DISCUSSION

SALECKER: Schopper has shown that there is a maximum slope in the $tg^2\,\theta/2$ diagrams, if one believes in the Rosenbluth formula. Is this maximum slope compatible with the slopes you find in the experiment?

WILSON: I believe that these are quite consistent with the maximum slope.

# MAGNETIC AND ELECTRIC FORM FACTORS

**R. G. Sachs**

University of Wisconsin, Madison, Wis.

*(presented by R. G. Sachs)*

I have been asked to explain our motivation for introducing the form factors $F_{\text{ch}}$ and $F_{\text{mag}}$ in place of the usual $F_1$ and $F_2$ [Ernst, Sachs and Wali, Phys. Rev. *119*, 1105 (1960)]. As you know, the form factors $F_1(q^2)$ and $F_2(q^2)$ are directly related to the matrix element of the current density operator for a transition between two momentum states of the proton, $\langle p'|j_\mu(x)|p\rangle$. The invariant momentum transfer is $q^2 = (p'-p)^2$. (The normalization I shall use is $F_2(0) = \mu_p$, the magnetic moment of the proton.) The charge and magnetic form factors used by the previous speaker, but with our normalization, are

$$F_{\text{ch}}(q^2) = F_1(q^2) - \frac{q^2}{2M} F_2(q^2)\,,$$

$$F_{\text{mag}}(q^2) = \frac{F_1(q^2)}{2M} + F_2(q^2)\,.$$

Since Foldy's first work on the electromagnetic properties of nucleons, the form factors have been interpreted in terms of a distribution of charge and magnetization. In order to determine such a distribution for a system, one may calculate the moments of the current density, and the distributions are determined if all moments of the distribution are given. In our case the matrix element of any moment of the current operator may be expressed directly in terms of $F_1$ and $F_2$. To obtain the equivalent of a classical charge and current distribution one need only specify the state of the system and then calculate the expectation value of every moment of the distribution in this state, namely

$$\langle \int d^3r\; x^\alpha y^\beta z^\gamma j_\mu(\mathbf{r})\rangle .$$

This would be an $(\alpha, \beta, \gamma)$ moment of the 4-current distribution.

The proper state to take for evaluating this expectation value is a wave packet. The wave packet is required in order to have a well-defined answer, but all of the specific moments due to its detailed shape are eliminated. Then at the end of the calculation of the moments the wave packet is taken to describe a particle at rest. The moments of the current density operator obtained in this way are found to be equal to those of the classical current densities defined by

$$J_4(\mathbf{r}) = i\rho(\mathbf{r}) = \frac{ie}{(2\pi)^3}\int d^3q F_{ch}(\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{r}}$$

$$\mathbf{J}(\mathbf{r}) = \frac{ie}{(2\pi)^3}\int d^3q (\boldsymbol{\sigma}\times\mathbf{q}) F_{mag}(\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{r}}.$$

Hence $F_{\text{ch}}$ is the Fourier transform of the equivalent classical charge density, and $F_{\text{mag}}$ is the Fourier transform of the equivalent density of magnetization.

It is important to notice here that $F_{\text{ch}}$ and $F_{\text{mag}}$ appear as functions of $|\mathbf{q}|^2$, which is just the three vector of the momentum transfer squared. The original expression for the form factor is a function of the single invariant variable $q^2$. $F(|\mathbf{q}|^2)$ is simply obtained by setting the fourth component, $q_0$, equal to zero in the general expression, corresponding to going into the Breit reference frame. It has recently been noted by Hand, Richard Wilson, and others, that there is no interference between $F_{\text{ch}}$ and $F_{\text{mag}}$ in electron-nucleon scattering since only $F_{\text{mag}}$ occurs in the spinflip term. This is suggested by the form of the classical currents given above and it goes back to an observation of Walecka that, in the Breit frame, the interaction energy divides naturally into two terms, one of them proportional to $F_{\text{ch}}$, the other proportional to $(\boldsymbol{\sigma}\times\mathbf{q})F_{\text{mag}}$.

Finally, I would like to remark on the high-energy behaviour of the form factors. On the basis of the physical interpretation of $F_{\text{ch}}$ and $F_{\text{mag}}$ given above, it follows that, as $q^2$ goes to infinity, $F_{\text{ch}}(q^2)\rightarrow Z_2Q$, $F_{\text{mag}}(q^2)\rightarrow Z_2Q/2M$, where $Q = 1$ for the proton, $Q = 0$ for the neutron. If there is a centre of charge in the nucleon it must appear in $F_{\text{ch}}$ because that is the charge distribution. Hence $F_{\text{ch}}$ should become the probability of finding a bare core in the nucleon, which is the renormalization constant $Z_2$, times the charge of the nucleon, while $F_{\text{mag}}$ should become $Z_2$ times $Q/2M$, because this is the Dirac moment of the core. This result may be used to eliminate one parameter in the analysis of the data. In terms of the form factors $G_{\text{ch}}$ and $G_{\text{mag}}$ used by Professor Wilson in the previous talk, the high-energy condition is $G_{\text{ch}}(q^2)-G_{\text{mag}}(q^2)\rightarrow 0$.

The measurement of $Z_2$ by means of the high-energy limit would be interesting because, if $Z_2$ turns out to be different from zero, we would be inclined to interpret the nucleon as a fundamental particle, since it would have an " indestructible " centre.

## DISCUSSION

FUBINI: Are these formulae for the asymptotic limit verified, for example, in perturbation theory?

SACHS: I do not think you can obtain the result for the magnetic form factor in standard perturbation theory. I should mention in this connection that it has been found by Hiida *et al.* and also by Gell-Mann and Zachariasen on the basis of dispersion relations that $F_1\rightarrow Z_2$ as $q^2$ goes to infinity. The Gell-Mann - Zachariasen result is really quite a different result; it has to do with whether you are above or below the cut-off of the strong interactions. The Hiida result is the same result as ours for $F_1$. What is different here is the high-energy behaviour of the magnetic form factor.

KÄLLÉN: I agree completely with what you just said about the appearance of $Z_2$ in the high-energy limit. Perhaps I can add the extra piece of information that if you do a perturbation theory calculation with a specific model *and introduce cut-offs in a gauge-invariant way* (e.g. by the Pauli-Villars regulators) the results comes out with $Z_2$ in just the way you indicated.

SACHS: Does this mean that you know what the correct value of $Z_2$ is?

KÄLLÉN: In a given perturbation theory order and a given cut-off method, yes.

BREIT: Referring to the paper by Ernst, Wali and Sachs of about a year ago, part of the motivation was a belief that it might eventually be more fruitful to have a classification not in terms of a division of effects according to a picture in which there is a bare Dirac particle surrounded by pions or other kinds of mesons, but rather in terms of total charge density of whatever origin and correspondingly for currents?

SACHS: I would not say that I would abandon the notion of understanding the charge distribution, but from the point of view of the phenomenological analysis I think it is important to realize that the interpretation of the form factors does not make reference to the origin of the charge distribution. When one does make a theoretical approach to the pionic origin of the charge distribution I believe that one must recognize that these form factors are the things that one calculates when one calculates the charge density or a density of magnetization. I think this has misled many of us. There are innumerable statements in the literature that the charge radius of the neutron is zero. This is not a correct statement. It is true that $F_1{}'_N$ (the derivative of $F_1$ with respect to $q^2$ at $q^2 = 0$) happens to be very small. That is not the charge radius but it has been interpreted as a charge radius in the past. Therefore there were a number of strained efforts to explain why the charge radius should be zero although the static model indicated that pions were spread over a pion Compton wavelength. Actually the static model gives a reasonable value of the true charge radius.

BREIT: Would it not be reasonable that for some purposes the Dirac-Pauli classification would be useful?

SACHS: I have no objection as long as one does not call it a charge density.

SALECKER: Well I should like to point out the possibility that the form factor is no longer valid in the very high-energy region where you can hope to determine the value of $Z_2$ experimentally.

SACHS: I believe you must be referring to the fact that radiative corrections will become important eventually for large $q^2$. This analysis is based on a hope that there is a region above which the strong interactions are cut off, but which is still not sufficiently high so that high order electromagnetic radiative corrections become important. There may not exist such a region. But if there exists such a region, then one can make an interpretation this way. If not, this interpretation, as you say, is nonsense. There is a theorem by Drell and Zachariasen, I believe, which says that if $Z_2$, the photon propagator renormalization, is finite, then $Z_2$ must be zero when you take into account all the electromagnetic radiative corrections. This theorem was actually given first by Evans, I believe, but maybe by Källén; everything was in Källén's article.

---

# THE $\pi^0$-PRODUCTION IN THE COULOMB NUCLEAR FIELD

**V. V. Barmin, Yu. S. Krestnikov, E. V. Kuznetsov, A. G. Meshkovski, Yu. P. Nikitin and V. A. Shebanov.**

Institute of Theoretical and Experimental Physics, Academy of Sciences of the USSR, Moscow

*(presented by V. V. Vladimirsky)*

In the theoretical papers by Pomeranchuk and Shmushkevich [1] and Good and Walker [2] the coherent Coulomb interactions of fast particles with a nucleus were considered. We have investigated experimentally a particular case of the coherent interaction

$$\pi^- + N_Z^A \rightarrow \pi^- + \pi^0 + N_Z^A \qquad (1)$$

As follows from ref. [1] at $q^2 \ll m^2/A^{2/3}$ ($q^2$ being the momentum transfer to the nucleus and $m$ being the mass of the pion) the differential cross section of the reaction has the shape

$$d\sigma_c = \frac{Z^2 d}{\pi} \cdot \frac{dW^2}{W^2 - m^2} \frac{dq^2}{q^4} \left[ q^2 - \left( \frac{W^2 - m^2}{2E_L} \right)^2 \right] \times$$
$$\times |F(q^2)|^2 \sigma_p(W) \qquad (2)$$

where $W$ is the total energy of two pions in their centre-of-mass system, $E_L$ is the energy of the incident negative pion in the laboratory coordinate system, $F$ is the form-factor of a nucleus and $\sigma_p$ is the cross section of the photoreaction

$$\gamma + \pi^- \rightarrow \pi^- + \pi^0 \qquad (3)$$

In the present work the reaction (1) on xenon nuclei ($Z = 54$, $A = 131$) was investigated at $E_L = 2.8$ GeV. At this comparatively low energy the charge distribution in the nucleus should be taken into account. We assume the form-factor of a nucleus to be

$$F(q^2) = \frac{1}{1 + q^2 \frac{R^2}{\sigma}} \qquad (4)$$

Here $R = \frac{A^{1/3}}{m}$ denotes the nuclear radius and $\hbar = c = 1$. The effective momentum transfer is restricted by the condition $q \leqq \frac{\sqrt{\sigma}}{R}$, that yields for a xenon nucleus the value $\sim m/2$, i.e. $\sim 70$ MeV/c. Assuming this value and integrating the expression (2) over the intervals $\left[\frac{(W^2 - m^2)}{2E_L}\right]^2 \leqq q^2 \leqq \frac{\sigma m^2}{A^{2/3}}$ and $4m^2 \leqq W^2 \leqq \left(\frac{2E_L m\sqrt{\sigma}}{A^{1/3}}\right)$ we have for xenon nucleus at our energy

$$\sigma_c \cong 7.5\, \bar{\sigma}_p \qquad (5)$$

where $\bar{\sigma}_p$ is the cross section of the reaction (3) averaged in the interval $4m^2 \leqq w^2 \leqq 21m^2$. At $w^2 = 21m^2$