universe

IMPACT FACTOR 2.5

CITESCORE 4.3

Article

Neutron Beta Decay and Exact Conservation of Charged Weak Hadronic Vector Current in the Standard Model

Derar Altarawneh, Roman Höllwieser and Markus Wellenzohn

MDPI

https://doi.org/10.3390/universe10110415 universe

MDPI

Article

Neutron Beta Decay and Exact Conservation of Charged Weak Hadronic Vector Current in the Standard Model

Derar Altarawneh [1,*], Roman Höllwieser [2] and Markus Wellenzohn [3,4]

[1] Department of Applied Physics, Tafila Technical University, Tafila 66110, Jordan
[2] Department of Theoretical Physics, Faculty of Mathematics and Natural Sciences, Wuppertal University, Gaußstraße 20, 42119 Wuppertal, Germany
[3] Department of Engineering, Applied Electronics and Technical Informatics, FH Campus Wien, University of Applied Sciences Vienna, 1100 Vienna, Austria
[4] Research Center IT-Security, Department of Engineering, FH Campus Wien, University of Applied Sciences Vienna, 1100 Vienna, Austria
* Correspondence: derar@ttu.edu.jo

Abstract: We investigate the reliability of the hypothesis of exact conservation of the charged weak hadronic vector current in neutron β^--decay with a polarized neutron and an unpolarized proton and electron. We calculate the contributions of the phenomenological term responsible for Exact Conservation of the charged weak hadronic Vector Current (or the ECVC effect) in neutron β^--decay, even for different masses of the neutron and proton, to the correlation coefficients, together with a complete set of contributions of scalar and tensor interactions beyond the Standard Model (SM). We argue that if the total contributions of scalar and tensor interactions beyond the SM fail to reconcile the experimental data on the correlation coefficients with the contributions of the ECVC effect, one may conclude that the charged weak hadronic vector current is not conserved in the hadronic transitions of weak processes with different masses of incoming and outgoing hadrons.

Keywords: neutron beta decay; ECVC

PACS: 12.15.Ff; 13.15.+g; 23.40.Bw; 26.65.+t

check for updates

Citation: Altarawneh, D.; Höllwieser, R.; Wellenzohn, M. Neutron Beta Decay and Exact Conservation of Charged Weak Hadronic Vector Current in the Standard Model. *Universe* **2024**, *10*, 415. https://doi.org/10.3390/universe10110415

Academic Editor: Máté Csanád

Received: 18 September 2024
Revised: 3 November 2024
Accepted: 4 November 2024
Published: 6 November 2024

1. Introduction

Nowadays, it is generally accepted that neutron β^--decay is a perfect laboratory for tests of the Standard Model (SM) [1–8]. The theoretical analysis of neutron β^--decay at the level of 10^{-3}, including the radiative corrections of order $O(\alpha/\pi)$, where α is the fine–structure constant [9], and corrections of order $1/M$, caused by the weak magnetism and proton recoil, where M is an averaged nucleon mass, provide a theoretical background at the level of 10^{-3} [6–8] for experimental searches of interactions beyond the SM at the level of 10^{-4} [7,8]. The calculation of the main contributions to neutron β^-–deca,y as well as corrections of order 10^{-3}, has been carried out in the $V - A$ theory of weak interactions and QED, where V and A are charged weak hadronic vector $V^{(+)}_\mu(x)$ and axial $A^{(+)}_\mu(x)$ currents, respectively. The contribution of the charged weak hadronic vector current to the matrix element of the hadronic $n \to p$ transition has been calculated at the standard assumption that local conservation of $V^{(+)}_\mu(x)$, i.e., a vanishing divergence $\partial^\mu V^{(+)}_\mu(x) = 0$, can be violated only by isospin breaking [10]. Another possibility for the contribution of the charged weak hadronic vector current to the matrix element of the hadronic $n \to p$ transition has been proposed by Leitner et al. [11]. According to Leitner et al. [11], the matrix element $\langle p(\vec{k}_p, \sigma_p)|V^{(+)}_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle$ of the hadronic $n \to p$ transition is defined by

$$\langle p(k_p,\sigma_p)|V^{(+)}_\mu(0)|n(k_n,\sigma_n)\rangle = \bar{u}_p(k_p,\sigma_p)\Big(\Big(\gamma_\mu - \frac{q_\mu \hat{q}}{q^2}\Big)\, F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M}\, F_2(q^2)\Big) u_n(k_n,\sigma_n), \tag{1}$$

where $F_1(q^2)$ and $F_2(q^2)$ are form factors dependent on the squared 4–momentum transferred $q^2 = (k_p - k_n)^2$, $\bar{u}_p(k_p,\sigma_p)$ and $u_n(k_n,\sigma_n)$ are the Dirac wave functions of the free proton and neutron, γ_μ and $\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$ are the Dirac matrices and $\hat{q} = \gamma_\nu q^\nu$, $M = (m_n + m_p)/2$, where m_n and m_p are masses of the neutron and proton, respectively [9]. The matrix element Equation (1) obeys the condition

$$q^\mu \langle p(k_p,\sigma_p)|V^{(+)}_\mu(0)|n(k_n,\sigma_n)\rangle = 0 \tag{2}$$

or $\langle p(k_p,\sigma_p)|\partial^\mu V^{(+)}_\mu(0)|n(k_n,\sigma_n)\rangle = 0$ even for different masses of the proton and neutron. As has been shown by Ivanov [12,13], the terms with the Lorentz structures γ_μ, $q_\mu\hat{q}$ and $i\,\sigma_{\mu\nu}q^\nu$ are induced by the first-class currents [14]. According to Ivanov [13], the matrix element of the hadronic $n \to p$ transition, caused by the charged weak hadronic vector current, has the following general structure

$$\langle p(k_p,\sigma_p)|V^{(+)}_\mu(0)|n(k_n,\sigma_n)\rangle = \bar{u}_p(k_p,\sigma_p)\Big(\gamma_\mu\, F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M}\, F_2(q^2) + \frac{q_\mu \hat{q}}{M^2}\, F_4(q^2) + \frac{q_\mu}{M}\, F_3(q^2)\Big) u_n(k_n,\sigma_n). \tag{3}$$

where $F_j(q^2)$ with $j = 1,2,3,4$ are form factors. The first three terms with the Lorentz structures γ_μ, $i\,\sigma_{\mu\nu}q^\nu/M$ and $q_\mu\hat{q}/M^2$ are induced by the first-class current [12,13], whereas the term with the Lorentz structure q_μ/M is a phenomenological contribution of the second-class one [13]. According to Weinberg [14], the contribution of the second-class currents should vanish. This gives $F_3(q^2) = 0$. Thus, the general structure of the matrix element of the hadronic $n \to p$ transition, caused by the contributions of the first-class current only, is

$$\langle p(k_p,\sigma_p)|V^{(+)}_\mu(0)|n(k_n,\sigma_n)\rangle = \bar{u}_p(k_p,\sigma_p)\Big(\gamma_\mu\, F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M}\, F_2(q^2) + \frac{q_\mu \hat{q}}{M^2}\, F_4(q^2)\Big) u_n(k_n,\sigma_n). \tag{4}$$

We would like to mention that for the first time, the term with the Lorentz structure $q_\mu\hat{q}/M^2$ was introduced by Berman and Sirlin [15] as a part of the Lorentz-invariant structure of the nucleon vector form factor, with initial and final nucleons on-mass and off-mass shell, respectively (see Equation (8) of Ref. [15]).

Of course, the form factor $F_4(q^2)$ is a new phenomenological function of q^2 with respect to the form factors $F_1(q^2)$ and $F_2(q^2)$, which are usually used for the description of the hadronic $n \longleftrightarrow p$ transitions. Moreover, it is not obvious that $F_4(q^2) = -(M^2/q^2)\, F_1(q^2)$. Such a relation we may call a realization of the hypothesis of Exact Conservation of the charged weak hadronic Vector Current (or the ECVC effect [16]). For the first time, the contribution of the term $-F_1(q^2)\, q_\mu\hat{q}/q^2$ or the contribution of the ECVC effect has been analyzed by Leitner et al. [11] in the quasi-elastic neutrino–neutron scattering $\nu_\ell + n \to p + \ell^-$, where (ν_ℓ, ℓ^-) are the neutrino and charged lepton with lepton flavours $\ell = e, \mu$, and so on. Such an analysis of the quasi-elastic neutrino–neutron scattering with the ECVC effect has been carried out by Leitner et al. [11] in connection with neutrino production of the resonance $P_{33}(1232)$ or the Δ-resonance, i.e., $\nu_\ell + N \to \Delta + \ell^-$.

The use of the ECVC effect in the quasi-elastic neutrino–neutron scattering in connection with the neutrino production of the resonance $P_{33}(1232)$ or the Δ-resonance is not a surprise. Indeed, it is well known that exact conservation of the charged weak hadronic vector current or the ECVC hypothesis is usually used for the analysis of neutrino production of baryon resonances by a nucleon N, i.e., in the reactions $\nu_\ell + N \to R + \ell^-$, [17–23] (see also [11]). Here, R is a baryon resonance such as $P_{33}(1232)$, $P_{11}(1440)$, and so on [21–23], where the numbers in parentheses define resonance masses m_R measured in MeV. In this case, the matrix elements $\langle R|\partial^\mu V^{(\pm)}_\mu(0)|N\rangle$ vanish in spite of the fact that masses m_N and

m_R are different, i.e., $m_N \neq m_R$, even in the limit of isospin symmetry. Recently, the ECVC effect or the contribution of the phenomenological term $-F_1(q^2)\, q_\mu \hat{q}/q^2$ to the cross section for the inverse β-decay $\bar{\nu}_e + p \to n + e^+$ has been analyzed by Ankowski [24].

It is very likely that scattering processes like quasi-elastic neutrino–neutron scattering and inverse beta decay are not sensitive enough to the contribution of the ECVC effect. Indeed, the contribution of the term $-F_1(q^2)\, q_\mu \hat{q}/q^2$ to the matrix element of the hadronic $n \to p$ transition of the quasi-elastic scattering $\nu_e + n \to p + e^-$ and the inverse β^--decay $\bar{\nu}_e + p \to n + e^+$ is equal to $\mp F_1(q^2)\, m_e \Delta/q^2$, where $m_e = 0.5110\,\mathrm{MeV}$ and Δ are the electron mass and the mass–difference $\Delta = m_n - m_p = 1.2934\,\mathrm{MeV}$, respectively, and $q^2 = m_e^2 - 2E_\nu E_e(1 - \beta \cos\vartheta_{e\nu})$ [25]. As we show in Appendix A, the cross sections for the quasi-elastic electron neutrino–neutron scattering and for the inverse β-decay are not sensitive to the contributions of the ECVC effect (see Figure A1). For the illustration of the sensitivity of the cross sections for the quasi-elastic electron neutrino–neutron scattering and for the inverse β-decay to the ECVC effect, we calculate in Appendix A, the cross sections for the neutrino and antineutrino energy regions $2\,\mathrm{MeV} \leq E_\nu \leq 8\,\mathrm{MeV}$ and $2\,\mathrm{MeV} \leq E_{\bar{\nu}} \leq 8\,\mathrm{MeV}$ [25], where E_ν and $E_{\bar{\nu}}$ are the neutrino and antineutrino energy, respectively. In these neutrino and antineutrino energy regions, the form factor $F_1(q^2)$ can be replaced by unity, i.e., $F_1(q^2) = 1$ [25].

In Figure A1 we plot the relative contributions of the ECVC effect to the cross sections for the reactions under consideration. One may see that the contributions of the ECVC effect decrease with the neutrino and antineutrino energies. The contribution of the ECVC effect to the cross section for the quasi-elastic electron neutrino–neutron scattering makes up about 0.7 % at $E_\nu \simeq 2\,\mathrm{MeV}$ and one order of magnitude smaller at $E_\nu \simeq 8\,\mathrm{MeV}$. In turn, the contribution of the ECVC effect to the cross section for the inverse β-decay is of about 3 % at $E_{\bar{\nu}} \simeq 2\,\mathrm{MeV}$ and two orders of magnitude smaller at $E_{\bar{\nu}} \simeq 8\,\mathrm{MeV}$. It is important to emphasize that the contribution of the ECVC effect to the cross section for the inverse β-decay is negative. This implies that such a contribution may only increase a deficit of positron [26]. Since the cross section for the inverse β-decay should be averaged over the reactor electron antineutrino–energy spectrum, which has a maximum at $E_{\bar{\nu}} \simeq 4\,\mathrm{MeV}$, one may expect an increase of the deficit of the reactor electron antineutrinos by about 0.5 %. Although this is important as a hint for a search of light sterile neutrinos with a mass $m_{\nu_s} \sim 1\,\mathrm{eV}$ [27–29], the contribution of the ECVC effect to the yield of positrons Y_{e^+} is smaller than the experimental error bars $Y_{e^+} = 0.943(23)$ [26]. So one may argue that the cross section for the inverse β-decay is insensitive to the contributions of the ECVC effect.

As has been shown in [16], neutron β^--decay and, namely, the neutron lifetime has turned out to be extremely sensitive to the ECVC effect. Indeed, the contribution of the ECVC effect changes the neutron lifetime by $\Delta\tau_n = 76.4\,\mathrm{s}$. Together with the theoretical value $\tau_n^{(\mathrm{SM})} = 879.6(1.1)\,\mathrm{s}$, calculated by Ivanov et al. [7], the ECVC effect increases the neutron lifetime equal to $\tau_n^{(\mathrm{eff})} = 950(1.1)\,\mathrm{s}$, which disagrees strongly with the world averaged value $\bar{\tau}_n = 880.2(1.0)\,\mathrm{s}$ [9] and recent experimental value $\tau_n^{(\mathrm{exp})} = 880.2(1.2)\,\mathrm{s}$ [30]. As has been shown in [16], since in the SM there are no interactions, which can cancel such a huge contribution of the ECVC effect, only scalar and tensor interactions beyond the SM can be used to reconcile the ECVC effect with the world averaged value $\bar{\tau}_n = 880.2(1.0)\,\mathrm{s}$ of the neutron lifetime [9]. In a linear approximation the contributions of scalar and tensor interactions beyond the SM reduce themselves to the neutron lifetime in the form of the Fierz interference term b_F. Cancelling the contribution of the ECVC effect one obtains $b_F = 0.1219(12)$ [16]. As has been shown in [16], taking into account the contributions of scalar and tensor interactions beyond the SM without any approximation, one may cancel the contribution of the ECVC effect without fixing the value of the Fierz interference term. The latter is because the equation which reduces $\tau_n^{(\mathrm{eff})}$ to $\bar{\tau}_n$, is a quadratic algebraical equation depending on four complex parameters, which are phenomenological coupling constants of scalar and tensor interactions beyond the SM [7]. Thus, the analysis of the contribution of the ECVC effect to the neutron lifetime, carried out in [16], has shown the form factor $F_4(q^2)$ cannot be equal to $-(M^2/q^2)\, F_1(q^2)$ in the SM. Moreover, in comparison with the results, obtained in [7,8], the contribution of the term $F_4(q^2)\, q_\mu \hat{q}/M^2$ to the neutron lifetime, calculated in the SM without contributions of interactions beyond the SM, should be of order $O(m_e\Delta/M^2)$ or even smaller.

Nevertheless, as has been pointed out by Ivanov et al. [16], in order to understand a validity of the hypothesis of exact conservation of the charged weak hadronic vector current or the ECVC effect, corresponding to a vanishing matrix element $\langle h'|\partial^\mu V^{(+)}_\mu(0)|h\rangle$, i.e., $\langle h'|\partial^\mu V^{(+)}_\mu(0)|h\rangle = 0$, of a hadronic $h \to h'$ transition for different masses of incoming h and outgoing h' hadrons, and to estimate the values of the phenomenological coupling constants of scalar and tensor interactions beyond the SM one has to investigate experimentally all asymmetries of neutron β^--decay, expressed in terms of the electron and proton energy and angular distributions, obtained in [7,8] and supplemented by the contributions of the ECVC effect and scalar and tensor interactions beyond the SM, calculated without any approximation.

This paper is addressed to the analysis of the contributions of the ECVC effect, i.e., the contributions of the term $-F_1(q^2)\, q_\mu \hat{q}/q^2$, to the matrix element $\langle p|V^{(+)}_\mu(0)|n\rangle$ of the hadronic $n \to p$ transition providing a vanishing matrix element $\langle p|\partial^\mu V^{(+)}_\mu(0)|n\rangle$ even for different masses of the neutron and proton. We calculate the contributions of the ECVC effect to the correlation coefficients of the electron energy and angular distributions of neutron β^--decay with a polarized neutron and unpolarized proton and electron.

The paper is organized as follows. In Section 2 we define the electron energy and its angular distribution in neutron β^--decay $n \to p + e^- + \bar{\nu}_e$. In Section 3 we calculate the correlation coefficient $\mathcal{A}(E_e)$, defining the electron asymmetry, with the contribution of the ECVC effect and the Fierz interference term in the linear approximation for scalar and tensor interactions beyond the SM. In Section 4 we discuss the obtained results, compare the theoretical correlation coefficient $\mathcal{A}(E_e)$ with the available experimental data. The correlation coefficient $\mathcal{A}(E_e)$ contains the contributions of (i) the correlation coefficient $A^{(\rm SM)}(E_e)$, calculated at the level of 10^{-3} in the SM [7], (ii) the ECVC effect and (iii) the Fierz interference term.

We propose for new experimental investigations of the contributions of the ECVC effect together with the contributions of scalar and tensor interactions beyond the SM, to use the effective expressions for the correlation coefficients of the neutron β^--decay. These are given in Equations (14) and (19) for a polarized neutron, unpolarized proton and electron and neutron lifetime $\tau^{(\rm eff)}_n$. They take into account the contributions of the SM, calculated at the level of 10^{-3}, the ECVC effect and scalar and tensor interactions beyond the SM, calculated to linear approximation for vector and axial vector interactions beyond the SM [7,16]. In Appendix A, we calculate the cross sections for the quasi-elastic electron neutrino–neutron scattering and for the inverse β-decay by taking into account the contributions of the ECVC effect in the laboratory frame and in the non-relativistic limit for the outgoing hadron. We show that the cross sections of the reactions under consideration are not sensitive to the contributions of the ECVC effect.

2. Electron Energy and Angular Distribution

Following [16], the amplitude of the neutron β^--decay $n \to p + e^- + \bar{\nu}_e$ is given in the standard form by

$$M(n \to p\, e^-\, \bar{\nu}_e) = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, \langle p(\vec{k}_p, \sigma_p)|J^{(+)}_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle \Big[\bar{u}_e(\vec{k}_e, \sigma_e)\gamma^\mu(1-\gamma^5)\, v_\nu(\vec{k}, +\frac{1}{2})\Big], \quad (5)$$

where G_F and V_{ud} are the Fermi weak coupling constant and the Cabibbo–Kobayashi–Maskawa (CKM) matrix element [9], $\bar{u}_e(\vec{k}_e, \sigma_e)$ and $v_\nu(\vec{k}_\nu, +\frac{1}{2})$ are the Dirac wave functions of the free electron and antineutrino with 3-momenta $\vec{k}_e$ and $\vec{k}_\nu$ and polarizations $\sigma_e = \pm 1$ and $+\frac{1}{2}$ [7,31], respectively, and γ^5 is the Dirac matrix. The matrix element of the charged weak hadronic $V - A$ current $\langle p(\vec{k}_p, \sigma_p)|J^{(+)}_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle$ is given by [16]

$$\langle p(\vec{k}_p,\sigma_p)|J^{(+)}_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle = \bar{u}_p(\vec{k}_p,\sigma_p)\Big[\Big(\gamma_\mu - \frac{q_\mu\hat{q}}{q^2}\Big) + \frac{\kappa}{2M}\,i\,\sigma_{\mu\nu}q^\nu + \lambda\,\Big(-\frac{2M\,q_\mu}{q^2-m^2_\pi} + \gamma_\mu\Big)\,\gamma^5\Big]\,u_n(\vec{k}_n,\sigma_n). \quad (6)$$

where $q = k_p - k_n$ is a 4-momentum transferred and $-q_\mu\hat{q}/q^2$ is the phenomenological term responsible for the ECVC in neutron β^--decay. Then, the term $\kappa/2M$, where $M = (m_n + m_p)/2$ is the averaged nucleon mass, defines the contribution of the weak magnetism, where $\kappa = \kappa_p - \kappa_n = 3.7058$ is the isovector anomalous magnetic moment of the nucleon, measured in nuclear magneton [9]. The contribution of the axial current is given by the last term in Equation (6), where $\lambda = -1.2750(9)$ is the axial coupling constant [1] (see also [7,31]) and m_π is the charged pion mass [9]. In the limit $m_\pi \to 0$ (or in the chiral limit) [32,33], the matrix element Equation (6) obeys the constraint $q^\mu\langle p(\vec{k}_p,\sigma_p)|J^{(+)}_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle = 0$, even for different masses of the neutron and proton.

Using the results obtained in [7,16], and skipping standard intermediate calculations, we arrive at the electron energy and angular distribution for the neutron β^--decay with a polarized neutron and unpolarized proton and electron

$$\frac{d^5\lambda_n(E_e,\vec{k}_e,\vec{k}_\nu,\vec{\xi}_n)}{dE_e d\Omega_e d\Omega_\nu} = (1+3\lambda^2)\,\frac{G^2_F|V_{ud}|^2}{32\pi^5}\,(E_0-E_e)^2\,\sqrt{E^2_e - m^2_e}\,E_e\,F(E_e,Z=1)\,\zeta(E_e)$$
$$\times\Big\{1 + \Big[b_F - \frac{1}{1+3\lambda^2}\Big(\frac{2m_e\Delta}{q^2} - \frac{m_eE_e\Delta^2}{q^4}\Big)\Big]\,\frac{m_e}{E_e} + \Big[-\frac{1}{1+3\lambda^2}\,\frac{m^2_e\Delta^2}{q^4} + a(E_e)\Big]\,\frac{\vec{k}_e\cdot\vec{k}_\nu}{E_eE_\nu}$$
$$+A(E_e)\,\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e} + \Big[-b_{ST}\,\frac{m_e}{E_e} - \frac{1}{2}\,(A_0+B_0)\,\frac{m^2_e\Delta}{q^2E_e} + B(E_e)\Big]\,\frac{\vec{\xi}_n\cdot\vec{k}_\nu}{E_\nu} + K_n(E_e)\,\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{k}_\nu)}{E^2_eE_\nu}$$
$$+Q_n(E_e)\,\frac{(\vec{\xi}_n\cdot\vec{k}_\nu)(\vec{k}_e\cdot\vec{k}_\nu)}{E_eE^2_\nu} + \Big[-\frac{1}{2}\,(A_0+B_0)\,\frac{\alpha\,m^2_e\Delta}{q^2k_e} + D(E_e)\Big]\,\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{k}_\nu)}{E_eE_\nu} - 3\,a_0\,\frac{E_e}{M}\,\Big(\frac{(\vec{k}_e\cdot\vec{k}_\nu)^2}{E^2_eE^2_\nu} - \frac{1}{3}\,\frac{k^2_e}{E^2_e}\Big)\Big\}, \quad (7)$$

where $\Delta = m_n - m_p$ and $E_\nu = E_0 - E_e$ with $E_0 = (m^2_n - m^2_p + m^2_e)/2m_n = 1.2927\,\mathrm{MeV}$ is the end-point energy of the electron energy spectrum, and $q^2 = m^2_e + 2\,k_e\cdot k_\nu = m^2_e + 2E_eE_\nu - 2\,\vec{k}_e\cdot\vec{k}_\nu$. All terms depending on Δ are caused by the ECVC effect. The correlation coefficients a_0, A_0 and B_0 are equal to [1]

$$a_0 = \frac{1-\lambda^2}{1+3\lambda^2}\;,\;A_0 = -2\,\frac{\lambda(1+\lambda)}{1+3\lambda^2}\;,\;B_0 = -2\,\frac{\lambda(1-\lambda)}{1+3\lambda^2}. \quad (8)$$

They are calculated to leading order in the large nucleon mass expansion. Then, $\vec{\xi}_n$ is a unit polarization vector of the neutron. The correlation coefficients $\zeta(E_e)$, $a(E_e)$, $A(E_e)$, and so on, have been calculated in [7] (see also [6]) within the SM at the level 10^{-3} by taking into account the $1/M$ corrections, caused by the weak magnetism and proton recoil, and radiative corrections to order $O(\alpha/\pi)$. The contributions of interactions beyond the SM are calculated to linear approximation (see Appendix G of Ref. [7]) and denoted by b_F, which is the Fierz interference term, and b_{ST}. The correlation coefficients b_F and b_{ST} are defined by [7]

$$b_F = \frac{1}{1+3\lambda^2}\,\Big(\mathrm{Re}(C_S - \bar{C}_S) + 3\lambda\,\mathrm{Re}(C_T - \bar{C}_T)\Big),$$
$$b_{\rm ST} = \frac{1}{1+3\lambda^2}\,\Big(\lambda\,\mathrm{Re}(C_S - \bar{C}_S) + (1-2\lambda)\,\mathrm{Re}(C_T - \bar{C}_T)\Big), \quad (9)$$

where C_S, $\bar{C}_S$, C_T and $\bar{C}_T$ are phenomenological coupling constants of scalar and tensor interactions beyond the SM [7]. The contribution of the ECVC effect to the correlation coefficient $D(E_e)$ is caused by the distortion of the electron wave function in the Coulomb field of the proton [34–36] (see also [8,37–40]).

3. Electron Asymmetry $A_{\rm exp}(E_e)$

The electron asymmetry, caused by the correlations between the electron 3-momentum and neutron spin, is defined by the electron energy spectrum and angular distribution Equation (7) integrated over the antineutrino 3-momentum $\vec{k}_{\nu}$ [7]. Having integrated over directions of $\vec{k}_{\nu}$ we get

$$\frac{d^3\lambda_n(E_e,\vec{k}_e,\vec{\xi}_n)}{dE_e d\Omega_e} = (1+3\lambda^2)\,\frac{G_F^2|V_{ud}|^2}{8\pi^4}\,(E_0-E_e)^2\sqrt{E_e^2-m_e^2}\,E_e\,F(E_e,Z=1)\,\zeta(E_e)\Big\{1+\Big[b_F-\frac{1}{1+3\lambda^2}\\ \times\frac{m_e}{\beta E_e}\,\frac{\Delta}{2E_{\nu}}\Big(1-\frac{\Delta}{4E_{\nu}}\Big)\,\ell n\Big(\frac{m_e^2+2E_eE_{\nu}(1+\beta)}{m_e^2+2E_eE_{\nu}(1-\beta)}\Big)-\frac{1}{1+3\lambda^2}\,\frac{\Delta}{2E_{\nu}}\,\frac{m_e^3\Delta}{(m_e^2+2E_eE_{\nu})^2-4E_e^2E_{\nu}^2\beta^2}\Big]\,\frac{m_e}{E_e}\\ +\Big[A^{(\rm SM)}(E_e)-(A_0+B_0)\,\frac{m_e^2\Delta}{8\beta E_e^3}\,\Big(\frac{m_e^2+2E_eE_{\nu}}{2\beta^2E_{\nu}^2}\,\ell n\Big(\frac{m_e^2+2E_eE_{\nu}(1+\beta)}{m_e^2+2E_eE_{\nu}(1-\beta)}\Big)-\frac{2E_e}{\beta E_{\nu}}\Big)\Big]\,\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e}\Big\}. \quad (10)$$

The correlation coefficient $A^{(\rm SM)}(E_e)$ is fully defined by the contributions of the SM interactions, including the $1/M$ corrections, caused by the weak magnetism and proton recoil [41], and radiative corrections of order $O(\alpha/\pi)$. It is equal to [7]

$$A^{(\rm SM)}(E_e) = A^{(\rm W)}(E_e)\,\Big(1+\frac{\alpha}{\pi}\,f_n(E_e)\Big), \quad (11)$$

where the correlation coefficient $A^{(\rm W)}(E_e)$ has been calculated by Wilkinson [41] (see also Equation (20) of Ref. [7]). The function $(\alpha/\pi)\,f_n(E_e)$ describes the radiative corrections [42] (see also [6,7]). Following [7], the expression of the electron asymmetry $A_{\rm exp}(E_e)$ is given by

$$A_{\rm exp}(E_e) = \frac{N^+(E_e)-N^-(E_e)}{N^+(E_e)+N^-(E_e)} = \frac{1}{2}\,\mathcal{A}(E_e)\,P\beta\,(\cos\theta_1+\cos\theta_2), \quad (12)$$

where $N^{\pm}(E_e)$ are the numbers of events of the emission of the electron forward $(+)$ and backward $(-)$ with respect to the neutron spin into the solid angle $\Delta\Omega_{12} = 2\pi(\cos\theta_1 - \cos\theta_2)$ with $0 \le \varphi \le 2\pi$ and $\theta_1 \le \theta_e \le \theta_2$. Then, $P = |\vec{\xi}_n| \le 1$ is the neutron polarization and $\mathcal{A}(E_e)$ is the correlation coefficient, taking into account the contribution of the ECVC effect and of the Fierz interference term. It is equal to

$$\mathcal{A}(E_e) = \Big\{A^{(\rm SM)}(E_e)-(A_0+B_0)\,\frac{m_e^2\Delta}{8\beta E_e^3}\Big[\frac{m_e^2+2E_eE_{\nu}}{2\beta^2E_{\nu}^2}\,\ell n\Big(\frac{m_e^2+2E_eE_{\nu}(1+\beta)}{m_e^2+2E_eE_{\nu}(1-\beta)}\Big)-\frac{2E_e}{\beta E_{\nu}}\Big]\Big\}\Big\{1+\Big[b_F-\frac{1}{1+3\lambda^2}\\ \times\frac{m_e}{\beta E_e}\,\frac{\Delta}{2E_{\nu}}\Big(1-\frac{\Delta}{4E_{\nu}}\Big)\,\ell n\Big(\frac{m_e^2+2E_eE_{\nu}(1+\beta)}{m_e^2+2E_eE_{\nu}(1-\beta)}\Big)-\frac{1}{1+3\lambda^2}\,\frac{\Delta}{2E_{\nu}}\,\frac{m_e^3\Delta}{(m_e^2+2E_eE_{\nu})^2-4E_e^2E_{\nu}^2\beta^2}\Big]\,\frac{m_e}{E_e}\Big\}^{-1}. \quad (13)$$

In Figure 1, we plot the correlation coefficients $\mathcal{A}(E_e)$, $A^{(\rm SM)}(E_e)$ in the electron energy region $m_e \le E_e \le E_0$ (left), and the denominator of the correlation coefficient $\mathcal{A}(E_e)$ for the Fierz interference term $b_F = 0.1219$ and $b_F = 0$, respectively (right). In Figure 2, we show $\beta\mathcal{A}(E_e)$ for $b_F = 0.1219$ and $b_F = 0$, respectively, and $\beta A^{(\rm SM)}(E_e)$. The vertical lines constrain the experimental electron energy region $0.761\,{\rm MeV} \le E_e \le 0.966\,{\rm MeV}$ [7].

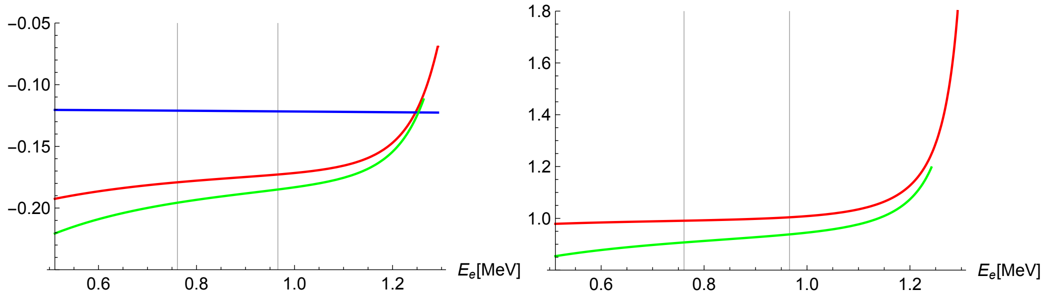

Figure 1. (**Left**) The correlation coefficient $\mathcal{A}(E_e)$, given by Equation (13) and calculated in the electron energy region $m_e \le E_e \le E_0$ for $b_F = 0.1219$ (red) and $b_F = 0$ (green), and the correlation coefficient $A^{(\mathrm{SM})}(E_e)$ (blue). (**Right**) The denominator of the correlation coefficient $\mathcal{A}(E_e)$, plotted in the electron energy region $m_e \le E_e \le E_0$ with $b_F = 0.1219$ (red) and $b_F = 0$ (green).

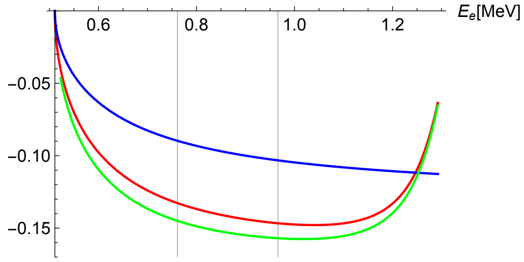

Figure 2. The correlation coefficient $\beta\mathcal{A}(E_e)$, given by Equation (13) and calculated in the electron energy region $m_e \le E_e \le E_0$ for $b_F = 0.1219$ (red) and $b_F = 0$ (green), and the correlation coefficient $\beta A^{(\mathrm{SM})}(E_e)$ (blue). The interval between lines $E_e = 0.761\,\mathrm{MeV}$ and $E_e = 0.966\,\mathrm{MeV}$ defines the experimental electron energy region.

We discuss the results of the theoretical analysis of the electron asymmetry in comparison with the experimental data in Section 4.

4. Discussions and Proposals

We have continued the analysis of the reliability of the hypothesis of exact conservation of the charged weak hadronic vector current in the framework of neutron β^--decay, even for different masses of the neutron and proton, which we have begun in [16]. The contributions of the phenomenological term in the matrix element of the hadronic $n \to p$ transition, which is responsible for the Exact Conservation of the charged weak hadronic Vector Current, are referred to as the ECVC effect. We have calculated the contributions of the ECVC effect together with the contributions of scalar and tensor interactions beyond the SM, taken in the linear approximation, to the correlation coefficients of the electron energy and angular distributions of neutron β^--decay with a polarized neutron and unpolarized proton and electron. We have analyzed the validity of the ECVC hypothesis using example of the electron asymmetry, which is caused by correlations of the electron 3-momentum and neutron spin. In Figures 1 and 2, we plot the correlation coefficient $\mathcal{A}(E_e)$ for $b_F = 0.1219(12)$ (red line), for $b_F = 0$ (green line) and also the correlation coefficient $A^{(\mathrm{SM})}(E_e)$ (blue line), defined by Equation (11). One may see that the correlation coefficient $A^{(\mathrm{SM})}(E_e)$ agrees well with the experimental values of the correlation coefficient A_0: (i) $A_0^{(\mathrm{exp})} = -0.11933(34)$ [1], (ii) $A_0^{(\mathrm{exp})} = -0.11996(58)$ [43], (iii) $A_0^{(\mathrm{exp})} = -0.11966 \pm 0.00089^{+0.00123}_{-0.00140}$ [44], and (iv) $A_0^{(\mathrm{exp})} = -0.11832(78)$ [45]. As has been shown in [7], the deviations of $A^{(\mathrm{SM})}(E_e)$ from A_0 are of order 10^{-3}. This explains a weak energy dependence of $A^{(\mathrm{SM})}(E_e)$ on the electron energy E_e. Unfortunately, the energy dependence of the correlation coefficient $\mathcal{A}(E_e)$, corrected by the contribution of the ECVC effect and the Fierz interference term, differs crucially from that of the correlation coefficient $A^{(\mathrm{SM})}(E_e)$. This also means that the correlation coefficient $\mathcal{A}(E_e)$, given by Equation (13), is unable to reproduce the experimental data of the electron asymmetry, giving the experimental values of the correlation coefficients A_0 [1,43–45].

At this point, we do not want to argue immediately that the obtained result means that (i) the ECVC hypothesis is not valid, even in the SM with scalar and tensor interactions beyond the SM; nor that (ii) the charged weak hadronic vector current is not conserved in the hadronic $h \to h'$ transition with different masses of incoming h and outgoing h' hadrons in the sense of a vanishing matrix element $\langle h'|\partial^\mu V^{(+)}_\mu(0)|h\rangle = 0$. Hence, we may argue that the account for the ECVC effect together with the contributions of scalar and tensor interactions beyond the SM calculated in the linear approximation does not support the ECVC hypothesis in the neutron β^--decay, at least using example of the electron asymmetry.

As a result, in order to avoid the problem of invalidity of the ECVC hypothesis with scalar and tensor interactions beyond the SM and the experimental data, we propose to take into account a complete set of contributions of scalar and tensor interactions beyond the SM, which include also the quadratic terms [7]. Using the results, obtained in [7], we get the following expressions for the neutron lifetime [16] and correlation coefficients

$$\begin{aligned}
\frac{1}{\tau_n^{(\rm eff)}} &= \frac{1}{\tau_n^{(\rm SM)}}\Big(1 + \frac{1}{2}\,\frac{1}{1+3\lambda^2}\,\Big(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2\Big) + \frac{\Delta f_n}{f_n} + b_F\,\Big\langle \frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big),\\
a_{\rm eff}(E_e,\vec{k}_e\cdot\vec{k}_\nu) &= \Big(a(E_e) - \frac{1}{1+3\lambda^2}\,\frac{m_e^2\Delta^2}{q^4} + \frac{1}{1+3\lambda^2}\,\frac{1}{2}\Big(|C_T|^2 + |\bar{C}_T|^2 - |C_S|^2 - |\bar{C}_S|^2\Big)\Big)/Y(E_e),\\
A_{\rm eff}(E_e,\vec{k}_e\cdot\vec{k}_\nu) &= \Big(A(E_e) - \frac{1}{1+3\lambda^2}{\rm Re}\Big(2C_T\bar{C}^*_T + (C_S\bar{C}^*_T + \bar{C}_S C^*_T)\Big)/Y(E_e),\\
B_{\rm eff}(E_e,\vec{k}_e\cdot\vec{k}_\nu) &= \Big(B(E_e) - \frac{1}{2}\,(A_0+B_0)\,\frac{m_e^2\Delta}{q^2E_e} - b_{ST}\,\frac{m_e}{E_e} - \frac{1}{1+3\lambda^2}{\rm Re}\Big(2C_T\bar{C}^*_T - (C_S\bar{C}^*_T + \bar{C}_S C^*_T)\Big)/Y(E_e),\\
D_{\rm eff}(E_e,\vec{k}_e\cdot\vec{k}_\nu) &= \Big(D(E_e) + \frac{2\,{\rm Im}\lambda}{1+3\lambda^2} + \frac{1}{1+3\lambda^2}\,\frac{1}{2}\,{\rm Im}\Big(C_S C^*_T + \bar{C}_S\bar{C}^*_T\Big)\Big)/Y(E_e),
\end{aligned} \tag{14}$$

with a common denominator

$$Y(E_e) = 1 + \frac{1}{2}\,\frac{1}{1+3\lambda^2}\,\Big(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2\Big) + \Big[b_F - \frac{1}{1+3\lambda^2}\,\Big(\frac{2m_e\Delta}{q^2} - \frac{m_eE_e\Delta^2}{q^4}\Big)\Big]\,\frac{m_e}{E_e}, \tag{15}$$

where $\langle m_e/E_e\rangle_{\rm SM} = 0.6556$ [7,16], calculated for the electron energy density Equation (D.59) of Ref. [7]. The correlation coefficients $a(E_e)$, $A(E_e)$, $B(E_e)$ and $D(E_e)$ are calculated in the SM by taking into account the $1/M$ corrections, caused by the weak magnetism and proton recoil, and radiative corrections of order $O(\alpha/\pi)$ [7]. Then, Δf_n is the phase-space factor of neutron β^--decay, caused by the ECVC effect. It is given by

$$\begin{aligned}
\Delta f_n &= \frac{1}{1+3\,\lambda^2}\int_{m_e}^{E_0} dE_e\,k_e\,(E_0-E_e)^2\,F(E_e,Z=1)\int\frac{d\Omega_{e\nu}}{4\pi}\Big\{-\frac{2\,m_e^2\,\Delta}{m_e^2 + 2\,E_e(E_0-E_e) - 2\,\vec{k}_e\cdot\vec{k}_\nu}\\
&+\frac{m_e^2\Delta^2}{\Big(m_e^2 + 2\,E_e(E_0-E_e) - 2\,\vec{k}_e\cdot\vec{k}_\nu\Big)^2}\Big(E_e - \frac{\vec{k}_e\cdot\vec{k}_\nu}{E_\nu}\Big)\Big\} = \int_{m_e}^{E_0} dE_e\,\sqrt{E_e^2 - m_e^2}\,(E_0-E_e)^2\,F(E_e,Z=1)\\
&\times\Big\{-\frac{1}{1+3\lambda^2}\,\frac{m_e^2}{\beta E_e}\,\frac{\Delta}{2E_\nu}\,\Big(1 - \frac{\Delta}{4E_\nu}\Big)\,\ell n\Big(\frac{m_e^2 + 2E_eE_\nu(1+\beta)}{m_e^2 + 2E_eE_\nu(1-\beta)}\Big) - \frac{1}{1+3\lambda^2}\,\frac{\Delta}{2E_\nu}\,\frac{m_e^4\Delta}{(m_e^2 + 2E_eE_\nu)^2 - 4E_e^2E_\nu^2\beta^2}\Big\}
\end{aligned} \tag{16}$$

and is equal to $\Delta f_n = -4.887\times 10^{-3}\,{\rm MeV}^5$ [16], where $f_n = 6.116\times 10^{-2}\,{\rm MeV}^5$ is the phase-space factor of neutron β^--decay, calculated to order $O(1/M)$ and $O(\alpha/\pi)$ caused by the contributions of the weak magnetism and proton recoil and the radiative corrections, respectively [7]. As we have shown in [16], the contribution of the term $\Delta f_n = -4.887\times 10^{-3}\,{\rm MeV}^5$, caused by the ECVC effect, changes the value of the neutron lifetime by $\Delta\tau_n = (-\Delta f_n/(f_n + \Delta f_n))\,\tau_n^{(\rm SM)} = 76.4\,{\rm s}$. Thus, fitting the neutron lifetime $\tau_n^{(\rm eff)} = \tau_n^{(\rm SM)}$ at the level of current experimental accuracy 1.2×10^{-3} or one standard deviation [16], where $\tau_n^{(\mathrm{SM})} = 879.6(1.1)$ s, calculated in [7], agrees well with the world averaged value $\tau_n = 880.2(1.0)$ s [9] and the experimental value $\tau_n^{(\exp)} = 880.2(1.2)$ s, we get

$$\frac{1}{2}\,\frac{1}{1+3\lambda^2}\Big(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2\Big) = -\frac{\Delta f_n}{f_n} - b_F\left\langle\frac{m_e}{E_e}\right\rangle_{\mathrm{SM}}. \tag{17}$$

Neglecting the contributions of the quadratic terms, we arrive at the equation [16]

$$\frac{\Delta f_n}{f_n} + b_F\left\langle\frac{m_e}{E_e}\right\rangle_{\mathrm{SM}} = 0, \tag{18}$$

fixing the value of the Fierz interference term $b_F = 0.1219(12)$ in terms of the contribution of the ECVC effect [16]. Plugging Equation (17) into Equation (14), we obtain

$$\begin{aligned}
a_{\mathrm{eff}}(E_e, \vec{k}_e\cdot\vec{k}_\nu) &= \Big(a(E_e) - \tfrac{1}{1+3\lambda^2}\,\tfrac{m_e^2\Delta^2}{q^4} + \tfrac{1}{1+3\lambda^2}\,\tfrac{1}{2}\Big(|C_T|^2 + |\bar{C}_T|^2 - |C_S|^2 - |\bar{C}_S|^2\Big)\Big)/Z(E_e),\\
A_{\mathrm{eff}}(E_e, \vec{k}_e\cdot\vec{k}_\nu) &= \Big(A(E_e) - \tfrac{1}{1+3\lambda^2}\mathrm{Re}\Big(2C_T\bar{C}_T^* + (C_S\bar{C}_T^* + \bar{C}_S C_T^*)\Big)\Big)/Z(E_e),\\
B_{\mathrm{eff}}(E_e, \vec{k}_e\cdot\vec{k}_\nu) &= \Big(B(E_e) - \tfrac{1}{2}\,(A_0 + B_0)\,\tfrac{m_e^2\Delta}{q^2 E_e} - b_{ST}\,\tfrac{m_e}{E_e} - \tfrac{1}{1+3\lambda^2}\mathrm{Re}\Big(2C_T\bar{C}_T^* - (C_S\bar{C}_T^* + \bar{C}_S C_T^*)\Big)\Big)/Z(E_e),\\
D_{\mathrm{eff}}(E_e, \vec{k}_e\cdot\vec{k}_\nu) &= \Big(D(E_e) - \tfrac{1}{2}\,(A_0 + B_0)\,\tfrac{\alpha\, m_e^2\Delta}{q^2 k_e} + \tfrac{1}{1+3\lambda^2}\,\tfrac{1}{2}\,\mathrm{Im}\Big(C_S C_T^* + \bar{C}_S\bar{C}_T^*\Big)\Big)/Z(E_e),
\end{aligned} \tag{19}$$

with a common denominator

$$Z(E_e) = 1 - \frac{\Delta f_n}{f_n} - b_F\left\langle\frac{m_e}{E_e}\right\rangle_{\mathrm{SM}} + \left[b_F - \frac{1}{1+3\lambda^2}\left(\frac{2m_e\Delta}{q^2} - \frac{m_e E_e\Delta^2}{q^4}\right)\right]\frac{m_e}{E_e}. \tag{20}$$

This is a complete set of correlation coefficients, which can be used for a fit of the experimental data on the electron energy and angular distributions of neutron β^--decay with a polarized neutron and unpolarized proton and electron.

Since our numerical analysis of the relative contributions of the ECVC effect to the cross sections for the quasi-elastic electron neutrino–neutron scattering and for the inverse β-decay, carried out in Appendix A (see Figure A1), shows that these processes are not sensitive to the ECVC effect, the experimental analysis of the correlation coefficients, given in Equation (19), is a matter of great importance for understanding whether the ECVC hypothesis is correct in the sense that a matrix element of the divergence of the charged weak hadronic vector current $\langle h'|\partial^\mu V_\mu^{(+)}(0)|h\rangle$ vanishes, i.e., $\langle h'|\partial^\mu V_\mu^{(+)}(0)|h\rangle = 0$, for the hadronic $h \to h'$ transitions, even for different masses of incoming h and outgoing h hadrons, or not. However, it is obvious that without contributions of interactions beyond the SM, such a hypothesis cannot be fulfilled.

If such an experimental analysis of the ECVC effect shows that the correlation coefficients Equation (19) are irrelevant, the contributions of the term $F_4(q^2)q_\mu\hat{q}/M^2$ to the neutron lifetime and correlation coefficients of neutron β^--decay should be of order $O(m_e\Delta/M^2)$ or even smaller in comparison with the results obtained in [7,8].

In turn, in case of positive results of the experimental analysis of the ECVC effect using the correlation coefficients Equation (19), they should be taken into account for the description of the quasi-elastic neutrino–neutron scattering, the inverse β-decay, as well as in the energy spectra of the neutrino production of nucleon resonances. The values of the scalar and tensor phenomenological coupling constants $C_S, \bar{C}_S, C_T$ and $\bar{C}_T$ together with the Fierz interference term b_F and the correlation coefficients b_{ST}, which can be obtained by means of the fit of the experimental data on different asymmetries of neutron β^--decay.

5. Conclusions

We are examining the validity of the hypothesis regarding the precise conservation of the charged weak hadronic vector current in neutron beta-decay, utilizing polarized neutrons and unpolarized protons and electrons. Our analysis involves the computation of contributions from the phenomenological term, which is responsible for the Exact Conservation of the charged weak hadronic Vector Current (ECVC effect) in neutron beta-decay, even when considering varying masses of the neutron and proton. We calculate correlation coefficients, incorporating the complete set of contributions from scalar and tensor interactions beyond the Standard Model (SM). We posit that if the cumulative effects of scalar and tensor interactions beyond the SM fail to reconcile with the experimental data on correlation coefficients alongside the contributions of the ECVC effect, it may be inferred that the conservation of the charged weak hadronic vector current is not maintained in the hadronic transitions of weak processes involving distinct masses of incoming and outgoing hadrons.

Author Contributions: D.A.: Conceptualization, Methodology, Data Curation, Formal analysis, Investigation, Writing—Original draft preparation. R.H.: Conceptualization, Formal analysis, Data Curation, Investigation, Methodology, Software, Validation, Writing—Original draft preparation. M.W.: Conceptualization, Formal analysis, Data Curation, Investigation, Methodology, Software, Validation, Writing—Original draft preparation. All authors contributed equally and agreed to the published version of the manuscript. The sole responsibility for the content of this publication lies with the authors.

Funding: The work of M. Wellenzohn was supported by MA 23 (p.n. 30-22).

Data Availability Statement: The data and illustrations presented in this study can be obtained directly from the equations. All data are available on request from the corresponding author.

Acknowledgments: We want to thank our dear colleague Andrey Nikolaevich Ivanov, who was the main investigator of this work until he sadly passed away on 18 December 2021. We see it as our professional and personal duty to honor his legacy by continuing to publish our collaborative work. Andrey was born on 3 June 1945 in what was then Leningrad. Since 1993 he was a university professor at the Faculty of Physics, named "Peter The Great St. Petersburg Polytechnic University" after Peter the Great. Since 1995 he has been a guest professor at the Institute for Nuclear Physics at the Vienna University of Technology for several years and has been closely associated with the institute ever since. This is also were we met Andrey and have been collaborating with him closely for more than 20 years, resulting in 40 scientific publications; see also [46–56]. We will miss Andrey as a personal friend and we will miss his immense wealth of ideas, scientific skills and his creativity. See also the official obituary (https://www.tuwien.at/en/phy/ati/news/test, accessed on 3 November 2024) for Andrey Nikolaevich Ivanov. The work of M. Wellenzohn was supported by MA 23 (p.n. 27-07 and p.n. 30-22). The sole responsibility for the content of this publication lies with the authors.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Cross Sections for the Quasi-Elastic Electron Neutrino–Neutron Scattering and the Inverse β-Decay

In this Appendix we calculate the cross sections for the quasi-elastic scattering $\nu_e + n \to p + e^-$ and the inverse β-decay $\bar{\nu}_e + p \to n + e^+$ by taking into account the contributions of the ECVC effect. Neglecting the contributions of the weak magnetism, recoil and radiative corrections and skipping intermediate standard calculations, we obtain the following cross sections for the quasi-elastic electron neutrino–neutron scattering $\sigma(E_\nu)$ and the inverse β-decay $\sigma(E_{\bar{\nu}})$:

$$\begin{aligned}\sigma(E_\nu) &= \sigma_0(E_\nu) + \frac{G_F^2|V_{ud}|^2}{2\pi}\,k_-E_-\Big\{-\frac{m_e^2\Delta}{k_-E_-E_\nu}\Big[\ell n\Big(1+\frac{2k_-E_\nu\beta_-}{m_e^2-2E_-E_\nu}\Big)-\ell n\Big(1-\frac{2k_-E_\nu}{m_e^2-2E_-E_\nu}\Big)\Big]\\ &+ \frac{2m_e^2\Delta^2}{(m_e^2-2E_-E_\nu)^2-4k_-^2E_\nu^2}-\frac{m_e^2\Delta^2}{4k_-E_-E_\nu^2}\Big[\ell n\Big(1+\frac{2k_-E_\nu\beta_-}{m_e^2-2E_-E_\nu}\Big)-\ell n\Big(1-\frac{2k_-E_\nu}{m_e^2-2E_-E_\nu}\Big)\\ &- \frac{4k_-E_\nu(m_e^2-2E_-E_\nu)}{(m_e^2-2E_-E_\nu)^2-4k_-^2E_\nu^2}\Big]\Big\},\end{aligned} \quad \text{(A1)}$$

where $E_- = E_\nu + \Delta$, $k_- = \sqrt{E_-^2 - m_e^2}$ and $\beta_- = k_-/E_-$ are the energy, momentum and velocity of the electron, and

$$\begin{aligned}\sigma(E_{\bar\nu}) &= \sigma_0(E_{\bar\nu}) + \frac{G_F^2|V_{ud}|^2}{2\pi}\,k_+E_+\Big\{\frac{m_e^2\Delta}{k_+E_+E_{\bar\nu}}\Big[\ell n\Big(1+\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Big) - \ell n\Big(1-\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Big)\Big]\\ &+ \frac{2m_e^2\Delta^2}{(m_e^2-2E_+E_{\bar\nu})^2-4k_+^2E_{\bar\nu}^2} - \frac{m_e^2\Delta^2}{4k_+E_+E_{\bar\nu}^2}\Big[\ell n\Big(1+\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Big) - \ell n\Big(1-\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Big)\\ &- \frac{4k_+E_{\bar\nu}(m_e^2-2E_+E_{\bar\nu})}{(m_e^2-2E_+E_{\bar\nu})^2-4k_+^2E_{\bar\nu}^2}\Big]\Big\},\end{aligned} \tag{A2}$$

where $E_+ = E_{\bar\nu} - \Delta$ and $k_+ = \sqrt{E_+^2 - m_e^2}$ are the energy and momentum of the positron. The cross sections $\sigma_0(E_\nu)$ and $\sigma_0(E_{\bar\nu})$ are given by [25]

$$\sigma_0(E_\nu) = (1+3\lambda^2)\,\frac{G_F^2|V_{ud}|^2}{\pi}\,k_-E_- \quad , \quad \sigma_0(E_{\bar\nu}) = (1+3\lambda^2)\,\frac{G_F^2|V_{ud}|^2}{\pi}\,k_+E_+. \tag{A3}$$

In the quasi-elastic electron neutrino–neutron scattering and the inverse β-decay the energies of neutrino and antineutrino vary in the regions $E_\nu \geq 0$ and $E_{\bar\nu} \geq (E_{\bar\nu})_{\rm thr} = ((m_n+m_e)^2 - m_p^2)/2m_p = 1.8061\,{\rm MeV}$ [25]. The terms dependent on Δ are caused by the ECVC effect. We define the relative contributions of the ECVC effect to the cross sections under consideration as follows

$$\begin{aligned}R_\nu(E_\nu) &= \frac{1}{2}\,\frac{1}{1+3\lambda^2}\Big\{-\frac{m_e^2\Delta}{k_-E_-E_\nu}\Big[\ell n\Big(1+\frac{2k_-E_\nu\beta_-}{m_e^2-2E_-E_\nu}\Big) - \ell n\Big(1-\frac{2k_-E_\nu}{m_e^2-2E_-E_\nu}\Big)\Big] + \frac{2m_e^2\Delta^2}{(m_e^2-2E_-E_\nu)^2-4k_-^2E_\nu^2}\\ &-\frac{m_e^2\Delta^2}{4k_-E_-E_\nu^2}\Big[\ell n\Big(1+\frac{2k_-E_\nu\beta_-}{m_e^2-2E_-E_\nu}\Big) - \ell n\Big(1-\frac{2k_-E_\nu}{m_e^2-2E_-E_\nu}\Big) - \frac{4k_-E_\nu(m_e^2-2E_-E_\nu)}{(m_e^2-2E_-E_\nu)^2-4k_-^2E_\nu^2}\Big]\Big\},\end{aligned} \tag{A4}$$

and

$$\begin{aligned}R_{\bar\nu}(E_{\bar\nu}) &= \frac{1}{2}\,\frac{1}{1+3\lambda^2}\Big\{\frac{m_e^2\Delta}{k_+E_+E_{\bar\nu}}\Big[\ell n\Big(1+\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Big) - \ell n\Big(1-\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Big)\Big] + \frac{2m_e^2\Delta^2}{(m_e^2-2E_+E_{\bar\nu})^2-4k_+^2E_{\bar\nu}^2}\\ &-\frac{m_e^2\Delta^2}{4k_+E_+E_{\bar\nu}^2}\Big[\ell n\Big(1+\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Big) - \ell n\Big(1-\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Big) - \frac{4k_+E_{\bar\nu}(m_e^2-2E_+E_{\bar\nu})}{(m_e^2-2E_+E_{\bar\nu})^2-4k_+^2E_{\bar\nu}^2}\Big]\Big\},\end{aligned} \tag{A5}$$

where $R_\nu(E_\nu) = \Delta\sigma(E_\nu)/\sigma_0(E_\nu)$, $R_{\bar\nu}(E_{\bar\nu}) = \Delta\sigma(E_{\bar\nu})/\sigma_0(E_{\bar\nu})$ with $\Delta\sigma(E_\nu) = \sigma(E_\nu) - \sigma_0(E_\nu)$ and $\Delta\sigma(E_{\bar\nu}) = \sigma(E_{\bar\nu}) - \sigma_0(E_{\bar\nu})$, respectively. The cross sections Equations (A1) and (A2) are calculated in the laboratory frame in the non-relativistic approximation for outgoing hadrons. Since the most important region of the antineutrino energies for the inverse β-decay is $2\,{\rm MeV} \leq E_{\bar\nu} \leq 8\,{\rm MeV}$ [25], in Figure A1, we plot $R_\nu(E_\nu)$ and $R_{\bar\nu}(E_{\bar\nu})$ for E_ν and $E_{\bar\nu}$ varying over the regions $2\,{\rm MeV} \leq E_\nu \leq 8\,{\rm MeV}$ and $2\,{\rm MeV} \leq E_{\bar\nu} \leq 8\,{\rm MeV}$, respectively.

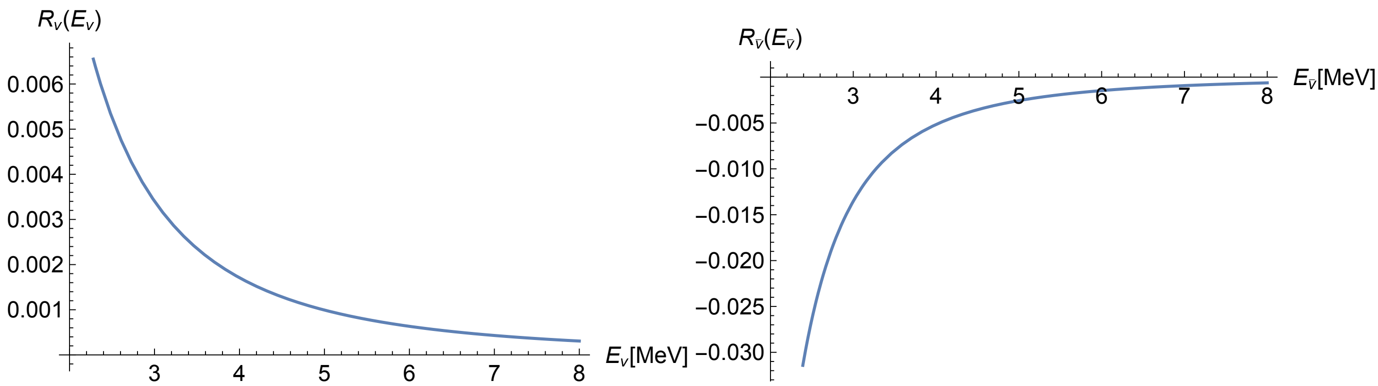

Figure A1. The relative contributions $R_\nu(E_\nu)$ (**left**) and $R_{\bar\nu}(E_{\bar\nu})$ (**right**) of the ECVC effect to the cross sections for the quasi-elastic electron neutrino–neutron and inverse β-decay in the neutrino and antineutrino energy regions $2\,{\rm MeV} \leq E_\nu \leq 8\,{\rm MeV}$ and $2\,{\rm MeV} \leq E_{\bar\nu} \leq 8\,{\rm MeV}$, calculated for $\lambda = -1.2750$ [25].

Our numerical examination of the relative impacts of the ECVC effect on the cross sections for both quasi-elastic electron neutrino–neutron scattering and inverse β-decay indicates that these processes exhibit low sensitivity to the ECVC effect. Specifically, the contribution of the ECVC effect to the cross section for quasi-elastic electron neutrino–neutron scattering is less than 0.7 % at $E_\nu \simeq 2\,\mathrm{MeV}$ and diminishes significantly by approximately two orders of magnitude at $E_\nu \simeq 8\,\mathrm{MeV}$. The cross-section analysis for inverse β-decay, utilized in examining the deficit of positrons induced by reactor electron antineutrinos [25,26], should be averaged over the reactor electron antineutrino energy spectrum, which peaks at $E_{\bar{\nu}} \simeq 4\,\mathrm{MeV}$. According to Figure A1, the contribution of the ECVC effect is expected to decrease the yield of positrons Y_{e^+} by about 0.5 %. Given that this contribution is smaller than the experimental error bars $Y_{e^+} = 0.943(23)$ [26], one could argue that the inverse β-decay is insensitive to the influence of the ECVC effect.

References

1. Abele, H. The neutron. Its properties and basic interactions. *Progr. Part. Nucl. Phys.* **2008**, *60*, 1. [CrossRef]
2. Nico, J.S. Neutron beta decay. *J. Phys. G Nucl. Part. Phys.* **2009**, *36*, 104001. [CrossRef]
3. Abele, H.; Dubbers, D.; Abele, H.; Bäßler, S.; Märkisch, B.; Schumann, M.; Soldner, T.; Zimmer, O. A clean, bright, and versatile source of neutron decay products. *Nucl. Instr. Meth. Phys. Res. A* **2008**, *596*, 238.
4. Konrad, G. NoMoS: Beyond the Standard Model Physics in Neutron Decay. PoS EPS-HEP2015, 592. In Proceedings of the 2015 European Physical Society Conference on High Energy Physics (EPS-HEP 2015), Vienna, Austria, 22–29 July 2015.
5. Abele, H. Precision experiments with cold and ultracold neutrons. Neutron β-decay and gravity resonance spectroscopy. *Hyperfine Interact.* **2016**, *155*, 237.
6. Gudkov, V.; Greene, G.I.; Calarco, J.R. General classification and analysis of neutron beta-decay experiments. *Phys. Rev. C* **2006**, *73*, 035501. [CrossRef]
7. Ivanov, A.N.; Pitschmann, M.; Troitskaya, N.I. Neutron beta decay as a laboratory for testing the standard model. *Phys. Rev. D* **2013**, *88*, 073002. [CrossRef]
8. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Precision analysis of electron energy spectrum and angular distribution of neutron $\beta-$decay with polarized neutron and electron. *Phys. Rev. C* **2017**, *95*, 055502; Erratum in *Phys. Rev. C* **2021**, *104*, 069901. [CrossRef]
9. Partignani C.; Particle Data Group. Review of Particle Physics. *Chin. Phys. C* **2016**, *40*, 100001.
10. Marshak, R.E.; Riazuddin; Ryan, C.P. *Theory of Weak interactions in Particle Physics*; Wiley-Interscience, a Division of John Wiley & Sons, Inc.: New York, NY, USA, 1969; p. 41.
11. Leitner, T.; Alvarez–Ruso, L.; Mosel, U. Charged current neutrino nucleus interactions at intermediate energies. *Phys. Rev. C* **2006**, *73*, 065502. [CrossRef]
12. Ivanov, A.N. Comment on "On the implementation of CVC in weak charged-current proton-neutron transitions" by C. Giunti. arXiv: 1602.00215 [hep-ph]. *arXiv* **2017**, arXiv:1705.09573.
13. Ivanov, A.N. Lorentz Structure of Vector Part of Matrix Elements of Transitions $n \longleftrightarrow p$, Caused by Strong Low-Energy Interactions and Hypothesis of Conservation of Charged Vector Current. *arXiv* **2017**, arXiv:1705.11102.
14. Weinberg, S. Charge Symmetry of Weak Interactions. *Phys. Rev.* **1958**, *112*, 1375. [CrossRef]
15. Berman S.M.; Sirlin, A. Some Considerations on the Radiative Corrections to Muon and Neutron Decay. *Ann. Phys. N. Y.* **1962**, *20*, 20. [CrossRef]
16. Altarawneh, D.; Höllwieser, R.; Wellenzohn, M. On the Hypothesis of Exact Conservation of Charged Weak Hadronic Vector Current in the Standard Model. *Preprints* **2024**, *1*, 2024102486. [CrossRef]
17. Dufner, A.J.; Tasi, Y.S. Phenomenological Analysis of the Gamma NN^* Form-factors. *Phys. Rev.* **1968**, *168*, 1801. [CrossRef]
18. Schreiner, P.A.; von Hippel, F.$\nu p \to \mu^- \Delta^{++}$ comparison with theory. *Phys. Rev. Lett.* **1973**, *30*, 339. [CrossRef]
19. Schreiner, P.A.; von Hippel, F. Neutrino production of the Delta (1236). *Nucl. Phys. B* **1973**, *58*, 333. [CrossRef]
20. Singh, S.K.; Vicenta–Vacas, M.J. Oset, E. Nuclear effects in neutrinoproduction of Delta at intermediate energies. *Phys. Lett. B* **1998**, *416*, 23. [CrossRef]
21. Lalakulich O.; Paschos, E.A. Resonance production by neutrinos: $J = 3/2$ resonances. *Phys. Rev. D* **2005**, *71*, 074003. [CrossRef]
22. Lalakulich, O.; Paschos, E.A.; Piranishvili, G. Resonance production by neutrinos: The Second resonance region. *Phys. Rev. D* **2006**, *74*, 014009. [CrossRef]
23. Lalakulich, O.; Paschos, E.A.; Piranishvili, G.Resonance production by neutrinos. *Nucl. Phys. B (Proc. Suppl.)* **2006**, *159*, 133. [CrossRef]
24. Ankowski, A.M. Improved estimate of the cross section for inverse beta decay. *arXiv* **2016**, arXiv:1601.06169v1. [CrossRef]
25. Ivanov, A.N.; Höllwieser, R.H.; Troitskaya, N.I.; Wellenzohn, M.; Zherebtsov, O.M.; Serebrov, A.P. Deficit of reactor antineutrinos at distances smaller than 100 m and inverse beta decay. *Phys. Rev. C* **2013**, *88*, 055501. [CrossRef]
26. Mention, G. The reactor antineutrino anomaly. *J. Phys. Conf. Ser.* **2013**, *408*, 012025. [CrossRef]

27. Abazajian, K.N.; Acero, M.A.; Agarwalla, S.K.; Aguilar-Arevalo, A.A.; Albright, C.H.; Antusch, S.; Arguelles, C.A.; Balantekin, A.B.; Barenboim, G.; Barger, V.; et al. Light Sterile Neutrinos: A White Paper. *arXiv* **2016**, arXiv:1204.5379.
28. Gariazzo, S.; Giunti, C.; Laveder, M.; Li, Y.F.; Zavanin, E.M.Light sterile neutrinos. *J. Phys. G* **2016**, *43*, 033001. [CrossRef]
29. Giunti, C. Sterile Neutrino Searches: Experiment and Theory. *Nucl. Part. Phys. Proc.* **2017**, *287–288*, 133. [CrossRef]
30. Arzumanov, S.; Bondarenko, L.; Chernyavsky, S.; Geltenbort, P.; Morozov, V.; Nesvizhevsky, V.V.; Panin, Y.; Strepetov, A. A measurement of the neutron lifetime using the method of storage of ultracold neutrons and detection of inelastically up-scattered neutrons. *Phys. Lett. B* **2015**, *745*, 79. [CrossRef]
31. Ivanov, A.N.; Pitschmann, M.; Troitskaya, N.I.; Berdnikov, Y.A. Bound-state beta decay of the neutron re-examined. *Phys. Rev. C* **2014**, *89*, 055502. [CrossRef]
32. Adler, S.L.; Dashen, R. *Current Algebras*; Benjamin: New York, NY, USA, 1968.
33. De Alfaro, V.; Fubini, S.; Furlan, G.; Rossetti, C. *Currents in Hadron Physics*; North–Holland Publishing Co.: Amsterdam, The Netherlands; London, UK; New York, NY, USA, 1973.
34. Jackson, J.D.; Treiman, S.B.; Wyld, H.W., Jr. Coulomb corrections in allowed beta transitions. *Nucl. Phys.* **1957**, *4*, 206. [CrossRef]
35. Jackson, J.D.; Treiman, S.B.; Wyld, H.W., Jr. Note on relativistic coulomb wave functions. *Z. Phys.* **1958**, *150*, 640. [CrossRef]
36. Konopinski, E.K. *The Theory of Beta Radioactivity*; Oxford at the Clarendon Press: Oxford, UK, 1966.
37. Ivanov, A.N.; Höllwieser, R.; Wellenzohn, M.; Troitskaya, N.I.; Berdnikov, Y.A. Internal bremsstrahlung of beta decay of atomic ${}^{35}_{16}$S. *Phys. Rev. C* **2014**, *90*, 064608.
38. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Neutron dark matter decays and correlation coefficients of neutron β^--decays. *Nucl. Phys. B* **2018**, *938*, 114–130. [CrossRef]
39. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Neutron Dark Matter Decays. *arXiv* **2018**, arXiv:1806.10107.
40. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Tests of the standard model in neutron β decay with a polarized neutron and electron and an unpolarized proton. *Phys. Rev. C* **2018**, *98*, 035503. [CrossRef]
41. Wilkinson, D.H. Analysis Of Neutron Beta Decay. *Nucl. Phys. A* **1982**, *377*, 474. [CrossRef]
42. Shann, R.T. Electromagnetic effects in the decay of polarized neutrons. *Nuovo Cimento A* **1971**, *5*, 591. [CrossRef]
43. Mund, D.; Maerkisch, B.; Deissenroth, M.; Krempel, J.; Schumann, M.; Abele, H.; Petoukhov, A.; Soldner, T. Determination of the weak axial vector coupling from a measurement of the beta asymmetry parameter A in neutron beta decay. *Phys. Rev. Lett.* **2013**, *110*, 172502. [CrossRef]
44. Mendenhall, M.P.; Pattie, R.W., Jr.; Bagdasarova, Y.; Berguno, D.B.; Broussard, L.J.; Carr, R.; Currie, S.; Ding, X.; Filippone, B.W.; Garcia, A.; et al. (UCNA Collaboration) Precision measurement of the neutron beta decay asymmetry. *Phys. Rev. C* **2013**, *87*, 032501. [CrossRef]
45. Maerkisch, B.; Abele, H. Measurement of the axial-vector coupling constant g_A in neutron beta decay. In Proceedings of the 8th International Workshop on the CKM Unitarity Triangle (CKM 2014), Vienna, Austria, 8–12 September 2014.
46. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M. Proton recoil energy and angular distribution of neutron radiative β-decay. *Phys. Rev.* **2013**, *D88*, 065026.
47. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Gauge Properties of Hadronic Structure of Nucleon in Neutron Radiative Beta Decay to Order O(α/π) in Standard V—A Effective Theory with QED and Linear Sigma Model of Strong Low–Energy Interactions. *Int. J. Mod. Phys. A* **2018**, *33*, 1850199. [CrossRef]
48. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Gauge and infrared properties of hadronic structure of nucleon in neutron beta decay to order $O(\alpha/\pi)$ in standard V—A effective theory with QED and linear sigma model of strong low-energy interactions. *Int. J. Mod. Phys. A* **2019**, *34*, 1950010. [CrossRef]
49. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Electrodisintegration of Deuteron into Dark Matter and Proton Close to Threshold. *Symmetry* **2021**, *13*, 2169. [CrossRef]
50. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Tests of the standard model in neutron beta decay with polarized electrons and unpolarized neutrons and protons. *Phys. Rev. D* **2019**, *99*, 053004; Erratum in *Phys. Rev. D* **2021**, *104*, 059902. [CrossRef]
51. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Corrections of order $O(E_e^2/m_N^2)$, caused by weak magnetism and proton recoil, to the neutron lifetime and correlation coefficients of the neutron beta decay. *Results Phys.* **2021**, *21*, 103806. [CrossRef]
52. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Radiative corrections of order $O(\alpha E_e/m_N)$ to Sirlin's radiative corrections of order $O(\alpha/\pi)$, induced by the hadronic structure of the neutron. *Phys. Rev. D* **2021**, *103*, 113007. [CrossRef]
53. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Theoretical description of the neutron beta decay in the standard model at the level of 10-5. *Phys. Rev. D* **2021**, *104*, 033006. [CrossRef]
54. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Structure of the correlation coefficients $S(E_e)$ and $U(E_e)$ of the neutron β decay. *Phys. Rev. C* **2021**, *104*, 025503. [CrossRef]

55. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. On the correlation coefficient $T(E_e)$ of the neutron beta decay, caused by the correlation structure invariant under discrete P, C and T symmetries. *Phys. Lett. B* **2021**, *816*, 136263. [CrossRef]
56. Ivanov, A.N.; Höllwieser, R.; Troitskaya, N.I.; Wellenzohn, M.; Berdnikov, Y.A. Precision analysis of pseudoscalar interactions in neutron beta decays. *Nucl. Phys. B* **2020**, *951*, 114891. [CrossRef]