Determination of the astrophysical factor of the $^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$ down to zero energy using the asymptotic normalization coefficient method.

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Abstract. The observation of neutrinos emitted in the $p-p$ chain and in the CNO cycle can be employed to test the Standard Solar Model. The $^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$ reaction is the first reaction of the 2nd and 3rd branch of the $p-p$ chain, so the indetermination of its cross section significantly affects the predicted $^{7}\mathrm{Be}$ and $^{8}\mathrm{B}$ neutrino fluxes. Notwithstanding its relevance and the great deal of experimental and theoretical papers, information of the reaction cross section at energies of the core of the Sun (15 keV - 30 keV) is sparse and additional experimental work is necessary to attain the target ($\sim$ 3%) accuracy. The precise understanding of the external capture component to the $^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$ reaction cross section is pivotal for the theoretical assessment of the reaction mechanism. In this work, the indirect measurement of this external capture component using the Asymptotic Normalization Coefficient (ANC) technique is discussed. To extract the ANC, the angular distributions of deuterons yielded in the ^{6}Li(^{3}He,d)^{7}Be α-transfer reaction were detected with high precision at $E_{^3He}$=3.0 MeV and 5.0 MeV. The ANCs were then deduced from the juxtaposition of DWBA and CC calculations with the experimental angular distributions and the zero energy astrophysical S-factor for ^{3}He$(\alpha,\gamma)^{7}$Be reaction was calculated to equal 0.534 ± 0.025 keVb. Both our experimental and theoretical approaches were tested through the analysis of the ^{6}Li(p,γ)^{7}Be astrophysical factor, with further interesting astrophysical implications.

1. The astrophysical background
The ^{3}He(α,γ)^{7}Be is an essential reaction in nuclear astrophysics. It is the first reaction of the 2^{nd} and 3^{rd} $p-p$ chain branch and, for this reason, the indetermination on its rate significantly affects the accuracy of the evaluated ^{7}Be and ^{8}B neutrino fluxes. While the measurement of the neutrino fluxes directly from the Sun core has progressively got more precise after the installation of larger and more efficient neutrino detection facilities, sensitive to a broader neutrino energy interval, the ^{3}He(α,γ)^{7}Be reaction has continued to be uncertain after decades, in spite of the great deal of experimental and theoretical works focused on its assessment.

In detail, the flux of the $p-p$ neutrinos was measured with a precision of about 3.4% by the BOREXINO, SNO and Super-Kamiokande detectors [1, 2, 3]. The accurate neutrino flux determinations are used to constrain the Standard Solar Model (SSM) and provide evaluations of the temperature of the core of the Sun; however, the important nuclear reaction cross sections should be known with analogous accuracy. Nonetheless, present-day errors on these input parameters are very large, typically of the order of 5-8% [4] at odds with the 3% target accuracy [5, 6]. Hence, an advance in the understanding of the low-energy cross section of the ^{3}He(α,γ)^{7}Be reaction would turn out in a significant decrease of the errors and might have major consequences for the SSM.

The chief source of indetermination is that the astrophysically important energy region, the so-called Gamow peak, is located between about 15 keV and 30 keV for a temperature of 15 MK, typical of the core of the Sun, and at these energies the ^{3}He$(\alpha,\gamma)^{7}$Be reaction cross section is so small to make direct measurements impossibile. Theory-based extrapolations are often adopted to attain the reaction rate [7, 8, 9]. Focusing on the experimental techniques latterly used, they can be divided into three categories: the detection of prompt γ rays [10, 11, 12, 13], the measurement of the ^{7}Be activity [14, 15, 16, 17, 18], and the counting of the recoiling ^{7}Be nuclei with a recoil mass separator [19].

About the theoretical models, several different approximations - such as external capture model (e.g. [20]), potential model (e.g. [21, 22]), modified two-body potential approach [23], resonating group calculation (e.g. [24]), *ab initio* model (e.g. [25, 26]) and R-matrix theory [27, 28] - were applied to calculated the reaction cross section. While the accuracy of the extrapolations is about 6-7%, the spread between the zero-energy ^{3}He(α,γ)^{7}Be astrophysical factors $S_{34}(0)$ is larger than about 10%. The estimated $S_{34}(0)$ factors are shown in Fig. 1. The figure demonstrates that the calculated $S_{34}(0)$ factors depend distinctly on the model selected for the extrapolation procedure and high accuracy experimental data is needed to constrain the theoretical models.

2. The ANC experiment
This paper epitomizes the results previously published in Refs.[32, 33]. The method therein described has made it viable to obtain the $S_{34}(0)$ factor of the ^{3}He(α,γ)^{7}Be reaction without extrapolation, by means of the asymptotic normalization coefficient (ANC) technique [35]. In detail, since the ^{3}He(α,γ)^{7}Be reaction at stellar energies is a pure external direct capture process

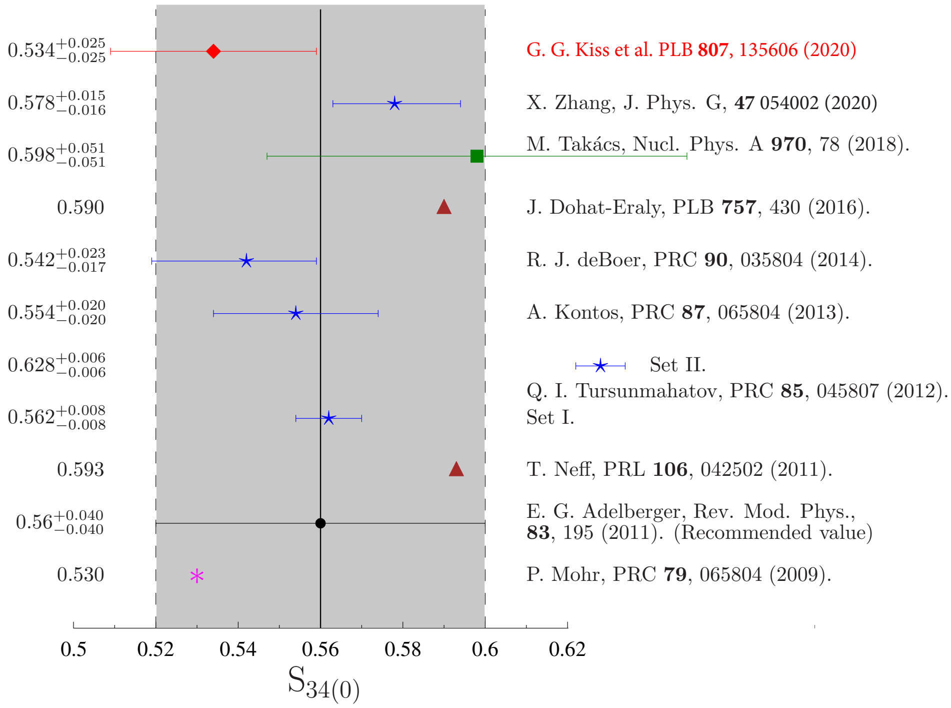

Figure 1. Résumé of the latest ^{3}He$(\alpha,\gamma)^7$Be $S_{34}(0)$ factors (fig.1 of [32]): deduced from the analysis of elastic-scattering angular distributions [22] (pink star), theoretical calculations [25, 30] (dark red triangle), extrapolations of experimental data sets [13, 23, 28, 29] (blue star), determination based on neutrino yield measurements [31] (green box) and obtained using the ANC technique [32] (red diamond). The solid central line shows the recommended value of [8], with its uncertainty range marked by the shaded area. For Tursunmahatov et al. [23], the $S_{34}(0)$ value deduced by fitting [10, 12, 14, 15] is given.

[8], it mostly takes place through the tail of the nuclear overlap function, with no sensitivity to nuclear structure features. The shape of the overlap function in the tail region is exclusively defined by the Coulomb interaction and, in turn, the amplitude of the overlap function fixes the rate of the capture reaction [36, 37]. Becaus the direct capture cross sections are proportional to the squares of the ANCs - which are deduced from transfer reactions - the investigation of the near barrier ^{6}Li(^{3}He,d)^{7}Be α particle transfer reaction makes it possible to obtain the ANCs for the ^{3}He(α,γ)^{7}Be reaction. This alternative experimental method, enhancing our knowledge of the low-energy behavior of this reaction, was so-far never adopted to study the ^{3}He(α,γ)^{7}Be reaction.

The transfer reaction was studied using the ^{3}He beams supplied by the 3.5 MV singletron accelerator of the Department of Physics and Astronomy (DFA) of the University of Catania (Italy) and the FN tandem accelerator at the John D. Fox Superconducting Accelerator Laboratory at the Florida State University (FSU), Tallahassee (FL), USA. More details about the experiments and the theoretical approach can be found in [32, 33]. To determine the ANCs, deuteron angular distributions were measured at two energies ($E_{lab.}$ = 3 MeV and $E_{lab.}$ = 5 MeV) over a wide angular range using silicon ΔE-E telescopes mounted on a movable arms. Monitor detectors were set at fixed angles with respect to the beam axis for absolute normalization. ^{6}LiF (enriched in ^{6}Li by 95%) and pure ^{6}Li targets (enriched in ^{6}Li by 98%) were employed. By using the ΔE-E particle identification technique [34] and thanks to the high-resolution attained, clear d_0 and d_1 loci, corresponding to ^{7}Be ground and first excited states, were observed. At the backward hemisphere the differential cross section (DCS) increases with increasing angles and this corroborates the occurrence of a dominant one-step α-particle exchange mechanism. Similarly, one-step proton transfer mechanism is observed to be the largest in the forwards hemisphere, with minor interference.

The ANCs for the $^{3}\mathrm{He} + \alpha \to {}^{7}\mathrm{Be}$ channel were determined by adopting the modified Distorted Wave Born Approximation (DWBA) [38] approach, hypothesizing one-step proton and α particle transfer [39]. By scaling the computed DCSs to the experimental ones for each experimental point ($\theta = \theta^{\mathrm{exp}}$) for the backward angle regions, the ANCs for $^{3}\mathrm{He} + \alpha \to {}^{7}\mathrm{Be}_{\mathrm{g.s}}$ and $^{3}\mathrm{He} + \alpha \to {}^{7}\mathrm{Be}(0.429$ MeV) (that is, ^{7}Be first excited state) channels were derived. The channels coupling effects (CCE) were deduced for each experimental point of θ^{exp} using the FRESCO code [40] by including one-step processes only, with proton stripping $^{6}\mathrm{Li}(^{3}\mathrm{He}, d)^{7}\mathrm{Be}$ and exchange mechanism with the α-particle cluster transfer $^{6}\mathrm{Li}(^{3}\mathrm{He}, {}^{7}\mathrm{Be})d$.

The weighed mean values of the square of the ANCs for the $^{3}\mathrm{He} + \alpha \to {}^{7}\mathrm{Be}$(g.s.) and $^{3}\mathrm{He} + \alpha \to {}^{7}\mathrm{Be}(0.429$ MeV) are equal to C^2= 20.84 ± 1.12 [0.82; 0.77] fm^{-1} and C^2= 12.86 ± 0.50 [0.35; 0.36] fm^{-1}, respectively, which are in very good agreement with those of [23], deduced from the analysis of the experimental S-factor of [10, 12, 14, 15]. The total errors shown here are calculated by summing the uncertainties in quadrature, taking into accout both experimental errors in the $d\sigma^{\mathrm{exp}}/d\Omega$ (first term in square parentheses) and the indetermination from the ANC for $d +^4 He \to {}^{6}$Li, and the model uncertainties (second term in square parentheses). Next, the direct capture term of the $^{3}\mathrm{He}(^{4}\mathrm{He},\gamma)^{7}\mathrm{Be}$ astrophysical factor at the Gamow energy of the core of the Sun was deduced using the modified two-body potential model (MTBPM) [41, 42], and the resulting $S_{34}(0)$ and S_{34}(23 keV) factors were established to be $S_{34}(0) = 0.534 \pm 0.025$ [0.015; 0.019] keVb and $S_{34}(23\ \mathrm{keV}) = 0.525 \pm 0.022$ [0.016; 0.016] keVb. The juxtaposition with the values in the literature shown in Fig.1 implies an increased accuracy in comparison with the present-day recommended value in ref.[8], though an indetermination still higher than the pursed value is apparent, calling for more measurements to further reduce it.

3. Independent validation of the approach: the $^{6}\mathrm{Li}(p, \gamma)^{7}\mathrm{Be}$ reaction

Because the one-step proton transfer is dominant at forward angles, the ANCs for the $^{6}\mathrm{Li} + p \to {}^{7}\mathrm{Be}$ channel was also derived, using a similar approach as sketched above by normalizing the forward hemisphere angular distributions computed in the post form of the modified DWBA [38] by means of the LOLA code [47]. As in [48], we focused on the first peak in the angular distribution, where the extraction of the ANC is most accurate since ANC can be deduced from peripheral transfer processes only. The weighted mean values of the square of the ANCs for $^{6}\mathrm{Li} + p \to {}^{7}\mathrm{Be}$ were calculated to be 4.81±0.38 fm^{-1} and 4.29±0.27 fm^{-1} for the ground and first excited states of ^{7}Be, respectively, for to their sum over $j_{^{6}\mathrm{Li}-p}$ ($j_{^{6}\mathrm{Li}-p} = 1/2$ and 3/2). The total error corresponds to the averaged squared of the different uncertainty sources, including both experimental uncertainties on $d\sigma^{\mathrm{exp}}/d\Omega$ and theoretical uncertainties from the DWBA analysis. Lastly, as discussed in detail in ref.[33], the $^{6}\mathrm{Li}(p, \gamma)^{7}\mathrm{Be}$ astrophysical S-factor was computed assuming $E1$ direct capture (DC) (green line in fig.2). At E=0, the indirect $S_{61}^{(\mathrm{DC})}(E)$ equals $65.781 \pm 5.227[3.380; 1.040; 3.859]$ eV·b and $30.675 \pm 1.957[0.464; 0.514; 1.828]$ eV·b for the ground and the first excited states of ^{7}Be, respectively, entailing a total S-factor value of 96.5±5.7 eV·b. This is in very good agreement with the extrapolated S-factor to zero energy (S(0) = 95 ± 9 eV·b) of [43], with an indetermination 1.6 times lower. While this conclusion does not support the presence of the 200 keV resonance highlighted in [44], such accord is a proof of the validity of the approach adopted for deriving the $^{3}\mathrm{He}(^{4}\mathrm{He},\gamma)^{7}\mathrm{Be}$ S-factor.

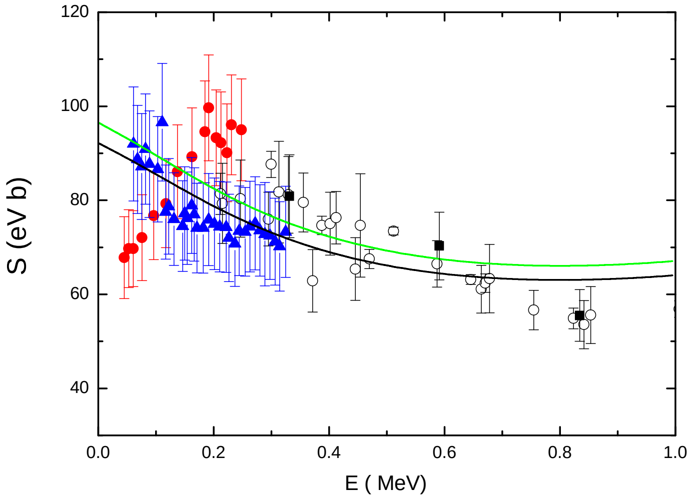

Figure 2. The experimental and computed astrophysical S-factor for the radiative-capture ^{6}Li$(p,\gamma)^7$Be reaction (fig.7 of ref.[33]). The solid green line is the direct part of the astrophysical S factor, calculated using the weighted average ANC values from the near-barrier proton transfer reaction ^{6}Li(^{3}He,d)^{7}Be at $E_{^3\mathrm{He}}$=3 and 5 MeV. The black line is the S factor calculated using the ANCs determined from the analysis of ^{6}Li$(p,\gamma)^7$Be directly-measured reaction [43]. Blue solid triangles mark the bare-nucleus astrophysical factor from ref.[43] (including systematic error), red filled circles are the experimental astrophysical factor in [44], empty circles are from [45] and black solid squares from [46].

An additional test of the used theoretical framework is given by the re-analysis of the ^{6}Li(p,γ)^{7}Be directly-measured astrophysical factor [43]. The ANCs for the $^6\mathrm{Li} + p \to {}^7\mathrm{Be}$(g.s.) and $^6\mathrm{Li}+p \to {}^7$Be(0.429 MeV) channels were deduced from the experimental total astrophysical S-factor and the branching ratios of ref. [43], within the MTBPM [42]. The values of the weighted average for the ANC values for ^{7}Be ground and first excited states deduced from all the experimental data in [43] equal $(C^{exp}_{1\,1/2+1\,3/2})^2 = 4.345 \pm 0.576\,[0.033;\,0.041;\,0.574]\,\mathrm{fm}^{-1}$ and $(C^{exp}_{1\,1/2+1\,3/2})^2 = 4.571 \pm 0.595\,[0.027;\,0.033;\,0.594]\,\mathrm{fm}^{-1}$, respectively. These ANCs are in very good accordance with the value of 4.81$\pm$0.38 fm^{-1} for the ground and 4.29$\pm$0.27 fm^{-1} for the first excited state of ^{7}Be, derived from the analysis of the ^{6}Li(^{3}He,d)^{7}Be transfer reaction. Furthermore, the ^{6}Li(p,γ)^{7}Be astrophysical factor was computed assuming $E1$ DC ending up in the black line in fig.2. The very good accord with the astrophysical factor in ref.[43] is an additional test of the used approach.

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