

The Eötvös experiment, GTR, and differing *gravitational* and *inertial masses*: Proposition for a crucial test of metric theories

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Abstract. The Eötvös experiment has been taken as basis for metric theories of gravity and particularly for the general theory of relativity (GTR), which assumes that gravitational and inertial masses are identical. We highlight the fact that, unlike the long lasting and reigning belief, the setup by Eötvös experiments and its follow-ups serve to demonstrate no more than a *mere linear proportionality* between said masses, and not ineludibly their *exclusive equality*. So much so that, as one distinct framework, Yarman–Arik–Kholmetskii (YARK) gravitation theory, where a purely metric approach is not aimed, makes the identity between inertial and gravitational masses no longer imperative while still remaining in full conformance with the result of the Eötvös experiment, as well as that of free fall experiments. It is further shown that Eötvös experiment deprives us of any knowledge concerning the determination of the proportionality coefficient coming into play. Henceforward, the Eötvös experiment and its follow-ups cannot be taken as a rigorous foundation for GTR. In this respect, we suggest a crucial test of the equality of gravitational and inertial masses via the comparison of the oscillation periods of two pendulums with different arm lengths, where the deviation of the predictions by GTR and by YARK theory represents a measurable value.

1. Introduction

It is known that the Einstein equivalence principle sets up an equality of gravitational and inertial masses, and that this represents the necessary condition to describe gravity as the alteration of the geometry of space-time. The identity of gravitational and inertial masses gave rise to the development of purely metric theories of gravity, where general theory of relativity (GTR) is the most recognized one (see, e.g. [1-4]). It is also widely accepted that the assumption about the equality of gravitational and inertial masses had been confirmed in the famous experiments by Eötvös, as well as in various modern experiments [5-7].

Besides, it is generally believed that non-metric theories of gravity – which, in general, do not necessarily require the equality of gravitational and inertial masses (see e.g. [8]) – ought to fail in providing a plausible description of gravitational phenomena.

These facts reinforced the strong belief that the experimental results ultimately reporting the equality of gravitational and inertial masses are in full harmony with contemporary presentations about gravity, and further experiments on this subject could rather be aimed for the search of a new kind of interaction (such as modern free fall experiments [9-12]). It is important to stress that free fall



experiments actually prove that the fall motion is independent of the mass of the object of concern, and this does not necessarily mean that gravitational mass and inertial mass are identical. If ever they were, the mass of the object would necessarily drop off of the equation of motion. At the same time, as we will elaborate on below, these mass components may well be not equal to each other.

Thus, the fact that *available experiments destined to show the identity of gravitational and inertial masses do not demonstrate anything other than that there is a linear proportionality of these masses to each other instead of their strict equality* (section 2). The proportionality choice for the given masses appears to be skipped, because GTR and the extended theories of gravity assume the coefficient of proportionality K to be exactly equal to unity; whereas, any non-metric theories (where K , in general, differs from unity) are blamed to be at odds with experimental facts.

This situation is drastically shaken with the development of Yarman-Arik-Kholmetskii (YARK) gravitation theory, which implies neither a purely metric theory, nor a purely dynamical theory, but rather combines the properties of both of them. At the moment, YARK remains the most successful theory in the explanation of both the old and modern results in cosmology, including those of them, which still did not find explanations under GTR. In addition, YARK represents the only alternative to GTR, which provides its own successful explanation of the origin of the GW150914 and GW151226 signals beyond the mechanism involving gravitational waves [13].

We emphasize that the description of gravity in YARK theory beyond a purely metric approach does not, in general, require the identity of gravitational and inertial masses; it is sufficient to stipulate only the linear proportionality between these mass components (see section 3). What is more, YARK theory provides an explicit expression for the coefficient of proportionality K between the gravitational and inertial masses, which, however, cannot be measured in the known setups, including the experiments by Eötvös. We further demonstrate that the null result of such experiments still admits the dependence of K on the *intensity of gravity* and the *velocity of the test mass*. In particular, we show in section 3 that, in YARK theory, the intensity of gravity affects the gravitational and inertial masses in exactly the same extent, and does not alter the value of the coefficient K .

We show below that for a moving test mass, the gravitational mass is inversely proportional to the Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$, while the inertial mass is linearly proportional to γ , and the coefficient K is determined by the inverse of γ^2 . We discuss the physical meaning of these results and possible ways of their experimental verification. In section 4 we propose a new test for the determination of the relationship between gravitational and inertial masses, which is based on the comparison of the oscillation periods of two pendulums with different arm-lengths. The difference of the given oscillation periods differ in GTR and in YARK at the value, which can be reliably measured under modern developments in experimental techniques. Thus, the proposed experiment can be considered as a qualitative new test of GTR. Finally, we conclude in section 5.

2. Relationship between gravitational and inertial masses via measurements

Measurements had been carried out since several centuries to check out the equality of gravitational and inertial masses (e.g., [5-7,9-12]). The fact remains that stating the motion of an object is independent of its proper mass, had been taken as a sound ground with regards to claiming the identity of gravitational and inertial masses. However, as we will see below, even though the ratio of the gravitational mass to the inertial mass of a given object can be different than unity; its proper mass still drops off from its equation of motion.

In the present contribution we do not review the mentioned experiments, and address only a typical and the most famous experiment on this subject performed by Eötvös [5].

The apparatus by Eötvös, features two test masses A and B fixed on the opposite sides of the arms of a torsion balance. The masses A and B in both arms are subject to a gravitational pull directed downward (the z axis) governed by Earth's local acceleration of free fall g at the given location (see Fig. 1). The related weights are denoted by m_{GAg} and m_{GBg} , where m_{GA} and m_{GB} are the respective gravitational masses. Both masses are subject to a centrifugal push of acceleration a on the surface of

Earth due to its self-rotation directed outward (the y axis) at the given latitude on the plane parallel to Earth's equator. The related forces are denoted by $m_{IA}a$ and $m_{IB}a$, where m_{IA} and m_{IB} are the respective inertial masses.

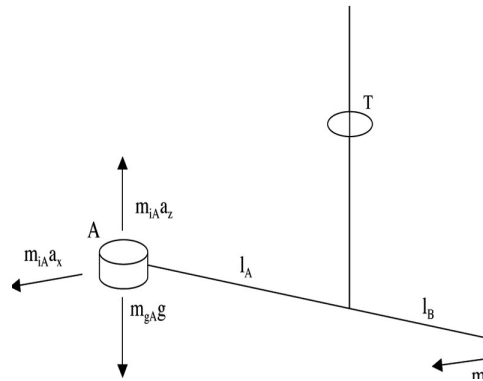


Figure 1 Forces acting on the masses A and B in the torsion balance by Eötvös [14]

Thus, the torque τ experienced by the balance is given by the equation

$$\tau = m_{IA} a_x l_A g \left(\frac{m_{BG}}{m_{BI}} - \frac{m_{AG}}{m_{AI}} \right) \left(\frac{m_{BG}}{m_{BI}} - a_z \right)^{-1}. \quad (1)$$

Hence, the condition of no torsion is to be written as the equality

$$\frac{m_{BG}}{m_{BI}} - \frac{m_{AG}}{m_{AI}} = 0, \quad (2)$$

and, astoundingly enough, it has been believed that this condition can only hold in the case

$$m_{AG} = m_{AI}, \quad (3)$$

$$m_{BG} = m_{BI} \quad (4)$$

However, one can see that the result by Eötvös (no torsion) does not necessarily mean the equality of the gravitational and inertial masses. Elementary mathematics already points to the general solution about the plain linear proportionality of the given pair of masses; i.e.,

$$m_G = K m_I. \quad (5)$$

Here K is some coefficient, which, by supposition, does not depend on m_G and m_I – and thus, represents a function of motional characteristics of the object under consideration (as a minimum, on its velocity) and the intensity of gravity at the location of apparatus. And there appears strictly no evidence whatsoever that K can be taken as unity.

Applying now eq. (5) to the Eötvös experiment where $m_{GA} = K_A m_{IA}$ and $m_{GB} = K_B m_{IB}$, we obtain from eq. (1)

$$\tau = m_{IA} a_x l_A \frac{g [K_B - K_A]}{g K_B - a_z}. \quad (6)$$

For any observer moving with respect to the Eötvös apparatus, the motional characteristic of the masses A and B (which rest with respect to each other) are the same. Besides, the intensity of gravity at points A and B is also practically the same with a very high precision. Hence, we get $K_A = K_B$, so that eq. (6) yields $\tau = 0$, which is in full agreement with the measured results. However, this by no means entails $K_A = K_B = K = 1$.

We emphasize that the possibility of $K_A = K_B \neq 1$ was well considered earlier by the scientific community. However, it was without verifiable foundation adopted that K must be a *universal constant*; and the only possibility that would fulfill this anticipation must have been $K = 1$ [15] according to purely metric gravitational theories; however, we again emphasize that the Eötvös experiment does not constitute any proof of the equality $K = 1$.

As we will see below, in YARK theory (which happens to be, in fact, the first successful non-metric theory of gravity) the parameter K is, in general, different than unity and depends on *the velocity of the object, which also well agrees with the result of the Eötvös experiment*.

In section 3 we remind cornerstone features of YARK theory and clarify its physical meaning, which is important for a better elucidation and discussion of the implications of the result of the Eötvös experiment.

3. Gravitational and inertial masses in YARK gravitation theory

In our papers [16-19], we already presented an introduction to YARK theory, which is based on the framework developed by Yarman [20-27] and advanced together with his colleagues [16-19,28-31]. For the sake of convenience, below we reproduce some important points of this theory by focusing our attention on its physical meaning.

The root postulate of YARK theory states that the overall energy of the object with the proper mass m_0 initially measured at an infinitely far away distance from all other masses in the presence of gravity acquires the form [22,23]

$$E = \gamma m_0 c^2 (1 - E_B / m_0 c^2), \quad (7)$$

where γ is the Lorentz factor associated with the motion of the object, and E_B represents the “*static binding energy*” defined by the work done to the object in order to bring it quasi-statically from infinity to the given location.

For our immediate purpose, we reproduce the motional equation for a test object with the rest mass m in the presence of immovable heavy host mass M (the one-body problem):

$$-G \frac{M m_0 e^{-\alpha} \mathbf{r}}{\gamma_0 r^3} = \frac{d}{dt} \gamma_0 m_0 e^{-\alpha} \mathbf{v}_0, \quad (8)$$

or

$$-G \frac{M \mathbf{r}}{\gamma_0^2 r^3} = \frac{d\mathbf{v}_0}{dt}, \quad (9)$$

where we have taken into account that, for the one-body problem, $\frac{d}{dt}(\gamma_0 e^{-\alpha}) = 0$ due to the energy conservation law. Here r is the distance between M and m , and γ_0 is measured by a local observer.

Eq. (9) indicates that the rest mass of the test particle drops off of the motional equation, so that YARK theory is fully compatible with the weak equivalence principle (WEP). Even so, YARK theory cannot be joined to metric theories, because the force of gravity remains “*real*” in any reference frame, including the proper frame of a particle in a free fall [32]. In the vicinity of such a particle, the metric tensor in its proper frame acquires the Minkowskian form; this result simply reflects the known mathematical theorem stating that any symmetric tensor with constant coefficients (defined at the

location of the particle) can be presented in a diagonal form via an appropriate transformation. However, the latter theorem has different physical interpretations in metric theories and in YARK respectively. Namely, according to the logic of any metric theory like GTR, the gravitational field, which characterizes the deviation of space-time geometry from the Minkowskian one, totally disappears in this frame along with the corresponding gravitational “force” and energy.

This GTR result concurrently signifies the impossibility to localize gravitational energy; which is reflected in the known fact that, in any metric theory, this energy cannot be described via a true energy-momentum tensor, but rather via a pseudo-tensor; where the latter obeys tensorial transformations only under linear space-time transformations.

However, these results are inapplicable to YARK theory where the force of gravity is “real” to the same extent as that of any other force in other areas of physics. And if it exists in one frame, it cannot disappear in another frame; including the frame of the free fall of the test particle. In other words, the particle continues to “sense” the gravitational field via the variation of its rest mass even in the state of a free fall. From the viewpoint of a distant inertial observer, the frame of free fall moves with a non-vanishing acceleration, and hence, the test object additionally experiences an action of a “fictitious” force – which is, in effect, capable of exactly counterbalancing the gravitational force.

These results signify that YARK theory, in fact, successfully combines the properties of metric and dynamical theories, and it is fully compatible with available observations in the limit of a weak gravitational field (gravitational redshift, precession of the perihelion of Mercury [22,23,25], gravitational lensing [34], Shapiro Delay [35,36]).

YARK theory also achieved considerable successes in the explanation of modern observations where the weak relativistic limit is abandoned (e.g., derivation of the alternating sign for the accelerated expansion of the Universe without the need to involve a notion of “dark energy”; presentation of the Hubble constant in an analytical form; elimination of the “information paradox” for black holes of the YARK type [19,37]). What is more, YARK theory remains the only alternative to GTR, which provides its explanation of the GW150914 and GW151226 signals of LIGO beyond the hypothesis about gravitational waves [13].

Besides these, we wish to spotlight two very recent experimental facts – the extra-energy shift between emission and absorption resonant lines in a rotating system [38-41], and the practically null bending of high-energy γ -quanta under Earth’s gravity [42] – both of which have found a successful explanation under YARK theory [16] while they still remain puzzling in GTR [32,43].

Finally, we stress that YARK theory of gravity is fully compatible with quantum mechanics [24]. Thus, YARK theory definitely reflects certain advantages along with combined metric and dynamical approaches in comparison with a purely metric approach such as that found in GTR.

It is worth reiterating the fact that such a combination of both metric and dynamic traits does not inevitably require the equality of gravitational and inertial masses, but only their linear proportionality, with the coefficient K , being in general, different than unity.

Next, we address to eq. (8) and remind that its lhs represents the gravitational force acting on the object, which should be sensitive to the gravitational mass of the particle m_G . On the other hand, the rhs of eq. (8) defines the total time derivative of the momentum of particle, which should include its inertial mass m_I .

At the same time, one can see that eq. (8) does not allow us to unambiguously establish a relationship between gravitational and inertial masses. In this situation, we have to revert to reasonable physical argumentations. In particular, in YARK theory, we can demand that for a particle at rest, both mass components coincide with each other and equal to [21]

$$m_G = m_I = m_0 e^{-\alpha}, \quad (10)$$

which means that they both obey the original postulate of YARK theory (1).

It is naturally to demand that the inertial mass of any object should be defined in such a way that this mass component acquires its standard relativistic form in the absence of gravity. This yield just a single definition of the inertial mass of a moving particle as

$$m_I = \gamma_0 m_0 e^{-\alpha}. \quad (11)$$

This definition of inertial mass, taken jointly with the motional equations (8), still leaves a freedom with regards to the setup of the gravitational mass; which, from a mathematical viewpoint, is restricted only by the requirement to keep the equivalence of eqs. (8) and (9). In particular, we can propose to formally define the gravitational mass as

$$m_G = \gamma_0^n m_0 e^{-\alpha}, \quad (12)$$

with the presentation of eq. (8) via the inertial and gravitational masses in the form

$$-G \frac{M m_G \mathbf{r}}{\gamma_0^{n+1} r^3} = \frac{d}{dt} m_I \mathbf{v}_0, \quad (13)$$

where n is a number to be fixed.

Then, one can scrutinize that the substitution of eqs. (11) and (12) into eq. (13) indeed yields the motional equation (9), which is free from any mass component.

Comparing eqs. (11) and (12), we see that the equality of m_I and m_G – as required by metric theories of gravity – can, as a choice, indeed be mathematically stated at $n = 1$ when

$$m_G = \gamma_0 m_0 e^{-\alpha}. \quad (14)$$

Hence, under the definition (14), eq. (13) takes the form

$$-G \frac{M m_G \mathbf{r}}{\gamma_0^2 r^3} = \frac{d}{dt} m_I \mathbf{v}_0. \quad (15)$$

Considering now the choice for a possible relationship between m_I and m_G in YARK theory, we are not in the least restricted by the equality $n = 1$. Under these conditions, one can assume that a “true relationship” between these mass components corresponds to the most compact form given by eq. (13). One can see that this is achieved at the choice $n = -1$, where eq. (13) then acquires the simplest Newtonian-like form

$$-G \frac{M m_G \mathbf{r}}{r^3} = \frac{d}{dt} m_I \mathbf{v}_0, \quad (16)$$

though its relativistic character is hidden in the definitions of m_I and m_G , i.e.

$$m_G = m_0 e^{-\alpha} / \gamma_0, \quad m_I = m_0 e^{-\alpha} \gamma_0. \quad (17a,b)$$

One can see that the substitution of eqs. (17a), (17b) into eq. (16) leaves unchanged the final motional equation (9). Formally other choices for n are allowed, they seem not plausible or even necessary. Further, it follows from eqs. (17a), (17b) that the ratio of gravitational and inertial masses in YARK theory becomes equal to

$$\frac{m_G}{m_I} = K = \frac{1}{\gamma_0^2} = 1 - \frac{v_0^2}{c^2}. \quad (18)$$

Thus, for the experiment by Eötvös (as well as for any other similar experiment where the test masses A and B are at rest with respect to each other); eq. (18) yields $K_A = K_B$, which, according to eq. (6), provides a vanishing torsion in the Eötvös balance.

But this, as we have elaborated previously, cannot be considered as a demonstration of $K=1$, and the claim about the necessity of metric theories of gravity becomes groundless.

Given such circumstances, two crucial questions emerge:

- what is the physical meaning of the definitions (17a), (17b)?
- could we distinguish the assumed expressions for the gravitational mass in metric theories (14) and in YARK theory (17a) in appropriate measurements?

In order to answer the first question, we remind the old problem of “harmony of phases” pointed out by de Broglie [44]. It originates from two different definitions of frequency based on some intrinsic periodic processes related to a uniform translational motion of a test particle with the rest mass m_0 . The first kind of frequency emerges in the equality

$$m_0 c^2 = h \nu_0, \quad (19)$$

which characterizes the frequency of the radiation with an energy equal to the rest energy of the particle. Hence, for a moving particle, we have

$$\gamma m_0 c^2 = h \nu_I, \quad (20)$$

so that eqs. (18), (19) provide the relationship

$$\nu_I = \gamma \nu_0. \quad (21)$$

On the other hand, due to the relativistic dilation of time, the frequency of any processes related to a moving particle decreases by γ times; i.e.

$$\nu_G = \nu_0 / \gamma. \quad (22)$$

Thus, comparing eqs. (17a), (17b) with eqs. (20) and (21), we get the relationships

$$\begin{aligned} m_G c^2 = h \nu_G, \quad m_I c^2 = h \nu_I; \text{ and} \\ K = m_G / m_I = \nu_G / \nu_I = 1 / \gamma^2. \end{aligned} \quad (23)$$

Eq. (23) discloses the physical meaning of the relativistic behavior of gravitational and inertial masses. Namely, we can suppose that an observer tracking the motion of the particle comes to conclude that, via the gravitational mass, a particle “senses” *the metric properties* of space-time related to the variation of its intrinsic frequency ν_G ; whereas via the inertial mass, the particle “senses” *its dynamical properties* characterized by the frequency ν_I .

One can add that the different dependence of gravitational and inertial masses on the factor γ according to eqs. (17a), (17b) had been first suggested by Mie [45-47], when he sought a compatibility between the STR and gravitation; though he could not capture the rest mass decreasing factor of $e^{-\alpha}$ specific to YARK theory in eqs. (17a), (17b) [48]. We can add that eq. (17a) signifies that the density

of the gravitational mass of any object does not depend on its velocity insofar as the volume of this object is reduced by γ times. A reasonable explanation of this fact is given by Mie on the basis of the Hamiltonian approach to the description of gravity [45-48]. After the development of GTR, this approach was, in point of fact, denied. However, with the development of YARK theory, some of the results of Mie could be topical again, along with the problem of seeking new approaches to measure the actual relationship between gravitational and inertial masses.

We emphasize again that the assumed inequality of gravitational and inertial mass components does not affect the motional equation (9) for the one-body problem. What is more, one can show (see, e.g., [21]) that the motional equation (9) for the one-body problem in YARK theory could differ from the corresponding motional equation for the one-body problem in GTR in the order c^{-2} and higher.

Therefore, a wide class of gravitational problems, which can be approximated by the one-body problem in the case of a weak gravitational field (e.g., the precession of the perihelion of Mercury [22]) has practically identical solutions in GTR and in YARK within the achieved measurement precision. Nevertheless, the relative measurement error of the order $(v/c)^2$, needed to distinguish the predictions of YARK and GTR, can be achieved via appropriate tests where some effects of the indicated order $(v/c)^2$, being tiny for non-relativistic objects, have the property to be accumulated with time via repeating measurements under identical conditions. For example, this can be made in experiments with a pendulum, if one aims to measure its oscillation period in the gravitational field of Earth. In the next section, we describe the essence of such an experiment and show that the difference between the predictions of YARK theory and GTR can indeed be reliably measured on the basis of high-quality pendulum systems with different arm lengths.

4. Proposal for a crucial test of GTR versus YARK theory: Comparing oscillations of pendulums with different arm lengths

First we show that the motional equations for a pendulum oscillating in a gravitational field, derived in GTR and in YARK theory correspondingly, differ from each other on the order of $(v/c)^2$, where v is a typical velocity of a suspended mass m of the pendulum. Below we derive the pendulum equations in YARK and in GTR, assuming that the mass of the arm of the pendulum is negligible in comparison with m , and the length of the arm is equal to L .

4.1. Pendulum equation in YARK theory

To proceed further, we introduce the angle θ between the axis y and the pendulum arm. Then, we consider the process where we move the pendulum mass quasistatically via pushing it along the circumference of radius L from its original angular position θ to the final position $\theta + d\theta$. Designating the infinitely short path $dl = Ld\theta$ of the pendulum mass, we obtain the variation of this mass dm due to the YARK postulate (7) as

$$mg \sin \theta = dmc^2. \quad (24)$$

Carrying out a corresponding integration between the angles θ_0 (the amplitude value) and θ , we obtain the rest mass of the pendulum as the function of θ :

$$m(\theta) = m(\theta_0) e^{\left[\frac{gL}{c^2}(\cos \theta - \cos \theta_0) \right]} = m_0 e^{-\alpha(\theta_0)} e^{\left[\frac{gL}{c^2}(\cos \theta - \cos \theta_0) \right]}. \quad (25)$$

Next, we take into account that, due to the energy conservation law, the total mass of the pendulum should remain constant, i.e.

$$m_0 e^{-\alpha(\theta_0)} c^2 e^{-\frac{gL}{c^2}(\cos\theta - \cos\theta_0)} \frac{1}{\sqrt{1-v^2/c^2}} = \text{constant}. \quad (26)$$

The pendulum equation of YARK theory is obtained via the differentiation of eq. (26):

$$\frac{d}{dt} \frac{e^{-\frac{gL}{c^2}(\cos\theta - \cos\theta_0)}}{\sqrt{1-v^2/c^2}} = 0, \text{ or } -gdl \sin\theta \sqrt{1-v^2/c^2} = \frac{v dv}{\sqrt{1-v^2/c^2}}. \quad (27)$$

Eq. (27) can be straightforwardly generalized to the vector form

$$\begin{aligned} -g \sin\theta \sqrt{1-v^2/c^2} \frac{\mathbf{l}}{l} &= \frac{1}{\sqrt{1-v^2/c^2}} \frac{d\mathbf{v}}{dt}, \text{ which leads to} \\ -g \sin\theta \sqrt{1-v^2/c^2} &= \frac{L}{\sqrt{1-v^2/c^2}} \frac{d^2\theta}{dt^2}. \end{aligned} \quad (28)$$

Further, we see that eq. (28) admits a presentation in the Newtonian-like form

$$-m_G g \sin\theta = m_I L \frac{d^2\theta}{dt^2} \quad (29)$$

along with the gravitational mass

$$m_G = m_0 e^{-\alpha(\theta_0)} (1-v^2/c^2)^{1/2} \quad (30)$$

and the inertial mass

$$m_I = m_0 e^{-\alpha(\theta_0)} (1-v^2/c^2)^{-1/2}, \quad (31)$$

yielding the relationship

$$m_G = m_I (1-v^2/c^2), \quad (32)$$

which coincides with eq. (18) obtained above.

Substituting eq. (32) into eq. (29), and using the approximation of small oscillations (where $\sin\theta \approx \theta$), which is sufficient for the present analysis, we obtain

$$\frac{d^2\theta}{dt^2} + \frac{g}{L} \theta - \frac{v^2}{c^2} \frac{g}{L} \theta = 0. \quad (33)$$

Here we omit the detailed solution of eq. (33), and present the final result

$$\omega_L = \sqrt{g_L/L_L} \sqrt{1-\langle v_L^2 \rangle/c^2}, \quad (34)$$

where $\langle v_L^2 \rangle$ is some averaged velocity of the pendulum mass over the period of oscillation as measured by the local observer. Therefore, the period of oscillation of the pendulum predicted in YARK theory is equal to

$$(T_L)_{YARK} = \frac{2\pi}{\omega} = 2\pi \sqrt{g_L/L_L} \left(1 - \langle v_L^2 \rangle / c^2\right)^{-1/2}. \quad (35)$$

4.2. Pendulum equation in GTR

In order to derive the motional equation of the pendulum in GTR, we shall start with the expression for the energy of a test particle in the Schwarzschild metric [1], i.e.

$$E_{GTR} = \gamma_L m_0 c^2 \sqrt{1 - 2\alpha}, \quad (36)$$

where the Lorentz factor as γ_L is defined for the local observer, and the factor α is written for the distant observer. Further, we take into account the fact that, during the oscillation of the pendulum, the tension force of the pendulum arm is always orthogonal to the velocity of the pendulum mass, so that this force does not do any work. Given these conditions, the energy (46) is conserved, and its time differentiation yields

$$d\alpha \left(1 - v_L^2 / c^2\right) = (1 - 2\alpha) \frac{v_L dv_L}{c^2}. \quad (37)$$

Defining the quantity $C = \left(1 - v_L^2 / c^2\right) (1 - 2\alpha)^{-1}$, we notice that it remains constant due to the constancy of the energy (36). In this case, eq. (37) becomes similar to the classical Newtonian pendulum equation where the constant parameter C is introduced. Hence, the period of oscillation of such a pendulum is equal to

$$T_{GTR} = 2\pi \sqrt{L/Cg}. \quad (38)$$

We express the constant C via the amplitude angle of the pendulum's oscillation θ_0 when $\alpha = \alpha(\theta_0) \equiv \alpha_0$, and $v_L(\theta_0) = 0$. Hence, $C = (1 - 2\alpha_0)^{-1}$, and eq. (38) acquires the form

$$T_{GTR} = 2\pi \sqrt{L/g} \sqrt{1 - 2\alpha_0}. \quad (39)$$

In order to compare the equation of YARK (35) with the equation of GTR (39), we have to express the ratio L/g via the corresponding local values. In the adopted approximation of small amplitude of pendulum oscillation, when the direction of its arm does not significantly deviate from the radial direction, we obtain in the Schwarzschild metric

$$L = L_L \sqrt{1 - 2\alpha_0}, \quad t = t_L / \sqrt{1 - 2\alpha_0}, \quad g = g_L (1 - 2\alpha_0)^{3/2}. \quad (40a,b,c)$$

Hence, substituting eqs. (40a) and (40c) into eq. (39), we get

$$T_{GTR} = 2\pi \sqrt{\frac{L_L (1 - 2\alpha_0)^{1/2}}{g_L (1 - 2\alpha_0)^{3/2}}} \sqrt{1 - 2\alpha_0} = 2\pi \sqrt{\frac{L_L}{g_L}}. \quad (41)$$

Thus, the GTR oscillation period for the local observer is equal to

$$(T_L)_{GTR} = \sqrt{g_{00}} T_{GTR} = \sqrt{1-2\alpha_0} T_{GTR} = 2\pi \sqrt{L_L/g_L} \sqrt{1-2\alpha_0}, \quad (42)$$

which is the final GTR expression for a local observer.

4.3. Comparison of the periods of oscillation of two pendulums with different arm lengths

Comparing now eqs. (35) and (42), we first estimate the corresponding numerical values for a terrestrial experiment, with $\alpha_0 = GM/Rc^2 \approx 0.7 \cdot 10^{-17}$, $v_L \approx 1$ m/s, and $v_L^2/c^2 \approx 10^{-17}$. Therefore, the latter ratio can be fully neglected, and the difference between the predictions of YARK theory and GTR with respect to the oscillation period of the pendulum is equal to

$$(T_L)_{YARK} - (T_L)_{GTR} = 2\pi \sqrt{\frac{L_L}{g_L}} - 2\pi \sqrt{\frac{L_L}{g_L}} \sqrt{1-2\alpha_0} = 2\pi \sqrt{\frac{L_L}{g_L}} (1 - \sqrt{1-2\alpha_0}) \approx 2\pi \alpha_0 \sqrt{\frac{L_L}{g_L}}. \quad (43)$$

At $L_L \approx 1$ m, and $g_L = 9.8$ m/s², we obtain

$$(T_L)_{YARK} - (T_L)_{GTR} \approx 1.5 \times 10^{-9}. \quad (44)$$

While this value lies within modern measurement capabilities, it still seems impractical to provide the corresponding accuracy of mechanical oscillations of any pendulum where the difference (44) can be reliably detected.

In order to overcome the indicated difficulty, we suggest to use two pendulums with different arm lengths, and to compare their oscillation periods.

Thus, for two pendulums with the length L_1 and L respectively, the difference of their periods of oscillation in YARK is equal to (for brevity, we omit below the subscript “L”):

$$(\Delta T_L)_{YARK} = 2\pi \sqrt{L_1/g} - 2\pi \sqrt{L/g} = 2\pi \sqrt{L/g} (\sqrt{k} - 1); \quad (45)$$

hereinafter we adopt $L_1 > L$, and designate the ratio $L_1/L = k > 1$.

The same expression in GTR reads as

$$(\Delta T_L)_{GTR} = 2\pi \sqrt{L/g} (\sqrt{k} - 1) \sqrt{1-2\alpha_0} \approx 2\pi \sqrt{L/g} (\sqrt{k} - 1) (1 - \alpha_0). \quad (46)$$

Hence, the difference in the oscillation periods of the two pendulums with different arm lengths, as predicted in YARK and in GTR, is equal to

$$(\Delta T_L)_{YARK} - (\Delta T_L)_{GTR} = 2\pi \alpha_0 \sqrt{L/g} (\sqrt{k} - 1). \quad (47)$$

During a chosen overall measurement time τ , this difference per unit cycle of oscillation accumulates and increases by the number N of the pendulums' total cycle of oscillations that take place during τ . Thus, the accumulated difference during the chosen time τ will be equal to

$$\Delta T = N 2\pi \alpha_0 \sqrt{L/g} (\sqrt{k} - 1) \approx (\tau / (T_0)_{YARK}) 2\pi \alpha_0 \sqrt{L/g} (\sqrt{k} - 1) = \alpha_0 \tau (1 - 1/\sqrt{k}), \quad (48)$$

where we have taken $N = \tau / (T_0)_{YARK}$ and used eq. (35), neglecting the term $\langle v_L^2 \rangle / c^2$.

Thus, the suggested “differential” method for the measurement of the difference of the periods of oscillations for two pendulums with different arm lengths yields the value (48), which can be measured via fixing, for example, the time moments of the pendulums at which they cross the minimum altitude during oscillation. For example, at $T = 10^3$ s (about 15 min), and $k = 2$, we get $\Delta T \approx 2 \times 10^{-7}$ s, which represents a readily measurable value.

Another point is that the quality of both pendulums must be very high (ensuring absence of friction in a high vacuum chamber, etc); however, this does not constitute a difficult problem with respect to quotidian achievements in experimental technique.

5. Conclusion

The principal outcome of the present paper is the explicit demonstration of the fact that the experiment by Eötvös, as well as other experiments aimed to check the *identity* between gravitational m_G and inertial m_I masses, all were *inconclusive*. They serve no more than displaying just an ordinary linear proportionality of the gravitational and inertial mass components to each other with some proportionality constant K . Thus, to the contrary to the widespread claim about the equality of the gravitational mass and inertial mass, which is strongly assumed in GTR and extended theories of gravity, is deprived of any reliable experimental evidence.

We would like to accentuate the fact that, within purely metric, or metric-affine gravitational theories, the coefficient K must be exactly equal to unity; this is the necessary condition to reduce gravity to the geometry of space-time. And the Eötvös experiment aimed to prove it, does not constitute, and in fact has never constituted, any rigorous proof of it.

By the same token, we underline the factuality that the Eötvös experiment and modern experiments on the same subject [6,7,9-12] cannot by any means be cited to condemn theories like YARK, which admits the inequality, yet linear proportionality, of gravitational and inertial masses.

With the ordinary linear proportionality condition of gravitation mass to inertial mass, YARK theory perfectly obeys the null result of any Eötvös type experiment; all the more so since YARK fully predicts the null result of it.

In these respects, we re-analyzed the problem of the determination of gravitational and inertial masses in YARK theory of gravity, and found that the coefficient proportionality K between m_G and m_I depends on the velocity of the object. We have demonstrated that the YARK motional equation of the test particle in the presence of gravity acquires the most compact form at $K = 1/\gamma^2$ (with γ being the Lorentz factor of that particle), along with the gravitational and inertial mass components defined via eqs. (17a), (17b). At the same time, the assumed equalities (17a), (17b) were conjectured through an inverse problem setup by Mie (except our exponential mass decreasing factors), who sought a full compatibility between STR and any theory of gravitation more than a 100 years ago [46]. We pointed out that Mie considered this, as an inverse problem unlike how it was done straightforwardly in YARK, where we landed at the given equations based on the law of energy conservation postulated in the form of eq. (7), and in full conformity with quantum mechanics and the WEP.

Furthermore, we advanced physical argumentations (e.g., the problem of the harmony of phases disclosed by de Broglie) that the inequality of gravitational and inertial masses could be tested in suitable quantum mechanical phenomena.

We brought up a crucial point, i.e. the indication of the possibility to test the equality (or inequality) of gravitational and inertial masses via a macroscopic pendulum experiment. In effect, we have found above that, for an ordinary pendulum with an arm length of about 1 m, the difference between YARK theory and GTR predictions with respect to the oscillation period is equal to only a few nanoseconds. However, it is difficult to believe that even high-quality pendulums could provide an accuracy of their motion where any effect of the mentioned order of magnitude could be measured. Under these conditions, we suggested a “differential” method where two high-quality pendulums are initially set to oscillate parallel to each other, and we proposed to measure the difference in periods of

time of the given oscillations; the difference evidently accumulates with time, making realistic the determination of the given difference for high-quality pendulum systems.

In this respect, we remind that, historically, the experiments for the comparison of periods of oscillations T of two pendulums had been already carried out since Newton's time. However, as elaborated above, the variable parameter throughout was the pendulum mass m , so that the revealed independence of T on m can be considered as one more successful test of WEP and nothing more.

Now we suggest comparing two pendulums with different arm lengths. Thereby, in the present contribution, next to our fundamental revelations with regards to the quandary about the identity of gravitational and inertial masses, we have shown that the expected difference of the oscillation periods of the given pendulums (48) can be reliably measured, and will represent a new crucial test of GTR versus YARK theory.

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