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Review of the application of quantum annealing-related technologies in transportation optimization

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Abstract

Traffic optimization remains a significant challenge in urban planning and transportation management. While efficient traffic optimization is crucial for enhancing urban mobility, reducing congestion, and promoting environmental sustainability, traditional computational methods often struggle with the complex, dynamic nature of traffic systems. Recent advances in quantum computing, particularly quantum annealing, offer promising new techniques that could revolutionize traffic flow optimization. This work systematically reviews the literature, starting with search term formulation and ending with the final set of articles. These articles are categorized into three groups: (1) traffic signal control, (2) traffic flow optimization, and (3) routing problems optimization (including vehicle routing problem and traveling salesman problem). The review critically examines current studies on quantum annealing-based traffic optimization, focusing on contributions, methods, solvers, problem suitability, key findings, benchmark fairness, and limitations. It identifies key challenges and provides recommendations for future research. Insights from this work offer researchers and practitioners a concise overview of current challenges and future directions in traffic optimization.

Keywords Quantum Annealing · Quantum Computing · QUBO · Traffic Flow · Traffic Optimization · Traveling Salesperson · Traffic Signal · Urban Mobility · Vehicle Routing

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1 Introduction

With the growing number of vehicles on the road, optimizing traffic flow has become a critical challenge for transportation engineers. Traditional solutions, such as road expansions, are often costly and impractical in dense urban areas. Instead, advanced optimization strategies offer a more effective approach to improving mobility, reducing travel time, and enhancing road safety. Recent technological advancements have enabled more sophisticated traffic management models that consider various factors, including traffic patterns, weather conditions, and driver behavior.

Urban population growth has intensified traffic congestion, resulting in significant economic losses. According to the INRIX Global Traffic Scorecard [1], in 2022, congestion cost US drivers \$81 billion, UK drivers £9.5 billion, and German drivers €3.9 billion. London, Chicago, and Paris were among the most congested cities, with London drivers losing 156 h annually. By 2023 [2], New York became the most congested city, with drivers losing 101 h, followed by Mexico City, London, Paris, and Chicago. The economic costs were \$70.4 billion in the USA, £7.5 billion in the UK, and €3.3 billion in Germany. While congestion increased in most urban areas, delays grew significantly in US cities, whereas downtown trips declined in the UK and Germany. These trends underscore the urgent need for intelligent transportation systems (ITS) to optimize traffic flow and minimize economic losses. Implementing real-time data analytics, adaptive signal control, and integrated public transportation can help mitigate congestion. Moreover, emerging technologies like quantum computing (QC), particularly quantum annealing (QA) and digital annealing (DA), offer promising approaches for solving complex traffic optimization problems, improving efficiency and reducing congestion.

Traffic flow optimization is a critical challenge in modern cities, impacting both transportation efficiency and urban quality of life [3]. Managing traffic in dense road networks requires continuous redistribution of vehicles [4], yet the complexity and dynamic nature of traffic patterns make optimization difficult. Traditional methods struggle to handle these challenges, necessitating ITS that integrate advanced sensors, communication networks, and data processing to analyze large-scale traffic data. This increasing computational demand calls for innovative architectures, among which QA has emerged as a promising solution [5].

In the past decade, QC has gained traction due to its potential to solve computationally intractable problems [6]. Recent studies propose QC, particularly QA, as a powerful tool for tackling large-scale, highly constrained optimization problems, including traffic optimization [7]. However, several challenges remain, such as scaling quantum algorithms for real-world applications, enhancing efficiency, and improving hardware performance [8]. Despite its potential, a comprehensive, up-to-date review of QA's role in traffic optimization is lacking. This paper aims to fill this gap by systematically examining its applications in traffic signal control (TSC), traffic flow optimization (TFO), and routing problems (RPs).

The paper's structure is as follows: In Sect. 2, the discussion will focus on related review papers that have concentrated on QA in diverse fields associated with optimization problems. This endeavor seeks to identify the gaps that the present review intends to tackle. Section 3 covers traffic management, offering a background on key elements and their interplay in addressing traffic optimization. Section 4 provides a concise overview of QC. Section 5 explains the key elements of optimizing traffic flow, emphasizing factors that require thoughtful consideration, and defines crucial traffic optimization terms and concepts. Section 6 focuses on the SLR, discussing research protocol, data analysis, and critical analysis of selected papers. Section 7 highlights challenges, recommendations, and future directions. In Sect. 8, the conclusion of this review paper is presented.

2 Existing review articles and current review contributions

In this section, related review articles will be explored, and the presented review paper's unique contributions to the field of QA-based traffic optimization will be highlighted. Traditional optimization methods have limitations in effectively addressing the complex and dynamic nature of traffic systems. QA, with its inherent ability to handle large-scale optimization problems, has emerged as a promising alternative. Several review papers have been published in the field of QC that are relevant to this review paper. Previous review papers have explored various aspects of QC and its applications, yet they often lack a specific focus on traffic optimization. For instance, reviews have covered quantum routing protocols and metrics [5], metaheuristic algorithms inspired by quantum mechanics [9], and the transition from classical to quantum-inspired metaheuristics [10]. Additionally, there are systematic studies of QC as it relates to routing problems. Two such reviews, one concentrating on QA and related implementations [7] and the other on TSP and VRP [4], are examples of this. [7]. Additional reviews have provided insights into adiabatic evolution [11] and examined the applications of QA in various industries [12]. Other reviews have looked at QC taxonomies and future directions [13], quantum optimization and learning algorithms [14], and specific applications such as vehicle routing with limited qubits [15]. Moreover, traditional TSC methods have been extensively reviewed [16], alongside general applications of QA in NP-hard problems [17], and the impact of QC on evolutionary algorithms [18]. However, these reviews generally focus on broader optimization problems and do not specifically address the application of QA for traffic optimization, leaving a significant gap in the literature, as stated in Table 1.

Previous review studies have explored various aspects of QC, including the TSP, VRP, quantum routing protocols, annealing, and metaheuristic algorithms. However, none has specifically addressed QA-based traffic optimization. This review makes a significant contribution by critically analyzing studies on QA applications in TSC, TFO, and RPs (e.g., VRPs and TSP). It examines current research, identifies challenges, and explores future directions while addressing potential integration of QA with RL for traffic optimization. The review contributes to the field by:

1) Providing a comprehensive analysis of QA-based traffic optimization, filling a critical gap in the literature, and offering a valuable resource for researchers and practitioners.

Table 1 Overview of Existing and Current Review Papers: Focus, Insights, and Gaps

Ref	Focus Areas	Contributions and Gaps Identified
[4]	QC, Routing Problems	• The focus was on applying QC to routing issues like TSP and VRP
[5]	Quantum Routing Protocols	• The research primarily investigated quantum routing protocols and metrics
[7]	QA, Optimization Problems	• The study was dedicated to QA and solving optimization problems
[9]	Quantum Metaheuristic Algorithms	• This work is centered on quantum metaheuristic algorithms
[10]	Quantum-Inspired Metaheuristics	• The study is mainly focused on the evolution from classical to quantum-inspired metaheuristics
[11]	QA, Adiabatic Quantum Computation	• The primary focus was on QA through adiabatic evolution for tough optimization problems
[12]	QA, Industry Applications	• The paper investigated the applications of QA across different industries
[13]	QC Taxonomy	• This study provided an extensive review of QC literature and its taxonomy
[14]	Quantum Optimization, Quantum Learning Algorithms	• The main subject was quantum optimization and learning algorithms
[15]	VRP, Quantum Path Integral	• The research focused on vehicle routing problems via quantum path integral
[17]	QA, NP-Hard Problems	• The emphasis was on QA applications for NP-hard problems
[18]	Quantum Genetic Algorithm Optimization	• The work centered on quantum genetic and evolutionary algorithms
Ours	QC-based traffic optimization	• This research investigates the use of QA to optimize traffic in four areas: TSC, TF, VRP, and TSP. It reviewed current research, discussed challenges and future directions, and compared QA with RL, evaluating their strengths, weaknesses, and appropriateness for different traffic optimization tasks

2) Highlighting future research directions to enhance QA's effectiveness in traffic management, guiding further advancements in this area, and addressing potential integration of QA with RL for traffic optimization.

By addressing these gaps, this review enhances current understanding and establishes a foundation for future research and practical implementations in traffic optimization using QA. The following section presents an overview of traffic management methods, forming the basis for integrating traffic engineering and quantum optimization techniques.

3 Traffic management: background

Traffic in the presented work refers to the movement of vehicles, bicycles, and pedestrians on roads and highways. It is an important aspect of transportation and is affected by various factors such as population growth, urbanization, and economic development. Traffic can be classified into different types, such as vehicular traffic, public transport, pedestrian traffic, and bicycle traffic. To manage traffic, various strategies and technologies are employed, such as traffic signals [16], roundabouts [19, 20], lane markings [21], and traffic cameras [22]. In this review, we categorize existing research on QA applications in traffic optimization into three main domains: TSC, TFO, and routing problems (RPs), the latter comprising the VRP and TSP.

3.1 Traffic signal control

TSC regulates the duration of green, yellow, and red lights at intersections to optimize traffic flow (TF), reduce congestion, and ensure safety for drivers, pedestrians, and cyclists. The timing of signals depends on various factors, including traffic volume, speed, pedestrian presence, road network topology, time of day, and day of the week. Traffic engineers continuously monitor and adjust TSC to maintain optimal performance. In real-world applications, TSC is complex, requiring expertise in both traffic engineering and optimization to balance efficiency and safety.

In addition, it is essential to emphasize that traffic lights operate in a cyclical manner, repeating their behavior at every predetermined duration of seconds. The entire cycle repeats, maintaining order at intersections and ensuring the safe movement of both vehicles and pedestrians. In this regard, TSC strategies are divided into fixed-time, adaptive, and actuated approaches. Fixed-time control uses predetermined timings all day. Adaptive control adjusts signal timings based on real-time traffic conditions, reducing congestion and enhancing TF [23, 24]. Actuated control, a form of adaptive strategy, detects vehicles with sensors and adjusts signals, especially effective in low-traffic-volume zones [25, 26]. Furthermore, self-organizing traffic signal algorithms aim to optimize TF during light traffic [26]. The following are the control parameters that are used in TSC [16]:

1) Green Time: it refers to the duration for the green signal, which is determined by traffic volume and intersection capacity.
2) Cycle Length: The time required to complete all signal phases at a junction, determined by green times and clearance intervals.
3) Phase Sequence: Order in which traffic movements receive the green signal, prioritizing major flows and demand.
4) Green Wave (Offset): It describes timing coordination between consecutive intersections, optimizing traffic progression.
5) Programs: Traffic light programs regulate signal timing based on traffic patterns throughout the day. Morning peak hours prioritize longer green lights for commuters, afternoons focus on school zones and pedestrian crossings, evenings balance commuter and leisure traffic, and nighttime programs optimize energy efficiency. Each program adjusts signal timings to improve traffic flow.

3.2 Traffic flow

TF is the movement of vehicles on roads and highways. It can be described by various characteristics such as density (the number of vehicles per unit length of road), speed, and flow [27] (the number of vehicles passing a point in each time period). It is a complex system that is affected by various factors such as road conditions, weather, and traffic volume, road design, traffic signals, and the presence of accidents or construction [3]. Understanding and managing TF is essential for reducing congestion, improving safety, and optimizing the use of resources. Various methods have been developed to study and improve TF, such as mathematical models, simulation tools, and traffic monitoring systems [28]. These tools and methods help traffic engineers and planners make informed decisions about road infrastructure and traffic management strategies [29].

In particular, TF models have been developed and used to understand, describe, and predict TF since the beginning of the twentieth century. TF can be classified into several categories based on different factors such as traffic density, traffic speed, or the relationship between them [30]. The choice of model depends on research objectives and the scale of the traffic system being studied. Here is an example of one possible taxonomy of TF as discussed in [31] and [32]:

Fundamental Diagram [33]: The fundamental diagram illustrates the relationship between TF, density, and speed within a system, divided into three regions: free flow, congested, and capacity drop.
Microscopic [34]: Microscopic models simulate individual vehicle behavior, considering factors like acceleration, deceleration, and lane changes.
Macroscopic [35]: Macroscopic models describe traffic dynamics at an aggregated level, focusing on variables like TF, density, and speed averaged over a road segment or network.
Mesoscopic [36]: Mesoscopic models act as intermediaries between microscopic and macroscopic models, grouping vehicles into clusters and studying their collective behavior.

3.3 Routing problems

RPs constitute a critical category in the domain of traffic and logistics optimization and include both VRP and its well-known special case, TSP. Although TSP is subsumed within the broader VRP framework, it is frequently examined independently in the quantum annealing (QA) literature due to its historical role as a canonical benchmark problem. The TSP's relatively simple structure and well-established QUBO formulations have made it a primary target for early experimental validation on quantum annealing hardware [70] [59]. In contrast, VRP encompasses a broader class of real-world routing scenarios involving multiple vehicles and additional constraints, thereby presenting increased complexity and practical relevance. By discussing these problems under a unified heading, we aim to maintain a conceptually coherent taxonomy while also acknowledging the methodological and contextual distinctions that justify the continued separate treatment of TSP in QA research.

VRP refers to the process of determining optimal routes for a fleet of vehicles to efficiently navigate or deliver goods and services to a set of destinations. It involves finding the most cost-effective and time-efficient paths for vehicles to follow [37], considering factors such as distance, traffic conditions, delivery time windows, vehicle capacity, and customer preferences. VRPs can vary in complexity, ranging from simple cases with a single vehicle and a few stops to more complex scenarios with multiple vehicles, multiple depots, and a large number of destinations [38]. In the optimization and transportation communities, routing problems have garnered considerable attention [39–41]. The VR domain has piqued interest compared to the aforementioned two traffic problems due to two primary factors, as stated in [4]. Firstly, routing problems exhibit high computational complexity, rendering them challenging to address even for medium-size instances [42]. Secondly, these problems have demonstrated their practicality in real-world business and logistics scenarios, as well as in leisure and tourism contexts [43]. In essence, advancements in the development of efficient algorithms for routing problems yield both economic and social advantages. Consequently, robust methods specifically tailored to tackle routing problems have experienced a surge in popularity. Various algorithms and optimization techniques, such as heuristic algorithms [44], genetic algorithms [45], and mathematical programming models [46], are used to solve VRPs and optimize the allocation of resources. Effective VR can reduce transportation costs, improve customer service, and enhance operational efficiency for businesses in industries such as logistics, transportation, and delivery services [43].

TSP is a prominent combinatorial optimization challenge that seeks to determine the shortest possible route for a salesman to visit each city exactly once and return to the starting point [47]. TSP is one of the most extensively studied problems in combinatorial optimization. It involves finding the shortest tour that visits each node in a given weighted complete graph exactly once and returns to the origin node. A notable variant of the TSP is the clustered TSP (CTSP) [48], which was first explored in [49]. CTSP is an extension of the TSP in which the cities are divided into clusters. In the CTSP, the salesman must visit all the cities within each cluster consecutively before moving on to the next cluster. When each cluster consists of only one city, the CTSP reduces to the standard TSP. The difficulty of TSP arises from its combinatorial nature, where the number of potential routes increases factorially with the number of cities, making it increasingly computationally intensive to solve as the number of cities grows. Unlike the VRP, which optimizes routes for multiple vehicles to service a set of locations with specific constraints (such as capacity and delivery windows), TSP focuses solely on finding the optimal route for a single traveler. VRP extends the TSP by adding layers of complexity, including multiple vehicles, varying demands at each location, and the need to optimize the overall fleet's efficiency [50]. Numerous studies have explored various approaches to address the complexity of TSP, leveraging diverse methodologies to find efficient solutions [51] [52]. The ongoing research and advancements in solving TSP and its variants demonstrate the problem's significance in both theoretical and practical applications, highlighting the continuous need for innovative optimization techniques. The next sections will explore the fundamentals of quantum computing, its classification, and the role of QA in optimization.

4 Foundations of quantum computing and its application in optimization

Quantum science spans quantum communication, sensing, and computing [53], all leveraging entanglement and superposition to surpass classical limitations. While quantum communication enhances encryption security and quantum sensing improves precision in measurements, QC revolutionizes computation by solving problems beyond classical capabilities, particularly in combinatorial optimization [53]. Utilizing qubits that exist in multiple states simultaneously, QC achieves quantum speedup [54] through superposition, entanglement, and reduced computational steps [55].

QC is classified into digital and analog models. Gate-based quantum computing (GBQC) operates through unitary gate manipulations, forming the foundation of quantum algorithms and quantum machine learning applications [56]. On the other hand, analog quantum computing relies on natural quantum state evolution, including adiabatic quantum computing (AQC), quantum simulation (QS), and QA. AQC optimizes solutions under adiabatic conditions [57] [58] [59]. Particularly, it utilizes the adiabatic theorem of quantum mechanics to evolve a physical quantum computer from an initial state that encodes the problem to a final state that provides the solution [60–62], which is particularly effective for NP-complete and NP-hard problems [59, 63]. QS enables material science and physics simulations [64], while QA, central to this discussion, excels in combinatorial optimization, leveraging quantum tunneling for efficiency beyond classical methods [57].

QA has emerged as a powerful technique for solving combinatorial optimization problems, leveraging quantum tunneling and thermal fluctuations to achieve faster convergence compared to classical approaches [57]. QA operates under less stringent adiabatic constraints, making it more feasible for current hardware implementations [12, 65]. Furthermore, QA is less sensitive to noise and decoherence than other quantum computing methods, making it particularly well suited for optimization tasks in traffic signal optimization, vehicle routing, and the TSP. The extensive potential of QA in solving complex optimization problems highlights its significance across various fields. Thus, further research is necessary to fully realize the potential of QA and its place in QC. [66, 67]. To effectively utilize QA, optimization problems must be reformulated into mathematical structures compatible with quantum hardware, primarily through (1) Ising model representation, where constraints are encoded into a quantum Hamiltonian function, and (2) quadratic unconstrained binary optimization (QUBO), which maps optimization problems onto binary variables with quadratic cost functions.

The Hamiltonian is a mathematical representation of a system's energy. In QA, optimization problems are mapped to a Hamiltonian, where the system's lowest energy state corresponds to the optimal solution [68]. The basic idea of QA is to encode the problem in such a way that it correlates with the Hamiltonian of the system, so that the states of the quantum system correspond to different expressions in the search space for the problem. Thereby, the system must be constructed in a way that solutions of the optimization problem correspond to energetically optimal states of the quantum system. To achieve this, the cost function of the optimization problem must be expressed in terms of the total energy of the quantum system [69, 70]. The Hamiltonian consists of a driver Hamiltonian and a problem Hamiltonian. The driver Hamiltonian [71] is a simple Hamiltonian that controls the time evolution of the quantum system in QA, which describes the system's energy landscape. The total Hamiltonian consists of a driver Hamiltonian (Hd) responsible for quantum fluctuations and a problem Hamiltonian (Hp), which encodes the optimization function, as presented in (1). The $s(t)$ is a function controlling the gradual shift from exploration to convergence. If this transition occurs slowly enough, the system remains in its lowest energy state, yielding an optimal solution. The system transitions from the driver to the problem Hamiltonian according to the annealing schedule:

$$H(t) = (1 - s(t))Hd + s(t)Hp \tag{1}$$

To represent problems in a form suitable for QA, they are often converted into a quadratic unconstrained binary optimization (QUBO). QUBO is a type of optimization problem represented by a quadratic objective function [72] [73] [74], involving binary variables as presented in (2). The x is a binary vector (i.e., each element is either 0 or 1), Q is a symmetric matrix of real-valued coefficients, and c is a constant. It aims to minimize the objective function while adhering to constraints that enforce the binary nature of the variables.

$$f(x) = x^T Qx + c \tag{2}$$

This formulation enables classical optimization problems to be efficiently mapped onto quantum hardware for computation. By leveraging the inherent parallelism of quantum systems, QA efficiently explores large solution spaces, making it a promising approach for traffic optimization problems such as vehicle routing, traffic flow control, and urban mobility planning.

The following section explores key elements and terminologies relevant to QA-based traffic optimization, emphasizing the factors that must be considered in problem formulation and implementation.

Mapping Optimization Problems to QUBO for Traffic Applications: QA to traffic optimization involves systematically reformulating real-world challenges into QUBO problems. Figure 1 illustrates this mapping process, which translates tasks such as TSC, vehicle routing, and the traveling salesperson problem into a computational framework compatible with quantum hardware. The process begins with identifying problem-specific inputs. For TSC, this includes intersection layouts, traffic flow data, and signal timings. VRPs require data on customer locations, delivery windows, and vehicle capacities, while the traveling salesperson problem relies on graph representations of cities, routes, and distances. These inputs define the objectives and constraints essential for optimization. Next, these inputs are formulated into a mathematical model, where the objective functions capture goals, such as minimizing travel time or congestion. Constraints, such as vehicle capacities or signal timing limitations, are incorporated to ensure practical solutions. This formulation is then encoded into a QUBO objective function, consisting of linear terms for individual variables and quadratic terms for interactions, with penalty terms enforcing constraints. The QUBO function is translated into a matrix representation, where diagonal elements represent

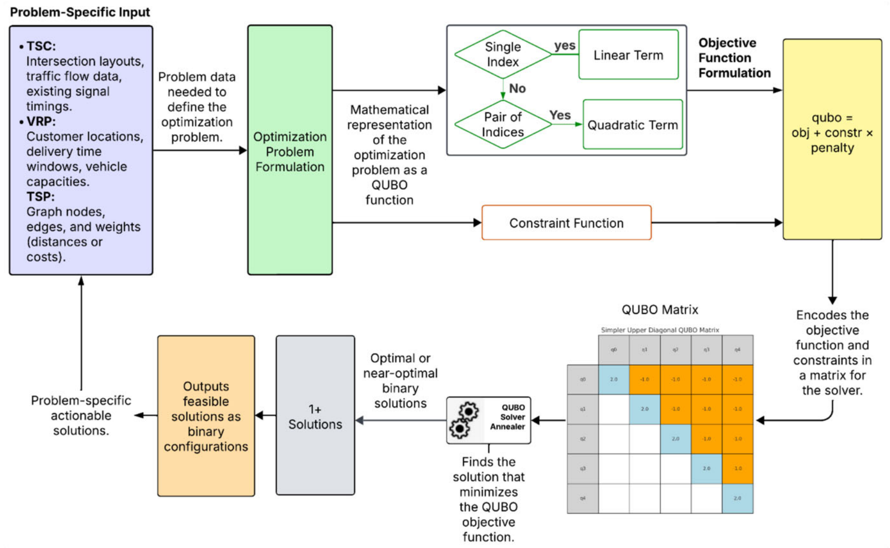

Fig. 1 Mapping Traffic Optimization Problems to QUBO Formulations

linear coefficients and off-diagonal elements encode pairwise interactions. This matrix is essential for compatibility with QA solvers, which minimize the QUBO objective function using quantum tunneling and parallelism. The resulting binary configurations represent actionable strategies, such as optimized traffic signal timings or efficient routing plans. By leveraging QA, this approach efficiently addresses the complexity of NP-hard traffic optimization problems. Figure 1 provides a visual representation of this workflow, linking theoretical principles with real-world applications in urban mobility.

As a practical example, the problem of optimizing traffic signals is considered. To encode the traffic signal optimization problem into a QUBO model, we define binary variables $x_{i,t} \in \{0,1\}$ indicating whether traffic phase i is active at time step t. The objective is to minimize total vehicle waiting time across all intersections, represented by $\sum_{i,t} w_{i,t} \bullet x_{i,t}$ where $w_{i,t}$ is the predicted waiting time if phase i is active at time t. Constraints such as "only one phase active at any time" are enforced using quadratic penalties like $\lambda_1 \sum_t \left(\sum_i x_{i,t} - 1 \right)^2$, and minimum green duration is enforced by penalizing phase switches within short intervals: $\lambda_2 \sum_{i,t} \left(x_{i,t} - x_{i,t+1} \right)^2$. The full QUBO objective is then expressed as a weighted sum of the linear cost function and all quadratic constraint terms, leading to a matrix Q suitable for quantum annealing. (C3).

5 Key elements and terminologies in quantum annealing and traffic optimization

The process of traffic optimization using QA involves various elements that require careful consideration to optimize traffic flow. These elements are not limited to the ones mentioned in Fig. 2, but they provide valuable insights to researchers who seek a comprehensive and clear understanding of the most critical factors to be considered, as shown in Fig. 2.

1. Problem Formulation: Problem formulation is a crucial step in optimizing traffic flow using QA. It involves defining: 1) The objective function may involve minimizing total travel time, the number of stops at traffic signals, maximizing vehicle throughput, or reducing congestion. 2) Constraints consider network physical limitations, environmental factors, and safety requirements. 3) Input data includes traffic flow rates, patterns, signal timings, and halting cars. 4) Solution representations can include binary variables for signal timings, continuous variables for flow rates, or combinations of both. 5) Problem size handling may involve breaking up large networks or optimizing regions separately. Thus, accurate problem definition enables effective optimization using QA.

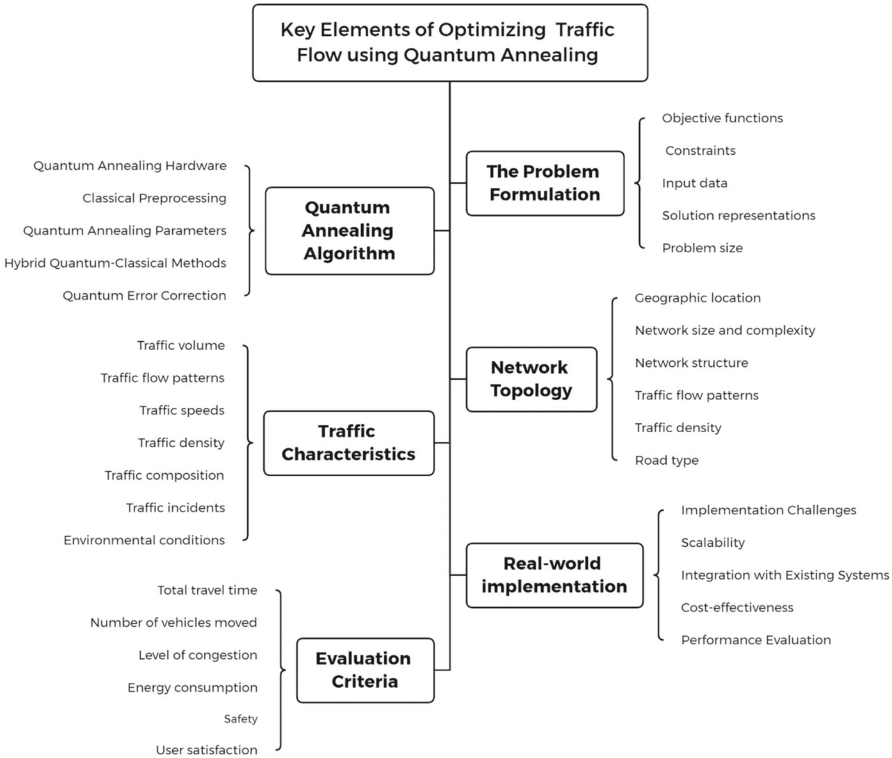

Fig. 2 Key Elements of Optimizing Traffic Flow

2. QA Algorithms: QA algorithms play a critical role in solving traffic optimization problems. It is made up of the following parts: 1) QA hardware aspects are crucial to take into account, including the particular QA processor, its architecture, and the number of qubits; 2) the problem is encoded into appropriate representations such as Ising or QUBO formulation as part of classical preprocessing; 3) QA parameters, such as annealing time, schedule, temperature, and repetitions, can be tuned for optimal results; 4) hybrid quantum–classical methods combine classical optimization techniques and quantum-inspired classical algorithms to enhance performance; and 5) quantum error correction techniques mitigate noise and errors through encoding into larger logical qubit spaces and error mitigation techniques.

3. Network Topology: Network topology encompasses factors like geographic location, network size and complexity, structure, traffic flow patterns, density, and road types. These factors differ based on urban, suburban, or rural settings, ranging from small-scale to large-scale networks, grid, hub-and-spoke, or ring structures, and unidirectional, bidirectional, or multi-directional flow. Traffic density varies from low to high, and road types include highways, arterial roads, and local roads. Considering these factors is vital for creating an effective traffic flow management system.

4. Traffic Characteristics: Traffic characteristics such as volume, flow patterns, speeds, density, composition, incidents, and environmental conditions require careful consideration for traffic optimization. Traffic volume fluctuates based on time and other factors. Flow patterns impact overall efficiency and can vary during the day. Speeds are influenced by congestion, road conditions, and driver behavior. Traffic density affects capacity and flow. Different vehicle types in traffic composition have unique characteristics and space requirements. Incidents disrupt traffic flow, and environmental conditions like weather and visibility play a role. Considering these characteristics helps create an effective traffic optimization strategy.

5. Real-World Implementation: Real-world implementation of traffic optimization using QA involves addressing critical elements for successful implementation. Thus, implementation challenges may include technical, regulatory, or logistical hurdles. Scalability considers the method's ability to adapt to different network sizes and complexities. Integration with existing traffic management systems necessitates addressing compatibility, data exchange protocols, and software and hardware configurations. Cost-effectiveness compares the proposed method to traditional control approaches. Performance evaluation involves testing the method on a real traffic network and assessing metrics like travel time, flow, and congestion levels.

6. Evaluation Criteria: Evaluation criteria for traffic optimization include total travel time, vehicle throughput, congestion levels, energy consumption, safety (accidents and collision severity), and user satisfaction. These criteria provide a holistic assessment of system performance.

In addition, Table 2 presents common terminologies related to QA and traffic optimization. Highlighting these terms aims to provide researchers with a deeper understanding of the intersection between QA and traffic optimization by defining the terminologies and their key aspects. The following section will cover the SLR, detailing the research protocol, data analysis, and critical analysis of the selected papers.

Table 2 Terminologies Related to Quantum Computing and Traffic Optimization in Transportation Engineering

Terminologies	Definition	Key Point
Ising Model	• Mathematical model used to describe the behavior of magnetic materials in statistical mechanics	Exploring Magnetic Behavior
Qubit	• Basic unit of QC, capable of storing and processing information in quantum states	Building Blocks of QC
Quantum Optimization	• Field of QC focused on finding optimal solutions to given problems, such as efficient routes in traffic optimization	Seeking Optimal Solutions
Annealing	• Method in QC and traffic optimization to find optimal solutions by gradually cooling the system to a low-energy state	Solving Complex Energy Landscapes
QA Machine	• Quantum computers are designed to solve optimization problems using the annealing algorithm, such as finding optimal routes in traffic optimization	Harnessing Quantum Optimization
Adiabatic QC	• Method of QC that uses the adiabatic theorem to solve optimization problems, such as finding optimal routes in traffic optimization	Leveraging the Adiabatic Theorem
Traffic Flow	• Movement of vehicles on a roadway system, measured by volume, speed, and density	Modeling Vehicle Movement
Traffic Congestion	• Occurs when demand for roadway capacity exceeds supply, resulting in slower speeds, longer travel times, and increased delays for drivers	Mitigating Congestion
Traffic Analysis	• Study of traffic patterns and behavior on a roadway system to identify trends and improve traffic flow	Studying Traffic Patterns
Capacity Analysis	• Study of maximum number of vehicles that can pass through a roadway section or network under different traffic conditions	Evaluating Roadway Capacity
Traffic Simulation	• Use of computer models to simulate vehicle movement on a roadway system, optimizing signal timing and design factors	Simulating Traffic Dynamics
Intersection Geometry	• Physical layout of an intersection, including lanes, turning lanes, islands, and medians, impacting traffic flow and safety	Analyzing Intersection Layout
Traffic Signal Detection	• Use of sensors to detect vehicle presence at an intersection, adjusting signal timing to optimize traffic flow and minimize delays	Monitoring Vehicle Presence

Table 2 (continued)

Terminologies	Definition	Key Point
Traffic signal control	• Management of traffic flow involves optimizing the timing of traffic signals at intersections to minimize congestion. This includes adjusting the duration of green, yellow, and red phases in the traffic signal cycle to enhance traffic flow and reduce delays	Managing Traffic Signal Operations
Signal Phasing	• Timing and sequencing of traffic signals at an intersection, determining right-of-way for different traffic directions	Coordinating Signal Sequences
Traffic Signal Cycle	• Sequence of signal phases occurring at an intersection over time, enabling all traffic movements to occur	Managing Signal Phase Sequences
Signal Coordination	• Process of synchronizing traffic signals at multiple intersections to create (e.g., green wave) for more efficient traffic movement	Synchronizing Traffic Signals
Traffic Signal Preemption	• System allowing emergency vehicles to change TSC, clearing a path through intersections for quicker and safer response	Prioritizing Emergency Vehicles

6 The Systematic literature review and its elements

This section presents a systematic review of the literature on optimizing traffic in transportation engineering using QA. The review followed a research protocol that involved searching for relevant articles, selecting studies based on specific inclusion criteria, and critically analyzing the quality of these studies.

6.1 Research protocol and the data analysis

This study follows a systematic literature review (SLR) approach to examine the role of QA in traffic optimization. The objective is to review and analyze recent research developments in traffic optimization using QA, identifying key contributions, trends, and challenges in the field. A structured research protocol was employed to ensure a comprehensive and rigorous review of relevant studies (see Fig A. in Appendix for an overview of the research protocol). To retrieve relevant articles, a keyword-based search strategy was implemented across four major scientific databases: IEEE Xplore, ScienceDirect, Web of Science, and Scopus. The search focused on peer-reviewed journal articles, conference proceedings, and review papers published between 2010 and 2024 to ensure coverage of recent advancements, while book chapters and non-English language publications were excluded. The primary search keywords used were designed to capture studies related to traffic optimization problems and their intersection with quantum computing. Specifically, the search included the terms: ***("Traffic***

Signal" OR "Vehicle Routing" OR "Traffic Flow" OR "Traveling Salesman") AND ("Quantum Computing" OR "QA" OR "QUBO" OR "DWAVE" OR "Qiskit").

To ensure a comprehensive review, additional searches were conducted using four inquiry-based search queries designed to further explore the intersection of traffic-related topics and quantum computing. The first query focused on traffic signal optimization by searching for studies related to ***("Traffic Signal" OR "Traffic Light" OR "Traffic Congestion") AND ("Quantum Annealing" OR "Adiabatic Quantum Computing" OR "Quantum Approximate Optimization" OR "Simulated Quantum Annealing" OR "Quantum Metropolis" OR "QUBO")***. The second query targeted traffic flow studies, using ***("Traffic Flow" OR "Traffic Congestion") AND ("Quantum Computing" OR "Quantum Annealing" OR "Adiabatic Quantum Computing" OR "Quantum Approximate Optimization" OR "Simulated Quantum Annealing" OR "Quantum Metropolis" OR "QUBO")***. The third query addressed vehicle routing problems, employing the keywords ***("Routing Problem" OR "Vehicle Routing Problem" OR "Optimal Route Planning" OR "Routing") AND ("Quantum Computing" OR "Quantum Annealing" OR "Adiabatic Quantum Computing" OR "Quantum Approximate Optimization" OR "Simulated Quantum Annealing" OR "Quantum Metropolis" OR "QUBO")***. The fourth and final query focused on the traveling salesman problem, using ***("Traveling Salesman" OR "Optimal Route Planning") AND ("Quantum Computing" OR "Quantum Annealing" OR "Adiabatic Quantum Computing" OR "Quantum Approximate Optimization" OR "Simulated Quantum Annealing" OR "Quantum Metropolis" OR "QUBO")***.

The retrieved articles were systematically screened by reviewing titles and abstracts to remove duplicates and irrelevant studies. The remaining articles underwent a full-text review and were classified based on their contributions to QA-based traffic optimization. A total of 73 research articles were included in the final dataset and divided into three main categories: TSC with 6 articles (8%), TFO with 11 articles (15%), and RPs with 56 articles (77%), which were further split into VRP with 33 articles (45%) and TSP with 23 articles (32%). The publication trend over the last 14 years (2010–2024) indicates a significant rise in research on QA-based traffic optimization, particularly in VRP and TSP applications. The highest number of articles were published between 2020 and 2021, reflecting the growing interest in leveraging quantum computing for transportation challenges.

6.2 Critical examination and discussion on related studies

In this section, an in-depth critical review of the related works is provided. According to the protocol of the systematic review followed in this paper, the classification of published articles on QA into three distinct groups highlights the different aspects of traffic optimization that researchers have been exploring: 1) TSC; 2) TFO; and 3) RPs.

These four categories demonstrate the potential of QA to revolutionize the transportation industry by improving the efficiency and sustainability of transportation systems. With the ability to analyze and optimize traffic patterns, route planning, and overall transportation networks, QA can enhance transportation efficiency, reduce travel times, and improve the overall quality of life for commuters.

6.2.1 Traffic signal control

TSC involves using QA to analyze real-time traffic data and adjust signal timings, resulting in improved TF and reduced congestion.

Few studies have proposed the use of QA for TSC and optimization. For example, in a study on TSC, Hussain et al. [75] have addressed the TSC problem using QA. They formulated the problem as a QUBO and used QA with D-Wave to solve the problem. At first, they have proposed six phases for an intersection, whereas the phase selection is based on the number of halting cars, with the goal of clearing the maximum number of vehicles waiting at any given time. The subsequent intersections were coordinated by examining the phase's influence j on intersection i and finding compatible phases of nearby intersections. Besides that, they have considered synchronizing nearby intersections by assigning coefficients λ_3 and $\lambda_{3\prime}$ to straight going and straight/right turning modes, respectively, to achieve signal coordination and create "green waves" for continuous TF. The travel time between intersections $\left(\Delta_{\tau_{ii'}}\right)$ for an average car is calculated based on road segment length and speed limit. Parameters $\tau_{ii'}$ ensure mode synchronizing only happens when a car has enough time to move from intersection i to i'. Mode synchronizing occurs when the car is close to the signal at the end of the segment, taking approximately 4 s to cross. Binary variables $\tau_{ii'}$ and C_{ij} were utilized, along with constant factors λ_2, λ_3, and $\lambda_{3\prime}$. To ensure at least one mode is active at each intersection, a constraint term was added. The objective function, as presented in (3), depended on the situation being modeled, allowing flexibility in assigning values to λ_3 and $\lambda_{3\prime}$.

$$
\begin{aligned}
Obj = -\lambda_1 \sum_{i=1}^{n}\sum_{j=1}^{6} c_{ij}x_{ij} - \lambda_2 \sum_{i=1}^{n}\sum_{j=1}^{6} c_{ij}x_{ij}[\tau_{i,a'}\lambda_3 C_{a',a}x_{a',a} + \tau_{i,b'}\lambda_{3\prime} C_{b',b}x_{b',b} \\
+ \tau_{i,c'}\lambda_3 C_{c',c}x_{c',c} + \tau_{i,d'}\lambda_{3\prime} C_{d',d}x_{d',d}] + \lambda_4 \sum_{i}\left[1 - \sum_{j=1}^{6} x_{ij}\right]^2
\end{aligned}
\tag{3}
$$

In contrast to the problem optimization of traffic signals, as discussed in [75], the issue addressed in [76] involved determining the optimal duration of each phase based on the number of vehicles instead of the optimal phase. To achieve this, the problem is also formulated as QUBO in [76]. At the beginning, the authors have used the QUBO formulation to select the optimal phase as same as [75]. Then, the optimal phase is compared to the current phase in order to determine the appropriate time duration. Within the study, two predefined time durations were assigned for each phase: 20 s and 40 s. For instance, if the current phase matches the optimal phase, the duration is set to 20 s, whereas if they differ, the duration is set to 40 s. The objective function, as expressed in (4), aims to minimize delay time and to maximize traffic flow. However, it is important to note that the time selection process is hardcoded and not determined through the QUBO algorithm. The role of the QUBO algorithm is solely to identify the optimal phase, while the determination of the appropriate duration is handled through the hardcoding process.

$$Obj = \sum_i \sum_j c_{ij} x_{ij} + \lambda \sum_i \left(1 - \sum_j x_{ij} \right)^2 \tag{4}$$

Inoue et al. [77] have attempted to address the problem of optimization of traffic signal operation in a large-scale urban city to minimize the imbalance of traffic flows in two orthogonal directions. In their study, a combinatorial optimization problem has been formulated as an Ising Hamiltonian, which is compatible with QA machines. This problem setting was basically following the study in [78]. The goal was to find the optimal solution that achieves a global balance of traffic flows throughout the entire city, which is a difficult problem due to the exponential computational complexity of the combinatorial optimization. The key variables that have been used to optimize the traffic signals are the traffic flow bias at each intersection, represented by the quantity$x_i(t)$, and the signal state at each intersection, represented by the vector $\sigma := \left[\sigma_1, ..., \sigma_{L^2}\right]^\top$. The traffic flow bias is computed using the difference equation $q_{\mathrm{ij}}(t+1) = q_{ij}(t) + \frac{s_{ij}}{2}\left(-\sigma + \alpha\sigma_j\right)$, where q_{ij} represents the number of vehicles in the traffic lane from intersection j to$i$, $\alpha := 2a - 1$ and $s_{ij} \in \{\pm 1\}$ is the direction of the lane from node j to$i$. The traffic signal state at each intersection is determined by minimizing the objective function $H(\sigma(t)) := x(t+1)^\top x(t+1) + \eta(\sigma(t) - \sigma(t-1))^\top(\sigma(t) - \sigma(t-1))$, where $x(t)$ is the flow bias vector and η is a weight parameter for determining the ratio between the two terms in the objective function. The objective function, as written in (5), is then minimized to find the traffic signal state σ¯(t) that suppresses the flow bias during the next time step at each intersection and prevents the traffic signal state from switching too frequently. However, the proposed method is demonstrated through numerical experiments, and a theoretical basis for the correspondence between the local and global control methods is established.

$$H(\sigma(t)) := x(t+1)^\top x(t+1) + \eta(\sigma(t) - \sigma(t-1))^\top(\sigma(t) - \sigma(t-1)), \tag{5}$$

Another study [79] developed Non-dominated Sorting Quantum Genetic Algorithm (NSQGA), an improved algorithm that integrates QC and self-adaptation, to optimize TSC plans at an isolated intersection and solve multi-objective optimization problems (MOOPs) in traffic systems. The existing NSGA-II algorithm cannot achieve satisfactory results in saturation or oversaturation conditions, where the demand for road space exceeds its capacity, causing a breakdown in traffic flow. NSQGA aims to address this issue. In their mathematical model for optimizing signal timing at an isolated two-phase intersection, four performance indexes are considered for optimization: traffic capacity, stop times, vehicle delay time, and non-motorized traffic delay time. A numerical simulation is conducted on MATLAB using the proposed NSQGA algorithm to find the optimal solution. The capacity of the intersection (Q) is expressed as a function of the saturation flow rate, effective green light time, and cycle length. The stop time (H) is a function of the green split, vehicle flow ratio, and vehicle arrival rate. Delay time (D) is divided into vehicle delay time (D_V) and non-motorized traffic delay time (D_{NMT}). D_V is a function of the vehicle arrival rate, green split, and saturation flow rate, while D_{NMT} is a function of the pedestrian and bicycle arrival rate, saturation flow, and green split. The optimization problem can be expressed as a multi-objective function, where the objective is to minimize D_V, D_{NMT}, H, and maximize $-Q$, as written in (6).

The problem involves optimizing TSCs subject to constraints such as minimum and maximum green times, green splits, yellow times, and cycle length. The key variables are g_{1min}, g_{2min}, g_{1max}, g_{2max} (minimum and maximum green time during phase i), λ_{1min}, λ_{2min} (minimum of the green split during each phase), C_{min}, C_{max} (minimum and maximum of the signal cycle), and λ_1, λ_2 (yellow time during each phase). For more information on the mathematical model and its implementation, please refer to Ref [79].

$$\min\begin{bmatrix} D_V(g_1, g_2, C), D_{\mathrm{NMT}}(g_1, g_2, C), H(g_1, g_2, C), \\ -Q(g_1, g_2, C) \end{bmatrix}$$
$$s.t.\begin{cases} g_{1min} \le g_1 \le g_{1max} \\ g_{2min} \le g_2 \le g_{2max} \\ g_1 \ge \lambda_{1min} C \\ g_2 \ge \lambda_{2min} C \\ C_{min} \le C \le C_{max} \\ g_1 + g_2 + \lambda_1 + \lambda_2 = C \end{cases} \tag{6}$$

Building on the previous points, it is essential to briefly conclude by considering the comparative analysis of quantum and classical approaches to TSC, which revealed significant advancements and persistent limitations. For instance, the study [79] employed a classical NSQGA solver, demonstrating a 5.45% improvement in traffic capacity and a 23.08% reduction in stop times. The benchmark strength was moderate, as NSGA-II was a robust multi-objective optimization algorithm, though not specifically tailored for TSC. However, the study was limited by its focus on isolated intersections and lack of real-world testing. Conversely, the study [75] leveraged D-Wave's 2000Q quantum annealer, achieving notable time savings with hybrid solvers. The benchmark strength lay in the weak performance of fixed signal cycles, which were not adaptive and failed to respond dynamically to fluctuating traffic conditions, leading to inefficiencies during peak times and underutilization during off-peak hours. Yet, this study's generalizability was restricted by assumptions of symmetric intersections and limited modes per intersection.

The [76] emphasized QUBO cycles' excellent flexibility with a 22.19% speed improvement, but fixed cycles demonstrated poor adaptability and C-QUBO performed moderately. However, it only included two intersection models and simulated data. In [77], multiple methods improved synchronization and signal correlation, with local control performing weakly and simulated annealing demonstrating substantial flexibility but constrained by simulation-based results and uniform traffic assumption. Finally, [80] highlighted QA's improved traffic reduction and energy efficiency for bigger networks, despite limited by quantum hardware scalability.

Research studies demonstrated the promise of QA for TSC, but they also pointed out its drawbacks. The complexity of traffic in the real world might not be captured by simulations, and the high error rates and qubit restrictions of quantum annealers present additional computational difficulties. Applicability is further limited by assumptions of uniform traffic flow, underscoring the necessity of investigating quantum-based TSC techniques for practical scenarios.

6.2.2 Traffic flow optimization

Traffic flow optimization is a multifaceted discipline that aims to enhance the efficiency of transportation systems and alleviate congestion. Quantum algorithms offer promising solutions to optimize traffic patterns and improve travel experiences. This section critically examines various computational optimization problems that address the challenges of traffic congestion, presenting a comprehensive review of related studies and articles that proposed innovative solutions to optimize traffic flow.

Optimizing bike sharing systems (BSS) and effectively tackling the bike sharing system rebalancing problem (BSS-RBP), for example, have garnered significant attention from researchers and practitioners alike. In [81], Harikrishnakumar and Nannapaneni employed the quantum approximate optimization algorithm (QAOA) to address the resource planning problem in transportation networks, specifically focusing on optimizing BSS and solving the BSS-RBP. The BSS-RBP involved a fixed number of vehicles (k) and considered demand at different bike stations. The authors formulated the problem as a QUBO and proposed an objective function, as in (7), to minimize the distance traveled by the vehicles across all supply and demand stations, with the aim of reducing travel time and congestion.

$$
\begin{aligned}
f(x) = Min \sum_{\substack{i \in I \\ j \in J}} D_{ij} x_{ij} + \sum_{\substack{i \neq k \\ i, k \in J}} D_{ik} x_{ik} \\
+ \sum_{\substack{j \neq h \\ j, h \in J}} D_{jh} x_{jh} + \sum_{i \in I} D_i \mu_i + \sum_{j \in I} D_j \eta_j
\end{aligned}
\tag{7}
$$

where are indices i, j, k, and h to represent supply and demand bike stations, binary variables x_{ij}, x_{ik}, and x_{jh} to indicate bike station serving and vehicle traveling, μ_i and η_j to represent the first supply bike station served and last demand bike station served before reaching the parking lot, and D_{ij}, D_{ik}, and D_{jh} to represent the distances between bike stations. In another study, Harikrishnakumar et al. [82] compared QAOA and QA for rebalancing optimization across three bike stations, highlighting the superiority of quantum optimization algorithms. However, the study acknowledged some limitations that needed to be addressed, including the investigation of error correction techniques and the conduct of a large-scale analysis on both QAOA and QA frameworks. These areas of improvement would enhance the understanding and applicability of quantum optimization algorithms in addressing resource planning challenges in transportation networks, particularly in the context of BSS.

Resource allocation optimization is another crucial optimization problem that plays a pivotal role in addressing the lane reservation problem, contributing to the effective management of traffic flow. The study by Wu et al. [83] has addressed the lane reservation problem in transportation, with a specific focus on large sporting events. The objective was to choose and reserve lanes in the existing transportation network exclusively for time-guaranteed transportation tasks during these events while minimizing the overall impact on normal traffic. Wu et al. have proposed a quantum evolutionary algorithm (QEA) that leverages the principles of QC and has demonstrated exceptional performance in solving combinatorial problems. The objective function was expressed as $\sum_{(i,j)\in A} c_{ij} z_{ij}$, which subjected several constraints, such as ensuring feasible paths, lane reservation restrictions, and preventing the total travel time from exceeding the deadline. For instance, the constraint $x_{ijs} \leq y_{ijs}, (i, j) \in A, \forall s \in S$ ensures that a reserved lane exists only if a travel path from the source node to the destination node passes through the arc (i, j). The decision variables, namely x_{ijs}, y_{ijs}, and z_{ij}, represent the choices regarding lane reservation and path selection. In their study [83], three scenarios with different destination nodes were tested. With a runtime of 520.167 s as opposed to 1442.709 s for IBM CPLEX, the QEA algorithm obtained a solution gap of 0.862%–3.344%. A simplistic model, static traffic assumptions, and a lack of validation were among the drawbacks, though. In order to compare different approaches for lane reservation at major athletic events and enhance efficiency, scalability, and practical application, more study is required.

Wang et al. [84] introduced an approach called QA and brain-inspired clustering algorithm (QABICA) to optimize urban taxi stand locations for better accessibility and traffic flow. Using location bias as a metric, QABICA outperformed K-means, reducing bias by 83% (weekdays) and 57% (weekends) in Xi'an, and 42% and 81% in Chengdu. Its advantage lies in avoiding local optima, improving clustering compared to K-means. However, computational complexity remains a challenge. Further research is needed to enhance its efficiency and impact on urban transportation.

In addition to that, Deng et al. [85] addressed the gate allocation problem in airports, aiming to optimize passenger walking distances, balance gate idle time, and maximize the use of large gates. They proposed an improved QEA, which combines niche co-evolution and enhanced particle swarm optimization (PSO) techniques, called improved population-based optimization quantum-inspired evolutionary algorithm (IPOQEA). The study focused on optimizing gate allocation in airports with multiple objectives and constraints. The method was validated using real data from Baiyun airport and proves effective in addressing passenger needs and optimizing gate allocation. The experimental comparison between ant colony optimization (ACO), improved ACO, e.g., stochastic ant colony optimization (SACO), quantum-inspired QEA, and the proposed IPOQEA revealed that the IPOQEA achieved the highest gate allocation rate (93.22%). It required the fewest iterations (average of 95) to find the optimal value and demonstrated superior stability with the lowest standard deviation. The proposed IPOQEA algorithm showed promising performance, but there are limitations to consider. For example, the study relied on specific objectives and real operation data from a single airport, which may limit generalizability to other airports. Future work should explore the dynamic nature of gate allocation and consider disruptions to enhance the method's robustness. Additionally, incorporating machine learning techniques and data-driven approaches could improve optimization and adaptability to diverse airport environments and changing operational requirements.

One crucial factor in mitigating traffic congestion is the effective management of visitor routes. By tackling this optimization problem, significant improvements can be made in traffic flow. For instance, a recent study utilized Ising machines to address the issue of recommending visiting routes in amusement parks [86]. This approach involved mapping the problem to a QUBO model and employing actual simulated annealing to generate efficient solutions. The primary goal was to enhance visitor satisfaction and experience by optimizing the attraction-visiting route, thereby minimizing both waiting and traveling times. Ultimately, this optimization would contribute to an overall improvement in visitors' experiences. The proposed method was evaluated using two samples based on real amusement parks. However, it is worth noting that the comparison between the samples was not entirely fair due to discrepancies in the experimental setups. Notably, the study did not consider waiting time as a time-dependent variable or explore its efficient QUBO mapping, despite its potential to enhance attraction experiences. These limitations must be carefully addressed and overcome to ensure the effectiveness and practicality of the approach.

The study [87] addressed the limited connectivity between artificial spins in QA on the chimera graph. To overcome this, the authors proposed an alternative approach using the Hubbard–Stratonovich transformation or its variants from statistical mechanics. This allows for solving large-scale optimization problems on the chimera graph without embedding. The proposed method provides non-trivial results for optimization problems with fully or partially connected spins. The approach was tested on partition problems and traffic flow optimization in Sendai and Kyoto cities in Japan. Traffic flow optimization was achieved through a combination of QUBO, neural networks, and the D-Wave 2000Q quantum annealer. Binary variables, $q_{\mu,i}$, represent route selection for each car i and route μ. The constraints for route selection were decomposed by substituting the quadratic term in $f_{0(\mathbf{q})} : f(q) - f_{0(q)} = \frac{\lambda}{2}\sum_i \Big(\sum_\mu q_{\mu,i} - 1\Big)^2$, where $f_{0(\mathbf{q})}$ is expressed as $f_{0(\mathbf{q})} = \frac{1}{2}\sum_e \Big(\sum_\mu \sum_i S_{e,\mu,i} q_{\mu,i}\Big)^2$. The original quadratic term, $\sum_e \Big(\sum_{\mu,i} S_{\mu,i,e} q_{\mu,i}\Big)^2$, was transformed into a decomposed quadratic term, $\sum_e \nu_e \Big(\sum_{\mu,i} S_{\mu,i,e} q_{\mu,i}\Big)$. Here, $S_{\mu,i,e}$ represented car i's occupation of road segment e for route μ. λ was a large, predetermined parameter, and ν_e denoted Lagrange multipliers. The effective Hamiltonian was simplified to an Ising model in local magnetic fields as written in (8).

The expected value of the effective Hamiltonian was obtained deterministically as $\langle q_{\mu,i} \rangle = \delta_{\mu=\mu_i^*}$ where $\mu_i^* = \mathrm{argmax}_\mu \big(h_{\mu,i}\big)$. Different sampling methods were compared to address the local minima issue, including the deterministic approach, Gibbs–Boltzmann distribution, and D-Wave 2000Q quantum annealer. With 350 cars and 3 candidate routes per car, the D-Wave 2000Q outperformed deterministic and classical methods, achieving lower-energy solutions. The method was tested in Sendai and Kyoto cities, surpassing the shortest path policy and yielding superior results compared to the Fujitsu digital annealer.

$$H(\mathrm{q}, \nu) = -\sum_{\mu,i} h_{\mu,i} q_{\mu,i} + \frac{\lambda}{2} \sum_{i} \left(\sum_{\mu} q_{\mu,i} - 1 \right)^2 \tag{8}$$

Expanding on the insights regarding traffic flow optimization, it is essential to briefly conclude by evaluating the implications of these findings and their potential impact on future developments. For instance, the study [83] employed a hybrid quantum—classical approach, demonstrating that while the QEA had a stable computation time compared to CPLEX, the average gap for QEA was 2.203% relative to CPLEX. The benchmark strength was strong due to CPLEX's established efficiency and accuracy, providing a robust baseline for evaluating QEA. However, the study lacked real-world validation and was specific to isolated intersections. In another study [88], the improved IQEA showed a reduced gap of 1.45% compared to CPLEX and outperformed both QEA and CPLEX in computation time. The benchmark strength was effective, as both QEA and CPLEX offered fair and reliable comparisons for assessing improvements. Despite this, the study faced implementation challenges and relied on static traffic assumptions.

Furthermore, studies such as [86] and [89] demonstrated significant improvements in solution quality, computation time, and feasibility probability over simulated annealing (SA). The benchmark strength was fair, given the similar conditions and standard metrics used for comparison. Nonetheless, the study did not consider waiting times, which affected the fairness of the comparison. In [84], the QABICA algorithm showed a 61.4% average improvement over the K-means algorithm in optimizing urban taxi stand locations. The benchmark strength was moderate, as K-means, while widely used, had limitations in handling complex spatial distributions. This study was limited by computational complexity and reliance on average location bias. Similarly, the study [90] utilized QA approaches from D-Wave, Fujitsu, and NEC, showing a 5% improvement in cargo load optimization and an 8% improvement in investment portfolio optimization. The benchmark strength was strong due to the significant improvements observed in solving complex optimization problems. Even so, the study faced emerging practical application limitations. In [82], the D-Wave hybrid solver consistently outperformed QAOA on IBM Qiskit, achieving a minimum cost of 11 in all tested scenarios. The benchmark strength was fair, as both quantum approaches addressed the same optimization problem under similar conditions. Nevertheless, the case study was limited to three bike stations.

Moreover, the study [81] employed IBM Qiskit, showing a 15% reduction in overall distance traveled compared to classical methods. The benchmark strength was strong due to well-established classical methods, with the hybrid QAOA approach demonstrating significant improvements in efficiency and distance minimization. This study did not address any specific limitations. Finally, [91] utilized Fixstars Amplify annealing engine and D-Wave advantage, achieving an 86% improvement in POI satisfaction and a 64% improvement in trip cost. The benchmark strength was robust, as Bao et al.'s [92]'s and Atobe et al.'s [93] methods provided rigorous standards for comparison.

However, the study was limited by input size constraints of Ising machines and dependence on subQUBO partitioning quality.

In conclusion, the reviewed studies demonstrated the potential of quantum algorithms in optimizing various aspects of transportation systems, as highlighted in Table III. QAOA and QA have shown promise in addressing resource planning challenges in BSS [81, 82], lane reservation problems [83], urban taxi stand locations [84], and gate allocation in airports [85]. These quantum optimization algorithms offer improved solutions compared to classical techniques, although further research is needed to address limitations such as error correction, computational efficiency, scalability, and practical implementation challenges. Additionally, studies leveraging Ising machines and QA have showcased their effectiveness in optimizing visitor routes and traffic flow [86, 87]. However, certain limitations, such as fair comparisons and the consideration of time-dependent variables, need to be addressed to ensure the practicality and effectiveness of these approaches. Overall, these studies provide valuable insights and lay the foundation for future research in QA for traffic flow optimization.

6.2.3 Routing problems

This section critically examines studies that apply QA to RPs, encompassing both VRP and TSP. Although TSP is a special case of VRP, it has been extensively investigated in QA research due to its algorithmic simplicity, benchmark status, and compatibility with QUBO formulations. Conversely, VRP instances present increased complexity and practical relevance, often involving multiple vehicles, capacity constraints, and dynamic conditions.

VRP is a highly challenging task in scheduling and optimization [94]. VRP is an NP-Hard optimization problem that has been widely studied over the last decades due to its wide practical applications. Researchers have been drawn to VRP due to its potential for significant cost reduction in transportation and logistics distribution [43, 95]. VRP aims to find the optimal route for a fleet of vehicles to serve a set of customers [96]. Various variations of VRP have been proposed, introducing constraints such as time [97], capacity [98], and distance [99]. While several algorithms have been proposed to solve VRP problems [37], classical approaches tend to be computationally intensive and inefficient for larger problem sizes. In recent years, there has been a growing interest in leveraging QC to address VRP [100, 101]. General-purpose quantum algorithms like variational quantum eigensolver (VQE) [102] and QAOA [100] have been investigated for solving VRP on quantum computers. Due to the complexity of the problem, solvers are usually designed for specific variants and do not generally scale well to larger problem instances [103].

In a study conducted by Syrichas and Crispin [104], QA was proposed as a solution for VRP. Their approach aimed to simplify the tuning process of control parameters by utilizing runtime behavior measurements. This empirical method successfully reduced the number of control parameters to just one and achieved new, best-known solutions for large-scale VRP instances. However, to enhance the applicability of their approach, further analysis of fitness clouds and exploration of heterogeneous fleets are necessary. By leveraging the power of QA, Syrichas and Crispin demonstrated the potential for addressing complex VRPs at scale. In another research paper by

Tlili et al. [105], a simulated annealing-based decision support system (SDSS) was developed to solve VRPs by integrating Geographic Information Systems (GIS) and optimization techniques. The architecture of the SDSS effectively combined GIS data with simulated annealing, resulting in favorable performance in terms of computation time and solution quality. By incorporating a mathematically formulated objective function and considering various constraints, the SDSS demonstrated robustness in solving VRPs. Moreover, the practical applicability of the SDSS was enhanced by implementing the simulated annealing algorithm on standard computing hardware. Tlili et al. highlighted the effectiveness of integrating GIS and simulated annealing for optimized vehicle routes, showcasing the potential of such approaches in solving VRPs efficiently.

The studies by Syrichas and Crispin [104] and Tlili et al. [105] represent two different approaches to tackling VRPs. While Syrichas and Crispin focused on the potential of QA for addressing large-scale VRP instances, Tlili et al. highlighted the effectiveness of SDSS that integrates GIS and optimization techniques. These studies contribute to the ongoing advancements in the field of VRPs by exploring diverse techniques and methodologies. Future research could explore the interplay between QA and simulated annealing-based approaches, considering their respective strengths and limitations, to develop comprehensive solutions for a wide range of VRP scenarios. In recent years, several studies have explored the application of QA techniques in solving network routing and optimization problems. One notable approach is the use of QUBO, which can be mapped to the Ising model. Irie et al. [106], Bao et al. [107], and Ahmed and Mahonen [108] have employed QUBO to tackle various routing problems, demonstrating the potential of QA in this domain. For instance, Bao et al. proposed an Ising machine-based algorithm to address the VRP with balanced pickup (VRPBP) in real-world postal item pickup services. Their algorithm consisted of load balancing and distance optimization phases, utilizing an extended knapsack problem, and solving the TSP. The study showcased successful load balancing, shorter route distances compared to baselines, and improved execution time. However, it encountered challenges with large-scale problems and routing time.

Suen et al. [109] addressed constrained routing problems in transportation and logistics using Fujitsu's quantum-inspired CMOS-based digital annealer (DA). They formulated the problem as a QUBO and compared the scalability and solution quality of the DA with an exact solver and a greedy heuristic. The DA showed promise as a more robust option than heuristics and a more scalable alternative to the exact solver. However, it had limitations regarding the complexity of problems it could handle, and the quality of solutions obtained for small-size instances. Another application of QA in routing problems was demonstrated by Nourbakhsh et al. [110]. They focused on minimizing the risk of COVID-19 exposure during city journeys in the context of the VRP. Using D-Wave solvers and various quantum techniques, they achieved solutions in significantly less time compared to classical solvers. While the proposed model resulted in a better distribution of vehicles on the road network, the study lacked real trajectory datasets and a comprehensive comparison with existing methods.

Chen et al. [111] explored the use of QA processors for energy-efficient routing in wireless sensor networks. Their comparison between D-Wave quantum processing units (QPUs) and classical solvers revealed superior runtime performance of the QPUs for small network routing problems. They also proposed strategies for embedding network optimization problems in quantum processors, highlighting the potential of QA in enhancing network algorithm efficiency. However, the study acknowledged limitations, such as the QPUs' inability to handle larger problems and the challenges posed by limited qubit availability. In the context of the airport taxiway routing problem, Qian et al. [112] presented a collaborative quantum-inspired ant colony algorithm (CQIACA) as a potential solution. They claimed that CQIACA outperformed traditional approaches in terms of efficiency and effectiveness but lacked detailed information on its implementation and performance in larger airport environments. Further research and empirical validation are needed to fully assess the capabilities of CQIACA. Overall, these studies highlight the progress made in utilizing QA for network routing and optimization problems. While the results are promising, there are still challenges to overcome, such as scalability, hardware limitations, and real-world applicability. Further research in these areas is necessary to fully leverage the potential of QA in network computation problems.

In addition to the aforementioned studies, another domain that has garnered considerable attention and interest in recent studies is the capacitated vehicle routing problem (CVRP). This particular problem poses its own unique set of challenges and complexities, further enriching the field of research. By exploring the intricacies of CVRP, researchers have sought to develop efficient and effective solutions that address the optimization of vehicle routes while considering capacity constraints. Thus, delving into the realm of CVRP provides valuable insights and contributes to the broader understanding of logistics and transportation optimization. CVRP is a fundamental variant of the VRP that has been extensively studied in the field of optimization. It involves determining optimal routes for a fleet of vehicles with limited capacity to deliver goods or services to a set of customers located at different locations. The objective is to minimize the total distance traveled while ensuring that each customer's demand does not exceed the vehicle's capacity. The CVRP is known to be an NP-Hard problem [38, 41, 94], and solving it efficiently poses significant challenges. In recent years, there has been growing interest in exploring quantum-inspired and QC approaches to tackle combinatorial optimization problems like CVRP. Researchers have proposed various algorithms and formulations leveraging quantum techniques to address the CVRP and its variants. These works aim to leverage the inherent parallelism and optimization capabilities of QC to potentially provide more efficient and effective solutions.

Harwood et al. [113] have proposed an approach that introduces four quantum algorithms specifically designed for solving the CVRP with time windows (CVRPTWs). These algorithms utilize different formulations, such as QUBO-based and variational methods, to handle CVRPTWs with different characteristics. While these quantum algorithms show promising developments, their effectiveness is currently limited by the available quantum hardware. The scalability and efficiency of these algorithms heavily depend on the capabilities of the quantum devices. Therefore, further research is needed to enhance algorithms, explore algorithmic improvements, advance the hardware capabilities, and validate their applicability on real-world CVRP instances. Another notable work by Xu et al. [103] has focused on leveraging the Fujitsu DA, a quantum-inspired specialized hardware, to address the scalability of solving the CVRP. By combining the capabilities of the DA with existing solvers, the authors demonstrated near-optimal solutions for significantly larger CVRP instances compared to traditional solvers. However, the limited scalability of the quantum-inspired digital annealer and the reliance on specialized hardware availability present practical implementation challenges. Additionally, the generalizability of this approach to other optimization problems requires further exploration. Future research directions should aim to improve the algorithm, advance hardware capabilities, explore hybrid approaches combining classical solvers and quantum-inspired hardware, and extensively validate the proposed method on real-world CVRP instances.

Furthermore, studies have also investigated the use of QA for solving the CVRP. One such study by [114] has proposed a QACVRP algorithm that utilizes the QA capabilities to generate near-optimal solutions. The experiments show that tuning the algorithm's parameters can significantly impact its performance, and a specific parameter setting yields a 100% success rate. However, the scalability of the QACVRP algorithm to larger CVRP instances and its practical applicability in real-world scenarios require further investigation. Future research should focus on evaluating the algorithm on a broader range of problem instances and exploring its practical usage. Additionally, other studies have explored alternative formulations, such as QUBO, to tackle the CVRP. The work presented by [106] introduced a novel QUBO formulation that incorporates time-related constraints and capacity adjustments. While successful results are obtained for small-size QUBO systems, the practical application of this formulation necessitates a larger number of logical qubits, which is currently limited by the capabilities of QA technology. Future research should aim to evaluate the formulation on digital Ising machines and develop efficient hybrid algorithms that combine classical, and quantum approaches to enhance their practical usage.

The work presented in [115] has introduced hybrid algorithms, including DBSCAN solver and solution partitioning solver, for solving VRP and its variant, CVRP, using QA. Experimental tests have demonstrated promising performance, with the hybrid methods achieving solutions of similar or better quality compared to classical algorithms. However, there are limitations, such as the restricted applicability of the full QUBO solver on QPU for larger test cases and inferior results from classical solver QBSolv as the number of vehicles increases. In summary, recent studies have made significant progress in leveraging quantum-inspired and QA techniques to address CVRP and its variants. These works demonstrated the potential of QA to provide more efficient and effective solutions for combinatorial optimization problems. Moreover, in the realm of optimization algorithms, QAOA [116] has emerged as a compelling area of research. The QAOA combines principles from QA with classical optimization techniques to tackle complex optimization problems. By leveraging the inherent properties of quantum systems, such as superposition and entanglement, the QAOA aims to find approximate solutions to optimization problems efficiently. QAOA leverages the variational principle and the mapping of NP-hard combinatorial optimization problems to the minimization of an Ising Hamiltonian, making it applicable to near-term quantum devices [117]. Its potential lies in its ability to handle combinatorial problems with a large solution space, where traditional classical algorithms often struggle to find optimal solutions within a reasonable time frame. Researchers are actively exploring different aspects of the QAOA, such as its performance under different problem settings, optimization of the algorithm's parameters, and integration with classical optimization approaches.

In a study by Azad et al. [100], QAOA was implemented on the IBM Qiskit platform to solve VRP within ITS and logistics management. The authors presented the QUBO formalism and Ising formulation for VRP and compared the performance of QAOA with classical solvers such as CPLEX on small VRP instances. They defined decision variables to represent the problem's solution and formulated VRP as a minimization problem. The findings highlighted the dependence of QAOA's performance on factors like the classical optimization routine, quantum circuit depth, parameter initialization, and specific problem instances. While QAOA showed potential in solving combinatorial optimization problems like VRP, challenges related to scalability and the need for further research and improvement were acknowledged. In another study by Alsaiyari and Felemban [102], the capability of state-of-the-art quantum computers in solving VRP was assessed by implementing two variational quantum algorithms, VQE and QAOA, using IBM Qiskit. The authors compared the performance of both algorithms in solving VRP instances of increasing complexity by systematically varying the size of the problem instances, configurations, and constraints. The evaluation focused on time complexity and revealed that current noisy intermediate-scale quantum (NISQ) devices can only handle small VRP instances, with QAOA outperforming VQE as the problem size increases. However, hardware limitations, such as the number of qubits and the lack of error correction, hinder scaling up for larger VRP cases. The authors emphasized the need for significant developments in both hardware and software to overcome these limitations and enhance current quantum algorithms for complex VRP scenarios.

Lately, in addition to these existing methods, many quantum-based optimization strategies are also being explored. Feld et al. [118] and Bennett et al. [119] utilized QA and quantum walks, respectively, to solve CVRPs. However, these quantum-based techniques are currently unsuitable for noisy quantum devices due to the large circuit depths required for their realization. As a result, there has been a significant effort in using hybrid quantum–classical optimization algorithms. For instance, Azad et al. [100] investigated the use of QAOA to solve the general-purpose variant of VRP, showcasing its potential in this domain.

Building on the examination of vehicle routing optimization, it is essential to summarize the key findings and their potential impact on future research directions. Vehicle routing optimization has also attracted researchers' attention, with intensive efforts to address the VRP in different studies using quantum and classical approaches, revealing notable improvements and specific limitations. For instance, IBM Qiskit was employed for VQE simulations, demonstrating that while QSVM effectively reached classical minima for VRP and outperformed conventional methods in some scenarios, it was efficient only for small-size QUBO systems [106]. The benchmark strength lays in evaluating the performance of various QSVM encoding schemes, though the study was limited by its complexity and hardware capabilities. Similarly, Fujitsu's CMOS digital annealer improved load balancing by up to 98.4% and significantly reduced route distances and execution times [107]. The benchmarks used, K-means with Clarke and Wright's savings algorithm and Christofides algorithm, were of moderate strength, suitable for initial comparisons but not representing the most advanced methods, although the routing phase may take longer for certain datasets.

The problem of scalability has been addressed by Fujitsu's digital annealer, which showed superior performance over CPLEX, particularly in larger instances [109]. While CPLEX provided better solutions for smaller instances, its performance dropped significantly with increasing problem size. The baselines, CPLEX and NN heuristics, offered moderate strength benchmarks, but the study was limited by hardware capabilities. Another evaluation using IBM Qiskit for VRP with VQE across various noisy quantum channels found that COBYLA was the most effective optimizer despite significant noise impacts [120]. Although this comparison of different noise models provided valuable insights, the study was restricted to smaller instances. Moreover, QAOA was assessed against CPLEX for VRP, showing that QAOA, especially at higher depth values, approached the performance of CPLEX, indicating its potential in solving VRP [100]. CPLEX served as a strong benchmark, known for its efficiency and accuracy, though QAOA exhibited limitations such as a lack of optimality guarantees and sensitivity to noise. Additionally, Fujitsu demonstrated strong scalability and near-optimal solution quality for CVRP and CVRPTW [103], using existing optimal solutions from CVRPLIB as strong baselines. However, the method was limited by the number of variables the hardware could handle. The D-Wave quantum annealer showcased significant speed and efficiency advantages over classical solvers like QBSolv, although the study was limited by the scalability of quantum solvers for larger problems [110]. Furthermore, for VRPTW, VQE generally outperformed QAOA in terms of success probability and feasibility, suggesting that VQE with constrained optimization by linear approximations (COBYLA) might be the most reliable approach [113]. However, this study was restricted by current quantum hardware capacities. In another case, Fujitsu's digital annealer demonstrated substantial improvements in optimality gaps for CVRP using a data-driven multi-start algorithm [121]. The strong baseline comparison with the standard DA algorithm indicated consistent improvements, yet performance enhancement was dependent on effective tuning of the algorithm's parameters.

Moreover, an evaluation of QAOA and VQE for VRP showed that QAOA outperformed VQE for VRP with larger variables due to better iteration and Ansatz (a parameterized quantum state used as an initial guess, optimized through quantum–classical iterations) handling [102]. Classical optimizers provided a moderate benchmark but were limited by NISQ devices. A QA algorithm using path integral Monte Carlo outperformed simulated annealing, improving success rates [114], though scalability remained a challenge. A study showed higher accuracy and scalability over classical methods, but complexity increased with problem size [15]. D-Wave's hybrid solver excelled in clustering and lower-energy solutions [122] and outperformed CPLEX in MDCVRPTW solution quality and processing time, though constrained by quantum hardware [123].

There have been notable efforts to address TSP using QA and optimization techniques. For instance, in one study, a modified quantum-inspired genetic algorithm was implemented on an IBM quantum computer to solve the TSP [124]. Another study focused on providing a benchmark specifically for quantum optimization algorithms applied to the TSP [125]. The Chinese postman problem (CPP) was investigated on a quantum annealer in a separate study [126]. Additionally, an improved quantum ant colony algorithm has been proposed for solving the TSP [127]. Another study aimed to design an improved quantum-inspired evolutionary algorithm for transportation problems in logistics systems [128]. Finally, a hybrid cellular genetic algorithm was developed for solving the TSP [129]. These studies contribute to the exploration and advancement of quantum-inspired and QA approaches for optimizing the TSP. On the other hand, several studies that have been concerned with optimizing the TSP have used QUBO as a mathematical optimization framework to solve the TSP. QUBO represents the problem as a quadratic function of binary variables and then solves the problem by finding the binary values that minimize this function. For instance, the study presented in [130] explored the mapping of TSP to D-Wave quantum machines and obtained optimal solutions using classical QUBO and simulated quantum environments. In [131], a QUBO formulation was proposed for the TSP with time windows (TSPTW), enabling its solution on D-Wave computers. Gonzalez-Bermejo et al. [132] aimed to solve the TSP by developing a QUBO formulation with minimal variables and qubits for implementation on a quantum computer. Jain [52] has presented a QUBO-based approach for solving TSP on a quantum annealer and discussed the challenges of execution time and accuracy.

A comprehensive analysis of various solvers for the symmetric TSP was presented in a case study conducted by Warren [51]. The study aimed to assess the performance of four software programs in solving the TSP using a quantum annealer. The authors successfully demonstrated the potential of leveraging both classical and QA platforms to optimize TSP solutions. To ensure consistency in the analysis, they introduced a uniform structure and designed a software experiment specifically for evaluating the effectiveness of the D-Wave quantum computer in achieving optimal TSP solutions. The study shed light on the challenges associated with variable embedding and emphasized the importance of incorporating applications for the asymmetric TSP in future experiments. These findings contribute to the understanding of the capabilities and limitations of different solvers for TSP optimization. In addition, Stogiannos et al. [133] have investigated the matrix formulation for the Hamiltonian cycle problem (HCP) and the TSP, contributing to the seamless integration of benchmark instances in quantum platforms and min–max normalization for TSP Hamiltonian coefficients. Quantum-inspired ant colony optimization (Qi-ACO) was employed in [134] to address the sustainable four-dimensional TSP and analyze carbon emissions based on conveyance type, route, and vehicle combination. Salehi et al. [135] have focused on the TSPTW and presented three different formulations, highlighting the advantages of both edge-based and node-based approaches. Bao et al. [92] proposed an Ising machine-based method for multi-day travel planning (MTPP), considering factors like point of interest satisfaction, travel expenses, and time limits. The successful application of hybrid quantum computing–tabu search algorithm (QTA) to solve the asymmetric TSP (ATSP) was discussed in [136, 137], marking the first QC-based solution for this problem.

TSP, considered a part of vehicle routing optimization in terms of finding the most efficient and cost-effective routes for salespersons, has also attracted researchers' attention. Intensive efforts to address TSP using quantum and classical methods have revealed notable progress and certain limitations. In light of these findings on TSP, it is important to briefly conclude by examining the implications of these advancements and their future potential. For instance, a study on TSPTW introduced a new QUBO formulation using D-Wave digital annealer, effectively addressing time constraints, though it was limited to small instances due to scalability issues [131]. Similarly, an improved noise QA method (INQA) showed enhanced search capabilities over standard QA, but its efficacy was dependent on precise tuning of noise parameters [131]. Mapping TSP to QUBO using D-Wave digital annealer demonstrated effective results, although it faced higher execution times for smaller graphs and size limitations [130]. Another study used Ising Hamiltonian on D-Wave digital annealer for solving TSP but underperformed compared to classical solvers, handling only small problems [52].

Additionally, research focused on multi-day travel planning using an Ising Machine, which improved solution quality and execution time but was restricted to small-scale applications [92]. Moreover, a quantum tabu search for asymmetric TSP efficiently handled larger ATSP instances, though it was limited to symmetric instances and small problem sizes [136, 137]. A new QUBO formulation for both TSP and VRP using D-Wave quantum annealers introduced an efficient model, yet it was not tested on gate-based quantum computers and primarily focused on variable reduction [132]. Another study integrated benchmark instances with a novel matrix formulation using D-Wave quantum annealers and hybrid solvers, highlighting the superior qubit utilization of the advantage system 4.1 but facing limitations with incomplete graphs and hardware capabilities [133].

A detailed analysis and quantum optimization for TSPTW on D-Wave quantum annealer demonstrated practical applications but was limited to small instances and faced scalability challenges [135]. Research on sustainable 4D TSP using a quantum-inspired heuristic (Qi-ACO) efficiently balanced cost and emissions but struggled with larger sizes and computational complexity [134]. A novel QUBO formulation for selective TSP (sTSP) using D-Wave 2000Q quantum annealer marked the first attempt to address sTSP within a QUBO framework, though scalability was constrained by current quantum hardware [138].

Further, a QUBO formulation and QAOA were used for m-TSP on NISQ devices, enhancing solution search capabilities but limited by NISQ hardware [139]. Quantum phase estimation and modified Grover algorithm applied to TSP via Qiskit simulation achieved quadratic speedup but faced limitations from current quantum hardware capabilities [140]. Finally, Grover's quantum algorithm for TSP and spanning tree problems was implemented on Qiskit, achieving a full circuit implementation, but its effectiveness was still bound by the current limits of QC [141].

6.3 Discussion summary of reviewed studies

This review systematically analyzed the literature across four key domains of QA-based traffic optimization, namely TSC, TFO, VRP, and TSP. The goal of this discussion is to consolidate findings, analyze development trends, and suggest future research directions that can further enhance the application of QA in traffic optimization. Table 3 summarizes the key findings, methods, solvers, benchmarks, and limitations of each study, illustrating the potential and challenges of using QA in traffic

Table 3 Brief summary of related works on Quantum Annealing-based traffic optimization

Year Ref	Contribution	Method	Solver	Suitability			
				TLC	TFO	VRO	TSP
2018 [79]	• Optimized signal timing at intersections	• NSQGA	• N/A	✓	✓	×	×
2020 [75]	• Minimized traffic congestion for dynamic signal control	• QA –QUBO	• D-Wave	✓	✓	×	×
2021 [76]	• Minimized congestion by optimizing traffic signals based on halted cars	• QA-QUBO	• D-Wave	✓	✓	×	×
2021 [77]	• Developed a global TSC method on a square lattice via QA	• QA—Ising Hamiltonian	• D-Wave 2000Q	✓	✓	×	×
2023 [80]	• Developed a QA model for urban traffic light control	• QA- QUBO	• D-Wave D-Wave Advantage (5000 + qubits)	✓	✓	×	×
2013 [83]	• Improved Quantum-Inspired Evolutionary Algorithm	• Quantum-Inspired Evolutionary Algorithm	• N/A	×	×	✓	×
2015 [88]	• Optimize lane allocation during events	• Quantum-Inspired Evolutionary Algorithm	• N/A	×	×	✓	×
2021 [86, 89]	• Improve visitor satisfaction and experience	• Ising machines by mapping problem optimization to QUBO model	• SA-based Ising machine • (64 bits of precision on the • quadratic term and 76 bits of precision on the linear term.)	×	✓	×	✓
2021 [84]	• Optimize urban taxi stand locations	• Hybrid QA and Cognitive Computing (QABICA)	• D-Wave 2000Q (2048 qubits),	×	×	✓	×

Table 3 (continued)

Year Ref	Contribution	Method	Solver	Suitability			
				TLC	TFO	VRO	TSP
2021 [90]	• Discuss QC applications	• QC, QA, VQE	• D-Wave (2048 qubits), Fujitsu, NEC (1024 qubits)	×	✓	✓	×
2022 [81, 82]	• Minimize distance traveled by vehicle carriers	• QAOA based on QUBO Formulation	• D-Wave 2000Q (2048 qubits), IBM Qiskit	×	✓	✓	×
2019 [106]	• QUBO formulation incorporating time constraints	• QA	• IBM Qiskit: VQE simulations • (6- and 12-qubit circuits)	×	×	✓	×
2021 [107]	• Multiple-objective functions for load balancing and travel distances	• QA	• Fujitsu's CMOS Digital Annealer (1,024-bit)	×	×	✓	×
2021 [109]	• Quantum-inspired algorithms for VSP	• Quantum-inspired QUBO	• Fujitsu Digital Annealer (8,192-bit)	×	×	✓	×
2021 [120]	• Resilience of hybrid algorithms to noisy quantum channels	• Hybrid Quantum Algorithms	• IBM Qiskit • (6- and 12-qubit circuits)	×	×	✓	×
2022 [100]	• Application of QAOA to solve VRP	• QAOA	• IBM Qiskit • (6-, 12-, and 15-qubit circuits)	×	×	✓	×
2022 [103]	• Scaling existing VRP solvers using hybrid QUBO method	• Quantum-inspired QUBO	• Fujitsu Digital Annealer	×	×	✓	×
2022 [110]	• Potential of quantum and hybrid solvers for large-scale VRP optimization to minimize COVID-19 exposure	• QA	• D-Wave advantage system 4.1 solver has 5760 qubits, from which 5619 qubits were active at the time of run	×	×	✓	✓
2022 [113]	• Efficient quantum formulations for routing problems	• QA	• IBM Quantum simulators and Qiskit	×	×	✓	×

Table 3 (continued)

Year Ref	Contribution	Method	Solver	Suitability			
				TLC	TFO	VRO	TSP
2022 [121]	• Data-driven multi-start algorithm for QUBO solver performance	• Quantum-inspired optimization	• Fujitsu Digital Annealer • The third-generation DA used in this study can solve binary quadratic programming (BQP) problems with up to 100,000 bits	×	×	✓	×
2023 [102]	• Evaluate VQE and QAOA for VRP using NISQ devices	• Variational Quantum Algorithms	• IBM Qiskit	×	×	✓	×
2023 [114]	• New QA with spin encoding scheme	• QA	• Using a classical simulation approach known as path integral Monte Carlo (PIMC) on a classical computer, not on actual quantum hardware	×	×	✓	×
2023 [15]	• Optimization with limited qubits using path integrals	• Quantum Path Integral	• N/A	×	×	✓	×
2023 [122]	• QUBO formulation for ARP in disaster scenarios	• QA	• D-Wave's hybrid solver service (HSS)/qubits-2024	×	×	✓	×
2024 [123]	• Classical vs quantum for node-based and time-expanded formulations	• QA and Classical Optimization	• D-Wave's (hybrid quantum–classical solver)	×	×	✓	×
2019 [131]	• First QUBO formulation for TSP with time windows (TSPTW)	• QUBO	• N/A	×	×	×	✓
2021 [52]	• QUBO formulation for TSP on a D-Wave quantum computer	• QUBO	• D-Wave Advantage 1.1 quantum annealer	×	×	×	✓

Table 3 (continued)

Year Ref	Contribution	Method	Solver	Suitability			
				TLC	TFO	VRO	TSP
2021 [92]	• Addressed the multi-day travel planning problem (MTPP) using Ising machines by mapping the problem onto QUBO forms	• Ising machine-based optimization	• Ising machines (QA)	×	×	×	✓
2021 [137]	• Addressing the problem of solving partitioning problems using a hybrid QTA by combining QA with classical tabu search	• Hybrid QC—Tabu Search Algorithm	• D-Wave Quantum Annealer and Tabu Search	×	×	×	✓

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2018 [79]	• 5.45% improvement in traffic capacity, 23.08% reduction in stop times, 5% reduction in vehicle delay time	• Moderate: NSGA-II is a robust multi-objective optimization algorithm but not specifically tailored for TSC	• Not tested in real-world settings; specific to isolated intersections with saturation conditions	• NSQGA shows notable improvements in traffic metrics but lacks real-world validation
2020 [75]	• Wasted Time (wasted time is the total waiting time behind red lights for all cars, weighted by the speed limit of the road each car will travel on after the light turns green): QBSolv saves 33.83 h, LEAP hybrid saves 33.70 h, no coordination saves 33.00 h (all compared to fixed cycles: 64.35 h); Solution Times is the total time taken: LEAP hybrid (3 s), QBSolv (0.5–2 s)	• Weak: fixed signal cycles are not adaptive and failed to respond dynamically to fluctuating traffic conditions, leading to inefficiencies during peak times and underutilization during off-peak hours	• Assumptions of symmetric intersection and six modes per intersection limit generalizability; no real-world implementation	• LEAP hybrid solver and QBSolv show almost similar time savings but face generalizability issues

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2021 [76]	• Average Speed: Fixed Cycles (6.85 m/s), C-0051UBO Cycles (5.97 m/s), QUBO Cycles (5.33 m/s); Increase in Average Flow Speed: C-QUBO Cycles (12.85%), QUBO Cycles (22.19%)	• Weak: fixed signal cycles are not adaptive and failed to respond dynamically to fluctuating traffic conditions, leading to inefficiencies during peak times and underutilization during off-peak hours	• Limited to two intersection models; results based on simulated traffic data	• QUBO cycles show significant speed improvements but are based on simulated data
2021 [77]	• Local Controller: Improved performance for small α values, where α is defined as $2a - 1$ and $\boldsymbol{a}$ represents the rate at which vehicles move straight but suffers from excessive signal switching and poor synchronization • Simulated Annealing: Better for scenarios prioritizing smooth traffic flow ($\eta < 0.5$), reduces signal switching frequency as α increases, where η is a weight parameter for determining the ratio of the two terms • QA: Superior performance for larger problem sizes and sparse matrices, better synchronization, and signal correlation, excels when preventing excessive signal switching is prioritized ($\eta > 0.5$)	• Weak: Local control optimizes based on local conditions only • Moderate: Simulated annealing handles optimization adaptively but is classical	• Simulation-based; lacks physical validation and assumes uniform traffic flow	• QA excels in larger, sparse matrices but lacks real-world validation

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2023 [80]	• Classical Method: Less efficient, leading to suboptimal, higher-energy solutions. SA: Comparable to QA but slower and machine dependent. QA: Superior traffic reduction and energy efficiency, faster for larger networks, though remote access time can impact performance	• Moderate: Classical Method is practical for real-time use and highlights improvements with QA and SA but lacks global optimization and scalability	• Effectiveness limited by current quantum hardware scalability	• QA shows superior traffic reduction and energy efficiency but faces scalability issues
2013 [83]	• The average gap for the QEA is 2.203% compared to CPLEX's. For computation time, set 16 shows CPLEX at 180.213 s and QEA at 319.043 s, while Set 20 shows CPLEX at 1880.480 s and QEA at 638.354 s. CPLEX's computation time increases exponentially with the number of nodes (|V|), whereas QEA's time remains stable and less sensitive to |V|	• Strong: CPLEX offers high-quality solutions with established efficiency and accuracy, making it a robust baseline and a widely recognized classical solver for optimization problems. Its robustness and widespread usage in solving complex optimization problems position it as an industry-standard benchmark to assess solution quality and computational efficiency. However, it lacks scalability	• Lack of real-world validation; specific to isolated intersections	• QEA shows stable computation time compared to CPLEX's exponential increase

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2015 [88]	• The QEA has an average gap of 2.09% compared to CPLEX optimal solutions, while the improved IQEA reduces this gap to 1.45%. In terms of computation time, IQEA significantly outperforms both QEA and CPLEX, taking 32% of QEA's time and just 9% of CPLEX's time on average. For problem sets with $\lvert N\rvert = 100$ and $\lvert K\rvert = 10$–60, CPLEX takes 848.3 s, while IQEA takes only 76.38 s. For larger sets with $\lvert N\rvert = 250$–500 and $\lvert K\rvert = 30$–50, IQEA averages 1443.88 s, compared to CPLEX, which often exceeds 18,000 s. IQEA's computation time increases more slowly with the number of nodes ($\lvert N\rvert$) and tasks ($\lvert K\rvert$), maintaining high efficiency even for large problems	• Strong: using QEA and CPLEX as benchmarks for evaluating IQEA is fair and effective. QEA provides a direct comparison to measure the improvements made, while CPLEX offers high-quality solutions with established efficiency and accuracy, making it a robust baseline and a widely recognized classical solver for optimization problems. Its robustness and widespread usage in solving complex optimization problems position it as an industry-standard benchmark to assess solution quality and computational efficiency. However, it lacks scalability	• Static traffic assumptions	• IQEA outperforms QEA and CPLEX in computation time and efficiency

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2021 [86, 89]	• The Ising Machine method significantly outperforms the SA benchmark in solution quality, computation time, and feasibility probability for both Sample A and Sample B, representing two real amusement parks. The mean best satisfaction values for the Ising Machine are 71.3 compared to SA's 70.4, and the average satisfaction values are 68.2 versus SA's 66.6. Computation times for the Ising Machine average around 274 ms, roughly one-tenth of SA's 2767 ms. The probability of obtaining feasible solutions with the Ising Machine is nearly 100%, compared to SA's 69.8%, demonstrating the Ising Machine's superior efficiency and effectiveness	• Moderate: Using simulated annealing as a benchmark is fair and appropriate for evaluating the Ising Machine method, as the comparison is conducted under similar conditions with standard metrics, ensuring a reliable assessment	• Comparison not entirely fair; did not consider waiting time	• Ising Machine shows superior efficiency and effectiveness compared to SA

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2021 [84]	• The QABICA algorithm significantly outperforms the K-means algorithm in optimizing urban taxi stand locations. For weekdays, QABICA reduces the average taxi stand location bias by 41.5%, and for weekends, by 81.2%, where the average taxi stand location bias measures the clustering results' bias under different sample datasets within the same time period and region. Overall, QABICA achieves an average improvement of 61.4% compared to the K-means algorithm	• Moderate: the K-means algorithm is considered a moderate benchmark for evaluating QABICA. While widely used and effective for clustering tasks, K-means has limitations in handling complex spatial distributions and constraints, making it less robust than advanced algorithms like QABICA	• Computational complexity; reliance on average location bias	• QABICA significantly reduces taxi stand location bias compared to K-means

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2021 [90]	• Cargo load optimization improvement of 5%, investment portfolio optimization improvement of 8%	• Strong: QA approaches by D-Wave, Fujitsu, and NEC show significant improvements in complex optimization problems	• Emerging practical application limitations	• QA shows notable improvements in cargo load and portfolio optimization
2022 [81, 82]	• In evaluating the bike sharing system rebalancing problem, the D-Wave Hybrid solver consistently outperformed QAOA on IBM Qiskit, achieving a minimum cost of 11 across all scenarios. QAOA only reached this best cost at B = 25. The D-Wave QPU also produced feasible solutions but with a slightly higher minimum cost in some scenarios. This demonstrates the superior efficiency and effectiveness of the D-Wave solvers, particularly the Hybrid solver, in solving the BSS-RBP compared to QAOA	• Comparing QAOA on IBM Qiskit and QA on D-Wave is fair, highlighting their strengths and effectiveness	• Case study limited to three bike stations	• D-Wave Hybrid solver shows superior efficiency in minimizing cost compared to QAOA

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2019 [106]	• The VQE simulation of QSVM for VRP evaluated four encoding schemes: amplitude encoding (AE), angle encoding (AgE), higher-order encoding (HOE), and IQP Encoding (IQPE) with optimizers COBYLA, L_BFGS_B, and SLSQP. AE achieved 100% accuracy for 6-qubit simulations but was limited by complexity. AgE showed high accuracy with 100% (COBYLA), 92% (SLSQP), and 88% (L_BFGS_B) for 12-qubit fixed Hamiltonians, though accuracy decreased with more layers. HOE had moderate performance, with significant drops in the second layer (34% for COBYLA). IQPE was the least accurate. COBYLA was the most efficient optimizer	• To evaluate the performance of various QSVM encoding schemes in solving the VRP using VQE on IBM's optimizers	• Works efficiently for small-size QUBO systems only	• Efficient for small QUBO systems; significant accuracy variations across encoding schemes • The results highlight that encoding and optimizer choice significantly affect performance and managing circuit complexity is crucial. QSVM effectively reaches classical minima for VRP, outperforming conventional methods in some scenarios

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2021 [107]	• The proposed method improved load balancing by up to 98.4%, reduced route distances by up to 3.8%, and decreased execution times by up to 74.1%, demonstrating its effectiveness in solving VRP compared to baseline methods	• Moderate: The benchmarks, Baseline 1 (K-means + Clarke and Wright's savings algorithm) and Baseline 2 (K-means + Christofides algorithm), are of moderate strength. They employ standard clustering and routing heuristics, suitable for initial comparisons, but do not represent the most advanced methods available	• Routing phase may take longer for certain datasets	• Improved load balancing and execution times; routing phase challenges
2021 [109]	• The Digital Annealer (DA) showed superior scalability over CPLEX, especially in larger instances. For Multi-destination vehicle sharing problem (MDVSP) with N = 40, DA achieved 100% feasibility, whereas CPLEX failed. In custom instances like sgu 40, DA improved by 79.36% over CPLEX, though in clustered 40, DA's route distance was 9.40% higher. DA consistently provided feasible solutions where CPLEX failed and often outperformed NN heuristics. Despite needing improvement for smaller instances, DA's robustness and scalability make it a strong tool for large-scale vehicle sharing problems	• Strong: CPLEX offers high-quality solutions with established efficiency and accuracy, making it a robust baseline and a widely recognized classical solver for optimization problems. Its robustness and widespread usage in solving complex optimization problems position it as an industry-standard benchmark to assess solution quality and computational efficiency. However, it lacks scalability while NN heuristics provide quick approximations but lack robustness	• Limited by hardware capabilities	• Scalable for large instances; struggles with smaller ones

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2021 [120]	• The study found COBYLA to be the most effective optimizer for VQE in VRP. Amplitude damping noise resulted in energy deviations within 25%-50% of the classical minimum, while bit-flip noise started at 50%-75%, deteriorating to nearly zero by the fourth layer. Bit-phase-flip noise had deviations near 100%, and phase-flip and depolarizing noise were the most detrimental, with values near zero. Simulations took 219 h of CPU time, highlighting the significant impact of noise on quantum algorithm performance	• The study compares the impact of different noise models on the VQE for solving VRP	• Limited to smaller instances	• Effective under noise; limited to small instances

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2022 [100]	• The study evaluated the performance of QAOA versus CPLEX on the VRP: For instances (4, 2), (5, 2), and (5, 3) → (n, k), where n is the number of locations and k is the number of vehicles): • QAOA with depth $p = 10$, using COBYLA and SPSA optimizers, approached the average costs of CPLEX • QAOA at lower p values performed poorly, but increasing to $p = 10$ significantly improved results, achieving costs near those of CPLEX	• Strong: CPLEX offers high-quality solutions with established efficiency and accuracy, making it a robust baseline and a widely recognized classical solver for optimization problems. Its robustness and widespread usage in solving complex optimization problems position it as an industry-standard benchmark to assess solution quality and computational efficiency. However, it lacks scalability	• Exhibits limitations such as lack of optimality guarantees, and sensitivity to noise	• QAOA's potential with careful optimization; noise sensitivity

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2022 [103]	• The Fujitsu Digital Annealer (DA) achieved strong results for CVRP and CVRPTW: • CVRP instance A-n60-k9 (60 customers, 9 vehicles): DA solution cost was 1403 compared to the optimal cost of 1354, an error difference of 3.6% • CVRPTW instance RC101 (100 customers, 25 vehicles): Hybrid method solution cost was 1674.4 compared to the optimal cost of 1619.8, an error difference of 3.4%	• Strong: Optimal solutions from CVRPLIB: Capacitated VRP library	• Limited by the number of variables hardware can handle	• Robust for large-scale problems; hardware variable limits
2022 [110]	• The study found that hybrid solvers like D-Wave Hybrid and Leap Hybrid returned solutions within 0.03 s of QPU time, compared to at least 5 s for the classical QBSolv solver • Hybrid solvers provided near-optimal solutions with lower energy levels, demonstrating significant speed and efficiency advantages over classical solvers	• The aim is to investigate different D-Wave solvers such as QBSolv, D-Wave hybrid, and Leap Hybrid solvers	• Limited scalability of quantum solvers for larger problems	• Fast hybrid solutions, scalability remains a challenge • This approach resulted in better vehicle distribution, reducing congestion and virus transmission risk, highlighting the promise of hybrid QC for real-time combinatorial optimization problems

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2022 [113]	• The study showed that VQE achieved a 100% success rate in both minimum-distance and minimum-time formulations, while QAOA only reached 33.33% in the minimum-time formulation despite increased circuit depth. Additionally, QAOA requires more iterations to converge. The solvers hardware-efficient Ansatz with constrained optimization by linear approximations (RY/COBYLA) and hardware-efficient Ansatz with Simultaneous Perturbation Stochastic Approximation optimizer (RY/SPSA) performed better in terms of average feasibility and success probabilities compared to QAOA/COBYLA and QAOA/SPSA. These results suggest that VQE with COBYLA might be the most reliable approach for this type of problem	• Highlighting the relative strengths of different quantum algorithms and their optimizers for addressing VRPTW	• Restricted by current quantum hardware capacities	• VQE's reliability; hardware limitations

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2022 [121]	• The study shows that a data-driven multi-start algorithm significantly improves the Fujitsu Digital Annealer's (DA) performance for solving the CVRP. By using a surrogate model to predict better initial states, the multi-start DA reduces the optimality gap from 24.28 ± 3.84% to 9.75 ± 3.23% for the A-n32-k5 instance (32 customers, 5 vehicles) without increasing computational time. This demonstrates the effectiveness of initial state adjustments in enhancing QUBO solver performance	• The study compares the performance of their multi-start DA algorithm against the standard DA algorithm. The DA is a well-established quantum-inspired solver known for its capability to handle combinatorial optimization problems effectively	• Performance enhancement dependent on effective tuning of algorithm's parameters	• Significant performance improvement with data-driven approach
2023 [102]	• For nodes = 3, performance is equal. For nodes = 4, QAOA and classical methods are 1.97% better than VQE. For nodes = 5, QAOA and classical methods are 99.16% better than VQE. Large VRP instances are challenging for current quantum computers, with QAOA performing better with larger VRP variables due to better iteration and Ansatz handling. Further studies are needed to enhance quantum algorithms for different problem sizes	• Moderate: Compared to classical optimizers	• Limited by current NISQ device capabilities, performance limitations with larger variables	• QAOA outperforms VQE for larger instances; NISQ limits

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2023 [114]	• The study evaluated a QA algorithm for the CVRP, showing superior performance with an average success rate of 80% compared to 71% for SA, a 12.68% improvement. The algorithm effectively finds the best-known solutions for all benchmark instances, demonstrating robustness and efficiency, despite longer computation times in some cases. This highlights the potential of quantum-inspired methods in enhancing solution quality for real-world optimization problems	• Moderated: The baseline SA is considered strong because SA is a well-established and widely used optimization method for solving complex NP-hard problems	• Scalability and practical application remain challenged	• QA's robustness; scalability and practical application issues
2023 [15]	• Demonstrated higher accuracy and scalability compared to classical methods, but specific numerical values are not provided	• Moderated: Compared to classical methods	• Complexity grows with problem size	• Accurate and scalable; complexity with size
2023 [122]	• The HSS achieved a 33.3% faster computation time (0.4 min) in clustering compared to SA (0.6 min) and produced low-energy solutions 70% of the time versus SA's 30%. However, SA excelled in the routing phase, completing tasks 46.6% faster (3.1 min) than HSS (5.8 min)	• Moderated: The baseline SA is considered strong because SA is a well-established and widely used optimization method for solving complex NP-hard problems	• Scalability and efficiency limited by quantum hardware capabilities	• Efficient clustering; routing needs improvement

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2024 [123]	• It achieved superior solution quality and processing time, solving a 40-node graph in 10 s and a 70-node graph in 1 min, while classical solvers struggled or failed within reasonable time frames. This showcases QC's potential to efficiently handle large-scale optimization problems	• Strong: CPLEX offers high-quality solutions with established efficiency and accuracy, making it a robust baseline and a widely recognized classical solver for optimization problems. Its robustness and widespread usage in solving complex optimization problems position it as an industry-standard benchmark to assess solution quality and computational efficiency. However, it lacks scalability	• Quantum solutions limited by current quantum hardware capabilities	• Effective for large problems; quantum hardware limitations
2019 [131]	• Achieved feasibility of solution formulation. Expected to handle small-scale instances effectively on the D-Wave platform. Demonstrated the potential to improve solution times and accuracy for TSPTW, though specific numerical results are not provided	• Strong: Benchmarked against classical TSP and VRP solutions. The QUBO approach shows promise in significantly reducing computation time and improving feasibility for small instances, surpassing conventional methods in these aspects	• No empirical results yet, theoretical formulation focus	• This paper presents the first QUBO formulation for TSPTW, enabling its potential solution on quantum computers like D-Wave, and lays the groundwork for future experimental implementation

Table 3 (continued)

Year Ref	Key Findings	Benchmark Strength	Limitations	Summary
2021 [52]	• Achieved feasibility for problem size up to 8 nodes. Quantum annealer took 2 ms for n = 8 compared to less than 1 ms for classical solver. Quantum solver provided suboptimal solutions for larger problem sizes	• Weak: Compared against a classical solver, the quantum solver showed inferior performance in terms of time and accuracy. Classical solver consistently found optimal solutions for n = 8 in less than 1 ms, whereas the quantum solver often failed to find optimal solutions and took longer	• Limited to 8 nodes. Suboptimal performance compared to classical methods	• The study demonstrates the potential and current limitations of using QA for TSP. The D-Wave quantum computer could handle small problem sizes but struggled with accuracy and scalability compared to classical solvers
2021 [92]	• The proposed method effectively solves the MTPP with better solution quality and execution time compared to conventional methods. Execution time reduced by up to 53.6%	• Moderate: Widely used and established but limited by high computation costs and scalability issues	• Limited by the scalability of the current Ising machines. Further research needed to handle larger-scale problems and additional practical constraints	• The study showcases the first application of Ising machines to MTPP, achieving significant improvements in solution quality and execution time, and suggests potential for QC in travel planning
2021 [137]	• QTA accessed the quantum device 40 times per execution, while QBSolv required an average of 259 to 474 accesses, reducing the quantum resource usage by approximately 84–91%	• Strong: QBSolv is widely recognized for its ability to handle large-scale optimization problems by partitioning them into smaller subproblems. It leverages QA for optimization, providing a robust and scalable approach. However, it requires significant quantum resources and can incur high economic costs due to frequent quantum device access	• Limited by current quantum hardware capabilities • Scalability to larger problem instances remains a challenge	• It highlights the potential for efficient quantum–classical hybrid algorithms in optimization problems

optimization. Therefore, the comparative analysis of quantum and classical approaches in TSC, vehicle routing optimization, the TSP, and traffic flow optimization reveals significant advancements as well as persistent challenges.

A critical evaluation of the reviewed TSC studies reveals varying levels of performance and scalability. Among the methods discussed, the study by Hussain et al. [75] demonstrated the most effective reduction in waiting times, achieving a 33.7% improvement compared to fixed-time signal cycles using D-Wave's hybrid solver. However, this approach assumes symmetric intersections and lacks real-world deployment. Singh et al. [76] focused on real-time control, increasing average flow speed by 22.19%, but its findings were based solely on simulations, limiting real-world applicability. In contrast, Inoue et al. [77] applied QA with Ising Hamiltonian to a multi-intersection network, showing superior signal synchronization and scalability. Although promising, this approach also lacks empirical validation. As a classical alternative, Qiao et al. [79] introduced the NSQGA, which improved traffic capacity by 5.45% and reduced vehicle stop times by 23.08%. This approach, while effective, is not quantum-based but remains a robust option. Considering these comparisons, the best approach for small-scale TSC applications appears to be Hussain et al., while Inoue et al. is more suited for multi-intersection scalability. However, to achieve further improvements, researchers should focus on integrating hybrid RL-QA methods to leverage RL's adaptability and QA's combinatorial optimization power. Additionally, future studies should emphasize large-scale implementations, real-world testing, and hybrid quantum–classical solutions that balance accuracy with computational efficiency.

In the domain of traffic flow optimization, the reviewed studies demonstrated that QA-based approaches are showing promise in optimizing bike sharing systems, lane reservations, and urban taxi stand placements. The study by Harikrishnakumar & Nannapaneni [81] applied QAOA for bike sharing system rebalancing, achieving a 15% reduction in overall vehicle travel distance. Wu et al. [83] implemented a quantum evolutionary algorithm (QEA) to optimize lane reservations for major events, reducing travel disruptions by 2.2% compared to CPLEX. Wang et al. [84] introduced the QABICA model, which optimized urban taxi stand placements and reduced taxi location bias by 61.4%. While these studies highlight the advantages of QA-based optimization, they also face practical implementation challenges due to hardware limitations and error rates. Moreover, RL-based adaptive traffic management has demonstrated superior real-time adaptability, whereas QA remains more effective in solving static combinatorial problems. Future research should prioritize hybrid quantum–classical solvers to improve scalability and adaptability, enabling real-time decision-making in large-scale urban networks.

VRPs have been widely explored in the context of QA due to the NP-hard nature of route optimization. The reviewed studies demonstrated that while QA significantly enhances logistics optimization, classical solvers such as CPLEX still perform better in large-scale cases. Among the notable findings, Suen et al. [109] showed that Fujitsu's Digital Annealer outperformed CPLEX for multi-destination vehicle sharing problems, achieving 100% feasibility while significantly reducing computation time. Azad et al. [100] evaluated QAOA for VRP optimization and found that, at higher quantum circuit depths ($p = 10$), QAOA provided solutions comparable to CPLEX, proving its potential for scalability. Xu et al. [103] implemented hybrid QA algorithms for solving CVRP and found near-optimal solutions for large datasets. Despite these advancements, scalability remains a key limitation, as current quantum hardware is not yet capable of handling large instances. Classical solvers such as CPLEX remain superior in larger datasets, and RL-based vehicle routing models provide better real-time adaptability. Future research should explore hybrid approaches that integrate QA with ML-based methods, ensuring optimal performance while addressing real-world routing constraints.

The TSP is another domain where QA has been applied to optimize route planning. Several studies have examined the feasibility of using QA-based solutions, but they remain constrained by the problem size. Gonzalez-Bermejo et al. [132] proposed a QUBO-based TSP formulation designed to minimize qubit usage on quantum processors, demonstrating feasibility on D-Wave systems for small problem sizes. Jain [52] compared D-Wave's quantum annealer with classical TSP solvers and found that the quantum approach produced suboptimal performance for larger problem instances due to hardware constraints. Osaba et al. [136, 137] introduced a hybrid QA–tabu search (QTA) algorithm, which reduced computational overhead by 84–91% compared to classical solvers. Despite these developments, the limitations of current quantum hardware prevent TSP solutions from being scalable for larger logistics applications. While QA shows promise in improving computational speedups, classical heuristic algorithms remain dominant in real-world routing tasks. Future studies should focus on developing noise-resistant quantum algorithms and better hardware scaling to enhance the applicability of QA for large-scale TSP optimization.

In conclusion, QA offers significant advantages in computational efficiency and combinatorial optimization, but its limitations require hybrid solutions incorporating classical and RL-based methods. Addressing scalability, real-time adaptability, and refining quantum algorithms will enhance QA's role in traffic optimization. In TSC, the main challenges include the lack of multi-intersection deployments and adaptability to dynamic traffic fluctuations. For TFO, high error rates in quantum computations affect lane allocation and urban taxi optimizations. In VRP, QA still lags behind CPLEX for large-scale route optimization, necessitating hybrid models. TSP applications face qubit constraints that limit scalability. Overcoming these challenges requires integrating quantum techniques with classical and RL-based models. Research trends indicate a growing shift toward hybrid approaches, citywide quantum traffic management models, and advancements in quantum error correction for scalability. Future studies should focus on hybrid RL-QA algorithms, real-world pilot projects, and integrating quantum optimization into ITS, ensuring its viability for large-scale traffic optimization.

7 Challenges, recommendations, and future directions

The field of traffic optimization presents a range of challenges that must be addressed to pave the way for more efficient and sustainable transportation systems. These challenges can be broadly categorized into general issues shared across various domains of traffic optimization, as well as specific problems unique to each area. By thoroughly understanding these barriers, researchers and practitioners can leverage emerging technologies, such as quantum algorithms (QA), to enhance traffic flow and overcome the limitations of traditional methods. Furthermore, identifying these challenges also opens up avenues for future research and innovation. This includes exploring advanced computational models, integrating real-time data analytics, and developing scalable solutions that can adapt to dynamic urban environments. In this section, the challenges and potential future directions within the domains of TSC, TFO, and RPs will be discussed.

7.1 Challenges

7.1.1 Scalability and hardware limitations

One of the primary categories of challenges in applying QA to traffic optimization relates to scalability and hardware limitations. Despite its potential, QA faces several limitations in traffic optimization. Scalability remains a key issue, as existing quantum hardware has limited qubits and connectivity, restricting the size of solvable problems [83, 85, 113]. While quantum annealing offers a promising approach for solving combinatorial optimization problems, current hardware such as the D-Wave Advantage system faces notable scalability constraints. The Advantage architecture supports over 5,000 qubits, yet due to the Pegasus topology and limited qubit connectivity (each qubit connects to at most 15 others), the effective problem size is significantly smaller. Studies show that embedding a fully connected graph with n logical variables require approximately $O(n^2)$ physical qubits, drastically reducing practical capacity [142]. Furthermore, the demand for logical qubits poses a general challenge, surpassing the current capabilities of QA technology and impeding the scalability and widespread adoption of the QUBO formulation. High computational costs and hardware accessibility remain significant barriers [84, 87], particularly when deploying QA-based solutions at scale. The scalability of quantum formulations, such as the QUBO formulation, also becomes a challenge as problem size increases, hindering the efficient discovery of solutions within reasonable timeframes. From an implementation perspective, current quantum models can handle small- to medium-scale VRP instances but struggle with citywide, real-time dynamic routing due to qubit limitations. TSC optimization studies are similarly constrained, as most focus on single-intersection optimization, limiting their applicability to citywide networks. Expanding QA techniques to multi-intersection traffic management requires more scalable quantum models capable of handling larger datasets and interdependencies across intersections.

7.1.2 Error Sensitivity and model stability

Another prominent set of issues involves error sensitivity and model stability. Error mitigation and noise sensitivity impact the reliability of QA-based solutions, necessitating advancements in error correction and noise-resilient algorithms to ensure stable performance [82, 84]. High error rates in quantum computation remain a fundamental issue, as quantum models are prone to noise and decoherence, which negatively impact real-time traffic flow predictions. Unlike classical solvers, quantum-based optimization models lack stability, leading to inaccuracies in traffic forecasting and flow adjustments. Limitations in quantum approximation algorithms, like QAOA, affect their ability to reach solutions close to the global optima due to their approximate nature, limited optimization steps, and issues with premature convergence, local minima, constraint violations, and noise-based errors [100].

7.1.3 Lack of real-time adaptability

A third major theme is the lack of real-time adaptability in QA-based systems. Many current QA studies rely on simplified problem models that fail to incorporate real-world complexities, such as dynamic congestion, multimodal transport, and unpredictable disruptions [83, 85, 87]. QA solutions are often precomputed, making them less adaptive to real-time traffic fluctuations compared to RL-based models. Real-time adaptability is a crucial limitation in TSC, where QA solutions typically rely on precomputed optimizations, making them less flexible than RL-based signal control models that dynamically adjust to changing traffic conditions. QA-generated routes in routing problems tend to be static, lacking the ability to adapt dynamically to real-time traffic conditions. Real-world transportation networks require continuous re-optimization based on fluctuating demand and congestion patterns.

7.1.4 Insufficient real-world validation

A further area of concern is the insufficient real-world validation of QA-based approaches. Most QA-based studies focus on narrow case applications, such as airport traffic [85] or amusement park logistics [86], limiting their applicability to broader urban settings. Ensuring real-world feasibility requires extensive benchmarking and pilot implementations across diverse traffic environments. The limited validation of QA-based models on real-world urban networks presents another major challenge, as most existing studies rely on synthetic traffic datasets, making it difficult to evaluate the feasibility of quantum-based approaches in actual urban environments. Field testing and integration with real-time sensor data are essential for improving the reliability and effectiveness of quantum-enhanced TFO models.

7.1.5 Optimization and algorithmic limitations

Optimization and algorithmic limitations also pose notable challenges. The effectiveness of QA is influenced by hyperparameter tuning, where manual selection remains inefficient, requiring the development of automated tuning techniques to enhance adaptability [75] [114]. Additionally, limitations in quantum approximation algorithms constrain their ability to efficiently explore complex solution landscapes, especially in large-scale applications. Existing classical heuristic algorithms, such as tabu search, simulated annealing, and genetic algorithms, still outperform QA when applied to complex TSP instances.

7.1.6 Integration and hybridization

Finally, there are challenges related to integration and hybridization with classical methods. Hybrid approaches that combine QA with classical solvers offer a promising direction but still face scalability bottlenecks and computational inefficiencies [115]. Future research should focus on optimizing hybrid models to balance computational power and adaptability, making them viable for large-scale applications such as TSC, VRP, and urban traffic management. Future research should also focus on developing hybrid quantum–classical VRP solvers, integrating QA with classical dynamic rerouting algorithms to improve adaptability and scalability in real-world applications. Similarly, developing hybrid quantum heuristics that combine classical evolutionary algorithms with quantum-inspired search techniques can improve the feasibility of QA for large-scale TSP applications.

In the aforementioned critical review and discussion, QC, specifically QA, was explored in the context of optimizing TSC, TF, and RPs, all of which are considered combinatorial problems. Combinatorial optimization, a subfield of mathematical optimization, focuses on finding the best solution from a finite set of possibilities, typically involving discrete or reducible sets of feasible solutions [143]. QA has demonstrated promising results in optimizing traffic flow, and numerous studies have been conducted in this domain. However, the majority of research in this area has primarily focused on vehicle routing optimization and TSP, with less emphasis on TSC. In the presented review study on QA-based traffic optimization, different challenges within this domain have been previously discussed, as well as some recommendations. Addressing the challenges in traffic optimization and considering the recommendations provided, several key future directions can be outlined. The efforts in addressing the aforementioned challenges will pave the way for reliable and practical solutions, such as optimizing traffic signals, improving vehicle routing, and enhancing overall traffic flow. The full potential of quantum-based traffic optimization can be unlocked, and the development of efficient and sustainable transportation systems can be driven by investing in these directions.

7.2 Case study: QA for traffic signal optimization

TSC is a critical aspect of urban traffic management, with significant implications for congestion mitigation and transportation efficiency. Traditional traffic signal optimization methods rely on heuristic or rule-based models, which often fail to adapt efficiently to dynamic traffic patterns. As an emerging computational paradigm, QA presents an alternative approach by leveraging quantum principles to solve combinatorial optimization problems more efficiently than classical methods. This case study examines the application of QA in TSC optimization, highlighting key findings, challenges, and future research directions.

A study conducted by Hussain et al. [75] utilized D-Wave's hybrid quantum–classical solver to optimize traffic signal timings. The TSC problem was formulated as a quadratic unconstrained binary optimization (QUBO) model, allowing it to be mapped onto QA hardware. The primary objective was to minimize vehicle waiting times and enhance overall traffic efficiency by identifying optimal signal configurations. The study demonstrated that the QA-based approach achieved significant improvements in computational performance and traffic flow optimization compared to conventional fixed-time control strategies.

7.2.1 Key findings and performance evaluation

The results of the study indicate that QA can provide meaningful enhancements in TSC. First, the QA-based approach led to a 33.7% reduction in vehicle waiting times compared to traditional fixed-time signal control methods, demonstrating its potential to optimize signal timing efficiently. Additionally, the hybrid quantum–classical model exhibited faster convergence rates toward near-optimal solutions compared to purely classical heuristic approaches. This acceleration in computational efficiency is particularly valuable in traffic scenarios where real-time decision-making is crucial. Furthermore, the simulation results highlighted notable improvements in traffic efficiency, particularly in congestion-heavy urban intersections, suggesting that QA could play a role in intelligent traffic management systems.

7.2.2 Challenges and limitations

Despite the promising results, several limitations hinder the large-scale deployment of QA in TSC applications. One of the primary challenges is scalability, as the study's approach was effective for optimizing single-intersection control but remains constrained when extended to multi-intersection networks due to the current limitations of quantum hardware. Furthermore, real-time adaptability is a critical concern, as QA solutions are typically precomputed and static, whereas RL-based models dynamically adjust to fluctuating traffic conditions. This lack of adaptability limits the effectiveness of QA in highly dynamic urban environments. Additionally, hardware accessibility and cost constraints present a barrier to practical implementation, as D-Wave's QA processors are not widely available and require significant computational resources. Lastly, the study suggests that a hybrid approach integrating QA with RL-based optimization techniques could enhance adaptability while retaining the combinatorial problem-solving advantages of quantum computing.

7.2.3 Future research directions

To address these challenges and advance the application of QA in traffic signal optimization, future research should focus on several key areas. First, scalability solutions must be explored to enable the deployment of QA-based TSC models across multi-intersection urban networks. This may involve partitioning large-scale traffic optimization problems into smaller subproblems that can be effectively processed by current quantum hardware. Second, the integration of hybrid RL-QA models should be investigated to enhance real-time adaptability while leveraging the optimization efficiency of QA. Third, research should emphasize hardware accessibility and cost reduction, as advances in quantum hardware and cloud-based quantum computing services could facilitate broader adoption of QA-based solutions. Finally, real-world pilot deployments should be conducted to validate the practical effectiveness of QA-based TSC systems and explore their integration into existing ITS.

7.2.4 Conclusion

This case study demonstrates the potential of QA in optimizing TSC by reducing vehicle waiting times, improving traffic flow, and enhancing computational efficiency. However, its current limitations, including scalability constraints, lack of real-time adaptability, and hardware accessibility issues, necessitate further research. Future advancements in hybrid quantum–classical models, hardware scalability, and real-world validation will be essential for enabling the widespread adoption of QA-based traffic optimization solutions. The integration of QA into citywide intelligent traffic management systems represents a promising avenue for future exploration in quantum-powered transportation research.

7.3 Future directions and recommendations

To address the challenges discussed in this section, several key research directions must be pursued to enhance the practical applicability of QA in traffic optimization. One critical area is the advancement of hybrid RL and QA models. While QA is highly effective in solving combinatorial optimization problems, RL excels in adapting to dynamic environments. Future research should focus on developing hybrid RL-QA frameworks that integrate the computational efficiency of QA with the adaptability of RL to optimize real-world transportation networks [144]. Another significant challenge lies in scalability and hardware limitations. Current quantum processors face constraints related to qubit count and noise interference, which hinder large-scale deployment. Future work should emphasize advancements in quantum hardware architectures, including improved error correction techniques and noise-resistant quantum circuits, to enhance QA's computational power and enable its use in large-scale traffic management applications [145].

A major limitation of existing QA-based traffic optimization models is their reliance on static precomputed solutions rather than real-time data streams. To improve real-world applicability, future research should explore the integration of real-time sensor data, adaptive decision-making mechanisms, and predictive traffic models. This would enable QA-based solutions to dynamically adjust to changing traffic conditions, making them more effective in smart city applications [17]. Despite promising advancements, much of the research on QA remains simulation-based, with limited real-world implementation. Bridging the gap between theory and practice requires pilot studies and real-world deployments in urban traffic networks. Collaborative projects between academia, industry, and government agencies could provide valuable insights into the feasibility, efficiency, and impact of quantum-based traffic optimization in real-world scenarios [146].

Furthermore, the development of more robust quantum algorithms is crucial for enhancing the efficiency and reliability of QA-based solutions. Future research should focus on designing noise-resistant algorithms, hybrid computational techniques, and scalable optimization frameworks that ensure the broader adoption of quantum approaches in transportation systems [146]. Ultimately, QA has the potential to reshape traffic flow optimization by offering superior computational power for TSC, vehicle routing, and urban congestion management. Recent studies highlight its role within ITS, where it could revolutionize path planning, traffic operation management, and infrastructure optimization. The ability of QA to efficiently solve large-scale optimization problems suggests that it could outperform classical methods in managing complex transportation networks, contributing to more adaptive and sustainable urban mobility solutions [147, 148]. Conversely, machine learning, particularly RL, has gained significant attention in ITS applications, particularly in TSC and adaptive routing strategies. While extensive research has been conducted in this area, existing studies exhibit methodological variations in intersection modeling, traffic monitoring technologies, and neural network architectures. Given the limitations of both traditional traffic optimization techniques and standalone RL models, there is a growing need to explore innovative approaches [146]. In light of the comprehensive review, it is clear that traditional methods face limitations in optimizing complex traffic systems. Consequently, the next section explores innovative approaches, specifically the integration of QA and RL, as a promising future direction in traffic optimization.

Toward a Hybrid QA-RL Framework for Traffic Optimization: The integration of QA and RL represents a promising frontier for traffic optimization. This section examines how combining QA and RL addresses modern traffic system complexities. Neumann et al. [149] compared QA, gate-based quantum computing (QC), and classical RL to identify optimal grid traversal policies. Using the D-Wave quantum annealer and Qiskit with deep RL, QA outperformed both classical and gate-based methods in efficiency. The upcoming analysis highlights key comparative aspects of QA and RL within traffic optimization, with a focus on traffic signal control (TSC) to illustrate their impact in intelligent transportation systems (ITS). In TSC, RL adapts signal timings to real-time traffic flow [150]. Effective RL deployment requires defining states (e.g., queue length, vehicle position/speed), actions, rewards, and network architecture. The control unit observes the current state, selects an action via a deep RL policy, receives feedback, and iteratively improves the policy to minimize intersection congestion. RL has yielded a variety of strategies for optimizing TSC.

Numerous studies have applied RL in traffic light control (TLC). For instance, the study [151] has introduced a policy gradient method to reduce vehicle distribution deviation. In [152], the off-policy rainbow algorithm utilized real-time data to optimize signal phases, outperforming traditional and deep RL methods. The double deep Q-network (DDQN) in [153] highlighted queue length importance in phase decisions with environmental considerations. The actor–critic model in [154] used real-time queues and temporal data to enhance phase transitions, contrasting fixed and self-organizing timings. Study [155] employed deep Q-network (DQN) and DDQN with vehicle position data for comprehensive phase control. Lastly, Meess et al. [156] focused on narrowing the domain gap to improve real-world applicability of RL in TSC. These studies collectively underscore the evolving role of RL in intersection management. Recent advancements in QA and RL have sparked considerable interest in their application across various optimization challenges. Adapting quantum

Boltzmann machines (QBMs) for Q-value approximation in multi-agent environments significantly reduces the convergence time compared to conventional RL methods, as demonstrated in [157]. Another approach involves a hybrid actor–critic model, which combines QC enhancements with RL algorithms to optimize the training of policy and value functions, as demonstrated at CERN's beam lines in [158]. Additionally, the use of Monte Carlo tree search algorithms augmented with neural networks for scheduling QA provides a novel way to enhance QA schedules, making the process more efficient, as discussed in [159]. Moreover, the study [160] demonstrated the effective employment of QUBO and RL in solving complex, multi-objective optimization problems in antenna scheduling at NASA's Deep Space Network. This success highlights the potential adaptability and scalability of these methods for traffic management optimization.

The techniques applied across domains, e.g., antenna scheduling, where QA and deep RL were used to manage multiple conflicting objectives efficiently, suggest a promising avenue for deploying hybrid quantum–classical computational strategies to enhance real-time decision-making and reduce congestion in urban traffic systems. Together, these studies provide a solid foundation for proposing new research directions in traffic management using advanced quantum and RL techniques. Although there are multiple studies that have integrated QA and RL across different domains, as illustrated in Table 4, these methodologies have yet to be implemented in traffic transportation optimization.

Roadmap for Implementing QA and RL in Traffic Systems: In addition, there is also a future direction to explore how to combine and utilize both approaches in a single approach. This integration of RL and QA can open up new directions for future research and development in the field of traffic management and optimization. Combining RL and QA might provide a comprehensive and robust solution that leverages the advantages of both approaches. RL algorithms excel in learning optimal policies through trial and error, adapting to changing environments, and exploring large action spaces. On the other hand, QA techniques focus on ensuring the quality and reliability of systems by detecting and mitigating errors or anomalies. For instance, one possible scenario for integrating RL and QA is to use RL as the main framework for learning and decision-making while incorporating QA techniques as a validation and safety mechanism. RL can be employed to learn traffic control policies based on rewards and interactions with the environment, considering factors such as traffic volume, congestion levels, and pedestrian flow. This RL-based system can continually adapt and improve its policies through iterative learning. To incorporate QA, the RL system can be equipped with monitoring and verification mechanisms. These mechanisms can continuously assess the performance of the RL policy, detect any anomalies or deviations from expected behavior, and trigger corrective actions if necessary. QA techniques can involve the use of rule-based checks, anomaly detection algorithms, statistical analysis, or even human-in-the-loop validation to ensure the system's reliability and safety. Thus, integrating RL and QA into a single approach for traffic flow optimization holds promise as a new direction for future research.

In RL, the actor–critic framework stands out as a popular architecture, consisting of two primary components: the actor and the critic. The actor assumes the responsibility for decision-making, while the critic evaluates the actor's actions, providing

Table 4 Integration of QA and RL Across Various Domains

Year	Optimization Problem	Methodology	Solver	RL Algorithm	Key Findings	Ref
2018	• Finite-Episode Games with Discrete State Spaces	• Used QA to optimize Monte Carlo policy iteration and state value function approximation	• D-Wave 2000Q	• Monte Carlo	• Enhanced state value function approximation and policy iteration using QA	[161]
2022	• Beam lines Optimization at The European Organisation for Nuclear Research	• Developed a hybrid actor–critic model using Deep Deterministic Policy Gradient (DDPG), combining a classical actor with a quantum Boltzmann machine (QBM)-based critic for continuous spaces	• QA, D-Wave hardware, • Simulated QA	• DDPG	• Demonstrated that quantum enhancements in RL can lead to improved sample efficiency and performance in complex, continuous control tasks	[158]
2022	• QA Schedule Optimization	• Enhanced QA schedules by integrating Monte Carlo tree search with neural networks to optimize decision-making processes	• D-Wave Quantum Annealer	• Monte Carlo Tree Search enhanced by Neural Networks	• More efficient scheduling of QA processes	[159]

Table 4 (continued)

Year	Optimization Problem	Methodology	Solver	RL Algorithm	Key Findings	Ref
2022	• Antenna Scheduling	• Utilized QUBO for quantum-based optimization, Deep RL for adaptive learning and policy development, and Mixed Integer Linear Programming (MILP) for traditional optimization techniques. Coordinated through a hybrid solver approach	• D-Wave Hybrid, Quantum-Inspired	• DeepRL	• Effective multi-objective optimization with competitive results	[160]
2022	• Post-error Correction for Quantum Annealers	• Combined QA for initial state preparation followed by RL to correct errors in quantum computation outputs	• D-Wave 2000Q	• Q-Learning	• Improved accuracy in quantum state identification post-annealing	[162]

feedback on their effectiveness. Its role is pivotal in guiding the actor toward improved decisions, with specific emphasis on long-term goals. The critic facilitates learning the value of various states and actions. Exploring an alternative direction in integrating QA and RL, one can consider utilizing QA as an actor within the RL (actor–critic) framework. In this scenario, QA serves as an actor, responsible for proposing actions in the environment. In contrast, the critic in the deep RL framework takes on the responsibility of adapting to real-world data and concentrating on long-term goals. This involves evaluating the actions suggested by the QA actor and guiding the learning process. The QA actor is specifically focused on maximizing the current traffic state based on the recommendations provided by the critic. For example, in the proposed hybrid framework, QA serves as a state space optimization layer that reformulates the RL environment into QUBO model. For each episode, the current state–action graph is converted into a QUBO matrix, and D-Wave's QA solver is employed to identify high-quality state–action sequences with minimized cumulative cost. These sequences inform the policy network in the RL agent by biasing action selection probabilities, effectively narrowing the exploration space. Conversely, the RL agent feeds back reward statistics and policy gradients, which dynamically update the penalty weights in the QUBO formulation. This closed-loop optimization ensures that QA sampling becomes progressively targeted toward policy-relevant regions.

In this context, QA proves particularly valuable for swiftly identifying optimal solutions for short-term traffic conditions. This synergy establishes a framework where the deep RL component excels in long-term planning and adapting to real-world data, while the QA component optimizes immediate traffic conditions based on the critic's guidance. In summary, this integration effectively harnesses the strengths of deep RL for long-term planning and adaptation to real-world data, with QA efficiently maximizing the current traffic state based on the critic's recommendations within the RL framework.

8 Conclusion

In conclusion, this systematic review has provided a comprehensive overview of the current research on utilizing QA for traffic optimization. The analysis of the literature has shed light on the theoretical foundations, methodological approaches, empirical findings, challenges, and future research opportunities in this emerging field.

The findings indicated that QA offers a unique and promising approach to addressing the complex computational problems associated with traffic congestion in urban areas. By leveraging the principles of quantum mechanics, QC can potentially optimize traffic flow, reduce congestion, and minimize emissions, contributing to the development of more sustainable and efficient transportation systems. However, several challenges and limitations have been identified that need to be addressed for the practical implementation of QA in traffic optimization. The scalability of QA systems, limited connectivity between qubits, and the current hardware constraints pose significant obstacles in handling larger-scale and more complex TSC problems. Efforts should be directed toward advancing QA technology, including the development of more powerful quantum processors with increased qubit counts and improved connectivity.

Moreover, the high error rates and calibration requirements of quantum devices necessitate the development of robust error mitigation techniques to ensure the reliability and accuracy of optimization results. The real-time and dynamic nature of traffic conditions require the design of efficient algorithms that can handle continuous data flow and make timely adjustments to signal timings. This calls for the integration of real-time data processing and the development of algorithms that provide near-optimal solutions within short time frames. Furthermore, the formulation of the optimization problem itself represents a critical aspect that needs further exploration. The current models used in the literature often oversimplify the complexity and heterogeneity of real-world traffic conditions. Future research should focus on developing more accurate and realistic models that consider various factors influencing traffic flow, such as different vehicle types, pedestrians, cyclists, and public transportation.

In terms of VRP, the application of QA techniques faces challenges related to scalability, efficiency, limitations of quantum algorithms, logical qubit demand, and parameter tuning. Addressing these challenges requires advancements in quantum hardware capabilities, the development of scalable algorithms specific to VRP, optimization of parameter tuning, and the exploration of hybrid algorithms that combine classical and quantum approaches.

To make significant progress in the field of quantum-based traffic optimization, collaboration between researchers, traffic engineers, and policymakers is crucial. Such collaboration will facilitate the effective implementation of quantum-based techniques in real-world scenarios, ensuring that the advancements in QA translate into practical benefits, including improved traffic flow, reduced congestion, and enhanced urban mobility. In conclusion, while there are still challenges to overcome, the utilization of QA techniques holds immense potential for revolutionizing traffic management and optimizing transportation systems. Future research should continue to explore and overcome the identified challenges, paving the way for more effective and sustainable traffic management strategies.

This review was intentionally focused on qualitative analyses of strategies designed to reduce traffic congestion and enhance traffic flow rather than on quantitative comparisons such as speed benchmarks or power consumption. This approach was rooted in the complexity and variability of traffic systems, where qualitative assessments were found to provide deeper insights into the effectiveness and practicality of various solutions. It is important to note that many of the studies reviewed were prioritized for their innovative approaches to improve traffic management, which are often difficult to quantify directly in terms of speed or power efficiency. The analysis was aimed at synthesizing these qualitative outcomes to offer a broad perspective on potential improvements in traffic systems, emphasizing the strategic value of the proposed solutions over numerical metrics. Future research is encouraged to explore both quantitative and qualitative impacts of these strategies to provide a more comprehensive understanding of their benefits.

Appendix

See appendix Fig. 3.

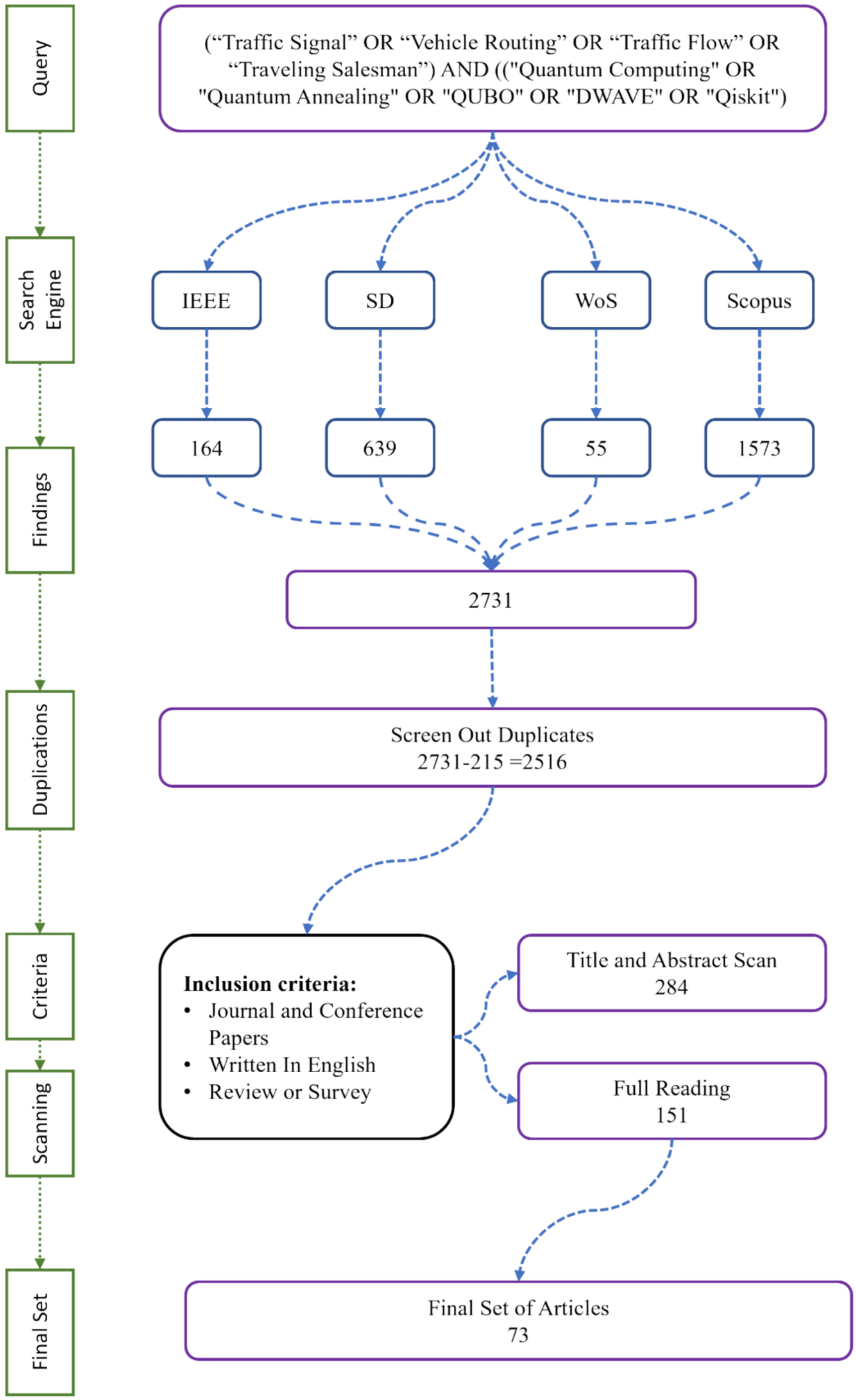

Fig. 3 Critical review protocol of presented paper

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