

CALCULATION OF FOCAL SPOT OF SECONDARY X-RAYS GENERATED BY HIGH-ENERGY ELECTRON BEAM BOMBARDING OF HEAVY METAL TARGETS

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Abstract

One of the main methods to generate X-rays is to bombard metal targets with electron beams. However, this process introduces uncertainty in the electron transport, which leads to uncertainty in the position and momentum of the secondary X-rays. As a result, the focal spot of the X-rays is larger than the electron beam. In this paper, we use the Monte Carlo software Geant4 to investigate the conditions for minimizing the X-ray focal spot size. We assign different weights to the X-rays according to their energy components, based on the actual application parameters, and calculate the focal spot size for three target materials: lead, copper, and tungsten, finding that when the incident electron energy is in the MeV range and the electron source radius is 1 μm , the mass thickness of the target of $1.935 \times 10^{-3} \text{g/cm}^2$ is the limit for achieving the smallest focal spot size.

INTRODUCTION

Nowadays, X-ray imaging technology is widely used in various fields such as medical treatment, security inspection, scientific research, etc. The pursuit of clearer and more detailed images in these fields is growing, and the imaging resolution of X-rays required is further improved, so how to produce X-rays with higher imaging resolution has become the next problem to be considered.

Due to an X-ray source's focal spot size largely determining the imaging resolution, many studies have been conducted on the factors affecting the focal spot size. For example, Edmond Sterpin *et al.* [1] investigated the distribution of X-ray energy injection under different conditions and explored the relationship between the spot size of the electron beam and the equivalent focal spot size and location of the focal spot. Jiayue Wang *et al.* [2] investigated the relationship between the X-ray tube point spread function (PSF) and the electron beam energy, target thickness, and the angle of incidence of the electron beam.

However, few studies of focal spots to date have taken the effects of the irradiated sample into account. Industrial CT generally explores larger, denser samples, so that the low-energy component of the X-rays decays quickly. In contrast, the high-energy component decays less, and more of the high-energy component ends up in the detector. In this paper, we propose a simple weighting method to take into account the influence of the sample and show the pattern of variation in

the size of the focal spot under different conditions on this basis.

DEFINITION OF FOCAL SPOT SIZE

The structure of the system studied in this paper is very simple, as shown in Figure 1, the electron source is a uniform surface source, the target is a cylindrical thin target, the electron beam is perpendicular to the bottom surface of the target, and is incident along the z-axis in the positive direction. The centers of the electron source, target, sample, and detector are all on the z-axis, and the whole system is rotationally symmetric with the z-axis as the axis of symmetry. Based on the rotational symmetry of the system, we choose the equivalent uniform focal spot size proposed by Kunio Doi *et al.* as the definition of the focal spot size [3].

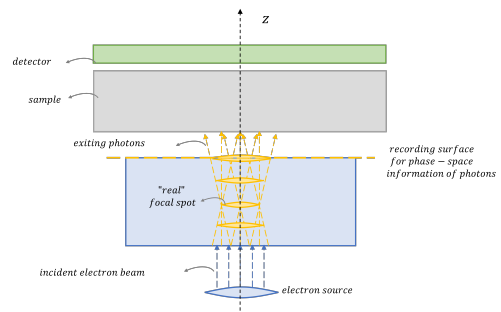


Figure 1: Cutaway of the system. Target, sample, and detector ratios do not represent actual ratios.

Edmond Sterpin *et al.* [1] point out that it is inaccurate to calculate the focal spot size directly on the photon exit plane of the target, which results in the size being unnecessarily enlarged. The correct way to do this is to back up the photons on the recording surface in their respective directions of motion, set up a plane perpendicular to the z-axis at regular intervals (the interval chosen in this paper is 1/30 of the target thickness), and recalculate the focal spot dimensions on the plane until a minimum value is found, which is the real focal spot size. A schematic of the backing-up process is also shown in Figure 1.

WEIGHTING METHOD

Taking container detection as an example, the container is used as a sample, its scale is usually in the order of tens

of meters, and the material is iron, removing the hollow part and simplifying the container to a uniform iron block of 1m thickness. Based on the two assumptions:

- Photons traveling through the sample are considered to be absorbed by the sample once the photoelectric effect, Compton scattering, or electron pair effect occurs;
- The direction of the photon exiting the target is approximately along the z axis, so that all photons travel the same distance through the sample before being recorded by the detector

the probability that a photon with energy E successfully passes through the sample and reaches the detector is given by

$$P(E) = e^{-\mu(E) \cdot \text{thickness}} \quad (1)$$

This i.e., the weights applied to the photons of different energies after considering the effect of the sample, where μ is the absorption coefficient of the photons in iron and thickness is the thickness of the sample.

FOCAL SPOT SIZE AND ERROR CALCULATION METHOD

Due to the limited space of the article, we have to omit the calculation formula and only show the calculation idea of focal spot size and error:

- The whole photons are assigned channels according to energy, and the energy of photons within each channel is regarded as the same;
- Assuming that the x-coordinate of the photon with the same energy obeys the Gaussian distribution, using the idea of parameter interval estimation can get such a random variable: obeys the cartesian distribution, and the observed value is the root mean square (RMS) of the x-coordinate of the photon. Calculate the confidence interval for this random variable and use half the length of the confidence interval as the error in the RMS;
- The previous step can only get the RMS and error of the x-coordinate of the photons within the same channel, this step uses the error transfer formula to get the RMS and error of the x-coordinate of the whole photons;
- Finally, multiplying by a factor of 12 yields the focal spot size and error for all photons. The reason for multiplying the coefficients is mentioned in literature [3].

RESULTS

We explored the variation of focal spot size with target thickness when the incident electron energy and target material are different. The comparison is made at the same mass thickness when the target material is different. In addition, the electron source is a circular uniformly distributed surface source and the target is a cylindrical target with uniform density distribution. The specific parameters are shown in Table 1.

Different Incident Electron Energy

The upper part of Figure 2 shows the variation of the focal spot size with the target thickness when the incident

Table 1: Electron Source and Target Specifications

Radius of Electron Source	Radius of Target	Materials of Target
1 μm	2 cm	Pb(11.34 g/cm ³) Cu(8.96 g/cm ³) W(19.35 g/cm ³) W(38.70 g/cm ³)

electron energy is different, and the target material is W with a density of 19.35 g/cm³. It can be observed that with the increase of the target thickness, the focal spot size firstly rises rapidly (the rising area), and then gradually tends to be saturated (the saturation area), which fluctuates within a certain range; the higher the incident electron energy is, the higher the upper limit of the focal spot size is. The lower part of Figure 2 is a local magnification, which shows that when the target thickness is very small, the higher the incident electron energy, the smaller the focal spot size is; at a target thickness of 1 μm , the focal spot size is the largest at the 1 MeV incident electron energy, which is 1.84 μm , which is only 6.49% larger than the focal spot size corresponding to the electron RMS (1.73 μm).

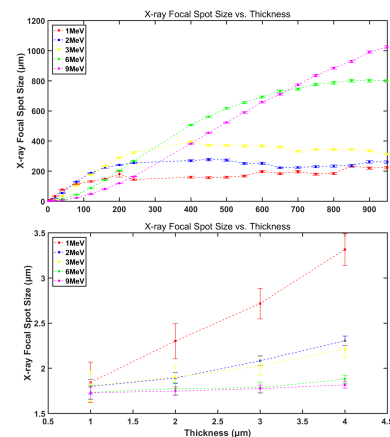


Figure 2: Focal spot size vs. target thickness for different incident electron energies.

A good explanation of this phenomenon can be provided by borrowing ideas from Edmond Sterpin *et al.* [1]. At this point, it is believed that photons are mainly produced in a small region near the bottom surface of the target. When the target thickness is large enough, the target can completely include this region, and then increase the target thickness, there will be almost no new photons generated in the newly added part, so the focal spot size is saturated; when the target thickness is small, the target can not yet completely include this region, this time to increase the target thickness, there will be new photons generated in the newly added part, so the focal spot size will be changed with the target thickness.

Different Materials of Target

The incident electron energy is set to 3 MeV, and the results, as depicted in Figure 3, illustrate the differences in focal spot size among the target materials Pb, Cu, and W. It can be seen that regardless of the mass thickness of the target, it is always the Pb target that has the largest focal spot size, followed by the Cu target, the low-density W target, and finally the high-density W target. If the incident electron energy is modified to 1 MeV, the focal spot size increases slightly, but the law of change remains unchanged, and at a target mass thickness of $1.935 \times 10^{-3} \text{ g/cm}^3$, it is still the Pb target that has the largest focal spot size of $2.07 \mu\text{m}$, which is only 20% larger than the focal spot size corresponding to electron source ($1.73 \mu\text{m}$). Therefore, for Pb, Cu, and W targets, with incident electron energies on the order of MeV and an electron source radius of $1 \mu\text{m}$, the mass thickness of the target of $1.935 \times 10^{-3} \text{ g/cm}^3$ (corresponding to the $1 \mu\text{m}$ target thickness of the low-density W target) can be considered as the mass thickness of the target at the limit of the focal spot size.

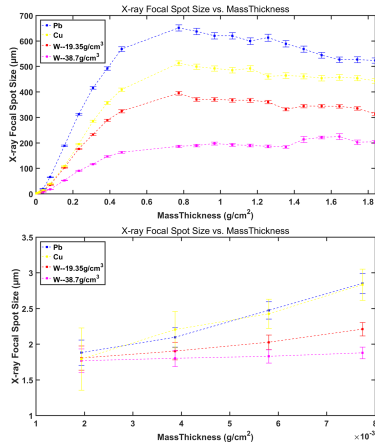


Figure 3: Focal spot size vs. target thickness for different materials of target.

We believe that the effect of atomic number and density should be considered to explain this phenomenon. For the two kinds of W targets with different densities, the high-density W target has more atoms per unit volume, which has greater obstruction to the lateral expansion of the incident electron beam, so the focal spot size is smaller under the same mass thickness. The density of Pb target is close to that of Cu target. At this time, the atomic number of Pb is larger, the atomic nucleus is more attractive to electrons, and the deflection of electrons is more intense, so the focal spot size is larger under the same mass thickness. The atomic number of Pb is close to W, but the density of Pb target is smaller, the number of atoms per unit volume is less, and the obstruction to the lateral expansion of electrons is smaller, so the focal spot size is larger under the same mass thickness; As for why the focal spot size of the Cu target is larger than that of the W target when the mass thickness is the same, I think this is the result of the comprehensive effect of the

atomic number difference and the density difference, and the final density difference has a greater impact.

Table 2 shows the focal spot sizes corresponding to W targets of different thicknesses when the electron source radius is $10 \mu\text{m}$ and $50 \mu\text{m}$, where the electron beam energy is set to 1 MeV. It can be seen from the data in the table that the size of the electron source only affects the focal spot size corresponding to the thin target thickness. When the target is thick enough, the influence of the electron source size on the focal spot size is negligible. Therefore, if the requirements for focal spot size are not particularly stringent, the constraints on the size of the electron source can be relaxed accordingly.

Table 2: Focal Spot Size at Different Electron Source Radius

Target Thickness	Radius at $1 \mu\text{m}$	Radius at $10 \mu\text{m}$	Radius at $50 \mu\text{m}$
$1 \mu\text{m}$	$1.84 \mu\text{m}$	$17.55 \mu\text{m}$	$87.90 \mu\text{m}$
$2 \mu\text{m}$	$2.30 \mu\text{m}$	$17.04 \mu\text{m}$	$87.00 \mu\text{m}$
$3 \mu\text{m}$	$2.72 \mu\text{m}$	$16.83 \mu\text{m}$	$85.25 \mu\text{m}$
$4 \mu\text{m}$	$3.32 \mu\text{m}$	$17.73 \mu\text{m}$	$85.67 \mu\text{m}$
$10 \mu\text{m}$	$13.27 \mu\text{m}$	$26.41 \mu\text{m}$	$92.32 \mu\text{m}$
$20 \mu\text{m}$	$34.61 \mu\text{m}$	$36.63 \mu\text{m}$	$109.48 \mu\text{m}$
$800 \mu\text{m}$	$184.29 \mu\text{m}$	$173.82 \mu\text{m}$	$188.96 \mu\text{m}$
$850 \mu\text{m}$	$234.17 \mu\text{m}$	$175.99 \mu\text{m}$	$186.89 \mu\text{m}$
$900 \mu\text{m}$	$219.97 \mu\text{m}$	$211.11 \mu\text{m}$	$221.63 \mu\text{m}$

CONCLUSION

In this paper, Monte Carlo simulation software Geant4 is used to simulate the process of electron bombardment of metal targets in the industrial CT energy zone (X-ray energy is MeV order), and the size of X-ray focal spot is calculated under different conditions after considering the influence of samples. The results show that for Pb, Cu, and W targets, with incident electron energies on the order of MeV and an electron source radius of $1 \mu\text{m}$, the mass thickness of the target of $1.935 \times 10^{-3} \text{ g/cm}^3$ can be considered as the mass thickness of the target at the limit of the focal spot size.

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