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Coulomb shift in two-center cluster systems

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Coulomb shift is analyzed for the mirror cluster systems of ^{18}O = α +^{14}C and ^{18}Ne = α + ^{14}O by applying the orthogonality condition model (OCM). The OCM calculation clearly predicts the suppressed excitation energy of the higher 0^+ states in the proton-rich system of ^{18}Ne. This results can be interpreted in terms of the extension of the Thomas-Ehrman shift (TES), which is discussed in ^{17}O = ^{16}O + N and ^{17}F = ^{16}O + P, to the cluster degrees of freedom.

KEYWORDS: α-cluster structure, mirror systems, Coulomb shift

1. Introduction

α-cluster structures are well known to appear in the excited states of light mass systems. α particle is a building block in constructing the intrinsic structures of nuclei because of its stable and inert property. The α-cluster structures have been extensively discussed in the $4N$ systems, which have the multiple mass number of the α particle [1], such as ^{8}Be = 2α, ^{12}C = 3α, ^{16}O = α + ^{12}C and ^{20}Ne = α + ^{16}O. These α-cluster structures mainly appear in the excited 0^+ states below 15 MeV in the excitation energy. In the α-cluster structure, the α particle and the residual nuclei are weakly coupled. This weak coupling feature leads to a large extension of the wave function of the cluster relative motion with a large mixture of the S-wave component. In this situation, the Coulomb interaction is not so effective, and the suppressed Coulomb shift, which is similar to the so-called Thomas-Ehrman shift (TES) discussed in ^{17}O – ^{17}F [2], is expected to be observed. Therefore, it is interesting to extend the viewpoint of TES, which is discussed in the single particle picture, to the α clustering phenomena. If TES occurs in the α cluster states, we will observe the compressed excitation energy of the excited 0^+ states in the proton-rich system. In fact, such the compression of the excitation energy for the excited 0^+ states has been briefly discussed in ^{10}Be – ^{10}C from the viewpoint of the covalent molecular picture [3].

In the present report, we extend the analysis of the Coulomb shift studied in ^{10}Be – ^{10}C to the heavier systems, such as ^{18}O – ^{18}Ne, which are handled by the cluster models of α + ^{14}C and α + ^{14}O, respectively. Since the full microscopic model is not necessarily successful in reproducing the energy levels in ^{18}O [4, 5], we introduce the semi-microscopic model, called the orthogonality condition model (OCM) [6, 7]. The OCM application to ^{18}O was done by Furutani *et al.* but the scattering boundary condition for the unbound continuum is not considered [7]. Here we employ the OCM plus absorbing boundary condituion (ABC) [8], which is powerful tool to handle the unbound continuum, and analyze the continuum states in the mirror cluster-systems of ^{18}O – ^{18}Ne. The results in the present report are compressed from Refs. [9, 10].

2. Framework

We apply the orthogonality condition model (OCM) to the α + ^{14}C and α + ^{14}O systems. In the following, we explain the OCM formulation for the α + ^{14}C system because the extension to the α + ^{14}O can be achieved in a straight forward manner. We construct the Pauli allowed states in a simple manner based on the Elliott's SU(3) algebra [11]. The internal configuration of ^{14}C is set to the lowest shell model configuration, such as $(\lambda, \mu) = (0, 2)$ in the SU(3) irreducible representation of the harmonic oscillator (HO) wave function [11]. The Pauli allowed states of the total system, which are composed of α and ^{14}C, must have the total HO oscillator quanta of $N \geq 6$ in the $\alpha - ^{14}$C relative motion. The $N = 6$ state corresponds to the partially allowed state, while the quantum states with $N \geq 8$ are the completely allowed states for the positive parity. The SU(3) representation for the partially allowed states can be generated according to the following coupling scheme of the SU(3) irreducible representation: $(0, 2) \otimes (6, 0) = (4, 0)$. The totally coupled state of $(4, 0)$ corresponds to the lowest shell model state of $^{18}\mathrm{O} = ^{16}\mathrm{O} \otimes \nu(1s0d)^2$.

After constructing the partially allowed state, the total wave function is expanded by the channel wave function of α + ^{14}C(0^+, 2^+), and the coupled-channel (CC) equations are solved. In the CC calculations, the nuclear potential of α and ^{14}C is calculated from the double folding (DF) model [12]. We use the matter density of the α particle, which reproduces the charge form factor of the electron scattering, while the density of ^{14}C is calculated from the HO-type distribution. As for the effective nucleon-nucleon interaction, we employ the DDM3Y (density dependent Michigan 3-range Yukawa) interaction [13] with the target density approximation (TDA) [14]. The DF+TDA potential nicely reproduces the angular distribution of the α + ^{14}C(0^+) elastic scattering [9].

We assume the same DF potential in the excited channel of α + ^{14}C(2^+) for simplicity. The coupling potential of ^{14}C(0^+) $\rightarrow$ ^{14}C(2^+) is constructed by the differential function of thfrom DF potential with the strength of -0.2 fm. Above the α decay threshold, the absorbing boundary condition (ABC) [8] is applied to identify the resonance parameters, such as the resonance energy E_R and the decay width Γ_R.

3. Results

The calculated energy spectra for the $J^\pi = 0^+$ state is shown in Fig. 1. The OCM calculation reproduces the binding energy of the ground 0^+_1 state (-6.2 MeV) with respect to the α decay threshold (dashed line). The first excited 0^+_2 state appears at the binding energy of $E = -1.95$ MeV, which corresponds to the observed 0^+_2 level (-2.60 MeV). The bound 0^+_1 and 0^+_2 states are reproduced but the 0^+_3 state is missing in the present calculation because this state has the $\nu(1s_{1/2})^2$ configuration with the ^{16}O core.

On the contrary, in the unbound region, two resonant 0^+ states are obtained in the present calculation: the resonances at $E_\mathrm{R} \sim 2.5$ MeV and at $E_\mathrm{R} = 9.3$ MeV. We label the former and latter resonances the 0^+_4 and 0^+_5 states, respectively, because of missing 0^+_3 in our calculation. The 0^+_4 level, which was not obtained in the previous calculation [7], exists around the Coulomb barrier of the α + $^{14}\mathrm{C}_{\mathrm{g.s.}}$ ($L = 0$) channel with a broad decay width of $\Gamma_\mathrm{R} \sim 2.5$ MeV. A broad 0^+ resonance is observed in the same energy region of the α + ^{14}C elastic scattering, which is plotted by 0^+ with a shade above the α threshold (Experiment) [15]; the observed energy and width are $E_\mathrm{R} = 3.7$ MeV ($E_\mathrm{ex} = 9.9$ MeV) and $\Gamma_\mathrm{R} = 3.2$ MeV, respectively. Moreover, a new resonant state has just been measured at the excitation energy close to the energy position of the 0^+_5 state in the reaction of ^{9}Be(^{13}C, ^{18}O)α [16].

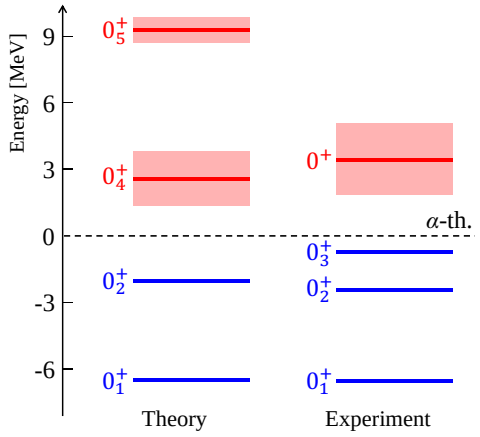

Fig. 1. (Color online) Energy spectra in ^{18}O = α + ^{14}C. The left and right levels are the results of the theoretical calculation and the experimental observations, respectively. The dashed line shows the α threshold, while the shades attached to the resonant levels represent the decay width. This figure is taken from Ref. [10]

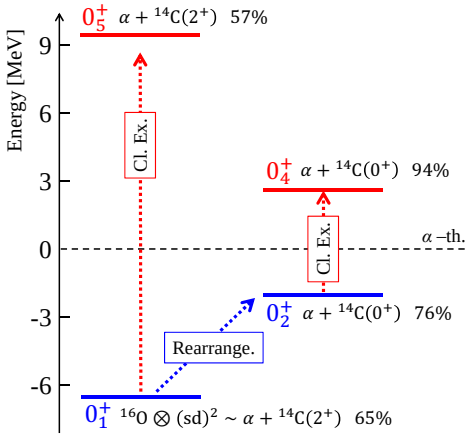

Fig. 2. (Color online) Excitation scheme in α + ^{14}C. The horizontal dashed-line shows the α threshold. The dotted arrows with the boxes represent the excitation schemes, such as the cluster relative excitation (Cl. Ex.) and the rearrangement (Rearrange.).This figure is taken from Ref. [10]

The excitation scheme in $J^\pi = 0^+$ is summarized in Fig. 2. Basically, the ground 0_1^+ state has a compact structure of the shell model state with $^{16}O \otimes \nu(1s0d)^2$ but it is equivalent to the cluster state with a dominance of the α + ^{14}C(2^+) channel (65%). The rearrangement (Rearrange.) of the main component occurs in the excited 0_2^+ state, in which the α + ^{14}C(0^+) channel is dominant (76%). The two resonant states of 0_4^+ and 0_5^+ are generated by the cluster relative excitation in α – ^{14}C (Cl. Ex.) from the low-lying 0_1^+ and 0_2^+ states, respectively.

In the resonant $0_{4,5}^+$ states, the relative wave functions have the one higher node in comparison to the bound states, and their tails are extended to the outer region. In this situation, the effect of the Coulomb interaction for the resonant states (0_4^+ and 0_5^+) is expected to be weaker than the effect for the bound states (0_1^+ and 0_2^+), in which the relative wave functions are confined in the region of the nuclear interaction. Therefore, it is interesting to analyze the Coulomb shift in connection to the development of the α-cluster degrees of freedom.

Table I. Energy shift in ^{18}O – ^{18}Ne systems. See text for details. This table is taken from Ref. 10

	$\Delta E_{\rm Coul}$		$\Delta E_{\rm ex}$	
	Expt.	Th.	Expt.	Th.
0_1^+ (B. S.)	1.12	1.12	0	0
0_2^+ (B. S.)	1.06	1.14	0.06	0.03
0_4^+ (Res.)	–	0.53	–	0.58
0_5^+ (Res.)	–	0.65	–	0.46

The energy shifts of ^{18}O – ^{18}Ne are summarized in table I. In this table, $\Delta E_{\rm Coul}$ represents the magnitude of the energy difference, $\Delta E_{\rm Coul} = |E_{\rm R}(^{18}{\rm O}) - E_{\rm R}(^{18}{\rm Ne})|$ with respect to the α decay threshold. As for the two bound states, 0_1^+ and 0_2^+, the experimental energy shifts (Expt.) are reproduced by the theoretical calculation (Th.), which amounts to about 1.1 MeV. The $\Delta E_{\rm Coul}$ for the unbound resonances, 0_4^+ and 0_5^+, are reduced to about half of the bound state, just about 0.6 MeV. This

reduction is originated from the extended and dilute structure of the $\alpha - {}^{14}\mathrm{C}$ relative wave function. This reduced energy shift with respect to the α threshold just corresponds to the reduction of the excitation energy, which is measured from the ground 0^+_1 state. In table I, the shift of the excitation energies are shown by ΔE_{ex}, which is defined by $\Delta E_{\mathrm{ex}} = |E_{\mathrm{ex}}({}^{18}\mathrm{O}) - E_{\mathrm{ex}}({}^{18}\mathrm{Ne})|$ with the excitation energy of $E_{\mathrm{ex}} = E(0^+_{\mathrm{ex}}) - E(0^+_1)$. ΔE_{ex} is almost negligible for the bound 0^+_2 state but it is enhanced to be about 0.5 MeV for the unbound resonant 0^+_4 and 0^+_5 states. The shift of the excitation energy corresponds to the so-called Thomas-Ehrman shift (TES), which is originally discussed in the single particle picture of ${}^{17}\mathrm{O} = {}^{16}\mathrm{O} + N$ and ${}^{17}\mathrm{F} = {}^{16}\mathrm{O} + P$ [2].

4. Summary

In sum, we have investigated the $J^\pi = 0^+$ energy spectra in ${}^{18}\mathrm{O} = \alpha + {}^{14}\mathrm{C}$ by applying the orthogonality condition model (OCM) under the absorbing boundary condition (ABC). The OCM + ABC calculation for ${}^{18}\mathrm{O}$ nicely reproduces the observed $J^\pi = 0^+$ levels from the bound to unbound region except for the 0^+_3 state, which is out of the α-cluster model space.

The OCM + ABC calculation is also extended to ${}^{18}\mathrm{Ne} = \alpha + {}^{14}\mathrm{O}$, and the Coulomb shift for the mirror systems of ${}^{18}\mathrm{O} - {}^{18}\mathrm{Ne}$ is investigated. The Coulomb shift, which is measured from the α decay threshold, is large for the bound 0^+_1 and 0^+_2 states but it is reduced to be half for the 0^+_4 and 0^+_5 states. This reduction of the Coulomb shift is originated from the extended and dilute structure in the $\alpha - {}^{14}\mathrm{C}$ (${}^{14}\mathrm{O}$) relative wave function, which involves a large mixture of S-wave component. The reduced Coulomb shift in the cluster states corresponds to the compression of the excitation energy in the proton-rich ${}^{18}\mathrm{Ne}$. The origin of this compressed energy is just similar to the Thomas-Ehrman shift (TES), which is discussed in ${}^{17}\mathrm{O} = {}^{16}\mathrm{O} + N$ and ${}^{17}\mathrm{F} = {}^{16}\mathrm{O} + P$ [2]. Thus, the energy compression predicted in the cluster system should be called "Cluster TES". We propose that Cluster TES is a new probe to identify the extended and dilute structure of the cluster configuration.

The present result about Cluster TES is also possible to occur in more general $N \neq Z$ systems. For example, it is interesting to consider the $4N$ nuclear system with two extra nucleons, such as ${}^{10}\mathrm{Be} - {}^{10}\mathrm{C}$, ${}^{14}\mathrm{C} - {}^{14}\mathrm{O}$, and ${}^{22}\mathrm{Ne} - {}^{22}\mathrm{Mg}$. In advancing the study of Cluster TES, it is essential to investigate the level structure, especially the unbound resonances, experimentally. The experimental analysis of the resonant structures in the proton-rich side is strongly desired.

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