

# Cosmological constant: the possible nature and relationship with quantum gravity and fundamental particle masses

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In order to reconcile the quantum mechanics and general relativity, the equivalence principle of quantum gravity is introduced for extending the equivalence principle of general relativity to the observer frames of reference which are in quantum mechanical motions. The equivalence principle of quantum gravity is that the laws of physics must be of such a nature that they apply to systems of reference in any kind of motions, both classical and quantum mechanical. Under such principle, the quantum gravity should be formulated in the quantum space-time-matter space with local conformal symmetry and the mathematical expressions for the cosmological constant as well as the masses of fundamental particles can be found by the theory.

*Keywords:* Cosmological constant, equivalence principle of quantum gravity, Higgs condensate, quantum space-time-matter geometry, local conformal symmetry, Weyl geometry.

## 1. Introduction

It is well known that the two main pillars of modern physics, the general relativity (GR) and quantum mechanics (QM), are fundamentally inconsistent with each other. In order to reconcile the QM and GR, the equivalence principle of quantum gravity is introduced for extending the equivalence principle of GR to the observer frames of reference which are in quantum mechanical motions<sup>1</sup>. The equivalence principle of quantum gravity is that the laws of physics must be of such a nature that they apply to systems of reference in any kind of motions, both classical and quantum mechanical. That means the quantum state of a particle is relative to the observer frame of reference which can also be itself in a quantum mechanical state. Under the principle, the quantum gravity should be formulated in the quantum space-time-matter (QSTM) space with local conformal symmetry. It was also expected that the ultimate theory of quantum gravity would explain the cosmological constant problem<sup>2,3</sup>.

## 2. Quantum space-time-matter geometry

In relativity, the space and time are defined by the coordinate system established by measuring-rods and synchronised clocks, which are in classical motions and their measuring results correspond to the points in the 4-dimensional spacetime coordinate system. However, the measuring-rods and clocks themselves are subject to the quantum uncertainty as well as quantum mechanical motions. Therefore, the space-time cannot simply be a 4-dimensional coordinate system with spacetime points but should be associated with the quantum states of measuring-rods and clocks specified

by their state parameters. The quantum state of the measuring-rod and clock, that is the quantum state of the spacetime by definition, can be defined<sup>4</sup> as  $|\vec{x}, \Delta \vec{x}\rangle$ , where  $\vec{x}$  is the classical spacetime vector and  $\Delta \vec{x}$  is the associated uncertainty. As the quantum uncertainties depends upon the energy scales, the modified definition of spacetime should therefore be observation energy scale dependent. The quantum state  $|\alpha\rangle$  of a particle can be projected to such quantum spacetime state to become a wavefunction  $\phi(\vec{x}, \Delta \vec{x})$  as  $\phi(\vec{x}, \Delta \vec{x}) = \langle \vec{x}, \Delta \vec{x} | \alpha \rangle$ . Furthermore, the particle quantum state can be observed in another quantum spacetime state  $|\vec{y}, \Delta \vec{y}\rangle$  which can be expressed in general as linear combination of the eigenstate of  $|\vec{x}, \Delta \vec{x}\rangle$ . The transformation can be expressed as

$$\langle \vec{y}, \Delta \vec{y} | \alpha \rangle = \int \langle \vec{y}, \Delta \vec{y} | \vec{x}, \Delta \vec{x} \rangle \langle \vec{x}, \Delta \vec{x} | \alpha \rangle \quad (1)$$

The integration sign denotes the summation or integration of all the possible eigenstates of  $|\vec{x}, \Delta \vec{x}\rangle$ . Actually, such notation can be omitted by identifying the operator  $|\vec{x}, \Delta \vec{x}\rangle \langle \vec{x}, \Delta \vec{x}|$  as the internal index of the summation or integration as analogous to the Einstein's convention.

For introducing the geometrical concept to QM and the equivalence principle of quantum gravity in quantum gravity, the spacetime should be merged with matter together to become the QSTM space with local conformal symmetry under the Weyl like geometry<sup>1,4</sup>. As a consequence, all the physical laws apply not just to the frames of reference in any kind of classical motion as in GR but also the same in the reference frames in quantum mechanical motions as well. It implies that the general physical laws of nature are required to be expressed by equations which hold good for all systems of quantum spacetime states which are covariant under the transformation in Eq (1) and the quantum mechanical motion is relative in nature. The QSTM state  $|\Psi\rangle$  is the combination of the quantum state of particle and spacetime as  $|\Psi\rangle = |\phi\rangle |\vec{x}, \Delta \vec{x}\rangle$ . The length of a general small line element  $dL$  can be defined in a small region of QSTM space as the inner product of  $\delta|\Psi\rangle$  as

$$dL^2 = \delta|\Psi\rangle \cdot \delta|\Psi\rangle = \delta(|\phi\rangle |\vec{x}, \Delta \vec{x}\rangle) \cdot \delta(|\phi\rangle |\vec{x}, \Delta \vec{x}\rangle) \quad (2)$$

where the inner product can be defined as the usual inner product used for quantum mechanical states and thus  $dL^2$  has a real value. Let us also introduce the operator  $\widehat{\delta \vec{x}}_{\Delta x}$ , which can extract the spacetime length from the quantum spacetime state, for small changes on spacetime state with the property as

$$\widehat{\delta \vec{x}}_{\Delta x} \delta |\vec{x}, \Delta \vec{x}\rangle = \delta \vec{x}_{\Delta x} \delta |\vec{x}, \Delta \vec{x}\rangle \quad (3)$$

When there is no local mixing between the quantum matter particle state and the quantum spacetime state, another line element  $dl$  can be defined in the QSTM

space and when expressing it in generalised coordinates  $\xi_m^\mu$  in the QSTM space gives

$$dl^2 = \delta |\phi\rangle \widehat{\delta \vec{x}}_{\Delta x} \delta |\vec{x}, \Delta \vec{x}\rangle \cdot \delta |\phi\rangle \widehat{\delta \vec{x}}_{\Delta x} \delta |\vec{x}, \Delta \vec{x}\rangle = G_{\mu\nu}^{mn} d\xi_m^\mu d\xi_n^\nu \quad (4)$$

where  $G_{\mu\nu}^{mn}$  is the combined metric of the QSTM space and the extracted spacetime length and the index  $\mu$  and  $\nu$  are the usual spacetime indexes in GR while  $m$  and  $n$  are the indexes associated with the quantum spacetime and matter states. Such formulation works well on discrete eigenstates as well as continuous eigenstates. Let us define the local conformal operator  $\Omega$  on the QSTM state as  $\delta |\Psi\rangle \rightarrow \Omega \delta |\Psi\rangle$ . By using the closest admissible action in Weyl geometry analogous to the Einstein's action for local conformal symmetry<sup>5</sup>,

$$S = \int \mathfrak{R}^2 \sqrt{-G} d^p \xi \quad (5)$$

where  $\mathfrak{R}$  is the curvature scalar defined for  $G_{\mu\nu}^{mn}$  with the QSTM and  $p$  is the total number of dimensions of it. It is a generalized action of quantum gravity in the formulation of the QSTM space. The variation of the action under a suitable local conformal transformation with imposing the gauge  $\mathfrak{R} \sim 4\Lambda$  for fixing the scale of the QSTM space, where  $\Lambda$  is a constant, gives

$$S = \int (\tilde{\mathfrak{R}} - 2\Lambda) \sqrt{-G} d^p \xi \quad (6)$$

In the special case that the QSTM space is a flat space and, for simplicity, assuming that the matter field is a scalar field, the Eq (4) can be written as

$$dl^2 = \langle \phi^2 \bar{g}(\Delta x)_{\mu\nu} \rangle dx^\mu dx^\nu \quad (7)$$

where  $\langle \phi^2 \bar{g}(\Delta x)_{\mu\nu} \rangle$  is the quantum probability weighted spacetime metric, which is specified with measurement uncertainty, combined with the matter field. Also, for a flat space, the metric  $G_{\mu\nu}^{mn}$  does not vary with  $\xi_m$  and the action can then be expressed in terms of  $\bar{g}_{\mu\nu}$  and  $\phi$  with variation on spacetime as

$$S = \int (\phi^2 \bar{R} - 2\Lambda \phi^4 + 6(\partial\phi)^2) \sqrt{-\bar{g}} d^4 x \quad (8)$$

Aligning the term  $(\partial\phi)^2$  as the kinetic term of the  $\phi$  field gives

$$S' = \int \left( \frac{1}{12} \phi^2 \bar{R} - \frac{1}{6} \Lambda \phi^4 + \frac{1}{2} (\partial\phi)^2 \right) \sqrt{-\bar{g}} d^4 x \quad (9)$$

If  $\bar{R}$  is a constant, the spontaneous symmetry breaking (SSB) on the  $\phi$  field gives the relation  $\bar{R}/4\Lambda = \phi_0^2$ . Defining the conformal operator  $\omega$ , which is associated with the change of scale of observation, in the spacetime on the metric and scalar field as  $\bar{g}_{\mu\nu} \rightarrow \omega^2 \bar{g}_{\mu\nu}$  and  $\phi \rightarrow \omega^{-1} \phi$ . The action in Eq (9) is invariant under

such conformal transformation. The quantum gravity therefore possesses a double conformal structures. This point is very crucial as it allows the change scale in one degree of freedom and then compensate by the other to make the physical laws invariant.

### 3. Particle masses and cosmological constant

Considering a shift field  $h$  around the minimum of the potential as  $\phi = \phi_0$  with an energy scale  $\phi_0^2 = 6m'^2$  in the SSB of the action in Eq (9), the equation becomes

$$S = \int \left( \frac{1}{2}m'^2\bar{R} - 3\Lambda m'^4 - \frac{1}{2}(\partial h)^2 + \frac{1}{2}\bar{R}h^2 + \dots \right) \sqrt{-\bar{g}}d^4x \tag{10}$$

Under the change the scale of observation by applying conformal transformation on the spacetime as  $\bar{g}_{\mu\nu} \rightarrow (m'/M_p)^2\bar{g}_{\mu\nu}$  and  $\phi \rightarrow (m'/M_p)^{-1}\phi$ , where the mass scale  $M_p$  is the reduced Planck mass, the equation can be expressed as

$$S = \int \left( \frac{1}{2}M_p^2\bar{R}' - 3\Lambda M_p^4 - \frac{1}{2}(\partial h')^2 + \frac{1}{2}\bar{R}'h'^2 + \dots \right) \sqrt{-\bar{g}'}d^4x \tag{11}$$

The term  $\bar{R}'h'^2$  acts as the mass term of the excitation field  $h'$ . Since  $\bar{R}'$  is the conformal transformation of the term related to the vacuum energy of spacetime in empty space and since  $\bar{R}\sqrt{-\bar{g}}$  should be transformed as  $\bar{g}$  and therefore  $\bar{R}'$  should be equal to  $(m'/M_p)^2\bar{R}$  which is the particle squared mass term. That imply the existence of a particle with mass  $(m'/M_p)\bar{R}^{\frac{1}{2}}$ . We can change the observer frame of reference for observing such particle due to the double conformal symmetry and this will lead to the change on the broken scale. Recalling that the scale of the QSTM space is fixed by the gauge  $\mathfrak{R} \sim 4\Lambda$  and let the factor for the associated scale change is  $\Omega^4$ , the scale of  $\Lambda$  is therefore changed as  $\Lambda \rightarrow \Omega^4\Lambda$ . The scale change of  $\Lambda$  implies that the SSB condition  $\bar{R}/4\Lambda = \phi_0^2$  requires to be varied so that the mass value  $(m'/M_p)\bar{R}^{\frac{1}{2}}$  in Eq (11) of the said particle will be changed also. As the mass value is determined by the ratio  $\bar{R}\sqrt{-\bar{g}}/\phi^2$  and under the change of scale for Eq (11) following the conformal transformation  $\omega$  on the spacetime and matter field induce the transformation as

$$\frac{\Omega^4\bar{R}\sqrt{-\bar{g}}}{\phi^2} \rightarrow \frac{\omega^4\Omega^4\bar{R}\sqrt{-\bar{g}}}{\phi^2} \tag{12}$$

Therefore, for keeping the mass unchanged through the change of observer frame of reference requires that  $\omega = 1/\Omega$ . The structure of the QSTM space under broken symmetry can be explored by repeatedly performing the above processes of changing observer frame such that in every changed frame the mass of the particle are still equal to  $m'/M_p$ , although the  $M_p$  varies with the transformed frame of reference. A set of scale factors can be found for the QSTM space scale

$$(\Omega)^{2n} = \left( \frac{m'}{M_p} \right)^2 \Rightarrow \Omega = \left( \frac{m'}{M_p} \right)^{\frac{1}{n}} \tag{13}$$

where  $n$  is the number of the said transformation to achieve the mass factor  $m'/M_p$  in a frame. Suppose  $m'/M_p \sim 10^{-15}$ , that is  $m' = 2.435\text{TeV}$ , and since the particle mass  $m_i$  is equal to  $\Omega_{mi}\overline{R}^{\frac{1}{2}}$ , if  $\overline{R}^{\frac{1}{2}} = 10\text{TeV}$ , a set of mass values can be found and compared with the lepton masses data of the Particle Data Group<sup>7</sup> as  $m_1 = 10^{-15}\overline{R}^{\frac{1}{2}} = 0.01\text{eV} \sim m_\nu?$ ;  $m_2 = 3.16 \times 10^{-8}\overline{R}^{\frac{1}{2}} = 316.23\text{keV} \sim m_e$ ;  $m_3 = 10^{-5}\overline{R}^{\frac{1}{2}} = 100\text{MeV} \sim m_\mu$ ;  $m_4 = 1.778 \times 10^{-4}\overline{R}^{\frac{1}{2}} = 1.778\text{GeV} \sim m_\tau$ , where  $m_\nu$ ,  $m_e$ ,  $m_\mu$  and  $m_\tau$  are respectively the mass of neutrinos, electron, muon and tau lepton. The actual neutrino masses might not necessary be degenerate as the radiative corrections is not considered in the above calculation. The  $m_2$  state is of the order of the electron mass and the difference could be due to the radiative correction to the QED vacuum. The  $m_3$  state is equal to the muon mass up to about 5 percent and the  $m_4$  state is even equal to the tau lepton mass up to only about 0.06 percent. For the mass states with  $n > 4$ , for instance, the mass value of  $m_5$  state is consistent with the dark matter candidate with a mass of 10 GeV as proposed by some researchers<sup>8</sup>. In extending the argument to half-integer value of  $n$ ,  $m_{4.5} = 4.64\text{GeV}$  and  $m_{8.5} = 171.9\text{GeV}$ . The values are very close to the mass values of bottom and top quarks. That means the half integer values of  $n$  may correspond to the quark masses. However, due to the QCD vacuum, the calculated values for the lighter quark masses would have greater discrepancy with the actual current quark masses.

As the Eq. (9) and Eq. (11) is related by the conformal transformation of spacetime and the matter field and the SSB which is determined by the observation scale. In fact, the Eq (9) resembles the following action of the Higgs potential when  $\overline{R}$  is a constant in one hand

$$S' = \int \left( \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4 + \frac{1}{2}(\partial\phi)^2 \right) \sqrt{-g}d^4x \quad (14)$$

On the other hand, it can become the action of gravitational field coupled to matter field after the local conformal transformation and SSB as discussed above. Comparing the coefficients between the Eq (9) and Eq (14) gives  $\overline{R}/12 = \mu^2/2$  and  $\Lambda/6 = \lambda/4$ . Furthermore, by comparing Equation (11) with the gravitational action and eliminating  $\Lambda$  in the above equations, we get  $3\Lambda M_p^2 = \Lambda_{cc}$  and  $\Lambda_{cc} = 4.5\lambda M_p^2$ , where  $\Lambda_{cc}$  is the cosmological constant. If then applying a conformal transformation on the QSTM space metric to change the observer frame of reference, under the same factor as the one in obtaining the fundamental particle masses with  $n = 2$ , and since the cosmological constant transforms as the square of the metric, that is conformal factor  $\Omega^4$ , with respect to the conformal factor of  $\Omega^2$  on the metric, we have

$$\Omega = \left( \frac{m'}{M_p} \right)^2 \Rightarrow \Lambda_{cc} = 4.5\lambda \frac{m'^4}{M_p^2} \left( \frac{m'}{M_p} \right)^4 \quad (15)$$

By putting  $\lambda = 0.258$  which is based on the 125 GeV Higgs particle mass and 246 GeV electroweak VEV value, we get  $\Lambda_{cc} = 6.49 \times 10^{-66}\text{eV}^2$ . Such calculated

cosmological constant value is in very good agreement with the observation value of  $4.33 \times 10^{-66} eV^2$  of Planck CMB probe when connecting to the fundamental particle masses.

#### 4. Discussions and conclusions

The theory of quantum gravity is formulated in the QSTM space with local conformal symmetry under the equivalence principle of quantum gravity. The QSTM space possesses a double conformal symmetry geometrical structure. Different exponent  $n$  values of the associated conformal factor  $\Omega = (m'/M_p)$  gives the fundamental particle mass values and the observed cosmological constant value with very good agreement. It indicates that the fundamental particle masses and the cosmological constant are related in the theory. Furthermore, the above mass formula allows the existence of some mass states that are so far not experimentally observed that might provide new opportunities for discussing the nature of such mass states.

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