=1}(S,m,g) \ + \\
&- \theta^2 \Gamma_2 \left[\frac{S}{m} \left(1 + \sum_{k=1}^{+\infty} \frac{3}{(k+1)!} \frac{\Gamma(\frac{3k}{2})}{\Gamma(\frac{k}{2})} \left(\frac{8g^2 S}{m^3} \right)^k \right) \right] \ + \\
&+ \theta^2 \Gamma_3 \left[\frac{S}{g} \left(\sum_{k=1}^{+\infty} \frac{2}{(k+1)!} \frac{\Gamma(\frac{3k}{2})}{\Gamma(\frac{k}{2})} \left(\frac{8g^2 S}{m^3} \right)^k \right) \right] \ + \\
&+ \theta^2 \Gamma_4 \left[\frac{m^4}{64 g^4} \sum_{k=2}^{+\infty} \frac{1}{k!} \left((k+1) \frac{\Gamma(\frac{1}{2}(3k-4))}{\Gamma(\frac{1}{2}k)} - 4 \frac{\Gamma(\frac{1}{2}(3k-1))}{\Gamma(\frac{1}{2}(k+1))} \right) \left(\frac{8g^2 S}{m^3} \right)^k \right] \ + \\
&- \ \theta^2 \sum_{j>4} \frac{\Gamma_j}{j} \left[\frac{g}{j!} \left(\frac{\partial^j}{\partial \xi^j} \frac{1}{\sqrt{(g+m\,\xi)^2 + f_0\,\xi^4 - 4gS\,\xi^3}} \right) \Bigg|_{\xi=0} + \right. \\
&\left. + \frac{m}{2(j-1)!} (1+Y) \left(\frac{\partial^{j-1}}{\partial \xi^{j-1}} \frac{1}{\sqrt{(g+m\,\xi)^2 + f_0\,\xi^4 - 4gS\,\xi^3}} \right) \Bigg|_{\xi=0} \right] ,
\end{aligned} \tag{7.26}$$

Y and f_0 are functions of $(S,\,m,\,g)$ whose expressions (C.19, C.25) are given in appendix A.

As a consistency check of our computation and focusing on the spurionic terms Γ_2 and Γ_3, we can compare the previous result with the mixed second derivatives of the $\mathcal{N}=1$ perturbative prepotential

$$\mathcal{F} = \frac{S^2}{2}\sum_{k=1}^{+\infty}\frac{1}{(k+2)!}\frac{\Gamma(\frac{3k}{2})}{\Gamma(\frac{k}{2}+1)}\left(\frac{8g^2S}{m^3}\right)^k \quad , \tag{7.27}$$

derived for the first time in [82] from the large-N matrix model. We find a complete agreement except the linear term ($\sim S$) in the series multiplied by Γ_2.
The appearance of the linear term can be explained in the following way. It is known [83, 70, 73] that the measure in the matrix model partition function and also the allowed divergent modes on the complex curve [71] give schematically a contribution like $(S - S\log(m\Lambda_0^2/S))$ where Λ_0 is a cut-off: this contribution together with the additive term $(2\pi i\tau S)$ in the effective superpotential gives the Veneziano–Yankielowicz superpotential [84]. The derivatives of this contribution w.r.t. the coupling m give exactly the linear term appearing in (7.26) which also agrees with what we have found using perturbative techniques (see Sec.6).
The supersymmetry breaking part coming from the quartic term (and also from the higher ones) can be checked by comparison with the $\mathcal{N}=1$ superpotential computed implicitly in [85] for a generic tree–level superpotential. By evaluating the derivative where all the coupling constants except $(m,\ g)$ are set to zero, we find agreement with their computation for all the finite order explicitly given by them.

In the case $U(N) \to U(N_1)\times U(N_2)$, we consider only $\Gamma_2,\ \Gamma_3,\ \Gamma_4$ in (7.23) as source of susy breaking. Then, using (7.13) we have

$$\begin{aligned}
-W_{eff} \;\; &= \;\; -W_{eff}^{\mathcal{N}=1}(S_1,S_2,m,g) \; + \\
+ \;\; &\theta^2\Gamma_2\Big[(2N_2-N_1)\frac{S_1}{m} + (2N_1-N_2)\frac{S_2}{m} + 30(N_1-N_2)\frac{g^2}{m^4}S_1S_2 + \\
&\quad + \; 3(5N_2-2N_1)\frac{g^2}{m^4}S_1^2 + 3(2N_2-5N_1)\frac{g^2}{m^4}S_2^2 + O(S^3)\Big] + \\
+ \;\; &\theta^2\Gamma_3\Big[-2N_2\frac{S_1}{g} - 2N_1\frac{S_2}{g} + 20(N_2-N_1)\frac{g}{m^3}S_1S_2 + \\
&\quad + \; 2(2N_1-5N_2)\frac{g}{m^3}S_1^2 + 2(5N_1-2N_2)\frac{g}{m^3}S_2^2 + O(S^3)\Big] + \\
+ \;\; &\theta^2\Gamma_4\Big[2N_2\frac{m}{g^2}S_1 + (2N_1+N_2)\frac{m}{g^2}S_2 + \frac{6}{m^2}(2N_1-3N_2)S_1S_2 + \\
&\quad + \; \frac{9}{2}(N_2-2N_1)\frac{S_2^2}{m^2} - \frac{3}{2}(N_1-4N_2)\frac{S_1^2}{m^2} + O(S^3)\Big] \; .
\end{aligned} \tag{7.28}$$

where we show terms up to the quadratic order in S; we give in Appendix A a sketch of the computation.

We can check also this case using the results of [71] for a quartic tree–level superpotential. Taking the derivatives of their results with respect to the coupling constants and then making the appropriate limit ($S_3 = 0$ and $g_4 \to 0$) we get exactly our supersymmetry breaking contributions. Observe that, also in this case, linear terms appear in the supersymmetry breaking series multiplied by Γ's. These can again be understood as coming from the Veneziano–Yankielowicz piece of the effective superpotential. In fact, the scales Λ_i associated with each unbroken gauge group sector $U(N_i)$ are functions of the coupling constants as a consequence of the threshold matching [86]; by taking derivatives w.r.t. the couplings we get exactly those linear contibutions appearing in (7.28).

Finally we note that, up to the quadratic order in $S \equiv S_1$, we can consistently get our first result (7.26) from the second one (7.28) simply by setting ($S_2 = 0$, $N_2 = 0$).

7.6 Perturbative arguments

In this section we exploit the perturbative approach [72] to discuss, from a field theoretical point of view, our use of the $\mathcal{N} = 1$ prepotential to study the low–energy glueball superpotential in the case with broken susy. We consider only the case of unbroken $U(N)$ gauge group and tree–level superpotential for the adjoint chiral superfields given by $W(\Phi) = \sum_{k=2}^{n+1} \frac{G_k}{k} \mathrm{Tr}\, \Phi^k$ where $G_k = g_k + \theta^2 \Gamma_k$ are the spurionic coupling constants.

We recall that, because of holomorphicity, in the $\mathcal{N} = 1$ case the effective superpotential is a function only of the coupling constants g_k and not of the $\overline{g}_k$ [80, 72, 73]. In our case susy is broken by the spurions and holomorphicity in the couplings is not any longer a property of the superpotential.

In a perturbative framework the spurions G_k can be thought as ordinary background chiral superfields. We can then think susy unbroken and the perturbative computations in a superspace approach go using the usual D–algebra [87]. The effective action will be schematically of the form

$$\int d^2\theta d^2\bar{\theta}\, \mathcal{K}(G_k, \overline{G}_k, D^2 G_k, \bar{D}^2 \overline{G}_k, \cdots, S, \overline{S}) + \int d^2\theta\, W_{eff}(G_k, S) + \text{h.c.} \quad , \tag{7.29}$$

where the superpotential $W_{eff}(G_k, S)$ is constrained to be a holomorphic function[4] of G_k.

If we choose the particular supersymmetric configuration in which all the chiral superfields G_k are equal to the constants g_k, without any dependence on θ_α, then the $\mathcal{N} = 1$ effective superpotential is $W_{eff}(g_k, S)$ and hence its holomorphicity [80].

If we choose instead the configuration $G_k = g_k + \theta^2 \Gamma_k$ ($\Gamma_k \neq 0$), we break susy and furthermore we will have two kind of contributions to the glueball superpotential.

The first ones come from $W_{eff}(g_k + \theta^2 \Gamma_k, S)$ in (7.29) and are the holomorphic ones we have studied in the previous sections. We call them the holomorphic contributions.

The others are D–terms contributions holomorphic in S but not necessarily in the coupling constants g_k, $\overline{g}_k$, Γ_k and $\overline{\Gamma}_k$ which come from particular contributions to $\mathcal{K}$ in (7.29) and which can be

[4]We are thinking about the case with masses in the Wilsonian approach for which the nonrenormalization argument works without IR patologies.

written as $\int d^2\theta$ integrals contributing to the glueball superpotential[5]. These terms in the $\mathcal{N}=1$ case ($\Gamma \to 0$) are zero.

Here we adopt a pragmatic attitude and we study only those contributions to the glueball superpotential which can be computed using the powerful perturbative techniques developed in [72] for the $\mathcal{N}=1$ case.

In [72] the perturbative series was generated using only the propagator of the chiral matter superfield sector and the antichiral superfield $\bar{\Phi}$ was integrated out. This was the central point for their simplifications. In order to be able to integrate out $\bar{\Phi}$ as in [72] we must have interactions only in terms of the chiral superfield Φ and then we consider the following UV action

$$
\begin{aligned}
S(\Phi,\bar{\Phi}) \;&=\; \int d^4x\, d^4\theta\ \mathrm{Tr}\ e^{-V}\bar{\Phi}e^{V}\Phi - \int d^4x\, d^2\theta\ \frac{m}{2}\mathrm{Tr}\,\Phi^2 - \int d^4x\, d^2\bar{\theta}\ \frac{\overline{m}}{2}\mathrm{Tr}\,\bar{\Phi}^2 + \\
&+\; \int d^4x\, d^2\theta\ \frac{1}{2}(\theta^2\Gamma_2)\mathrm{Tr}\,\Phi^2 + \int d^4x\, d^2\theta\ \sum_{k=3}^{m}\frac{1}{k}(g_k+\theta^2\Gamma_k)\mathrm{Tr}\,\Phi^k \quad , \qquad (7.30)
\end{aligned}
$$

where all the antiholomorphic interactions $\int d^2\bar{\theta}\left[\frac{1}{2}(\bar{\theta}^2\overline{\Gamma}_2)\mathrm{Tr}\,\bar{\Phi}^2 + \sum_{k=3}^{m}\frac{1}{k}(\overline{g}_k+\bar{\theta}^2\overline{\Gamma}_k)\mathrm{Tr}\,\bar{\Phi}^k\right]$ are neglected. Furthermore, since we are interested in the glueball superpotential it is also possible to do the usual simplifications of [72, 73] finding as the relevant action[6]

$$
\int d^4x\, d^2\theta \left\{\frac{1}{2\overline{m}}\Phi[\Box - i\mathcal{W}^{\alpha}\partial_{\alpha} - m\overline{m}]\Phi + W^{int}_{tree}(\Phi)\right\} \quad , \qquad (7.31)
$$

where W^{int}_{tree} in our susy broken case consists in the second line of (7.30). The difference with respect to [72] is that the tree–level superpotential is now defined in terms of spurionic coupling constants.

Now, from (7.31), it is clear that the glueball superpotential we are going to compute will be holomorphic in S and in all the coupling constants except, at most, for the mass. In particular we observe (we refer to Appendix B for the details) that we can have contributions only of the following form

$$
\int d^2\theta \left\{W_{eff}(G_k,S) + \frac{1}{\overline{m}^2}\sum_{l}\mathcal{B}_l(g_k,\Gamma_k,\theta)S^l\right\} \quad . \qquad (7.32)
$$

$W_{eff}(g_k+\theta^2\Gamma_k,S)$ is the holomorphic contribution we have already defined. Instead, the second part of (7.32) is a particular subclass of the D–term contributions discussed before where $\mathcal{B}_l$ are holomorphic in all g_k, Γ_k and possibly depend also on θ^2.

[5]For example, the reader could think about two terms like (with $W^{\alpha} = i\bar{D}^2(e^{-V}D^{\alpha}e^{V})$ [87]) $\int d^2\theta d^2\bar{\theta}\, G(g,\overline{g},\Gamma,\overline{\Gamma},\theta^2)\bar{\theta}^2\overline{\Gamma}\, S^p = \int d^2\theta\, G\,\overline{\Gamma}S^p$ or $\int d^2\theta d^2\bar{\theta}\, H(g,\overline{g},\Gamma,\overline{\Gamma},\theta^2)\mathrm{Tr}[i(e^{-V}D^{\alpha}e^{V})W_{\alpha}]\Gamma S^q = \int d^2\theta\, H\,\Gamma S^{q+1}$ where G and H are functions of $g,\overline{g},\Gamma,\overline{\Gamma},\theta^2$ and not $\bar{\theta}^2$.

[6]$\mathcal{W}_{\alpha} = [W_{\alpha},\cdots\}$ is the spinorial gauge field strength adapted to the action, as a graded–commutator, on the adjoint representation of the $U(N)$ gauge group.

A careful perturbative analysis of (7.32) shows that all the coefficients $\mathcal{B}_l = 0$ vanish $\forall l$ and that the contributions to the glueball superpotential we are computing have the following form

$$\int d^2\theta \left[N\theta^2\Gamma_2 \frac{S}{m} + N\frac{\partial \mathcal{F}_0}{\partial S} \right] \qquad \text{with} \qquad \mathcal{F}_0 = \sum_l \mathcal{F}_{0,l}(g_k + \theta^2\Gamma_k)S^l \quad . \tag{7.33}$$

We refer the interested reader to Appendix B for the technical details of our perturbative computations.

In (7.33) $\mathcal{F}_{0,l}$ are the planar amplitudes with l index loops of the dual matrix model [70, 72] where the coupling constants are in this case the spurions $G_k = g_k + \theta^2\Gamma_k$. The first term in (7.33) is given by a 1–loop diagram with one vertex $\frac{1}{2}\theta^2\Gamma_2 \mathrm{Tr}\,\Phi^2$. This term is associated with the 1–loop matter contribution to the Wilsonian beta function for the gauge kinetic term which is implicit in the nonperturbative Veneziano–Yankielowicz superpotential. In the previous section we have seen that this term is also given by the geometrical methods.

We conclude that, within our stringent approximations and in the case of unbroken $U(N)$, the effective glueball superpotential in the presence of spurions (7.33) can still be deduced from the $\mathcal{N} = 1$ holomorphic superpotential supporting the results of the previous sections.

Conclusions

The Dijkgraaf–Vafa conjecture with supersymmetry breaking is the subject of this work. We have considered the simple case of $U(N)$ gauge theory with massive adjoint chiral matter multiplet with a polynomial tree–level superpotential. We have studied the case where supersymmetry is broken in the tree–level superpotential by promoting the coupling constants to chiral spurions. We have considered their F–components as non–supersymmetric small perturbations of the $\mathcal{N} = 1$ gauge theory and we have discussed how holomorphy can still play a role. The non–supersymmetric holomorphic contributions to the effective low–energy glueball superpotential have been derived with geometrical methods embedded in the Whitham framework as well as with techniques of superfield formalism with spurionic fields.

Non–holomorphic D–terms, soft breaking via gaugino mass, low energy vacua are open to investigation. This goes beyond the information encoded in the holomorphic matrix model that we have used so far.

Chapter 8

Supersymmetric D-branes on $SU(2)$ structure manifolds

Besides addressing the problem of unification, string theory can give an insight in the search of duals of different gauge theories. This fact is related to an older proposal by t'Hooft. He pointed out that the Feynman diagrams of a $U(N)$ gauge theory can be rearranged as a sum over the genus of the surfaces in which the diagrams can be drawn. This is similar to the computation of string amplitudes, where there is a sum over the genus of the possible worldsheets. Then, there is a gauge/string duality, at least in some regime of parameters. At weak gauge coupling a convenient description of the theory involves conventional perturbative methods; at strong coupling, where such methods are intractable, the dual string description simplifies and gives exact information about the theory. The objects called D-branes play here and important role. D-branes are non perturbative solitonic objects that can be identified with hyperplanes where open strings can end. The dynamics of the D-branes can be described by the physics of the open strings, giving rise to a gauge theory living on the worldvolume of the brane. However, D-branes also act as sources of closed strings. From this point of view, branes are objects that modify the gravitational background, i.e. the geometry of space-time. Therefore, this open/closed string duality leads to a gauge/gravity duality. This notion has opened new and amazing possibilities.

In 1997, Maldacena has proposed a specific gauge/string duality. The statement is that type IIB string theory living on $AdS_5 \times S^5$ is exactly dual to four dimensional $\mathcal{N} = 4$ super Yang-Mills theory with $SU(N)$ gauge group (which is called AdS/CFT duality since the gauge theory is conformal). Although a strict proof has not been given, the duality has overcome a large number of tests. The duality between two such different theories was reached by looking at the dual open/closed string descriptions of the near horizon limit of a stack of N $D3$-branes. The low energy limit of string theory yields a supergravity theory, and hence the duality can be phrased as a gauge/gravity duality between type IIB supergravity on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM in its non-perturbative regime. The relation between the two theories that are supposed to be equivalent is holographic. This means that the number of dimensions in which they live is different and, that, somehow, the physics on the boundary of a space encodes all the bulk information.

Following these ideas, a lot of work has been devoted to the research on other possible dualities involving more realistic gauge theories. In particular, one would like to have less supersymmetry and break conformal invariance. The final goal is to find a gravity dual of QCD, at least for the limit with large number of colors.

Generalizations of the correspondence with additional structure added to both sides are inherently quite interesting, and potentially have much more to teach us about field theory dynamics, the nature of string theory and how holography relates them. A natural extension of the correspondence would be the inclusion of matter (quark) fields in the fundamental representation. This is equivalent to adding open string degrees of freedom to the supergravity side of the correspondence and can be achieved by adding D-branes to the supergravity background. Another possible generalization is related to the observation that spatial defects may be introduced into conformal field theories, reducing the total symmetry but preserving conformal invariance. A potential gravity dual was proposed as probe D-branes which share only a submanifold of dimension lower than four (the defect) with the D-branes that have generated the background.

For these reason the investigation of supersymmetric configurations of probe D-branes in supergravity backgrounds is relevant for enlarging the class of theories with gauge/gravity duality.

8.1 Introduction

Strings and supergravity backgrounds with non trivial RR and NS fluxes are intensively studied in the AdS/CFT correspondence [89] and in string compactification (see [90] and reference therein), in order to find string models holographically dual to more realistic gauge theories or to obtain sensible phenomenology from compactification. Here D-branes are successfully used as probes to explore the geometric properties of known backgrounds, and to provide further insights in the gauge/gravity duality. We focus on type IIB supergravity solutions which preserve four dimensional Poincaré invariance and $\mathcal{N} = 1$ supersymmetry. They correspond to a warped product of the four dimensional Minkowski spacetime and an internal six dimensional manifold $\mathcal{M}$, which can support fluxes. In the presence of non trivial background fluxes, the back-reacted internal manifold $\mathcal{M}$ is no longer Calabi Yau. There are special classes of solutions [91] where the internal manifold is conformal Calabi Yau, but in general [92, 93] the internal manifold with fluxes can be far different from the Calabi Yau case. The formalism of G-structures [94] and Generalized Complex Geometry (GCG) [95, 96, 92, 93] provide powerful tools to describe such manifolds. In GCG the basic objects are pure spinors, formal sums of even and odd forms. Their existence imposes topological constraints on the tangent and cotangent bundles of the internal manifold. Supersymmetry requires that the internal manifold has a $SU(3) \times SU(3)$ structure on $T_M \oplus T^*_M$, which may be further restricted to $SU(3)$ or $SU(2)$ structures on T_M. The $SU(3)$ structure has been much studied, e.g.[97], while the $SU(2)$ case has been explored in [98] and, using GCG, in [99].

As a matter of fact, supergravity solutions with fluxes dual to massive and marginal deformations of superconformal gauge theories are expected to be described by $SU(2)$ structure manifolds. Such manifolds are characterized by the existence of a globally defined nowhere vanishing vector field.

In the GCG language the preservation of $\mathcal{N} = 1$ supersymmetry is achieved by imposing a pair of differential equations for the pure spinors. The authors of [99] made an ansatz for pure spinors of $SU(2)$ structure manifolds and performed a detailed analysis of these pair of supersymmetry equations. Their ansatz covers a large class of solutions. In particular the Pilch Warner [100] and the Lunin Maldacena [101] ones are included: they are the gravity duals of the single mass deformation and of the beta marginal deformation of $\mathcal{N} = 4$ SYM, respectively.

In the GCG framework the supersymmetry conditions for D-branes probing $SU(3) \times SU(3)$ backgrounds have been established in [102, 103] (see also [104]). They are a set of constraints on the pull back of the pure spinors on the world volume of the D-brane. In [103] the supersymmetry conditions were given for D-branes filling Minkowski space time (space time filling), filling three space time directions (domain walls) and two space time directions (effective strings).

The addition of D-brane probes to the class of solutions of [99] can provide other interesting tests of the AdS/CFT correspondence. Supersymmetric configurations of D-branes can identify the moduli space of vacua of the dual gauge theory, in both the abelian and the non abelian branches. $D5$ domain wall like configurations can lead in the dual description to three dimensional defects, interacting with the conformal four dimensional gauge theory; the defect gauge invariant operators can then be mapped into the Kaluza Klein modes of the wrapped brane [105]. The addition of space time filling $D7$-branes corresponds to adding massless or massive flavours [106] and their fluctuations give the meson spectrum of the dual flavoured gauge theory.

In [99] the space time filling $D3$-brane configurations have been analyzed and it was shown that the supersymmetry conditions for such branes reproduce the mesonic moduli space of vacua of the dual field theory. Moreover the $D5$-brane configuration with world volume flux, related to the non abelian phase of the beta deformed gauge theory [101, 107], was recovered.

In this chapter we investigate new supersymmetric D-brane configurations in the class of $SU(2)$ structure manifolds of [99], and we propose the dual gauge theory interpretation as well as possible applications of the results.

We look for supersymmetric $D5$ domain wall like configurations finding a supersymmetric embedding which can be used to holographically study three dimensional defects coupled to the massive deformation of $\mathcal{N} = 4$ SYM.

We study a supersymmetric embedding of space time filling $D5$-branes with non trivial world volume flux in the Pilch Warner solution.

We explore different $D7$ supersymmetric embeddings suitable for adding flavour to the whole class of solutions, suggesting in each case the dual flavored gauge theory. These embeddings identify supersymmetric four cycles. Although the formalism we adopt does not apply to the non static case, these supersymmetric four cycles should be mapped, with a strategy similar to [108], to non static configurations of $D3$ branes (giant gravitons) in this class of backgrounds[1] .

Finally, we find supersymmetric configurations of $D3$ and $D7$ branes which behave as effective strings in the four dimensional gauge theory description.

The chapter is organized as follows. In section 9.2 we outline the spinor ansatz for $SU(2)$ structure manifolds [99] and in section 9.3 the GCG supersymmetry conditions for D-branes [103]. In section 9.4, after a brief survey of the supersymmetric family of backgrounds which includes the PW flow, we look for supersymmetric embeddings of D-branes. We present different D-brane configurations and we solve their supersymmetry conditions, identifying supersymmetric embeddings. We give some details on the computations and we interpret the supersymmetric configurations in the dual gauge theory. The same analysis is carried out for D-brane probes in the LM geometry in section 9.5. In the appendices D.1 D.2 we recall some useful definitions.

8.2 $SU(2)$ structure manifolds and pure spinors

The ten dimensional metric is

$$ds_{10}^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu + ds_6^2 \tag{8.1}$$

where the warp factor A is a function of the internal coordinates. The internal six dimensional manifold has $SU(2)$ structure. An $SU(2)$ structure is characterized by two nowhere vanishing spinors which are never parallel

$$\eta_+ \qquad \chi_+ = \frac{1}{2} z \cdot \eta_- \tag{8.2}$$

where η_- is the complex conjugate of η_+ and we denote with $\cdot$ the Clifford multiplication $z_m\gamma^m$. The six dimensional chiral spinors $\eta^i_\pm$, which are the supersymmetry parameters, are then constructed

$$\eta^1_+ = a\eta_+ + b\chi_+ \qquad \eta^2_+ = x\eta_+ + y\chi_+ \tag{8.3}$$

with a, b, x, y complex functions of the internal coordinates. The ten dimensional supersymmetry parameters can be written as

$$\epsilon_1 = \zeta_+ \otimes \eta^1_+ + \zeta_- \otimes \eta^1_- \tag{8.4}$$

$$\epsilon_2 = \zeta_+ \otimes \eta^2_+ + \zeta_- \otimes \eta^2_- \tag{8.5}$$

where $\zeta_\pm$ are four dimensional chiral spinors. Given the never vanishing spinors just introduced, a $SU(2)$ structure manifold admits the following globally defined forms built as bilinears in the spinors

$$j = \frac{i}{2}\chi^\dagger_+\gamma_{mn}\chi_+ dx^m \wedge dx^n - \frac{i}{2}\eta^\dagger_+\gamma_{mn}\eta_+ dx^m \wedge dx^n \tag{8.6}$$

$$\omega = -i\chi^\dagger_+\gamma_{mn}\eta_+ dx^m \wedge dx^n \tag{8.7}$$

$$z = -2\chi^\dagger_-\gamma_m\eta_+ dx^m \tag{8.8}$$

[1] For giants in the beta deformed background see [109].

where z is a complex 1-form, j a real 2-form, and ω a (2,0)-form satisfying

$$\omega \wedge j = 0 \qquad j \wedge j = \frac{1}{2}\omega \wedge \bar{\omega} \qquad z \llcorner j = z \llcorner \omega = 0 \tag{8.9}$$

The 1-form z is the globally defined complex vector characterizing the $SU(2)$ structure.

In GCG the relevant equations can be written in terms of poliforms with definite parity, the pure spinors. They are bispinors built by tensoring the supersymmetry parameters of the internal manifold

$$\Phi_1 = \eta^1_+ \otimes \eta^{2\dagger}_+ \tag{8.10}$$

$$\Phi_2 = \eta^1_+ \otimes \eta^{2\dagger}_- \tag{8.11}$$

and are annihilated by six combinations of Clifford(6,6) gamma matrices. From (8.3) they read

$$\begin{aligned} \Phi_1 &= \frac{1}{8}[a\bar{x}e^{-ij} + b\bar{y}e^{ij} - i(a\bar{y}\omega + \bar{x}b\bar{\omega})] \wedge e^{z\wedge\bar{z}/2} \\ \Phi_2 &= \frac{1}{8}[i(by\bar{\omega} - ax\omega) + (bxe^{ij} - aye^{-ij})] \wedge z \end{aligned} \tag{8.12}$$

The $SU(3)$ structure case is for $b = 0 = y$.

The ansatz used in [99] for the six dimensional supersymmetry parameters is the following

$$\eta^1_+ = a\eta_+ + b\chi_+ \qquad \eta^2_+ = -i(a\eta_+ - b\chi_+) \tag{8.13}$$

where the functions of (8.3) are parametrized as

$$a = ix = ie^{A/2}\cos\phi\, e^{i\alpha} \qquad b = -iy = -ie^{A/2}\sin\phi\, e^{i\beta} \tag{8.14}$$

Here $\cos\phi$, $\sin\phi$, α and β are functions of the internal coordinates. The two supersymmetry parameters η^1_+, η^2_+ can be brought to the form (8.13) if and only if $\mathrm{Re}(a\bar{x} + b\bar{y}) = 0$. This corresponds to admit a non trivial mesonic moduli space of vacua [99].

We are interested in D-branes probing the class of backgrounds specified by the ansatz (8.13), (8.14). This contains a family of supersymmetric backgrounds with constant dilaton (which itself includes the PW flow), and the gravity dual of beta deformation. Since the norms of the spinors η_1 and η_2 are equal, supersymmetric D-branes are admitted [103].

8.3 Supersymmetry conditions for probe D-branes

In GCG the main tool to analyze supersymmetric embeddings of D-branes is the generalized calibration introduced in [102, 103]. We will consider space time filling branes (STF), domain walls

(DW) and effective strings (ES) wrapping a submanifold Σ of the internal manifold. The supersymmetry conditions for these extended objects in terms of the pure spinors and their projection on the world volume read[2]

$$P_\Sigma[\mathrm{Im}(ie^{i\theta}\Phi_a)] \wedge e^{\mathcal{F}} = 0 \tag{8.15}$$
$$P_\Sigma[(i_n + g_{nm}dx^m\wedge)\Phi_b] \wedge e^{\mathcal{F}} = 0 \qquad a,b=1,2 \tag{8.16}$$

where g_{nm} is the internal metric, i_n and $dx^m\wedge$ are the usual operators mapping a p form in a $p-1$ and $p+1$ form respectively, and finally [3] $\mathcal{F} = F - P_\Sigma[B]$, where F is the world volume flux. The pullback on the world volume of the D-brane is denoted by P_Σ. Space time filling branes, domain walls and effective strings are summarized in Table 1, where θ_{DW} is an arbitrary constant [103].

	θ	a	b
STF	0	1	2
DW	θ_{DW}	2	1
ES	$-\frac{\pi}{2}$	1	2

Table 1

The same dictionary of [103] is used to label the possible embeddings. However, since the internal manifold is non compact, we should distinguish between the cases when the wrapped submanifold Σ is itself compact or non compact. We will comment on this point where needed.

8.4 D-branes on the family of supersymmetric backgrounds

8.4.1 The family of supersymmetric backgrounds

We now briefly review the family of supersymmetric backgrounds analyzed in [99] which includes the PW flow [100]. The PW solution is the gravity dual of the massive deformation of $\mathcal{N}=4$ SYM

$$W = h\mathrm{Tr}\Phi_3[\Phi_1,\Phi_2] + m\mathrm{Tr}\Phi_3^2 \tag{8.17}$$

which flows in the IR to a non trivial fixed point [110]. The gravity dual is asymptotically AdS in the UV and warped AdS in the IR. It is included in the following more general ansatz [99] which is a family of supersymmetric backgrounds

$$ds_6^2 = e^{-2A}\left(\eta_i A_{i\bar{j}}\bar{\eta}_{\bar{j}} + z\bar{z}\right) \qquad i,j=1,2 \tag{8.18}$$

where z is the globally defined vector characterizing the $SU(2)$ structure. The matrix $A_{i\bar{j}}$ is hermitian, and the vielbeins are defined in terms of local complex coordinates z_i

$$z_1 = \rho_1 + i\sigma_1 \qquad z_2 = \rho_2 + i\sigma_2 \qquad z_3 = \log u + i\sigma_3 \tag{8.19}$$
$$\eta_1 = dz_1 + \alpha_1 dz_3 \qquad \eta_2 = dz_2 + \alpha_2 dz_3 \qquad z = \sqrt{a_3}u dz_3 \tag{8.20}$$

[2]We do not consider the orientation conditions on these objects.

[3]We are using the conventions of [92, 93, 99] which differs for an H_{NS} sign with [103].

with a_3 real and α_i complex functions of z_i. The globally defined two forms are

$$j = \frac{i}{2} A_{i\bar{j}}\, \eta_i \wedge \eta_{\bar{j}} \tag{8.21}$$

$$\omega = i\sqrt{\det A}\, \eta_1 \wedge \eta_2 \tag{8.22}$$

There are also non trivial RR and NS fluxes

$$*F_5 = -e^{-4A} d(e^{4A} \cos 2\phi) \tag{8.23}$$

$$C_2 = \mathrm{Re}[\frac{2ie^{i(\alpha-\beta)}\sqrt{\det A}}{e^{2A}\sin 2\phi}(dz_1 \wedge dz_2 - \sin^2\phi\, \eta_1 \wedge \eta_2)] \tag{8.24}$$

$$B_2 = -\mathrm{Im}[\frac{2ie^{i(\alpha-\beta)}\sqrt{\det A}}{e^{2A}\sin 2\phi}(dz_1 \wedge dz_2 - \sin^2\phi\, \eta_1 \wedge \eta_2)] \tag{8.25}$$

The dilaton is constant, parametrising the RG line of dual conformal gauge theories.

The supersymmetry equations for this background [99] imply that $\alpha = \frac{1}{2}(\sigma_1 + \sigma_2 + 3\sigma_3)$, $\beta = -\frac{1}{2}(\sigma_1 + \sigma_2 - \sigma_3)$ and that the functions $a_3, \alpha_i, A_{i\bar{j}}$ can be obtained as derivatives of a single function $F(z_i, \bar{z}_{\bar{j}})$. These are all real for the subclass of this family of backgrounds which have an $U(1)^3$ symmetry, i.e. when the function $F(z_i, \bar{z}_{\bar{j}})$ does not depend on the phases σ_i. We call this the *toric subclass*; the PW flow belongs to it.

The detailed expressions for the family of backgrounds and how to recover the PW flow are reported in the Appendix D.1.

The pure spinors (8.12) are constructed with the rescaled forms $z \to e^{-A}z$ and $(j, \omega) \to (e^{-2A}j, e^{-2A}\omega)$ which refer to the complete six dimensional metric (8.18).

We look for supersymmetric embeddings of Dp-branes (with world volume coordinates ξ_a $(a = 0, \ldots, p)$) in this family of supersymmetric backgrounds, allowing in one case for non trivial world volume gauge flux. The main tools are the conditions (8.15),(8.16).

Even if the family of backgrounds is larger, we shall take the PW solution as a paradigm for the gauge theory dual interpretation of the brane configurations.

8.4.2 $D5$ domain walls

We study now a supersymmetric D-brane probe placed at $x_3 = 0$ and which fills three space time dimensions $(\xi_0, \xi_1, \xi_2) = (x_0, x_1, x_2)$. It can be viewed as a domain wall solution separating supersymmetric vacua. However, when the wrapped cycle is non compact, the domain wall interpretation would imply an infinite potential barrier. Instead in the AdS/CFT interpretation it is a three dimensional defect coupled to the four dimensional dual gauge theory.

In the $AdS_5 \times S^5$ case there are non trivial supersymmetric embeddings where a $D5$-brane wraps an AdS_4 inside the AdS_5 plus a trivial 2-sphere inside the S^5 [111]. The $D5$ brane should shrink around this 2-sphere but the correspondent tachionic mode does not lead to instability because its mass is above the BF bound [112]. This configuration has been studied in [105] as a three dimensional defect in $\mathcal{N}=4$ SYM.

We look for similar configurations of $D5$-brane in the family of supersymmetric backgrounds of section 8.4.1. We attempt the following three cycle embedding

$$z_k = e^{i\tau_k}(\xi_{k+2} + ic_k) \qquad \bar{z}_k = e^{-i\tau_k}(\xi_{k+2} - ic_k) \qquad k = 1, \ldots, 3 \tag{8.26}$$

with τ_k and c_k constants, and with no world volume flux, $F=0$. This ansatz covers for example the real slice ($\tau_k = 0$, $\forall k$) and the imaginary slice ($\tau_k = \frac{\pi}{2}$, $\forall k$).

We restrict ourselves to the *toric subclass*. The complex functions $\alpha_i, A_{i\bar{j}}$ characterizing the metric are then real and the computations simplify. We compute the supersymmetry conditions (8.15) and (8.16) in the DW case of Table 1.

The supersymmetry condition (8.15) results

$$P_\Sigma[\mathrm{Im}(ie^{i\theta_{DW}}\Phi_2)] \wedge e^{\mathcal{F}} = \frac{1}{8}\mathrm{Im}[e^{-2A}\sqrt{a_3}u\sqrt{\det A}\, e^{i(\theta_{DW}+2\beta-\tau_1-\tau_2+\tau_3)}]\, d\xi_3 \wedge d\xi_4 \wedge d\xi_5 \tag{8.27}$$

where the functions are intended evaluated on the world volume. A choice of the constant phase θ_{DW} can make it vanish only if the phase factor β does not depend on the embedding coordinates ξ_{k+2}. This can be achieved taking the real slice ($\tau_k = 0$, $\forall k$), such that $\beta = -\frac{1}{2}(c_1 + c_2 - c_3)$. Then we choose $\theta_{DW} = -2\beta$ and the expression (8.27) vanishes.

For the real slice ($\tau_k = 0$, $\forall k$), a detailed analysis shows that the supersymmetry conditions (8.16) are satisfied provided $\alpha = \beta + \frac{\pi}{2}$. This implies the following relation between the constants c_k

$$c_1 + c_2 + c_3 = \frac{\pi}{2} \tag{8.28}$$

Hence we conclude that for the *toric subclass* a $D5$ brane embedded as in (8.26) with $\tau_k = 0$, with the constants c_k satisfying (8.28) and with $\theta_{DW} = (c_1 + c_2 - c_3)$ is supersymmetric. In particular, such $D5$ brane is supersymmetric in the PW flow, since it belongs to the *toric subclass*. In the PW geometry (see the appendix D.1) the $D5$ brane fills the three radial directions.

This embedding can be used to study three dimensional defects in the massive deformation of $\mathcal{N}=4$. The c_i give the distance between the supersymmetric $D5$-brane and the D-branes which generate the background. They represent masses for the 3D hypermultiplet of the defect theory.

8.4.3 Spacetime filling D-branes

In this section we study D-brane probes filling all the Minkowski directions $\xi_\mu = x_\mu$ ($\mu = 0, \ldots, 3$). The supersymmetry conditions are (8.15) and (8.16) in the STF case of Table 1. We analyze here supersymmetric $D5$-brane embeddings with world volume flux, and $D7$ flavour branes.

$D5$-branes

We take the following two cycle embedding Σ for a $D5$ brane probing the background of section 8.4.1

$$z_k = e^{i\tau_k}(\xi_{k+3} + ic_k) \qquad k = 1, 2 \qquad z_3 = c_3 + ic_4 \tag{8.29}$$

with c_k and τ_k real constants. We allow for a generic world volume flux F. The only non trivial supersymmetry conditions for this configuration are the (8.15) and the z component of (8.16), since $\Phi_2 = \cdots \wedge z$ and $P_\Sigma[z] = 0$ from (8.29). The first one reads

$$P_\Sigma[\mathrm{Im}(i\Phi_1)] \wedge e^{\mathcal{F}} = -\frac{ie^{-A}}{16}(A_{1\bar{2}}e^{i(\tau_1-\tau_2)} - A_{2\bar{1}}e^{-i(\tau_1-\tau_2)})\, d\xi_4 \wedge d\xi_5 \tag{8.30}$$

and does not depend on the two form flux $\mathcal{F} = F - P[B]$ since $P_\Sigma[\mathrm{Im}(i\Phi_1)]|_0 = 0$. This expression cannot be made vanishing in general by a simple choice of the phases τ_1, τ_2. However, if we restrict ourselves to the *toric subclass* the matrix $A_{i\bar{j}}$ is real and symmetric, and $A_{1\bar{2}} = A_{2\bar{1}}$. If we then choose $\tau_1 = \tau_2$ the expression (8.30) vanishes.

We compute the z component of the second supersymmetry condition

$$P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2] \wedge e^{\mathcal{F}} = -\frac{ie^{-2A}}{8}(F_{\xi_4\xi_5}e^{2A}e^{i(\alpha+\beta)}\sin 2\phi + \sqrt{\det A}e^{-i(\tau_1+\tau_2-2\beta)})\, d\xi_4 \wedge d\xi_5 \tag{8.31}$$

where $F_{\xi_4\xi_5}$ is the world volume flux. The expression (8.31) vanishes if we turn on

$$F = -e^{-i(\tau_1+\tau_2+\alpha-\beta)}\frac{\sqrt{\det A}}{e^{2A}\sin 2\phi} d\xi_4 \wedge d\xi_5 \tag{8.32}$$

which for consistency should be real. The choices

$$\tau_1 = \tau_2 = 0 \qquad \alpha - \beta = c_1 + c_2 + c_3 = 0 \tag{8.33}$$

make the flux (8.32) real, since the phase factor in (8.32) is now independent of the embedding coordinates ξ_{k+3} and moreover it vanishes. We conclude that the choices (8.32) and (8.33) make the $D5$ brane configuration (8.29) supersymmetric in the *toric subclass*.

However particular care is needed in considering this embedding; indeed we observe that the $D5$ brane wraps a non compact submanifold and then the flux F is along non compact coordinates (see for example the coordinates for the PW geometry in appendix D.1).

$D7$ flavour branes

Here we look for supersymmetric $D7$-brane embeddings suitable for adding flavours to the family of backgrounds of section 8.4.1. The $D7$ branes should wrap a non compact four cycle in order to make the flavour symmetry group global. Adding N_f $D7$ branes on this non compact four cycle is dual to add N_f flavours with symmetry group $SU(N_f)$ to the $SU(N_c)$ gauge theory provided

$N_f < N_c$, so that the back-reaction of the $D7$-branes can be neglected. The shape of the $D7$ supersymmetric embedding sets the interaction terms in the superpotential between the flavours and the chiral superfields of the dual gauge theory as well as possible masses for the flavours.

In a $SU(2)$ structure manifold the globally defined vector z naturally identifies a four dimensional submanifold Σ where $P_\Sigma[z] = 0$. Thus we attempt the embedding with $P_\Sigma[z] = 0$, i.e. we place $D7$ branes as

$$\begin{aligned} &x_\mu = \xi_\mu \qquad \mu = 0, \ldots, 3 \\ &z_k = \xi_{k+3} + i\xi_{k+5} \qquad k = 1, 2 \qquad z_3 = \log m_0 \end{aligned} \tag{8.34}$$

with no world volume flux, $F = 0$, and where m_0 is an arbitrary constant. The first supersymmetry condition (8.15) can be analyzed by keeping the $4, 2, 0$ forms of the pulled back pure spinor Φ_1

$$\begin{aligned} i\Phi_1|_0 &= -\frac{e^A}{8}(\cos^2\phi - \sin^2\phi) \\ i\Phi_1|_2 &= \frac{ie^{-A}}{8}(j + \cos\phi\sin\phi(e^{i(\alpha-\beta)}\omega - e^{-i(\alpha-\beta)}\bar{\omega})) \\ i\Phi_1|_4 &= \frac{e^{-3A}}{16}(\cos^2\phi - \sin^2\phi) j \wedge j \end{aligned}$$

Taking the imaginary part of these expressions we obtain

$$P_\Sigma[\mathrm{Im}(i\Phi_1)] \wedge e^{-P[B]} = -\frac{e^A}{8} P[j] \wedge P[B] = 0 \tag{8.35}$$

This vanishes given the explicit expressions of j (8.21) and B (8.25) and reminding $P_\Sigma[z] = 0$. The only non trivial supersymmetry condition of (8.16) is on the z component. The projection on the pure spinor Φ_2 is

$$\begin{aligned} P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2] \;&=\; \frac{1}{8}(-ie^{i(\alpha+\beta)}\sin 2\phi + e^{-2A}e^{2i\alpha}\cos^2\phi\,\omega + \\ &\quad + e^{-2A}e^{2i\beta}\sin^2\phi\,\bar{\omega} + \frac{i}{2}e^{-4A}e^{i(\alpha+\beta)}\sin 2\phi\, j \wedge j) \end{aligned}$$

The pullback of the NS two form (8.25) is

$$\begin{aligned} P_\Sigma[B] \;&=\; -\frac{\sqrt{\det A}\cos^2\phi}{e^{2A}\sin 2\phi}\big(\, e^{i(\alpha-\beta)}(d\xi_4 + id\xi_6) \wedge (d\xi_5 + id\xi_7) + \\ &\quad + e^{-i(\alpha-\beta)}(d\xi_4 - id\xi_6) \wedge (d\xi_5 - id\xi_7)\big) \end{aligned} \tag{8.36}$$

We then compute the terms which contribute to the z component of (8.16)[4]

$$
\begin{aligned}
P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2]|_4 &= \frac{ie^{i(\alpha+\beta)}}{e^{4A}16} \det A \cos\phi \sin\phi \; d\mathrm{Vol}_\Sigma \\
P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2]|_2 \wedge (-P_\Sigma[B]) &= \frac{ie^{i(\alpha+\beta)}}{16} \frac{\cos\phi \det A}{e^{4A} \sin\phi} (\cos^2\phi - \sin^2\phi)\, d\mathrm{Vol}_\Sigma \\
P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2]|_0 \wedge \frac{1}{2} P_\Sigma[B] \wedge P_\Sigma[B] &= -\frac{ie^{i(\alpha+\beta)}}{16} \frac{\cos^3\phi \det A}{e^{4A}\sin\phi}\, d\mathrm{Vol}_\Sigma
\end{aligned}
$$

Adding these three contributions we conclude that

$$
P_\Sigma[(i_z + g_{z\bar{z}}\bar{z}\wedge)\Phi_2] \wedge e^{-P[B]} = 0 \tag{8.37}
$$

Then the configuration (8.34) is supersymmetric for the whole family of backgrounds considered in section 8.4.1, not only the *toric subclass*.

Other flavour embeddings We look also for other $D7$ brane embeddings which preserve supersymmetry in the supersymmetric family of backgrounds of sec 8.4.1. The computations of the supersymmetry conditions (8.15) and (8.16) are less easy but can be done with the same procedure outlined above. We list the relevant results.

We can place the $D7$ brane orthogonal to one of the other complex coordinates

$$
z_k = \log m_0 \qquad z_j = \xi_4 + i\xi_5 \qquad z_3 = \xi_6 + i\xi_7 \qquad k \neq j = 1,2 \tag{8.38}
$$

and after a long computation we find that this is a supersymmetric configuration, satisfying (8.15) and (8.16).

Other possible embeddings are submanifolds like the one suggested in [106], with chiral symmetry breaking. We observe that the complex coordinates we are using (see the Appendix D.1) are the exponential of the usual complex coordinates which are in correspondence with the chiral adjoint fields. Hence we consider embeddings like $e^{z_i}e^{z_j} = m_0^2$. We have to distinguish between two different cases. The first one involves the z_3 component

$$
\begin{aligned}
&e^{z_j}e^{z_3} = m_0^2 \\
&z_k = \xi_4 + i\xi_5 \qquad z_j = \xi_6 + i\xi_7 \qquad z_3 = \log m_0^2 - (\xi_6 + i\xi_7) \qquad k \neq j = 1,2
\end{aligned}
$$

This configuration turns out to be non supersymmetric.

The second case does not involve the z_3 coordinate

$$
\begin{aligned}
&e^{z_1}e^{z_2} = m_0^2 \\
&z_1 = \xi_4 + i\xi_5 \qquad z_2 = \log m_0^2 - (\xi_4 + i\xi_5) \qquad z_3 = \xi_6 + i\xi_7
\end{aligned} \tag{8.39}
$$

and it results supersymmetric.

[4] We denote the volume on the wrapped cycle with $d\mathrm{Vol}_\Sigma = (-4d\xi_4 \wedge d\xi_5 \wedge d\xi_6 \wedge d\xi_7)$.

The dual flavoured gauge theory The $D7$ supersymmetric embeddings presented here (8.34), (8.38), (8.39) can be used to add flavours to the PW flow.

If we add N_f $D7$-branes in the configuration (8.34) the dual gauge theory is $\mathcal{N}=1$ SYM with three chiral adjoint fields and N_f massive flavours with mass m_0, with superpotential

$$W = W_{\mathcal{N}=4} + m\mathrm{Tr}\Phi_3^2 + \mathrm{tr}\ Q\Phi_3\tilde{Q} + m_0\,\mathrm{tr}\ Q\tilde{Q} \tag{8.40}$$

where the first two terms are the mass deformation of $\mathcal{N}=4$ SYM (8.17). Since we are neglecting the back-reaction of the $D7$ branes, the geometry filled by the $D7$-branes in the IR is warped AdS_5 and the theory flows to the same IR fixed point. For $m_0 \neq 0$, the $D7$-branes end before reaching the IR.

If we add N_f $D7$-branes as in (8.38) the gauge theory dual is again $\mathcal{N}=1$ SYM with three chiral adjoint fields and N_f massive flavours, with superpotential

$$W = W_{\mathcal{N}=4} + m\mathrm{Tr}\Phi_3^2 + \mathrm{tr}\ Q\Phi_k\tilde{Q} + m_0\,\mathrm{tr}\ Q\tilde{Q} \qquad k=1,2 \tag{8.41}$$

The flavours $Q\tilde{Q}$ now couple to the massless adjoint field Φ_k.

Finally, if we add N_f $D7$-branes embedded as (8.39) the dual flavoured gauge theory is $\mathcal{N}=1$ SYM with three chiral adjoint fields and two different N_f massive flavours, with superpotential

$$W = W_{\mathcal{N}=4} + m\mathrm{Tr}\Phi_3^2 + \mathrm{tr}\ Q_1\Phi_1\tilde{Q}_1 + \mathrm{tr}\ Q_2\Phi_2\tilde{Q}_2 + m_0\,\mathrm{tr}\ (Q_1\tilde{Q}_2 + Q_2\tilde{Q}_1) \tag{8.42}$$

where Q_1 and Q_2 denote the two flavours. This configuration can be interpreted as two sets of N_f $D7$-branes at $e^{z_1} = m_0$ and $e^{z_2} = m_0$ respectively, each supporting different flavours, which are joint smoothly into one set of N_f $D7$ branes wrapped on $e^{z_1}e^{z_2} = m_0^2$ [106]. On the dual gauge theory picture there are two flavour groups $SU(N_f)_1 \times SU(N_f)_2$ broken to the diagonal subgroup by the mass term m_0.

8.4.4 Effective Strings

We take D-branes that fill two coordinates in the Minkowski space time, for example at $x_2 = x_3 = 0$, filling $\xi_0 = x_0, \xi_1 = x_1$. They can be viewed as propagating strings in the four dimensional description. However, when the wrapped cycle of the internal manifold is non compact, the effective string tension in the four dimensional picture diverges. The supersymmetry conditions are the pair (8.15) and (8.16) in the ES case of Table 1. We find supersymmetric embeddings of both $D3$ and $D7$ branes which involve non compact cycles in the internal manifold. The $D3$ brane wraps a two cycle, whereas the $D7$ brane fills the whole internal manifold. Our analysis concern the whole family of backgrounds presented in section 8.4.1.

$D3$ **effective strings** We place $D3$-brane probes filling two directions in the internal space. We fix the z_3 coordinate, i.e. $z_3 = c_3 e^{i\tau_3}$ and we look for supersymmetric embeddings filling z_1 and

z_2. The embedding along the two complex coordinates, $z_k = e^{i\tau_k}(\xi_{k+1} + ic_k)$ for $k = 1, 2$ results non supersymmetric.

On the other hand, the non compact embedding where we identify z_1 and z_2 except for constant phases and shifts

$$z_1 = e^{i\tau_1}(\xi_2 + c_1 + i(\xi_3 + c_2)) \qquad z_2 = e^{i\tau_2}(\xi_2 - c_1 + i(\xi_3 - c_2)) \qquad z_3 = c_3 e^{i\tau_3} \tag{8.43}$$

results supersymmetric for any choice of the phases τ_k and of the real constants c_k.

$D7$ effective strings We probe the geometry with $D7$-brane covering the whole internal space

$$z_k = \xi_{k+1} + i\xi_{k+4} \qquad k = 1, \ldots, 3 \tag{8.44}$$

By a long but straightforward computation we find that this is a supersymmetric embedding, which satisfies the supersymmetry conditions.

8.5 D-branes on the beta deformed background

8.5.1 Beta deformation of $\mathcal{N} = 4$ SYM and its gravity dual

The $\mathcal{N} = 1$ beta deformed gauge theory is a marginal deformation [110] of the $\mathcal{N} = 4$ SYM, with superpotential

$$W_\beta = h\, Tr(e^{i\pi\beta}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\beta}\Phi_1\Phi_3\Phi_2) \tag{8.45}$$

where Φ_i are the three chiral adjoint superfields, and β a complex constant. We consider β to be real; in this case it is usually denoted as γ. Besides the $U(1)_R$ symmetry, this theory has two global symmetries $U(1)_a \times U(1)_b$ with charges

	Φ_1	Φ_2	Φ_3
$U(1)_a$	0	1	-1
$U(1)_b$	-1	1	0

These two global symmetries were crucial in the generating solutions technique of [101], where the supergravity background dual to such gauge theory has been obtained. This background has been analyzed using generalized complex geometry in [99]. The ten dimensional metric is

$$ds^2 = e^{2A} ds^2_{Mink} + ds^2_6, \qquad ds^2_6 = e^{-2A} d\tilde{s}^2_6 \tag{8.46}$$

where $\tilde{ds}^2_6$ is the rescaled internal metric. The internal $SU(2)$ structure manifold can be described by local complex coordinates

$$\begin{aligned} z_1 &= r\mu_1 e^{i\sigma_1} = r\cos\alpha e^{i(\psi-\varphi_2)} \\ z_2 &= r\mu_2 e^{i\sigma_2} = r\sin\alpha\cos\theta e^{i(\psi+\varphi_1+\varphi_2)} \\ z_3 &= r\mu_3 e^{i\sigma_3} = r\sin\alpha\sin\theta e^{i(\psi-\varphi_1)} \end{aligned} \tag{8.47}$$

The almost complex structure can be expressed [99] in terms of 1-forms (for details see the Appendix D.2) which give the rescaled metric a simple expression

$$d\tilde{s}_6^2 = x_1^2 + x_2^2 + G(y_1^2 + y_2^2) + z\bar{z} \tag{8.48}$$

where

$$G = \frac{1}{1+\gamma^2 g} \qquad z = \frac{d(z_1 z_2 z_3)}{r^2\sqrt{g}} \qquad g = \mu_1^2\mu_2^2 + \mu_2^2\mu_3^2 + \mu_3^2\mu_1^2 \qquad e^{2A} = r^2 \tag{8.49}$$

The background has non trivial dilaton, RR and NS fluxes

$$e^{\phi} = \sqrt{G} \tag{8.50}$$

$$B_2 = \gamma\sqrt{g}G\frac{y_1 \wedge y_2}{r^2} \tag{8.51}$$

$$F_3 = 12\gamma\cos\alpha\sin^3\alpha\sin\theta\cos\theta d\psi \wedge d\alpha \wedge d\theta \tag{8.52}$$

$$F_5 = 4(\text{vol}_{AdS_5} + *\text{vol}_{AdS_5}) \tag{8.53}$$

This solution differs from the family of backgrounds reviewed in section 8.4.1, for example the dilaton is not constant here. However it is an $SU(2)$ structure manifold which can be described by the ansatz (8.13) and (8.14) for the spinors [99]. The 1-form z in (8.48) is a globally defined vector. The 2-forms j and ω are

$$j = \sqrt{G}(x_1 \wedge y_1 + x_2 \wedge y_2) \tag{8.54}$$

$$\omega = i(x_1 + i\sqrt{G}y_1) \wedge (x_2 + i\sqrt{G}y_2) \tag{8.55}$$

and

$$a = ix = ie^{A/2}\cos\phi = \frac{i}{\sqrt{2}}e^{A/2}(1+\sqrt{G})^{\frac{1}{2}} \tag{8.56}$$

$$b = -iy = -ie^{A/2}\sin\phi = \frac{i}{\sqrt{2}}e^{A/2}(1-\sqrt{G})^{\frac{1}{2}} \tag{8.57}$$

The phases α and β in (8.14) are vanishing, $\alpha = \beta = 0$. Once again the pure spinors (8.12) are constructed with the rescaled forms $(j,\omega) \to (e^{-2A}j, e^{-2A}\omega)$ and $z \to e^{-A}z$ which refer to the complete six dimensional metric (8.46).

We look for supersymmetric embeddings of D-branes in this background employing the conditions (8.15) and (8.16).

8.5.2 $D5$ domain walls

We look for $D5$-brane embeddings filling three directions in the internal manifold and placed in Minkowski at $x_3 = 0$ with ($\xi_\mu = x_\mu$, $\mu = 0,1,2$). We choose the following ansatz, which is supersymmetric in the undeformed $\gamma = 0$ case ($AdS_5 \times S^5$),

$$z_k = e^{-i\tau_k}(\xi_{k+2} + ic_k) \qquad \bar{z}_k = e^{i\tau_k}(\xi_{k+2} - ic_k) \qquad k = 1,\dots,3 \tag{8.58}$$

where τ_k, c_k are arbitrary real constants. Computing the supersymmetry conditions (8.15) and (8.16)[5] this embedding results non supersymmetric for any choice of the constants τ_k, c_k. For instance in the simple case ($\tau_k = 0, c_k = 0$) the z and $\bar{z}$ components of the supersymmetry conditions (8.16) can be computed

$$\frac{1}{3}P_\Sigma[(g^{\bar{z}z}i_z + \bar{z}\wedge)\Phi_2] \wedge e^{-P[B]} = P_\Sigma[(g^{z\bar{z}}i_{\bar{z}} + z\wedge)\Phi_2] \wedge e^{-P[B]} = -\frac{i}{16}e^{-A}\gamma\sqrt{g\,G} \tag{8.59}$$

where the functions (A, g, G) are intended evaluated on the world volume. The result (8.59) cannot vanish unless $\gamma = 0$, i.e. the undeformed case; hence the embedding (8.58) is not supersymmetric in the beta deformed background.

8.5.3 $D7$ flavour branes

We look for supersymmetric $D7$ configurations filling the Minkowski space time $\xi_\mu = x_\mu$ ($\mu = 0, \ldots, 3$) and wrapped on a non compact four cycle in the internal manifold, suitable for adding flavour to the beta deformed theory. As already observed, an $SU(2)$ structure manifold is characterized by a globally defined vector (z), and a natural four cycle Σ is where $P_\Sigma[z] = 0$. In the beta deformed background the vector z is (8.49), and the condition $P_\Sigma[z] = 0$ implies, in complex coordinates,

$$z_1 z_2 z_3 = m^3 \tag{8.60}$$

with m constant.

We then take the following four cycle embedding for $D7$-branes

$$z_k = \xi_{k+3}e^{i\xi_{k+5}} \qquad k = 1, 2 \qquad\qquad z_3 = \frac{m^3}{\xi_4 e^{i\xi_6}\xi_5 e^{i\xi_7}} \tag{8.61}$$

with no world volume flux, i.e. $F = 0$. By direct inspection we find that this embedding satisfies the conditions[6] (8.15) and (8.16), and hence is supersymmetric. It preserves the translational invariance of φ_1 and φ_2. We then expect the $U(1)_a$ and $U(1)_b$ symmetries to be preserved in the dual gauge theory description.

This embedding and the dual flavoured gauge theory can be explained as follows. We have three sets of N_f $D7$ branes located at $z_1 = m$, $z_2 = m$, $z_3 = m$ respectively, each one supporting a flavour group $SU(N_f)$. We can join these branes à la Karch and Katz [106] and obtain one single set of N_f $D7$ branes located as in (8.61). These $D7$-branes terminate before reaching the IR region and the conformal invariance is explicitly broken by the mass m, which also breaks the flavour groups $SU(N_f) \times SU(N_f) \times SU(N_f)$ to the diagonal subgroup.

In order to deduce the superpotential of the dual gauge theory we observe that the same configuration can be realized in the undeformed ($\gamma = 0$, $AdS_5 \times S^5$) case; here the superpotential

[5] In the DW case of Table 1.

[6] In the STF case of Table 1.

is the following[7]

$$W = W_{N=4} + \text{tr}\ Q_1\Phi_1\tilde{Q}_1 + \text{tr}\ Q_2\Phi_2\tilde{Q}_2 + \text{tr}\ Q_3\Phi_3\tilde{Q}_3 + m\,\text{tr}\ (Q_1\tilde{Q}_2 + Q_2\tilde{Q}_3 + Q_3\tilde{Q}_1) \qquad (8.62)$$

Note that the massive flavours preserves the $U(1)_a \times U(1)_b$ symmetry, assigning the charges as in Table 2.

	Φ_1	Φ_2	Φ_3	Q_1	$\tilde{Q}_1$	Q_2	$\tilde{Q}_2$	Q_3	$\tilde{Q}_3$
$U(1)_a$	0	1	-1	1	-1	0	-1	1	0
$U(1)_b$	-1	1	0	0	1	-1	0	-1	1

Table 2

Now, for N_f $D7$ branes embedded as (8.61) in the beta deformed background, the dual gauge theory is beta deformed $\mathcal{N} = 1$ SYM coupled to three different massive flavours. The resulting phase factors of the terms in the superpotential (8.62) can be easily obtained following the prescription of [101] with the charges in Table 2, having

$$\begin{aligned} W &= W_{\beta=\gamma} + e^{-i\pi\gamma}\,\text{tr}\ Q_1\Phi_1\tilde{Q}_1 + e^{i\pi\gamma}\,\text{tr}\ Q_2\Phi_2\tilde{Q}_2 + e^{-i\pi\gamma}\,\text{tr}\ Q_3\Phi_3\tilde{Q}_3 + \\ &\quad + m\,\text{tr}\ (Q_1\tilde{Q}_2 + Q_2\tilde{Q}_3 + Q_3\tilde{Q}_1) \end{aligned} \qquad (8.63)$$

Note that the flavour mass terms are not affected by the beta deformation.

Other $D7$ embeddings If we do not require the $U(1)_a$ and $U(1)_b$ global symmetries to be preserved we can try to embed the $D7$ branes in other submanifolds, with vanishing world volume flux. The computations of the supersymmetry conditions (8.15) and (8.16) get more complicated.

We take the embeddings

$$\begin{aligned} &\xi_\mu = x_\mu \qquad \mu = 0, \dots, 3 \\ &z_i = \xi_4 e^{i\xi_6} \qquad z_j = \xi_5 e^{i\xi_7} \qquad z_k = m_0 \qquad\qquad i \neq j \neq k = 1, 2, 3 \end{aligned} \qquad (8.64)$$

A long computation shows they are supersymmetric for any choice of the mass m_0. Here the dual gauge theory is beta deformed $\mathcal{N} = 1$ SYM plus N_f flavours[8] which couple with the adjoint field Φ_k.

Finally, after a long computation, we find that the following $D7$ embeddings with chiral symmetry breaking are supersymmetric

$$\begin{aligned} &\xi_\mu = x_\mu \qquad \mu = 0, \dots, 3 \\ &z_i = \xi_4 e^{i\xi_6} \qquad z_j = \xi_5 e^{i\xi_7} \qquad z_k = \frac{m_0^2}{\xi_5 e^{i\xi_7}} \qquad\qquad i \neq j \neq k = 1, 2, 3 \end{aligned} \qquad (8.65)$$

[7] We set the couplings to one for simplicity.

[8] N_f is the number of $D7$ branes.

The dual gauge theory is beta deformed $\mathcal{N} = 1$ SYM with two kinds of N_f massive flavours Q_1 and Q_2, which couple to Φ_j and Φ_k, respectively. The mass m_0 breaks the flavour groups $SU(N_f)_1 \times SU(N_f)_2$ to the diagonal subgroup.

For these additional $D7$ embeddings the superpotential terms and their phase factors can be obtained with the same procedure followed in the derivation of (8.63), by starting from the $\mathcal{N} = 4$ case (i.e. $\gamma = 0$).

8.5.4 Effective Strings

Finally we take D-branes that fill just two coordinates in the Minkowski space time ($\xi_0 = x_0, \xi_1 = x_1$). We place them at $x_2 = x_3 = 0$. We do not find supersymmetric configurations of $D3$ or $D5$ branes. We instead find that a $D7$-brane covering the whole internal space

$$z_k = \xi_{k+1} + i\xi_{k+4} \qquad k = 1, \ldots, 3 \tag{8.66}$$

is supersymmetric.

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Riassunto

La teoria delle stringhe rappresenta il pricipale candidato per l'unificazione delle forze fondamentali. La consistenza di questa teoria implica l'esistenza di una nuova simmetria, la supersimmetria, che trasforma bosoni in fermioni e viceversa. D'altro canto, per soddisfare ai vincoli dettati dalle osservazioni sperimentali, la supersimmetria deve essere rotta a basse energie se si vuole costruire modelli fenomenologicamente sensati. Nella mia attività di ricerca ho studiato aspetti della rottura di supersimmetria in teorie di gauge supersimmetriche.

L'analisi non perturbativa delle teorie di gauge supersimmetriche ha raggiunto negli ultimi anni notevoli traguardi, e.g. la dualità elettrico/magnetica. Nella prima parte della tesi si descrivono alcuni elementi di base per lo studio non perturbativo di teorie di gauge supersimmetriche e per l'analisi della rottura di supersimmetria. Questi argomenti sono sparsi nella letteratura più o meno recente e si è cercato di riassumere gli ingredienti principali per rendere la tesi più completa.

Nel primo capitolo si descrivono gli aspetti fondamentali della rottura di supersimmetria, spontanea o esplicita, e si spiega l'importanza della rottura dinamica. Viene in seguito largamente trattata la dualità elettrico magnetica, concentrandosi sulla dualità in presenza di campi chirali nella rappresentazione aggiunta.

Nel secondo capitolo viene descritto la teoria di Intriligator Seiberg e Shih per la rottura dinamica di supersimmetria in vuoti metastabili. Questi autori hanno mostrato l'esistenza di vuoti metastabili con rottura dinamica della supersimmetria nella descrizione a basse energie della SQCD (teoria supersimmetrica di YM $SU(N_c)$ con N_f sapori) nell'intervallo $N_c + 1 \leq N_f < \frac{3}{2}N_c$.

L'esistenza di vuoti metastabili in teorie di gauge supersimmetriche con maggior contenuto in campi e con molteplici gruppi di gauge è stato il soggetto principale dei miei studi. Le motivazioni sono molteplici. Un obiettivo è quello di individuare questi vuoti in teorie di gauge supersimmetriche che abbiano una naturale immersione in teoria di stringa così da fornire un meccanismo di rottura della supersimmetria nel principale candidato teorico per l'unificazione, i.e. la teoria delle stringhe. Un altro scopo è inserire queste teorie con rottura dinamica della supersimmetria in scenari di mediazione di gauge, con il ruolo di settore nascosto.

Per questo motivo nel terzo capitolo vengono introdotte alcune nozioni di modelli con mediazione di gauge, concentrandosi sulla generazione di massa per il gaugino.

Dal quarto capitolo inizia la parte originale di questa tesi. Nel capitolo quattro, basato su [1] in collaborazione con A. Amariti e L. Girardello, studiamo teorie che contengono campi carichi nella rappresentazione aggiunta. L'interesse in queste teorie risiede nel fatto che campi nella rappresentazione aggiunta sono presenti nella maggior parte dei modelli con origine in teoria di stringa. Inoltre il maggior contenuto rende necessario l'utilizzo di estensioni della dualità elettrico-magnetiche alla Seiberg, che danno origine a molteplici gradi di libertà macroscopici, e dunque ad una dinamica a basse energie più ricca di quella della SQCD. L'analisi dettagliata della dinamica infrarossa ha evidenziato la presenza di vuoti metastabili con rottura di supersimmetria. A livello classico esiste in realtà un landscape di vuoti con medesima energia. Tale degenerazione è modificata dalle correzioni quantistiche che selezionano un solo minimo locale. Questo vuoto ha inoltre la proprietà di non possedere R simmetria. Questa caratteristica, estremamente rara in teorie con rottura spontanea della supersimmetria, rende il modello particolarmente adatto come settore nascosto in uno scenario di mediazione di gauge.

Nel capitolo cinque, infatti, basato sul lavoro [2], viene studiata la trasmissione della rottura di supersimmetria ad un'estensione supersimmetrica del modello standard attraverso la mediazione di gauge diretta, mostrando che viene generata massa per il gaugino.

Il capitolo sei si basa su un recente lavoro [3], in collaborazione con A. Amariti e L. Girardello, dove analizziamo vuoti metastabili in teorie di quiver A_n con n arbitrario. Queste teorie hanno una descrizione in teoria di stringa di tipo IIB come $D5$-brane parzialmente avvolte su varietà non compatte di Calabi-Yau (ottenute come fibrazioni di un piano su singolarità di tipo A). Si studia una dualizzazione alla Seiberg di molteplici nodi (in posizione alternata) nel quiver. Nella teoria risultante a basse energie mostriamo la presenza di vuoti metastabili con rottura spontanea della supersimmetria. Per garantire la consistenza di questa procedura, è necessaria un'approfondita analisi dei flussi di rinormalizzazione delle costanti d'accoppiamente dei diversi gruppi di gauge. Questa analisi ha portato alla determinazione di precise gerarchie da imporre tra le scale di accoppiamento forte. Infine anche per questa classe di modelli viene mostrato come inserirli in uno schema di mediazione di gauge.

Nel capitolo sette viene descritta una teoria in cui la rottura di supersimmetria è invece di tipo esplicito, introducendo quindi termini di rottura soffice nella lagrangiana. Il contesto è quello della congettura di Dijkgraaf e Vafa; questa stabilisce che il superpotenziale efficace di glueball per teorie di gauge supersimmetriche $\mathcal{N} = 1$ può essere ottenuto mediante un appropriato modello di matrici hermitiane. In tale modello il potenziale per le matrici ha la medesima espressione del superpotenziale per il supercampo chirale nella rappresentazione aggiunta. Nel lavoro [4] in collaborazione con L. Girardello e G. Tartaglino-Mazzucchelli, su cui si basa questo capitolo, si analizza e si estende tale congettura nel caso di rottura esplicita (soffice) di supersimmetria. La rottura esplicita è introdotta promuovendo a spurioni le costanti d'accoppiamento del superpotenziale per il campo chirale nell'aggiunta. La rottura di supersimmetria può essere interpretata nella curva algebrica che racchiude le informazioni della teoria efficace come una deformazione alla Whitham, studiata nell'ambito della teoria dei sistemi integrabili. Attraverso questa interpretazione si può ricavare nella teoria di basse energie, i.e. nel superpotenziale di glueball, i nuovi termini olomorfi derivanti dalla rottura di supersimmetria. Questo approcio viene poi verificato con un calcolo perturbativo.

Infine la mia attività di ricerca ha compreso anche argomenti più direttamente legati alla teoria delle stringhe e alla dualità tra teorie di gauge e gravità (AdS/CFT). Nella dualità AdS/CFT giocano un ruolo centrale le D-brane e la loro dinamica. La ricerca di configurazioni supersimmetriche (e dunque stabili) di D-brane in background di supergravità è il contesto della mia pubblicazione [5] e del capitolo otto. Nella compattificazione con flussi, ma anche nella AdS/CFT, il formalismo della geometria complessa generalizzata (GCG) si è rivelato estremamente utile per l'analisi di soluzioni di supergravità con flussi. In particolare, è possibile descrivere con questo linguaggio geometrie che sono duali a teorie $\mathcal{N} = 1$ ottenute come deformazioni marginali o massive della teoria di SYM $\mathcal{N} = 4$. In questo capitolo studio configurazioni supersimmetriche di brane in questa classe di soluzioni di supergravità tramite il formalismo della GCG, suggerendone l'interpretazione nella teoria di gauge duale. Ad esempio trovo diverse configurazioni supersimmetriche di D7 brane e propongo il superpotenziale di interazione nella teoria di campo duale.

L'argomento trattato in questo ultimo capitolo non è direttamente connesso con la rottura di supersimmetria. D'altra parte un possibile metodo per rompere la supersimmetria in teoria di gravità consiste nell'introduzione di nuovi gradi di libertà, e.g. brane, che non preservino la supersimmetria del background. Con questa finalità, dunque, l'analisi delle configurazioni supersimmetriche è in ogni caso il primo passo.

Appendix A

Computation of the gaugino mass

In the derivation of (3.6) we have assumed that the interaction superpotential is diagonal (3.3). This could be not the case, especially in models of direct gauge mediation, where the supersymmetry breaking sector coincides with the messenger sector and hence its form is constrained by the dynamics. The procedure is then to diagonalize the mass matrix of the messenger fields in order to obtain a diagonal superpotential, and then follow the recipe given in the previous section. This procedure can have some subtleties.

We show how to proceede with a simple and clarifing example, with two families of messengers transforming in the fundamental and antifundamental of the SM gauge group. Suppose having the following superpotential for the messenger fields

$$W = \begin{pmatrix} \Phi_1 & \Phi_2 \end{pmatrix} \begin{pmatrix} X & \frac{Y}{2} \\ \frac{Y}{2} & 0 \end{pmatrix} \begin{pmatrix} \tilde{\Phi}_1 \\ \tilde{\Phi}_2 \end{pmatrix} \tag{A.1}$$

where, for instance, only the field X get a non trivial expectation values for its F_X component. The diagonalization brings to the following eigenvalues for the mass matrix

$$M = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(X - \sqrt{X^2+Y^2}) & 0 \\ 0 & \frac{1}{2}(X + \sqrt{X^2+Y^2}) \end{pmatrix} \tag{A.2}$$

so now the superpotential (A.1) can be rewritten as

$$W = \sum_{i=1}^{2} \beta_i \Phi'_i \tilde{\Phi}'_i \tag{A.3}$$

where the fields Φ'_i are redefinitions of the original fields Φ_i whose precise expression is not needed here. The superpotential is now in the diagonal form and we can compute using (3.6) tha gaugino mass. We observe that now both the coefficient of the $\Phi'_i\tilde{\Phi}'_i$ get non trivial F-term due to the non vanishing F_X, indeed

$$F_{\beta_1} = F_X \frac{\partial \beta_1}{\partial X} = \frac{F_X}{2}\left(1 - \frac{X}{\sqrt{X^2+Y^2}}\right) \tag{A.4}$$

$$F_{\beta_2} = F_X \frac{\partial \beta_2}{\partial X} = \frac{F_X}{2}\left(1 + \frac{X}{\sqrt{X^2+Y^2}}\right) \tag{A.5}$$

The gaugino mass is then

$$M_a \simeq \frac{\alpha_a}{4\pi}\left[\frac{|F_{\beta_1}|}{|\beta_1|} g\left(\frac{|F_{\beta_1}|}{|\beta_1|^2}\right) + \frac{|F_{\beta_2}|}{|\beta_2|} g\left(\frac{|F_{\beta_2}|}{|\beta_2|^2}\right)\right] \tag{A.6}$$

Inserting the expression (A.4,A.2) and the expansion for $g(x)$ (3.7) at second order in x, we obtain[1]

$$M_a \sim \frac{2\alpha_a}{3\pi}\,\frac{F_X^3 X}{X^2Y^4+2Y^6} \tag{A.7}$$

Observe that the gaugino mass is generated only at third order in the F_X parameter. Indeed there is a cancellation at linear order F_X. This is due to the fact that the first element of the inverse mass matrix for the messengers is vanishing $(M^{-1})_{11} = 0$ [63]. The third order contribution is relevant only for enough large F_X. Hence particular care is needed in computing gaugino mass in models where the interaction superpotential for the messenger fields is highly non trivial.

[1] For simplicity we assumed all the vacuum expectation values to be real and positive.

Appendix B

Appendices for metastable A_n quivers

B.1 Goldstone bosons

The analysis we made in the A_3 theories started from the limit $|\mu_1| > |\mu_3|$. Also the opposite limit can give meta-stable vacua. To understand the differences among the various choices, we have to study the classical masses acquired by the fields expanding them around their vevs.

We study the case with ranks $N_1 < \widetilde{N}_2 < N_3$. Since the flavor symmetry is $U(N_1) \times U(N_3)$, and not $U(N_1 + N_3)$, the linear terms of the mesons are different. We are still free to choose the hierarchy between them. We here analyze the breaking of the global symmetries taking $|\mu_1| > |\mu_3|$. Treating the gauge symmetry as a global one, and rearranging the quarks in the form

$$\langle q \rangle = \begin{pmatrix} q_{1,2} \\ q_{3,2} \end{pmatrix} = \begin{pmatrix} \mu_1 \mathbf{1}_{N_1} & 0 \\ 0 & \mu_3 \mathbf{1}_{\widetilde{N}_2 - N_1} \\ 0 & 0 \end{pmatrix} \quad \langle \widetilde{q}^T \rangle = \begin{pmatrix} q_{2,1} \\ q_{2,3} \end{pmatrix} = \begin{pmatrix} \mu_1 \mathbf{1}_{N_1} & 0 \\ 0 & \mu_3 \mathbf{1}_{\widetilde{N}_2 - N_1} \\ 0 & 0 \end{pmatrix} \tag{B.1}$$

we see that the global symmetry breaks as

$$U(N_1) \times U(\widetilde{N}_2) \times U(N_3) \longrightarrow U(N_1)_D \times U(\widetilde{N}_2 - N_1)_D \times U(N_1 + N_2 - \widetilde{N}_2) \tag{B.2}$$

This implies that the Goldstone bosons are $\widetilde{N}_2^2 + 2(\widetilde{N}_2 - N_1)(N_1 + N_3 - \widetilde{N}_2)$. The first $\widetilde{N}_2^2$ Goldstone bosons come from the upper $\widetilde{N}_2 \times \widetilde{N}_2$ block matrices in the quark fields, exactly the same as in ISS. The second part is a bit different. In fact in ISS, with equal masses, the Goldstone bosons which come from the lower $(N_1 + N_3 - \widetilde{N}_2) \times \widetilde{N}_2$ sector in the quarks matrices, are $2\widetilde{N}_2(N_3 + N_1 - \widetilde{N}_2)$. In this case, since we started with lesser flavor symmetry, there are $2N_1(N_3 + N_1 - \widetilde{N}_2)$ massless Goldstone bosons fewer than in ISS. We have to control the other directions. From the scalar potential we have to compute the masses that the fields acquire expanding around the vacuum. The relevant expansions for the potentially tachyonic directions are the ones around the vevs of the quarks

$$
\begin{aligned}
q_{12} &= \begin{pmatrix} \mu_1 + \phi_1 & \phi_2 \end{pmatrix} \quad q_{21} = \begin{pmatrix} \mu_1 + \widetilde{\phi}_1 \\ \widetilde{\phi}_2 \end{pmatrix} \\
q_{23} &= \begin{pmatrix} \phi_3 & \mu_3 + \phi_4 \\ \phi_5 & \phi_6 \end{pmatrix} \quad q_{32} = \begin{pmatrix} \widetilde{\phi}_3 & \widetilde{\phi}_5 \\ \mu_3 + \widetilde{\phi}_4 & \widetilde{\phi}_6 \end{pmatrix}
\end{aligned}
\tag{B.3}
$$

The relevant terms of the scalar potential come from the F-terms of the mesons

$$
V = |F_{M_{11}}|^2 + |F_{M_{13}}|^2 + |F_{M_{31}}|^2 + |F_{M_{33}}|^2 \tag{B.4}
$$

If we study the mass terms of the fields ϕ_5 and $\widetilde{\phi}_5$ we note that they are not zero, since $\mu_1 \neq \mu_3$. In fact their mass matrix is[1]

$$
\begin{pmatrix} \phi_5 & \widetilde{\phi}_5^\dagger \end{pmatrix} \begin{pmatrix} \mu_1^2 & -\mu_3^2 \\ -\mu_3^2 & \mu_1^2 \end{pmatrix} \begin{pmatrix} \phi_5^\dagger \\ \widetilde{\phi}_5 \end{pmatrix} \tag{B.5}
$$

with eigenvalues $\mu_1^2 \pm \mu_3^2$. A minimum of the scalar potential without tachyonic directions imposes a constraint on the masses, $\mu_1 > \mu_3$, consistent with the analysis of ISS.

We can ask now what happens if $\mu_1 < \mu_3$. The vacua we studied before are not true vacua any longer, but they have tachyonic directions in the quark fields. The meta-stable vacua are obtained choosing the vevs of $q_{1,2}$ and $q_{2,1}$ to be zero, and the vevs of the other quarks to be

$$
q_{3,2} = q_{2,3}^T = \begin{pmatrix} \mu_3 \mathbf{1}_{\widetilde{N}_2} \\ 0 \end{pmatrix} \tag{B.6}
$$

The differences in the two cases are the value of the scalar potential and the pseudo-moduli. In fact in the first limit $V_{vac} = (N_1 + N_3 - \widetilde{N}_2)|h\mu_3^2|^2$, and in the second limit the scalar potential is $V_{vac} = (N_3 - \widetilde{N}_2)|h\mu_3^2|^2 + N_1|h\mu_1^2|^2$. Since we choose the masses to be different, but of the same order, both cases have long lived meta-stable vacua. As far as the pseudo-moduli are concerned, in the case analyzed during chapter 7, they come out from a block of the $M_{3,3}$ meson, and in this case they come out from the whole M_1 meson and from a diagonal block $(N_3 - \widetilde{N}_2) \times (N_3 - \widetilde{N}_2)$ of the $M_{3,3}$ meson.

B.2 Hierarchy of scales

One of the main approximation we used to find metastable vacua has been to neglect the fact that the odd nodes are gauge nodes. In order to treat them as flavours groups in the region of

[1] From now on we will consider all the mass terms as real.

interest, it is necessary that their gauge couplings are lower than the couplings of the even nodes. We can treat the odd groups as flavour groups only if this relation holds.

In order to substantiate this idea we have to relate the electric scale of the flavour group to the other scales of the theory. The latter ones are the strong coupling scale of the gauge theories, Λ_{2i}, and the supersymmetry breaking scale μ, which is the value of the linear term in the dual version of the theory.

We must impose the groups related to the flavour/odd nodes to be less coupled than the gauge/even groups in the magnetic region. A similar analysis was performed in [40].

There are six possibilities, shown in Figure 1 in section 6.5. We have already discussed what happens in all these different cases. We will now show how to derive the formulas (6.29) and (6.33).

Let's denote by f all the objects related to the flavour group, and by g all the objects related to the gauge group. We have to distinguish four different cases, all with $\widetilde{b}_f > \widetilde{b}_g$ [2]. In fact the flavours can be IR free or UV free in the electric description (i.e. above the scale Λ_{2i}) and also UV free or IR free in the magnetic description.

We start studying a single case, and then we will comment about the others. Let's study the case (2) in Figure 1, where the flavours are UV free in the electric and IR free in the magnetic description, i.e. $b_f > 0$ and $\widetilde{b}_f < 0$.

We require that after Seiberg duality the gauge coupling g_g is larger than the flavour coupling g_f. More precisely we require that this happens at the supersymmetry breaking scale μ

$$\frac{1}{g_f^2(\mu)} > \frac{1}{g_g^2(\mu)} \qquad \Rightarrow \qquad \widetilde{b}_f \log\left(\frac{\widetilde{\Lambda}_f}{\mu}\right) < \widetilde{b}_g \log\left(\frac{\widetilde{\Lambda}_g}{\mu}\right) \tag{B.7}$$

from which follows

$$\widetilde{\Lambda}_f > \left(\frac{\widetilde{\Lambda}_g}{\mu}\right)^{\frac{\widetilde{b}_g-\widetilde{b}_f}{\widetilde{b}_f}} \widetilde{\Lambda}_g > \widetilde{\Lambda}_g \tag{B.8}$$

The scale matching relation coming from Seiberg duality

$$\Lambda_g^{3n_g-n_f}\widetilde{\Lambda}_g^{2n_f-3n_g} = \hat{\Lambda}_g^{n_f} \tag{B.9}$$

fixes $\Lambda_g = \widetilde{\Lambda}_g$, if we choose the intermediate scale to be $\hat{\Lambda}_g = \Lambda_g$.

For the flavour scale we observe that, at the scale Λ_g, where we perform Seiberg duality, the coupling in the electric description for the odd node is the same that the coupling of the magnetic description, and this implies

$$g_f = \widetilde{g}_f \quad \rightarrow \quad \left(\frac{\Lambda_f}{\Lambda_g}\right)^{b_f} = \left(\frac{\widetilde{\Lambda}_f}{\Lambda_g}\right)^{\widetilde{b}_f} \tag{B.10}$$

[2] The opposite inequality do not require this analysis, since at low energy the flavours are always less coupled than the gauge.

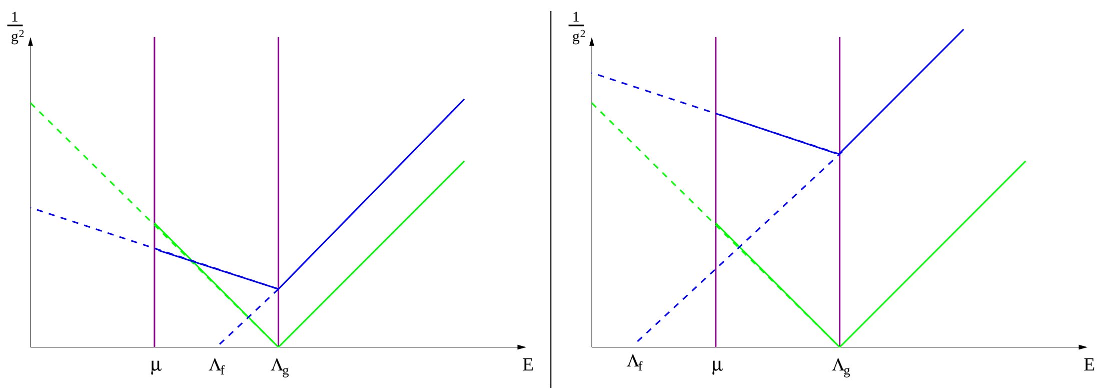
We can now write (B.8) in term of the electric scales (Λ_f and Λ_g) using (B.10), and we obtain

$$\Lambda_f < \mu^{\frac{\tilde{b}_f - \tilde{b}_g}{b_f}} \Lambda_g^{\frac{\tilde{b}_g - \tilde{b}_f + b_f}{b_f}} \tag{B.11}$$

Since the exponent of μ is positive we have

$$\frac{\tilde{b}_f - \tilde{b}_g}{b_f} > 0 \quad \rightarrow \quad \Lambda_f < \left(\frac{\mu}{\Lambda_g}\right)^{\frac{\tilde{b}_f - \tilde{b}_g}{b_f}} \Lambda_g \ll \Lambda_g \tag{B.12}$$

This imposes a stronger constraint on the scale of the flavour group Λ_f. In fact it is not enough to choose it lower than the gauge strong coupling scale Λ_g. It is also constrained by (B.12). The next figure explains what happens



In the first picture the scale Λ_f is lower than Λ_g but not enough: at the breaking scale it is not possible to neglect the contribution coming from $\tilde{g}_f$. Instead, if we constrain the scale Λ_f using (B.12), we obtain the runnings depicted in the second picture: here the flavour groups are less coupled than the gauge groups at the supersymmetry breaking scale.

As explained above there are four different possibilities. The second possibility is that the flavours are UV free both in the electric description and in the magnetic description, with $\tilde{b}_f > 0$. The analysis is the same as before, and we obtain the same inequality as (B.12). However this situation requires a more careful analysis, since in the infrared the gauge coupling associated to the flavour group develops a strong dynamics which has to be taken under control.

For the other two possibilities, where $b_f < 0$, one finds

$$\Lambda_f > \left(\frac{\Lambda_g}{\mu}\right)^{\frac{\tilde{b}_g - \tilde{b}_f}{b_f}} \Lambda_g \gg \Lambda_g \tag{B.13}$$

The general recipe we learn from this analysis can be summarized in three different cases

- If the inequality $\widetilde{b}_f < \widetilde{b}_g$ holds one has simply to choose $\Lambda_f \ll \Lambda_g$ or $\Lambda_f \gg \Lambda_g$ if $b_f > 0$ or $b_f < 0$ respectively as in (6.25,6.26).
- If $\widetilde{b}_f > \widetilde{b}_g$ we can still distinguish two cases
  - In the first case $b_f > 0$, and we have to constraint Λ_f with (B.12).
  - In the second case $b_f < 0$, and we have to constraint Λ_f with (B.13).

B.3 A_5 classification

We study A_5 quiver gauge theories obtained gluing all the possible combinations of A_3 which present metastable vacua, i.e. the one of section (6.4)

We analyze the beta function coefficients for these A_5 quiver gauge theories, with gauge group $U(N_1) \times U(N_2) \times U(N_3) \times U(N_4) \times U(N_5)$. The even nodes are in the IR free window

$$N_2 < N_1 + N_3 < \frac{3}{2}N_2 \qquad N_4 < N_3 + N_5 < \frac{3}{2}N_4 \tag{B.14}$$

We write in the table the beta coefficients of the third node of the A_5, specifying the range, compatible with (B.14), when this node is UV free or IR free in the electric and in the magnetic descriptions, respectively. The table classifies the possible A_5 quiver gauge theories which present alternate Seiberg dualities and which have metastable vacua.

As explained in section 6.6 we can obtain an A_n quiver gauge theory by gluing the A_3 patches. For the renormalization group, the internal flavour nodes of the A_n chain behave as the third node of the A_5 patches.

The table does not say anything about the external nodes of the A_n. In the electric theory one has $b_1 = 3N_1 - N_2$ and $b_n = 3N_n - N_{n-1}$; after duality, in the low energy description we have $\widetilde{b}_1 = N_1 + N_2 - N_3$, and $\widetilde{b}_n = N_n + N_{n-1} - 2N_{n-2}$. The possible values for $\widetilde{b}_1$ and $\widetilde{b}_n$ have to be studied separately.

Ranks of A_5	Further condition(I)	Further condition(II)	electric $b-$factor	magnetic $b-$factor
$N_1 < N_2 \leq N_3 < N_4 \leq N_5$		$N_2 + N_4 < 3N_3$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$3N_3 < N_2 + N_4$	$b_3 < 0$	$\widetilde{b}_3 < 0$
$N_1 < N_2 > N_3 < N_4 \leq N_5$			$b_3 < 0$	$\widetilde{b}_3 < 0$
$N_1 \geq N_2 > N_3 < N_4 \leq N_5$			$b_3 < 0$	$\widetilde{b}_3 < 0$
$N_1 < N_2 > N_3 < N_4 > N_5$	$N_3 < N_1 + N_5$	$N_2 + N_4 < 3N_3$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$3N_3 < N_2 + N_4$	$b_3 < 0$	$\widetilde{b}_3 < 0$
	$N_3 > N_1 + N_5$	$N_2 + N_4 < N_3 + 2N_1 + 2N_5$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$N_3 + 2N_1 + 2N_5 < N_2 + N_4$	$b_3 > 0$	$\widetilde{b}_3 > 0$
$N_1 < N_2 \leq N_3 \geq N_4 > N_5$		$N_2 + N_4 < N_3 + 2N_1 + 2N_5$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$N_3 + 2N_1 + 2N_5 < N_2 + N_4$	$b_3 > 0$	$\widetilde{b}_3 > 0$
$N_1 < N_2 \leq N_3 < N_4 > N_5$	$N_3 < N_1 + N_5$	$N_2 + N_4 < 3N_3$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$3N_3 < N_2 + N_4$	$b_3 < 0$	$\widetilde{b}_3 < 0$
	$N_3 > N_1 + N_5$	$N_2 + N_4 < N_3 + 2N_1 + 2N_5$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$N_3 + 2N_1 + 2N_5 < N_2 + N_4$	$b_3 > 0$	$\widetilde{b}_3 > 0$

In the first column we report all the possible inequalities among the A_5 rank numbers consistent with (B.14). Moving from left to right the further condition fix the signs of b_3, $\widetilde{b}_3$.

Appendix C

Details on non-perturbative and perturbative computations

C.1 Solution of the broken superpotential

In this appendix we show the main tools and details used for the computation in the cubic tree–level superpotential of section 7.5.

We have already written the genus one Riemann surface characterising the solution (7.24, 7.25). We observe that there is one holomorphic differential $\frac{dx}{y}$ defined on this surface. This differential can be expanded around the point at infinity in powers of $\xi = 1/x$

$$\frac{dx}{y} = \sum_{k=0}^{\infty} R_k \xi^k d\xi \quad , \qquad R_k = \frac{-1}{k!} \frac{\partial^k}{\partial \xi^k} \left(\frac{1}{\sqrt{(g + m\ \xi)^2 + f_0\ \xi^4 + 2gt_0\ \xi^3}} \right) \Bigg|_{\xi=0} \quad , \tag{C.1}$$

where R_m are functions of g_k, t_0, f_0 and can be simply computed by power expansion of y. The normalized holomorphic differential $d\omega$ is then

$$d\omega = \frac{1}{h_0} \frac{dx}{y} \quad , \tag{C.2}$$

where we have introduced the following quantities[1]

$$h_m = \oint_\alpha \frac{x^m dx}{y} \ . \tag{C.3}$$

The meromorphic differentials $d\Omega_k$ are defined by[2]

$$d\Omega_0 = \frac{\partial dS}{\partial t_0} = g \frac{x dx}{y} + \frac{1}{2} \frac{\partial f_0}{\partial t_0} \frac{dx}{y} \ , \qquad d\Omega_k = \frac{\partial dS}{\partial g_k} \qquad k = 2, 3 \ , \tag{C.4}$$

[1]The α–cycle encircle counterclokwise the second cut accordingly to our conventions.
[2]We denote $g_2 = m$ and $g_3 = g$ of (7.22).

and are completely fixed by the normalization constraints

$$\oint_\alpha d\Omega_0 = 0 \quad , \qquad \oint_\alpha d\Omega_k = 0 \qquad k = 2, 3 \ . \tag{C.5}$$

Then $d\Omega_0$ results to be

$$d\Omega_0 = \left(gx - g\frac{h_1}{h_0} \right) \frac{dx}{y} \ , \tag{C.6}$$

where we used the first normalization condition of (C.5) which implies $\frac{\partial f_0}{\partial t_0} = -2g\frac{h_1}{h_0}$. Collecting these formulas we can express the second derivatives of the prepotential characterizing the susy breaking terms in the effective superpotential (7.13) for the case under consideration as follow

$$\frac{\partial^2 \mathcal{F}}{\partial t_0 \partial g_k} = Res_0 \left(\frac{\xi^{-k}}{k} d\Omega_0 \right) = \frac{1}{k} \left(gR_k - g\frac{h_1}{h_0} R_{k-1} \right) \ , \tag{C.7}$$

$$\frac{\partial^2 \mathcal{F}}{\partial s_2 \partial g_k} = Res_0 \left(\frac{\xi^{-k}}{k} d\omega \right) = \frac{R_{k-1}}{k} \frac{1}{h_0} \ , \tag{C.8}$$

where the R_k are defined in (C.1).

In the case of unbroken gauge group ($U(N) \to U(N)$) the cut associated with the s_2 variable degenerate to a point with $s_2 \to 0$ and the only variable is t_0. The curve (7.24) can be written as

$$y^2 = g^2 (x - x_1)^2 (x - x_3)(x - x_4) \ , \tag{C.9}$$

where $x_3, \ x_4$ are the extremal points of the first cut and x_1 is the double zero of the curve where the second cut degenerates. Following [65] it is useful to introduce the quantities

$$\Delta_{43} = \tfrac{1}{2}(x_4 - x_3) \quad , \quad \Delta = (a_1 - a_2) = \tfrac{m}{g} \ , \tag{C.10}$$

$$Q = \tfrac{1}{2}(x_4 + x_3 + 2x_1) = (a_1 + a_2) = -\tfrac{m}{g} \ , \tag{C.11}$$

$$I = \tfrac{1}{2}(x_4 + x_3 - 2x_1) = \sqrt{\Delta^2 - 2\Delta_{43}^2} \ , \tag{C.12}$$

$$x_1 = \tfrac{Q-I}{2} \quad , \quad \alpha = \tfrac{g^2}{m^3} \quad , \quad \sigma = 8\alpha S \ . \tag{C.13}$$

The above relations can be proved comparing (7.24) and (C.9). We have also directly introduced the physically relevant variable $S \equiv S_1 = -\frac{t_0}{2}$.
Being interested in finding $\frac{\partial^2 \mathcal{F}}{\partial g_k \partial t_0}$ as in (C.7) we have evaluated h_1/h_0 in this case

$$\frac{h_1}{h_0} = x_1 = \frac{Q - I}{2} \ . \tag{C.14}$$

Then we find

$$\frac{\partial^2 \mathcal{F}}{\partial t_0 \partial g_k} = \frac{g}{k} \left(R_k - \frac{h_1}{h_0} R_{k-1} \right) = \frac{gR_k}{k} + \frac{mR_{k-1}}{2k}(1 + Y) \ , \tag{C.15}$$

where Y is defined as

$$Y = \frac{I}{\Delta} = \sqrt{1 - \frac{2\Delta_{43}^2}{\Delta^2}} \ . \tag{C.16}$$

Then $I \equiv I(\Delta_{43}^2)$ is a function of Δ_{43}^2. Written in terms of x_i the variable t_0 is

$$\begin{aligned} t_0 = -Res_\infty(dS) &= -Res_\infty[g(x-x_1)\sqrt{(x-x_3)(x-x_4)}\,dx] = \\ &= \frac{g}{16}(2x_1 - x_3 - x_4)(x_4 - x_3)^2 = -\frac{g}{2}\Delta_{43}^2 \ I \ . \end{aligned} \tag{C.17}$$

From (C.12, C.17) we find

$$\sigma = (1 - Y^2)Y \ , \tag{C.18}$$

which gives Y, and then $\frac{\partial^2 \mathcal{F}}{\partial g_k \partial t_0}$, as a function of $\sigma = 8\alpha S$. Solving (C.18) and taking the appropriate branch we obtain

$$Y = \frac{2^{\frac{1}{3}}}{3\left(\sqrt{\sigma^2 - \frac{4}{27}} - \sigma\right)^{\frac{1}{3}}} + \frac{\left(\sqrt{\sigma^2 - \frac{4}{27}} - \sigma\right)^{\frac{1}{3}}}{2^{\frac{1}{3}}} = 1 - \frac{1}{2}\sum_{k=1}^{+\infty} \frac{(8\alpha S)^k}{k!} \frac{\Gamma(\frac{1}{2}(3k-1))}{\Gamma(\frac{1}{2}(k+1))} \ . \tag{C.19}$$

Once R_k $(k = 1, 2, 3)$ are found from (C.1) we have the first two softly broken terms of (7.26)

$$\frac{\partial^2 \mathcal{F}}{\partial t_0 \partial m} = -\frac{S}{m}\left[1 + 3\sum_{k=1}^{+\infty} \frac{(8\alpha S)^k}{(k+1)!} \frac{\Gamma(\frac{3k}{2})}{\Gamma(\frac{k}{2})}\right] \ , \tag{C.20}$$

$$\frac{\partial^2 \mathcal{F}}{\partial t_0 \partial g} = \frac{2S}{g}\sum_{k=1}^{+\infty} \frac{(8\alpha S)^k}{(k+1)!} \frac{\Gamma(\frac{3k}{2})}{\Gamma(\frac{k}{2})} \ . \tag{C.21}$$

For the third term of (7.26) the computation is a little more involved. The derivative to be computed is

$$\frac{\partial^2 \mathcal{F}}{\partial t_0 \partial g_4} = \frac{g}{4}R_4 + \frac{m}{8}R_3(1 + Y) \ . \tag{C.22}$$

The coefficients R_3 and R_4 are

$$R_4 = \frac{f_0}{2g^3} + \frac{6m}{g^3}S - \frac{m^4}{g^5} \ , \qquad R_3 = \frac{m^3}{g^4} - \frac{2S}{g^2} \ , \tag{C.23}$$

where the unknown function f_0 appears. To compute it we integrate in S the equation that can be obtained from the first constraint in (C.5) and from (C.14)

$$\frac{\partial f_0}{\partial t_0} = -2g\frac{h_1}{h_0} = -2g\frac{Q - I}{2} = m + gI \ . \tag{C.24}$$

We then have

$$f_0[S] = -2mS - 2g\int \frac{m}{g}Y[S]dS = c_1 - 4mS + \frac{m^4}{8g^2}\sum_{j=2}^{+\infty}\frac{(8\alpha S)^j}{j!}\frac{\Gamma(\frac{1}{2}(3j-4))}{\Gamma(\frac{1}{2}j)} \ , \tag{C.25}$$

where c_1 is a function only of the couplings m and g. Using the other constraints in (C.5) that define the derivatives of f_0 with respect to the couplings (m, g) it can be proven that c_1 vanishes. Finally, we use the formulas (C.22, C.23, C.25) with $c_1 = 0$ and find

$$\frac{\partial^2 \mathcal{F}}{\partial t_0 \partial g_4} = \frac{m^4}{64g^4}\sum_{k=2}^{+\infty}\frac{(8\alpha S)^k}{k!}\Big((k+1)\frac{\Gamma(\frac{1}{2}(3k-4))}{\Gamma(\frac{1}{2}k)} - 4\frac{\Gamma(\frac{1}{2}(3k-1))}{\Gamma(\frac{1}{2}(k+1))}\Big) \ . \tag{C.26}$$

We observe that with these ingredients one has formally all the needed quantities to compute in a closed form, as power series of S, all the susy breaking terms in the effective superpotential (7.13) coming from higher order supersymmetry breaking deformation in the tree–level superpotential (7.11). These are functions only of $Y(S)$ and $f_0(S)$ as shown in (C.15). In fact the coefficient R_k are defined as (C.1) and are functions only of $t_0 = -2S$, of the couplings, and of $f_0(S)$ which is known (C.25). At the end the result is (7.26).

The computation in the case of broken gauge group ($U(N) \to U(N_1) \times U(N_2)$) uses a procedure as [65], making the calculation as a power series in the width of the cuts. Our aim is again to compute the susy breaking terms appearing in (7.13) using the formulas (C.7, C.8). The main difference with the unbroken gauge group case is that now the curve does not degenerate

$$y^2 = g^2(x-x_1)(x-x_2)(x-x_3)(x-x_4) \ . \tag{C.27}$$

We introduce quantities analogous as before[3]

$$\Delta_{43} = \tfrac{1}{2}(x_4 - x_3) \quad , \quad \Delta_{21} = \tfrac{1}{2}(x_2 - x_1) \ , \tag{C.28}$$

$$Q = \tfrac{1}{2}(x_4 + x_3 + x_2 + x_1) = (a_1 + a_2) = -\tfrac{m}{g} \ , \tag{C.29}$$

$$I = \tfrac{1}{2}(x_4 + x_3 - x_2 - x_1) = \sqrt{\Delta^2 - 2\Delta_{43}^2 - 2\Delta_{21}^2} \ . \tag{C.30}$$

We don't have anymore the simplification (C.14) and we have to write the integrals h_0 and h_1 as power series in the widths of the cuts ($O(\Delta_{ab}^3)$). We then find the inverse expression of the widths of the cuts Δ_{ab} as a functions of (s_2, t_0) and obtain (h_0, h_1) in terms of (s_2, t_0).
We have also to evaluate the parameters R_k. They have the form (C.1) but now f_0 has to be understood as a function of two variables and $t_0 = -2(S_1 + S_2)$. Precisely f_0 is a function of t_0 and s_2 which are the independent variables and it is determined through the relations

$$\frac{\partial f_0}{\partial t_0} = -2g\frac{h_1}{h_0} \quad , \quad \frac{\partial f_0}{\partial s_2} = \frac{1}{\frac{\partial s_2}{\partial f_0}} = \frac{1}{\oint_\alpha \frac{dx}{2y}} = \frac{2}{h_0} \ . \tag{C.31}$$

[3]Note that as concern the classical roots there are no modifications.

The first equation comes from the normalization condition (C.5) while the second one is a consequence of the definition (7.5) of the variable s_2 using the explicit form (7.25) of the differential dS.
We then compute directly the second derivatives of the prepotential, the susy breaking terms (7.13), using (C.7, C.8). At the end of the computation we change variables (7.8) to express the superpotential in terms of the physical glueball superfields S_i. What we find is (7.28).

C.2 Details on the perturbative approach

In this appendix we explore the details which give (7.33) from (7.30). We will use and extend to spurions the method developed in [72, 88] also reviewing, for reader convenience, their basic steps.

Starting from (7.30), the propagator is the same as in [72]

$$< \Phi\Phi >= \frac{-\overline{m}}{\Box - m\overline{m} - i\mathcal{W}^\alpha\partial_\alpha} \quad . \tag{C.32}$$

The gauge field strength is considered constant then the bosonic and fermionic integrations completely decouple in the computation [72]. To compute contributions to the glueball superpotential, we will use the usual chiral–ring properties of W_α [72, 73]. For example, we will use $\mathrm{Tr}(W_\alpha)^n = 0$ with $n > 2$.

Using the double line notation, a Riemann surface (oriented for $U(N)$) with genus g is associated to each topologically relevant diagram with L momentum loop and l index loop, so that $L = l + 2g - 1$. The D–algebra is exactly as in [72]. The only difference is that performing the D–algebra some ∂_α can act on the θ of the spurions $G_k = g_k + \theta^2\Gamma_k$ giving new terms.
It is possible to do some general considerations using the constraints given by the D–algebra structure, the properties of W_α and the geometry of the diagrams in the amplitudes.
We fix a diagram with L momentum loops and $l = L-2g+1$ index loops. From the W_α properties it follows that, for a relevant amplitude for the glueball superpotential, the maximal number of allowed W_α is $2l$ otherwise we would have at least one index loop with more than three W_α. Furthermore, in order to perform the fermionic loop integrations it is necessary to have at least $2L$ ∂_α and then at least $2L$ W_α. The number of W_α ($\#W_\alpha$) in a non–trivially zero diagram then satisfies the inequality

$$2L \leq \#W_\alpha \leq 2l = 2L + 2 - 4g \quad . \tag{C.33}$$

This implies that $g \equiv 0$ and the only relevant diagrams to be considered are planar. Moreover the relevant contributions to the glueball superpotential have $\#W_\alpha = 2L, 2L+2$.

We consider first the case $\#W_\alpha = 2L$ i.e. $\#\partial_\alpha = 2L$. In this case the D–algebra has to be done only inside the fermionic loops and then no derivative acts on the background spurions. This is equivalent to say that these contributions are insensible to the θ dependence of the G_k. Then for these kind of terms we can reabsorb the quadratic vertex $\frac{1}{2}\theta^2\Gamma_2\mathrm{Tr}\,\Phi^2$ into the propagator (C.32) by simply doing the redefinition $m \to m + \theta^2\Gamma_2$. This is clearly true exept a 1–loop amplitude with one vertex $\frac{1}{2}\theta^2\Gamma_2 \mathrm{Tr}\,\Phi^2$ contracted with the propagator (C.32) which gives the first term of (7.33). The resulting contribution to the glueball superpotential with $\#W_\alpha = 2L$ is then given by (7.33) and, except the linear term in S, these term are computed perturbatively using the dual matrix model of the $\mathcal{N} = 1$ case.

Now, we consider the other case $\#W_\alpha = \#\partial_\alpha = 2L + 2$. All index loops are now saturated and we have a contribution proportional to $S^{(L+1)}$.

These contributions are nonholomorphic since they are proportional to $\overline{m}^{-2}$. In fact, the corresponding diagrams have a multiplicative factor $\overline{m}^{P_i}$ from the numerator of the P_i propagators (C.32). Expanding the propagators (C.32) at order $(\mathcal{W}^\alpha \partial_\alpha)^{(2L+2)}$, we have $P_f = P_i + 2L + 2$ bosonic propagators $\frac{1}{p^2 + m\overline{m}}$ expressed in momentum space. By redefining the bosonic loop momentum variables $p^2 \to \overline{m}p^2$, from the bosonic Jacobian we are left with a contribution $\overline{m}^{2L}$ while from the denominator of the bosonic propagators we have a term $\overline{m}^{-(P_i+2L+2)}$. Summarizing we have $[\overline{m}^{P_i}][\overline{m}^{2L}][\overline{m}^{-(P_i+2L+2)}] = \overline{m}^{-2}$. Therefore, along the calculation we will set $\overline{m} \equiv 1$ and multiply the final result by $\overline{m}^{-2}$.

In performing the calculation it is convenient to express the propagator (C.32) in the Schwinger variables

$$\int_0^\infty ds_i \;\exp\left[-s_i(p_i^2 + i\mathcal{W}_i^\alpha \partial_\alpha + m)\right] . \tag{C.34}$$

As in [72], the bosonic contribution is given by

$$Z_{boson} = \frac{1}{(4\pi)^{2L}} \frac{1}{(\det M(s))^2} \quad , \quad M_{ab}(s) \equiv \sum_i s_i L_{ia} L_{ib} \quad , \quad p_i = \sum_a L_{ia} k_a \quad . \tag{C.35}$$

We note that $M(s)$ is an $L \times L$ matrix and then the denominator of Z_{boson} (C.35) is a homogeneous polynomial of degree $2L$ in s_i. Furthermore, we have, from the fermionic integrations of these diagrams, $2L + 2$ $s_i\mathcal{W}_i^\alpha$ terms. Then, at the numerator we have a homogeneous polynomial of degree $2L + 2$ in s_i. The degree in s_i of the numerator results to be greater than the denominator degree. Thus, for the class of diagrams with $\#W_\alpha = 2L + 2$ (certainly) there is no cancellation between the bosonic and fermionic integrations in contrast with the case $\#W_\alpha = 2L$ [72].

Performing the D–algebra we realize that there are two distinct possibilities depending on the way the two extra ∂_α are distributed on the external spurionic terms. The first possibility is when two ∂_α act on one spurionic constant $\theta^2\Gamma_k$. This contribution would have a multiplicative factor $[(\Gamma_k)\prod_{v=1}^{(V-1)}(g_{k_v} + \theta^2\Gamma_{k_v})]$ (V is the number of vertices of the considered diagram).

The second possibility is when the two ∂_α act on two different spurions. In this case we have a multiplicative term $[(\theta^\alpha\Gamma_k)(\theta_\alpha\Gamma_{k'})\prod_{v=1}^{(V-2)}(g_{k_v} + \theta^2\Gamma_{k_v})]$.

Summarizing the previous considerations, the general structure of the glueball superpotential, due to the integration of the matter fields considering only the holomorphic part of the interaction vertices as in (7.30), is

$$\int d^2\theta \left\{ N\theta^2\Gamma_2 \frac{S}{m} + N\sum_l \mathcal{F}_{0,l}(g_k + \theta^2\Gamma_k) l S^{l-1} + \frac{1}{\overline{m}^2}\sum_l \mathcal{B}_l(g_k, \Gamma_k, \theta) S^l \right\} \quad . \tag{C.36}$$

The $\mathcal{B}_l$ are holomorphic in all g_k, Γ_k and possibly depend also on θ^2. $\mathcal{B}_l$ are analytic in all the variables except m. We will show that $\mathcal{B}_l = 0\ \forall l$ justifying (7.33).

To compute $\mathcal{B}_l$ we must perform the D–algebra and treat the group theoretical factor. As in [72] to simplify the fermionic integrations we can use the fermionic Fourier momentum representation. The novelty in the computation is due to the fact that now there is also the θ^2 from the spurionic vertices to be Fourier transformed. In particular, we have

$$\theta^2 = -\delta^{(2)}(\theta) = -\int d^2\pi e^{i\pi^\alpha\theta_\alpha} \quad . \tag{C.37}$$

We focus on a planar diagram with $L = l-1$ bosonic momentum loops with P propagators and V vertices. In particular we consider the case with only one spurionic constant $\theta^2\Gamma_k$ on which the D–algebra acts nontrivially. The other case is analogue.

The fermionic contribution results to be[4]

$$\begin{aligned}
Z_{fermion} &= \int \prod_{v=1}^{V} d^2\theta_v\, \theta_1^2 \prod_{i=1}^{P}\left[e^{-s_i\mathcal{W}_i^\alpha i\partial_\alpha}\delta^{(2)}(\theta_{v_i}-\theta_{v_i'})\right] \\
&= -\int \prod_{i=1}^{P} d^2\pi_i d^2r \prod_{v=1}^{V} d^2\theta_v \prod_{i=1}^{P}\left[e^{-s_i\mathcal{W}_i^\alpha\pi_{i\alpha}}\right] e^{i(\sum_{j_1=1}^{k_1}\pi_{j_1}+r)^\alpha\theta_{1\alpha}} \prod_{v=2}^{V} e^{i(\sum_{j_v=1}^{k_v}\pi_{j_v})^\alpha\theta_{v\alpha}} \\
&= -\int d^2\theta \int \prod_{a=1}^{P-V+2} d^2\kappa_a d^2r\, \delta^{(2)}\Big(\sum_{j_1=1}^{k_1}\pi_{j_1}+r\Big)\prod_{i=1}^{P}\left[e^{-s_i\mathcal{W}_i^\alpha\pi_{i\alpha}}\right] e^{i(\sum_{j_V=1}^{k_V}\pi_{j_V})^\alpha\theta_\alpha} \\
&\Longrightarrow -\int d^2\theta \int \prod_{a=1}^{l} d^2\kappa_a \prod_{i=1}^{P}\left[e^{-s_i\mathcal{W}_i^\alpha\pi_{i\alpha}}\right] \quad .
\end{aligned} \tag{C.38}$$

In the second line π_{j_v} are the spinorial momuntum connected to the v–th θ–vertex and satisfy $\pi_{j_v} \equiv \pm\pi_i$ where the sign is $+$ $(-)$ if the spinorial momentum is going outside (inside) the vertex v. In the last two line we exploit the relations $\sum_{j_v=1}^{k_v}\pi_{j_v} \equiv 0$ $(v = 2,\cdots,V-1)$ with which we define the remaining l spinorial variables κ_a from the independent π_i. Furthermore, in the last line we have used the fact that we are searching for a contribution to $\mathcal{B}_l$ which has $\#\mathcal{W}_\alpha = 2l = 2L+2$. Then, in the expansion of $e^{-i(\sum_{j_V=1}^{k_V}\pi_{j_V})^\alpha\theta_\alpha}$ we can keep only 1, the term independent of θ_α. This is equivalent to say that only the term in which θ^2 of the spurion is killed by two ∂_α contributes to $\mathcal{B}_l$.

At the end of the above manipulations we remain with l fermionic integrations over the indipendent variables κ_a and the π_i are linear combinations of them

$$\pi_{i\alpha} \equiv \sum_{a=1}^{l}\widetilde{L}_{ia}\kappa_{a\alpha} \quad . \tag{C.39}$$

[4] The index j_v, depending on $v = \{1,\cdots,V\}$, runs from 1 to k_v which is the degree of the interaction vertex v as: $\mathrm{Tr}\,\Phi^{k_v}$.

As in [72] we can implement the requirement of having two insertions of $\mathcal{W}^\alpha$ for each index loop introducing $2l$ auxiliary grassmanian variables $\mathcal{W}_m^\alpha$ adapted to the action on the adjoint representation with

$$\mathcal{W}_i^\alpha \equiv \sum_{m=1}^{l} K_{im}\mathcal{W}_m^\alpha \quad . \tag{C.40}$$

The matrix K is defined so that for each oriented i–th propagator the m–th index loop can coincide and be parallel giving $K_{im}=1$; or coincide and be anti–parallel giving $K_{im}=-1$; or not coincide giving $K_{im}=0$.

Summarazing we find from the fermionic integration

$$\begin{aligned}
&(16\pi^2 S)^l \int \prod_{a,m=1}^{l} d^2\kappa_a d^2\mathcal{W}_m \;\exp\left[-\sum_i s_i\left(\sum_{a,m}\mathcal{W}_m^\alpha K_{mi}^T \widetilde{L}_{ia}\kappa_{a\alpha}\right)\right]\\
&= (16\pi^2 S)^l \int \prod_{a,m=1}^{l} d^2\kappa_a d^2\mathcal{W}_m \;\exp\left[-\sum_{a,m}\mathcal{W}_m^\alpha \widetilde{N}(s)_{ma}\kappa_{a\alpha}\right]\\
&= S^l(4\pi)^{2l}(\det\widetilde{N}(s))^2 \quad ,
\end{aligned} \tag{C.41}$$

with

$$\widetilde{N}(s)_{ma} \equiv \sum_i s_i K_{mi}^T \widetilde{L}_{ia} \quad . \tag{C.42}$$

The relevant fact is that, for our class of diagrams which has an S^2 topology, the matrix K has a nontrivial kernel. In fact, for example, the vector b_m, whose components are all equal to one, belong to the kernel of K_{im}[5]. This is simply due to the fact that in the case we are studying all momentum propagator lines have exact two index loop passing through them with opposite orientation; then, $\forall i$ there will be only one $K_{im'}=1$ and one $K_{im''}=-1$ $(m'\neq m'')$ and $\sum_m K_{im}b_m = 1-1 = 0$. It follows that also the matrix $[\widetilde{N}(s)]^T_{am} = \sum_i s_i \widetilde{L}^T_{ai}K_{im}$ has a nontrivial kernel indipendently of the explicit form of $\widetilde{L}$ which we have not analyzed in detail. Then $\det(\widetilde{N}(s)) \equiv 0$. This imply that $\mathcal{B}_l \equiv 0$ $\forall l$ as claimed before.

[5]Our susy broken case is similar to the situation which appears in the study of the perturbative reduction to matrix models for the case of $\mathcal{N}=1$ supersymmetry and SU/SO/Sp(N) gauge groups developed in [88] extending [72].

Appendix D

Details on $SU(2)$ structure manifolds

D.1 The supersymmetric family of backgrounds and IR PW

The supersymmetry equations for the ansatz (8.13,8.14) was studied in [99]. They imply, for complex solutions with constant dilaton, that the geometrical quantities can be expressed as derivatives of a single function F. If the background does not depend on σ_3 we have

$$A_{i\bar{j}} = \frac{\partial^2 F}{\partial z_i \partial \bar{z}_j} \quad i, j = 1, 2 \tag{D.1}$$

$$A_{i\bar{j}}\bar{\alpha}_j = \frac{\partial^2 F}{\partial z_i \partial \bar{z}_3}\,, \tag{D.2}$$

$$\alpha_i A_{i\bar{j}} = \frac{\partial^2 F}{\partial \bar{z}_j \partial z_3}\,, \tag{D.3}$$

$$u^2 a_3 \cos 2\phi + \alpha_i A_{i\bar{j}}\bar{\alpha}_j = \frac{\partial^2 F}{\partial z_3 \partial \bar{z}_3}\,. \tag{D.4}$$

$$a_3 u^2 \sin^2\phi = -\frac{\partial}{\partial z_3} F\,. \tag{D.5}$$

The infrared geometry of the PW flow can be reconstructed in this family of supersymmetric backgrounds as follows [99]. Choose coordinates

$$\begin{aligned} e^{z_1} &= r^{3/4}\cos\theta\cos\varphi e^{i\sigma_1}\,, \\ e^{z_2} &= r^{3/4}\cos\theta\sin\varphi e^{i\sigma_2}\,, \\ e^{z_3} &= r^{3/2}\sin\theta e^{i\sigma_3}\,. \end{aligned}$$

The generalized Kahler potential F is

$$F = \frac{3}{4} r^2 (1 - 2\sin^2\theta)\,, \tag{D.6}$$

and the warp factor

$$e^{2A} = r^2\sqrt{\frac{3}{4}(1+\sin^2\theta)} \tag{D.7}$$

The other quantities are determined, for example

$$\begin{aligned}
\sin 2\phi &= \frac{\sin\theta\sqrt{2+\sin^2\theta}}{1+\sin^2\theta} &\text{(D.8)}\\
A_{1\bar{1}} &= r^2\left(\cos^2\theta\cos^2\varphi + \frac{\cos^4\theta\cos^4\varphi}{3+3\sin^2\theta}\right) &\text{(D.9)}\\
A_{1\bar{2}} = A_{2\bar{1}} &= \frac{r^2\cos^4\theta\sin^2\varphi\cos^2\varphi}{3+3\sin^2\theta} &\text{(D.10)}\\
A_{2\bar{2}} &= r^2\left(\cos^2\theta\sin^2\varphi + \frac{\cos^4\theta\sin^4\varphi}{3+3\sin^2\theta}\right) &\text{(D.11)}\\
a_3 &= \frac{1+\sin^2\theta}{4r(2+\sin^2\theta)} &\text{(D.12)}
\end{aligned}$$

D.2 Beta deformed gravity dual

We have already introduced the complex coordinates z_i (8.47); the one forms appearing in (8.48) are defined as [99]

$$x_1 + iy_1 = e^{-i\sigma_1}\sqrt{\frac{g}{\mu_1^2(\mu_2^2+\mu_3^2)}}(dz_1 - \frac{\bar{z}_2\bar{z}_3 z}{r^2\sqrt{g}}) \tag{D.13}$$

$$x_2 + iy_2 = e^{-i\sigma_2}\sqrt{1+\frac{\mu_3^2}{\mu_2^2}}(dz_2 - \frac{\bar{z}_1\bar{z}_3 z}{r^2\sqrt{g}}) + \frac{\mu_3^2 e^{-i\sigma_1}}{\mu_1\sqrt{\mu_2^2+\mu_3^2}}(dz_1 - \frac{\bar{z}_2\bar{z}_3 z}{r^2\sqrt{g}}) \tag{D.14}$$

$$z = \frac{d[z_1 z_2 z_3]}{r^2\sqrt{g}} \tag{D.15}$$

The internal metric (8.48) gives then [101]

$$d\tilde{s}_6^2 = dr^2 + r^2\left(\sum_{i=1}^{3}(d\mu_i^2 + G\mu_i^2 d\sigma_i^2) + \gamma^2 G\mu_1^2\mu_2^2\mu_3^2(d\sigma_1+d\sigma_2+d\sigma_3)^2\right) \tag{D.16}$$

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