New Results for Reggeons using FRG

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Abstract. In this paper we extend our recent non perturbative functional renormalization group analysis of Reggeon Field Theory to the interactions of Pomeron and Odderon fields. We establish the existence of a fixed point and its universal properties. This analysis, allows to connect the nonperturbative infrared region (large transverse distances) with the UV region of small transverse distances where the high energy limit of perturbative QCD applies. We discuss the implications of result for the existence of an Odderon in high energy scattering.

1 Introduction

The D0 and TOTEM collaborations have recently published the observation of the Odderon [1]. Collaborations compared their pp and $p\bar{p}$ [2] elastic cross sections, and found they differ with a significance of $3.4\,\sigma$, where the protons and antiprotons are intact after interaction and scattered at very small angle. The fact that they are intact in the final state means that it is due to the exchange of colorless objects (Pomeron and Odderon). From the point of view of QCD, the Pomeron is made of an even number of gluons (two), which leads to a positive parity whereas the Odderon is made of an odd number of gluons (three) corresponding to a negative charge (C) parity. This leads to the evidence of a t-channel [3] exchanged Odderon and this result has been widely accepted by a majority of the particle physics community. It is the first experimental observation of the Odderon which was proposed by Lukaszuk and Nicolescu [4] in 1973. In the early 80‘s, using perturbative QCD the BFKL-Pomeron [5] was found, which describes the composite state of two reggeized gluons and it is characterized by its Pomeron Intercept α_P and slop α'_P. It was realized that this picture can be generalized to composite states of three (and more) reggeized gluons, the so-called BKP states [6–8]. A first solution of the three gluon problem was found by Janik and Wosiek [9] and its intercept was found to be $\alpha_O = 1 - 0.24717\alpha_s \frac{N_c}{\pi}$, which for a realistic $\alpha_s = 0.2$ yields $\alpha_O = 0.96$. In 1999 another solution was found by Bartels, Lipatov, and Vacca [10] with intercept exactly at one, $\alpha_O = 1$, independent of the value of α_s. These perturbative results, calculated in the UV region cannot directly be applied to soft hadron-hadron scattering. However, in

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recent years some progress has been made in analyzing the transition from the perturbative BFKL Pomeron/Odderon to the soft region or IR. It is of great importance in this context to study possible Odderon effects in reactions other than proton-proton elastic scattering. For example, the diffractive production of two ϕ -meson [11] seems to offer a good possibility to identify and study the Odderon exchanges. They found that various values of the Odderon intercept α_O can describe the data, and we need more information about this parameter. For a review of the Odderon, see, e.g., [12].

In our papers [13, 14] we have started an analysis of the flow equations of reggeon field theory (RFT) [15–17], following the idea that RFT might provide a good description of strong interactions in the Regge limit and infrared region. We have used the Wetterich formulation of the functional renormalization group equations [18, 19] to study directly the problem in two transverse dimensions. As our main result we have established the existence of a critical theory (fixed point) in this multidimensional space of the parameters of the effective potential: there exists one direction which is UV attractive (IR repulsive), whereas all other directions are IR attractive. We have verified that such a fixed point belongs to the universal behavior of the percolation model in statistical physics [20]. This result suggests an approach, in the space of 2 + 1- dimensional reggeon field theories, for an interpolation between the two domains: for long transverse distances the Pomeron field has intercept very close to unity and a nonzero t-slope, for short transverse distances the BFKL intercept is significantly above one, and the slope is very small.

Within such a program in [13, 14] we have restricted our analysis to the Pomeron field. Whereas other secondary reggeons (e.g. ρ, ω, or φ) have intercepts well separated from the Pomeron and, can therefore safely be neglected, there exists one other Regge singularity for which this is not the case, the Odderon with intercept at or near one. In the perturbative region the existence of the Odderon is well-established [21] and the Odderon intercept has been found to be exactly at one.

The existence of the pertubative region motivates interest in the question whether the Odderon exists also in the nonperturbative region. In an analysis which is set up to explore the connection between the UV region and the nonpertubative IR region, the Odderon has to be included: the IR fixed point structure should confirm whether the Odderon survives the flow from UV to IR. Also, such an analysis should provide some information on the interaction between Odderon and Pomeron. Self interactions of the Pomeron [10, 22–24] as well as interactions between Pomeron and Odderon naturally appear in perturbative QCD analyses [25, 26]. Analogous results are obtained also in the Color Glass Condensate and dipole approach [27–29].

In this paper we report our investigations [13], which establish the existence of a new critical theory (fixed point) which includes both Pomeron and Odderon fields. Again this investigation can have implications in the statistical physics of generalized multicomponent directed percolation models.

This work is organized as follows. We first describe the general setup, then we present results of our fixed point analysis. In our final part, we compute trajectories of physical parameters: Pomeron and Odderon intercept and derive first hints for a physical interpretation.

2 The general setup

Our main tool of investigating RFT, is the functional renormalization group approach (for a review see [30–32]), which has successfully been applied to numerous problems in statistical mechanics, in particle physics, and in quantum gravity. In short, in this approach we study the effective action Γ_k of a sequence of RFTs as a function of an infrared regulator R_k which controls the range of modes which are integrated out. The dependence on k is captured by the flow equations which have to be solved by suitable approximations. One main result will be the existence of a fixed point which we will analyse and discusse the possible physical significance of this fixed pont.

2.1 The Action and the flow equation

The effective action which describes the interactions between Pomeron and Odderon fields has the form:

$$\Gamma_k[\psi^\dagger,\psi,\chi^\dagger,\chi] = \int d^2x\, d\tau \Big(Z_P(\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial_\tau}\psi - \alpha'_P \psi^\dagger \nabla^2 \psi) + Z_O(\frac{1}{2}\chi^\dagger \overleftrightarrow{\partial_\tau}\chi - \alpha'_O \chi^\dagger \nabla^2 \chi) + V_k[\psi,\psi^\dagger,\chi,\chi^\dagger]\Big). \quad (1)$$

where $\psi, \psi^\dagger$ denote the Pomeron field, and $\chi, \chi^\dagger$ the Odderon fields. For the lowest truncation the potential V_k takes the form:

$$V_3 = -\mu_P \psi^\dagger\psi + i\lambda\psi^\dagger(\psi+\psi^\dagger)\psi - \mu_O\chi^\dagger\chi + i\lambda_2\chi^\dagger(\psi+\psi^\dagger)\chi + \lambda_3(\psi^\dagger\chi^2 + {\chi^\dagger}^2\psi). \quad (2)$$

For the quartic truncation we add the following terms:

$$V_4 = \lambda_{41}(\psi\psi^\dagger)^2 + \lambda_{42}\psi\psi^\dagger(\psi^2 + {\psi^\dagger}^2) + \lambda_{43}(\chi\chi^\dagger)^2 + i\lambda_{44}\chi\chi^\dagger(\chi^2 + {\chi^\dagger}^2) + i\lambda_{45}\psi\psi^\dagger(\chi^2 + {\chi^\dagger}^2) + \lambda_{46}\psi\psi^\dagger\chi\chi^\dagger + \lambda_{47}\chi\chi^\dagger(\psi^2 + {\psi^\dagger}^2). \quad (3)$$

As described in [13, 14], for the pure Pomeron case the couplings are real-valued for even powers of the Pomeron fields, whereas odd powers require imaginary couplings. This is a consequence of the even-signature of the Pomeron exchange. The Odderon has negative signature, then t-channel states with an odd number of Odderons never mix with pure Pomeron channels, the transition $P \to OO$ is real valued, the transition $O \to OP$ has to be imaginary. As a result, all triple couplings are imaginary, except for the real-valued transition $P \to OO$.

In the sector of quartic couplings, all couplings involving Pomerons only are real-valued. Once the Odderon is included, again most quartic couplings remain real, but there are two exceptions: the transitions $O \to OOO$ and $P \to P + OO$ are imaginary. In the perturbative region, the transition $P \to OO$ has been computed [25, 26] and found to be nonzero.

Next we introduce dimensionless variables. The field variables are rescaled as follows:

$$\tilde{\psi} = Z_P^{1/2} k^{-D/2}\psi, \quad \tilde{\chi} = Z_O^{1/2} k^{-D/2}\chi. \quad (4)$$

For the potential we introduce $\tilde{V}_k = \frac{V}{\alpha'_P k^{D+2}}$. Finally, the couplings are rescaled in the following way:

$$\tilde{\mu}_P = \frac{\mu_P}{Z_P \alpha'_P k^2}, \quad \tilde{\mu}_O = \frac{\mu_O}{Z_O \alpha'_P k^2}, \quad \tilde{\lambda} = \frac{\lambda}{Z_P^{3/2}\alpha'_P k^2} k^{D/2}, \quad \tilde{\lambda}_{2,3} = \frac{\lambda_{2,3}}{Z_O Z_P^{1/2} \alpha'_P k^2} k^{D/2}. \quad (5)$$

This choice implies that we introduce the dimensionless ratio $r = \frac{\alpha'_O}{\alpha'_P}$, and the Odderon slope α'_O will be written as $\alpha'_O = r\,\alpha'_P$. Moreover we define the anomalous dimensions:

$$\eta_P = -\frac{1}{Z_P}\partial_t Z_P, \quad \eta_O = -\frac{1}{Z_O}\partial_t Z_O, \quad \zeta_P = -\frac{1}{\alpha'_P}\partial_t \alpha'_P, \quad \zeta_O = -\frac{1}{\alpha'_O}\partial_t \alpha'_O. \quad (6)$$

In order to find the flow equation of the effective action, we need to compute the r.h.s of Wetterich's equation: $\partial_t \Gamma_k = \frac{1}{2} Tr[\Gamma_k^{(2)} + \mathbb{R}_k^{-1}]\partial_t \mathbb{R}_k$, where $t = ln\, k/k_0$, the trace on the r.h.s extends over a 4×4 matrix and the optimized regulator $\mathbb{R}_k$ [33] are chosen as follows:

$$R_P(q^2) = Z_P \alpha'_P (k^2 - q^2)\Theta(k^2 - q^2), \quad R_O(q^2) = Z_O \alpha'_O (k^2 - q^2)\Theta(k^2 - q^2), \quad (7)$$

which allows for a simple analytic integration in a closed form. The propagator matrix can be written the following form:

$$\Gamma^{(2)} + \mathbb{R} = \begin{pmatrix} \Gamma_P^{(2)} & \Gamma_{PO} \\ \Gamma_{OP} & \Gamma_O^{(2)} \end{pmatrix}, \tag{8}$$

where the 2×2 block matrices are:

$$\Gamma_P^{(2)} = \begin{pmatrix} V_{\psi\psi} & Z_P(-i\omega + \alpha'_P q^2) + R_P + V_{\psi\psi^\dagger} \\ Z_P(i\omega + \alpha'_P q^2) + R_P + V_{\psi^\dagger\psi} & V_{\psi^\dagger\psi^\dagger} \end{pmatrix}, \tag{9}$$

$$\Gamma_O^{(2)} = \begin{pmatrix} V_{\chi\chi} & Z_O(-i\omega + \alpha'_O q^2) + R_O + V_{\chi\chi^\dagger} \\ Z_O(i\omega + \alpha'_O q^2) + R_O + V_{\chi^\dagger\chi} & V_{\chi^\dagger\chi^\dagger} \end{pmatrix}, \tag{10}$$

$$\Gamma_{PO} = \begin{pmatrix} V_{\psi\chi} & V_{\psi\chi^\dagger} \\ V_{\psi^\dagger\chi} & V_{\psi^\dagger\chi^\dagger} \end{pmatrix}, \quad \Gamma_{OP} = \begin{pmatrix} V_{\chi\psi} & V_{\chi\psi^\dagger} \\ V_{\chi^\dagger\psi} & V_{\psi\psi^\dagger} \end{pmatrix}. \tag{11}$$

After the momentum integrals, the energy integral is done by complex integration, employing a weak field polinomial expansion and the use of dimensionless variables, we can obtain the beta functions, taking derivatives with respect to the field variables to the Wetterich-equation and subsequently set the fields equal to zero.

2.2 β functions

We begin with the lowest (cubic) truncation. The beta functions in the region $r - \mu_O > 0$ which is the physical relevant region are:

$$\begin{aligned}
\dot{\mu}_P &= (-2 + \eta_P + \zeta_P)\mu_P + 2A_P\frac{\lambda^2}{(1-\mu_P)^2} - 2A_O r\frac{\lambda_3^2}{(r-\mu_O)^2} \\
\dot{\mu}_O &= (-2 + \eta_O + \zeta_P)\mu_O + 2(A_P + A_O r)\frac{\lambda_2^2}{(1+r-\mu_P-\mu_O)^2} \\
\dot{\lambda} &= (-2 + D/2 + \zeta_P + \frac{3}{2}\eta_P)\lambda + 8A_P\frac{\lambda^3}{(1-\mu_P)^3} - 4A_O r\frac{\lambda_2\lambda_3^2}{(r-\mu_O)^3} \\
\dot{\lambda}_2 &= (-2 + D/2 + \zeta_P + \frac{1}{2}\eta_P + \eta_O)\lambda_2 \\
&\quad + \frac{2\lambda\lambda_2^2(6A_P + 5A_O r) + 4\lambda_2^3(A_P + A_O r) - 4\lambda_2\lambda_3^2(A_P + 2A_O r)}{(1+r-\mu_P-\mu_O)^3} \\
&\quad + \frac{2A_P\lambda\lambda_2^2(r-\mu_O)^2}{(1-\mu_P)^2(1+r-\mu_P-\mu_O)^3} - \frac{4A_O r\lambda_2\lambda_3^2(1-\mu_P)^2}{(r-\mu_O)^2(1+r-\mu_P-\mu_O)^3} \\
&\quad + \frac{2\lambda\lambda_2^2(3A_P + A_O r)(r-\mu_O)}{(1-\mu_P)(1+r-\mu_P-\mu_O)^3} - \frac{4\lambda_2\lambda_3^2(A_P + 3A_O r)(1-\mu_P)}{(r-\mu_O)(1+r-\mu_P-\mu_O)^3} \\
\dot{\lambda}_3 &= (-2 + D/2 + \zeta_P + \frac{1}{2}\eta_P + \eta_O)\lambda_3 \\
&\quad + \frac{2\lambda_2^2\lambda_3(A_P + 2A_O r)}{(r-\mu_O)(1+r-\mu_P-\mu_O)^2} + \frac{4\lambda\lambda_2\lambda_3(2A_P + A_O r)}{(1-\mu_P)(1+r-\mu_P-\mu_O)^2} \\
&\quad + \frac{2\lambda_2^2\lambda_3 A_O r(1-\mu_P)}{(r-\mu_O)^2(1+r-\mu_P-\mu_O)^2} + \frac{4\lambda\lambda_2\lambda_3 A_P(r-\mu_O)}{(1-\mu_P)^2(1+r-\mu_P-\mu_O)^2}.
\end{aligned} \tag{12}$$

Here we have defined $A_P = N_D A_D(\eta_P, \zeta_P)$, $A_O = N_D A_D(\eta_O, \zeta_O)$. For the next truncation, the quartic approximation, the results for the beta functions are already lengthy and will not be listed here. For the truncations of fourth order and beyond we have used Mathematica.

2.3 Anomalous dimensions

In order to obtain the anomalous dimensions we first define the two-point vertex functions:

$$\Gamma_P^{(1,1)}(\omega,q) = \frac{\delta^2\Gamma}{\delta\psi(\omega,q)\delta\psi^\dagger(\omega,q)}|_{\psi=\psi^\dagger=\chi=\chi^\dagger=0},\quad \Gamma_O^{(1,1)}(\omega,q) = \frac{\delta^2\Gamma}{\delta\chi(\omega,q)\delta\chi^\dagger(\omega,q)}|_{\psi=\psi^\dagger=\chi=\chi^\dagger=0}, \tag{13}$$

and then taking derivatives with respect to energy and momentum:

$$Z_P = \lim_{\omega\to 0, q\to 0} \frac{\partial}{\partial(i\omega)}\Gamma_P^{(1,1)}(\omega,q),\quad Z_O = \lim_{\omega\to 0, q\to 0} \frac{\partial}{\partial(i\omega)}\Gamma_O^{(1,1)}(\omega,q) \tag{14}$$

and

$$Z_P\alpha'_P = \lim_{\omega\to 0, q\to 0} \frac{\partial}{\partial q^2}\Gamma_P^{(1,1)}(\omega,q),\quad Z_O\alpha'_O = \lim_{\omega\to 0, q\to 0} \frac{\partial}{\partial q^2}\Gamma_O^{(1,1)}(\omega,q). \tag{15}$$

Finally, we obtained (see [14]):

$$\eta_P = -\frac{2A_P\lambda^2}{(1-\mu_P)^3} + \frac{2A_O r\lambda_3^2}{(r-\mu_O)^3},\quad \eta_O = -\frac{4(A_P + A_O\, r)\lambda_2^2}{(1+r-\mu_P-\mu_O)^3},\quad A_D(\eta,\zeta) = \frac{1}{D} - \frac{\eta_k+\zeta_k}{D(1+D)} \tag{16}$$

and

$$\eta_P+\zeta_P = -\frac{N_D\lambda^2}{D(1-\mu_P)^3} + \frac{N_D r^2\lambda_3^2}{D(r-\mu_O)^3},\quad \eta_O+\zeta_O = -\frac{4N_D\lambda_2^2}{D(1+r-\mu_P-\mu_O)^3},\quad N_D = \frac{2}{(4\pi)^{D/2}\Gamma(D/2)} \tag{17}$$

With the anomalous dimensions definition, the evolution equation for r then becomes: $\dot r = r\,(-\zeta_O + \zeta_P)$, which tells that at criticality the Pomeron and Odderon transverse space scaling laws do coincide.

3 Numerical results

Let us now focus on the physical case of $D = 2$ transverse dimensions. Our analysis is essentially in the LPA approximation with the addition of an extra coupling r, depending on the anomalous dimensions ζ_P and ζ_O, which we have evaluated at the lowest order.

In the cubic truncation we find a fixed point solution with the following values: $\mu_P = 0.111111, \mu_O = 0.110753, \lambda = 1.05034, \lambda_2 = 1.44665, \lambda_3 = 0, r = 0.921810$. The stability properties indicates that this fixed point has three negative and three positive eigenvalues, i.e. there are three relevant directions. Two of the negative eigenvalues $\lambda_O = -1.9398$ and $\lambda_P = -1.8860$ are associated to the $\nu_P \simeq 0.73$ and $\nu_O \simeq 0.6$ critical exponents, respectively. The third negative eigenvalue $\lambda^{(3)} = -0.0916$ is close to zero and with an eigenvector mainly associated to the r coupling. For the anomalous dimensions we find $\eta_P \simeq -0.33$, $\eta_O \simeq -0.35$ and $\zeta_P = \zeta_O \simeq +0.17$.

In this solution we observe a 'decoupling' of the two sectors: compared to the pure Pomeron case, the Pomeron is not affected by the presence of the Odderon, whereas the Odderon 'feels' the Pomeron. This decoupling is due to the vanishing of the triple coupling λ_3. We remind that in the UV region where perturbative QCD applies, the coupling $P \to OO$ is nonzero [25].

All these features also appear in the following solution obtained in the quartic truncation: ($\mu_P = 0.274381$, $\mu_O = 0.26979$, $\lambda = 1.34738$, $\lambda_2 = 1.79401$, $\lambda_3 = 0.$, $\lambda_{41} = -2.88712$, $\lambda_{42} = -1.27076$, $\lambda_{43} = -0.83228$, $\lambda_{44} = 0.$, $\lambda_{45} = 0.$, $\lambda_{46} = -5.2784$, $\lambda_{47} = -2.2078$, $r = 0.88018$). The stability properties are the same as in the cubic case: three negative eigenvalues (-1.8159, -1.6751 and -0.20957). The Pomeron and Odderon sectors are decoupled,

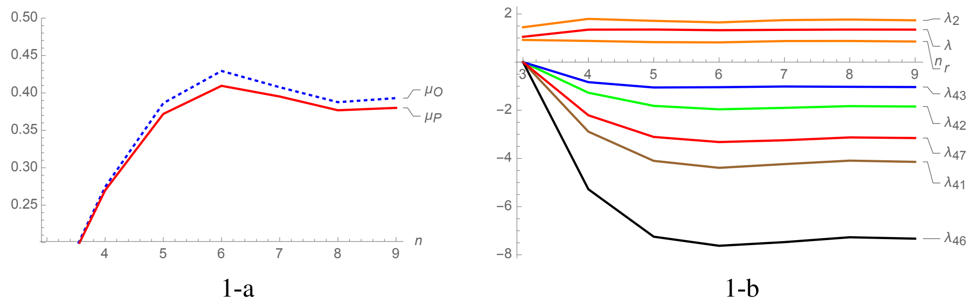

Figure 1. 1-a:Values of the parameters of the fixed point solution of the LPA truncations for different orders n of the polynomial ($3 \leq n \leq 9$). The masses (which equal intercept minus one) μ_P (red curve) and μ_O (blue dotted curve) for the Pomeron and Odderon fields are in the left panel. 1-b The first non zero couplings $\lambda, \lambda_2, \lambda_{41}, \lambda_{42}, \lambda_{43}, \lambda_{46}, \lambda_{47}, r$ are reported on the right panel.

since the couplings $\lambda_3, \lambda_{44}, \lambda_{45}$ vanish. The Pomeron parameters are the same as in the pure Pomeron case at the corresponding order. There exist three eigenvectors which span the subspace of the three 'exceptional ' couplings $\lambda_3, \lambda_{44}, \lambda_{45}$, which have negative eigenvalues, i.e. this subspace is part of the 10-dimensional critical subspace. Inside this subspace they are orthogonal to all other 7 eigenvectors with positive eigenvalues.

All this leads to the conclusion that near this fixed point the 'exceptional' couplings define a subspace inside the critical subspace which is orthogonal both to the remaining part of the critical subspace and to the three relevant directions. This subspace decouples from the other part. We do not find any other physical critical solution with all couplings being nonzero.

We then extend the analysis for this special fixed point solution up to order 9 in the polynomial expansion. We collect the results found in two figures in order to show the convergence with respect to the order of the truncation. In Fig. 1 we show on the left side the values for μ_P and μ_O while on the right side we give the values of the non zero couplings which characterize the truncation up to order fourth, for all the orders n between 3 and 9. We note that $\mu_P > \mu_O$ in all truncations. We see how at order 9 a good stability is reached. We stress that all the quantities reported in this figure are not universal.

3.1 Pomeron and the Odderon intercept evolution

Our fixed point analysis was done in the space of dimensionless parameters (cf. section 2), and the flow of the physical (i.e. dimensionful) parameters can be quite different. In particular, when $k \to 0$, the nonvanishing fixed point values values of the (dimensionless) Pomeron and Odderon masses lead to vanishing physical masses, quite in the same way as in the pure Pomeron case discussed in [14].

For the Pomeron-Odderon system we have performed numerical studies of the flow for the dimensionless and dimensionful parameters. If we start, in the UV region, inside the critical subspace, we end up, in the IR limit, at the fixed point. At this fixed point, both the Pomeron and the Odderon have intercept one. From (5) we see that near the fixed point both intercepts, $\alpha_P(0) - 1 = \mu_P/Z_P \sim k^{2-\zeta}\tilde{\mu}_P$ and $\alpha_O(0) - 1 = \mu_O/Z_O \sim k^{2-\zeta}\tilde{\mu}_O$ go to zero as k becomes small. Since the fixed point value of $\tilde{\mu}_P$ is slightly larger than $\tilde{\mu}_O$ we conclude that, for small but nonzero values of k, the Pomeron intercept is larger than the Odderon intercept. In Fig. 2 a first study of the flow equations (in the cubic truncation) further away from the fixed point shows that most trajectories belong to μ_P above and μ_O below its fixed

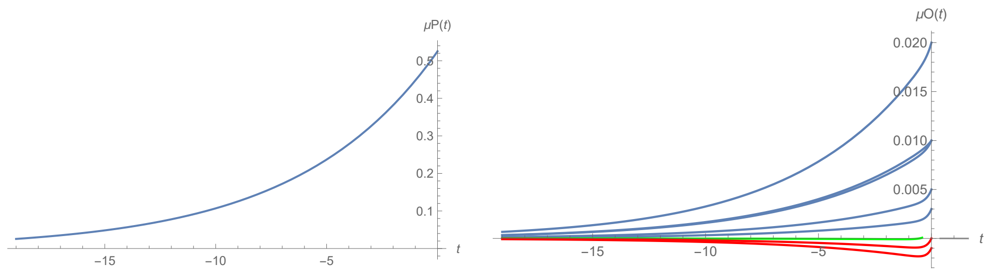

Figure 2. 2-a: We show the trayectories of the dimensionful parameters $\mu_P(t)$ in term of t-evolution, 2-b the trayectories of the dimensionful parameters $\mu_O(t)$ in term of t-evolution

point values. This is consistent with our expectations for the starting points in the UV region; the Pomeron value μ_P should be positive, whereas the Odderon mass μ_O shoul be at (or close to) zero. However, the most important conclusion to be drawn from this fixed point analysis is that the Odderon exists in the IR limit and does not die out with energy.

A flow starting outside the critical subspace may also lead to finite values of μ_O or μ_P which can be positive or negative. Detailed features of such flows require further studies.

4 Conclusion

In this paper we have extended our previous fixed point analysis of Pomeron reggeon field theory to a system of interacting Pomeron and Odderon fields in the infrared limit. As the main result, we have found a fixed point with three relevant directions: these directions are UV stable (i.e. IR unstable), whereas all other eigenvalues belong to infrared stable directions. For such a fixed point, at first sight, the situation looks as follows. In the parameter space of the effective potential, the relevant directions define an orthogonal subspace which we name 'critical subspace'. If one starts, at $k \neq 0$, within this subspace one ends up, for $k \to 0$, at the infrared stable fixed point. On the other hand, if one starts at a generic value outside the critical subspace (not too far away from the fixed point) the flow will eventually be attracted by the relevant direction and move away from the fixed point.

There is another interesting feature of the fixed point which we have found. Namely, in the critical regime there are no transitions from pure Pomeron states to states containing Odderons, the coupling POO (which was found to be nonzero in pQCD) vanishes.

In our analysis which explore the connection between the UV region and the nonpertubative IR region, the IR fixed point structure confirm that the Odderon survives the flow from UV to IR. Also, such an analysis provide information on the interaction between Odderon and Pomeron,e.g. a suppression of the Odderon exchange.

For the ratio of the Odderon and Pomeron slopes we find a fixed point value slightly below one. Phenomenologically, not much is known about the Odderon slope [12, 34], and our result might be seen as an asymptotic prediction which can be used in the study of Odderon effect in Diffractive process.

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