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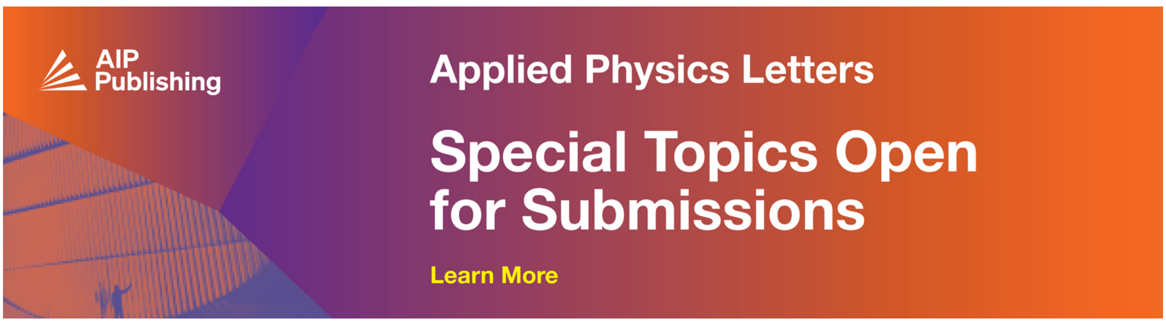

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ABSTRACT

The transfer of quantum entangled states is of fundamental interest in quantum physics and plays an important role in quantum information processing, quantum communication, and quantum technology. Here, we propose a scheme to transfer quantum entangled states of two photonic qubits by utilizing four microwave cavities coupled to a superconducting qutrit (a three-level quantum system). The photonic qubits are encoded using two orthogonal eigenstates of the photon-number parity operator with eigenvalues ± 1, which allows for various encodings for the photonic qubits. The employment of four cavities at distinct frequencies effectively reduces the inter-cavity crosstalk. The utilization of only a single superconducting qutrit as the coupler significantly reduces the circuit resources. The entanglement transfer can be completed in just one step, making this scheme remarkably efficient. During the state transfer process, the third energy level of the coupler qutrit remains unoccupied, and thus decoherence from this level is diminished. Our numerical simulations demonstrate that within current circuit quantum electrodynamics technology, one can achieve high-fidelity transfer of the entangled states of two photonic qubits encoded via squeezed vacuum states and cat states. Our scheme possesses generality and can be applied to accomplish the same task in a variety of physical systems.

Circuit quantum electrodynamics (QED) is an advanced field that delves into the interplay between light and matter, leveraging the interaction between superconducting (SC) qubits and microwave cavities to explore the fundamental physical principles. Over the past decade, circuit QED has emerged as one of the promising platforms for quantum computation and quantum information processing (QIP).[1–13] SC qubits have experienced significant advancements in recent years, as evidenced by remarkable improvements in their coherence time[14–18] and the flexibility of their energy-level rapid tuning.[19–21] These advancements have made SC qubits as key elements in building quantum computers based on circuit QED. Over the last decades, a number of theoretical proposals have been put forward for transferring quantum states among SC qubits based on circuit QED.[1,22–26] Experiments have not only demonstrated the successful transfer of quantum coherent states between two SC qubits via a microwave cavity[27] but also have achieved the transfer of quantum states in SC qubits chain.[28] Moreover, they have successfully implemented the quantum entanglement swapping in a SC circuit.[29]

Recently, the application of photonic qubits has attracted widespread attention within the fields of quantum computing and communication. Experimental realization of a microwave cavity with a high-quality factor has been achieved. Specifically, a one-dimensional microwave cavity achieves a quality factor of approximately $Q \sim 10^6$,[30–35] whereas the quality factor of a three-dimensional microwave cavity is about $Q \sim 3.5 \times 10^7$.[36–38] The lifetime of photons in microwave cavities is comparable to that of SC qubits.[39] Therefore, high-quality-factor microwave cavities or resonators serve as an integral component in QIP and play dual roles as an efficient quantum channel for information transfer[1–3,40] and as a robust quantum memory.[39,41]

Photonic qubits, in contrast to SC qubits, are characterized by a more expansive Hilbert space, allowing for a variety of encoding methods. The expanded Hilbert space of photonic qubits significantly augments their ability to represent a broader array of computational basis states and enhances their storage capacity for quantum information. Different encodings offer distinct advantages, each optimized for various applications within the field of QIP. Photonic qubits can be encoded in a variety of ways, including being encoded by a single-photon state, a vacuum state, a coherent state, or a cat state. The photonic qubits encoded by single-photon and vacuum states are easy to operate;[42] they exhibit robustness against single-photon loss with encoding via coherent states,[43] and can be used for quantum error correction through encoding based on cat states.[44]

Numerous methods have been proposed to implement single-qubit,[45–50] two-qubit,[51–55] and even multi-qubit gates with photonic qubits.[56–66] Moreover, various methods have been proposed for the preparation of different types of entangled states using photonic qubits.[67–84] Additionally, significant progress has been made in transferring entangled states between matter qubits and photonic coherent-state qubits,[85] and between photonic discrete-variable qubits and photonic continuous-variable qubits.[86] In experiments, successful simulations of coherent transfer of single photons have been achieved.[87]

The transfer of quantum entangled states is of fundamental interest in quantum physics and plays an important role in quantum information processing, quantum communication, and quantum technology. Previous studies[85,86] have employed coherent states, vacuum states, and single-photon states for encoding photonic qubits in the entanglement transfer. Quantum communications and QIP can benefit from the different encodings of photonic qubits in the entanglement transfer. The photonic qubits considered in this paper, which are encoded via two arbitrary orthogonal eigenstates $|\varphi_e\rangle$ and $|\varphi_o\rangle$ of the photon-number parity operator $\hat{\pi} = e^{i\pi\hat{a}^\dagger\hat{a}}$ can offer a remarkably rich array of encodings, including both discrete-variable encoding and continuous-variable encoding. Here, $\hat{a}^\dagger$ ($\hat{a}$) is the photon-creation (annihilation) operator.

With the above-mentioned encoding, we propose a one-step approach for transferring quantum entangled states of two photonic qubits from two microwave cavities to the other two microwave cavities, i.e.,

$$\begin{aligned}&(a|\varphi_e\rangle_1|\varphi_e\rangle_2 + b|\varphi_o\rangle_1|\varphi_o\rangle_2)|0\rangle_3|0\rangle_4\\ &\quad\rightarrow |0\rangle_1|0\rangle_2(a|\varphi_e\rangle_3|\varphi_e\rangle_4 + b|\varphi_o\rangle_3|\varphi_o\rangle_4),\end{aligned} \tag{1}$$

where $|0\rangle$ is the vacuum state, the subscripts 1 and 2 represent two microwave cavities, while the subscripts 3 and 4 represent the other two microwave cavities. Here, a and b are normalized complex numbers with $|a|^2 + |b|^2 = 1$, and the transferred state is a maximally entangled state for $a = b = \frac{1}{\sqrt{2}}$. The state transfer is achieved by four microwave cavities coupled to a SC qutrit. This proposal has several advantages: (i) Since the photonic qubits are encoded via two arbitrary orthogonal eigenstates of the photon-number parity operator, this proposal allows for various encodings for the photonic qubits. (ii) The employment of four cavities with different frequencies effectively reduces the inter-cavity crosstalk. (iii) The utilization of only a single SC qutrit as the coupler significantly reduces the circuit resources. (iv) The entanglement transfer can be completed in just one step, making this scheme remarkably efficient. (v) During the state transfer process, the third energy level of the coupler qutrit remains unoccupied and thus decoherence from this level is diminished.

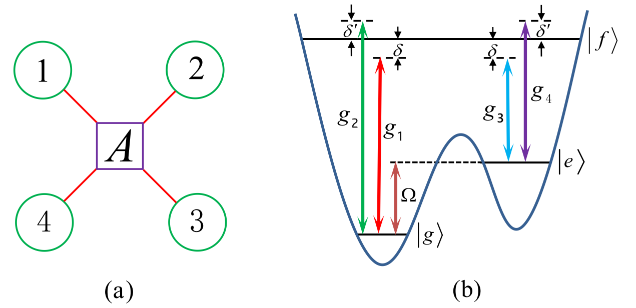

FIG. 1. (a) Schematic diagram of four microwave cavities and a SC qutrit. Each circle represents a microwave cavity, while the square A in the middle represents the SC qutrit. (b) Cavity 1 (2) is dispersively coupled to the $|g\rangle \leftrightarrow |f\rangle$ transition of the qutrit with coupling constant g_1 (g_2) and detuning δ (δ'), while cavity 3 (4) is dispersively coupled to the $|e\rangle \leftrightarrow |f\rangle$ transition of the qutrit with coupling constant g_3 (g_4) and detuning δ (δ'). Meanwhile, a microwave pulse with Rabi frequency Ω is applied to the qutrit, resonant with the $|g\rangle \leftrightarrow |e\rangle$ transition of the qutrit.

Consider a system consisting of four cavities coupled to a common SC qutrit [Fig. 1(a)]. The three levels of the qutrit are labeled as $|g\rangle$, $|e\rangle$, and $|f\rangle$. Assume that cavity 1 (2) is dispersively coupled to the $|g\rangle \leftrightarrow |f\rangle$ transition of the qutrit with coupling constant g_1 (g_2) and detuning δ (δ'), while cavity 3 (4) is dispersively coupled to the $|e\rangle \leftrightarrow |f\rangle$ transition of the qutrit with coupling constant g_3 (g_4) and detuning δ (δ'). Meanwhile, a microwave pulse with Rabi frequency Ω is applied to the qutrit, resonant with the transition between the two levels $|g\rangle$ and $|e\rangle$ [Fig. 1(b)].

The Hamiltonian, in the interaction picture and after applying the rotating-wave approximation, is described by (assuming $\hbar = 1$)

$$\begin{aligned}H_\mathrm{I} = {}& g_1 e^{i\delta t}\hat{a}_1|f\rangle\langle g| + g_2 e^{i\delta' t}\hat{a}_2|f\rangle\langle g| + g_3 e^{i\delta t}\hat{a}_3|f\rangle\langle e|\\ &+ g_4 e^{i\delta' t}\hat{a}_4|f\rangle\langle e| + \Omega|e\rangle\langle g| + \mathrm{H.c.},\end{aligned} \tag{2}$$

where $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_3$, and $\hat{a}_4$ represent the photon annihilation operators of cavities 1, 2, 3, and 4, respectively; $\delta = \omega_{fg} - \omega_1 = \omega_{fe} - \omega_3 > 0$ and $\delta' = \omega_{fg} - \omega_2 = \omega_{fe} - \omega_4 < 0$. Here, ω_{fg}, ω_{fe}, and ω_{eg} are, respectively, the $|g\rangle \leftrightarrow |f\rangle$ transition frequency, the $|e\rangle \leftrightarrow |f\rangle$ transition frequency, and the $|g\rangle \leftrightarrow |e\rangle$ transition frequency of the qutrit; while ω_1, ω_2, ω_3, and ω_4 are, respectively, the frequencies of cavities 1, 2, 3, and 4.

Under the large detuning conditions $\delta \gg g_1, g_3$ and $\delta' \gg g_2, g_4$, the Raman coupling between the qutrit's energy levels $|g\rangle$ and $|e\rangle$ can be induced by the cavity pairs (1, 3) and (2, 4) due to the adiabatic elimination of the intermediate energy level $|f\rangle$. When $\frac{|\delta-\delta'|}{|\delta^{-1}+\delta'^{-1}|} \gg (g_1g_2, g_1g_4, g_2g_3, g_3g_4)$, there is no interaction between the cavity pairs (1, 2), (1, 4), (2, 3), and (3, 4) induced by the qutrit. Additionally, assume that the detunings δ and δ' significantly exceed the Rabi frequency Ω, such that the influence of the pulse on the Raman transition is negligible. Thus, the Hamiltonian (2) will be[88]

$$\begin{aligned}H = {}& -2G_1\hat{a}_1^\dagger\hat{a}_1|g\rangle\langle g| - 2G_2\hat{a}_2^\dagger\hat{a}_2|g\rangle\langle g|\\ &- 2G_3\hat{a}_3^\dagger\hat{a}_3|e\rangle\langle e| - 2G_4\hat{a}_4^\dagger\hat{a}_4|e\rangle\langle e|\\ &- 2G_{13}(\hat{a}_1\hat{a}_3^\dagger|e\rangle\langle g| + \hat{a}_1^\dagger\hat{a}_3|g\rangle\langle e|)\\ &- 2G_{24}(\hat{a}_2\hat{a}_4^\dagger|e\rangle\langle g| + \hat{a}_2^\dagger\hat{a}_4|g\rangle\langle e|) + \Omega\sigma_x,\end{aligned} \tag{3}$$

where $\sigma_x = |e\rangle\langle g| + |g\rangle\langle e|$, $G_1 = g_1^2/(2\delta)$, $G_2 = g_2^2/(2\delta')$, $G_3 = g_3^2/(2\delta)$, $G_4 = g_4^2/(2\delta')$, $G_{13} = g_1 g_3/(2\delta)$, and $G_{24} = g_2 g_4/(2\delta')$. In Eq. (3), the first (second) line describes the Stark shifts of the energy level $|g\rangle$ ($|e\rangle$), which are dependent on the photon number. Meanwhile, the third line and the fourth line describe the coherent coupling between $|g\rangle$ and $|e\rangle$, caused by the cavity pair (1, 3) and the cavity pair (2, 4), respectively.

The operators corresponding to the SC qutrit in Eq. (3) can be expressed as $|g\rangle\langle g| = (I + \tilde{\sigma}^\dagger + \tilde{\sigma}^-)/2$, $|e\rangle\langle e| = (I - \tilde{\sigma}^\dagger - \tilde{\sigma}^-)/2$, $|e\rangle\langle g| = (\tilde{\sigma}_z + \tilde{\sigma}^\dagger - \tilde{\sigma}^-)/2$, $|g\rangle\langle e| = (\tilde{\sigma}_z - \tilde{\sigma}^\dagger + \tilde{\sigma}^-)/2$, and $\sigma_x = \tilde{\sigma}_z$, where $\tilde{\sigma}_z = |+\rangle\langle +| - |-\rangle\langle -|$, $\tilde{\sigma}^\dagger = |+\rangle\langle -|$, and $\tilde{\sigma}^- = |-\rangle\langle +|$. Here, $|+\rangle = (|g\rangle + |e\rangle)/\sqrt{2}$ and $|-\rangle = (|g\rangle - |e\rangle)/\sqrt{2}$. Using the above-mentioned expressions, Eq. (3) can be rewritten as

$$\begin{aligned} H = &-G_1\hat{a}_1^\dagger\hat{a}_1(I + \tilde{\sigma}^\dagger + \tilde{\sigma}^-) - G_2\hat{a}_2^\dagger\hat{a}_2(I + \tilde{\sigma}^\dagger + \tilde{\sigma}^-) \\ &- G_3\hat{a}_3^\dagger\hat{a}_3(I - \tilde{\sigma}^\dagger - \tilde{\sigma}^-) - G_4\hat{a}_4^\dagger\hat{a}_4(I - \tilde{\sigma}^\dagger - \tilde{\sigma}^-) \\ &- G_{13}\hat{a}_1\hat{a}_3^\dagger(\tilde{\sigma}_z + \tilde{\sigma}^\dagger - \tilde{\sigma}^-) - G_{13}\hat{a}_1^\dagger\hat{a}_3(\tilde{\sigma}_z - \tilde{\sigma}^\dagger + \tilde{\sigma}^-) \\ &- G_{24}\hat{a}_2\hat{a}_4^\dagger(\tilde{\sigma}_z + \tilde{\sigma}^\dagger - \tilde{\sigma}^-) - G_{24}\hat{a}_2^\dagger\hat{a}_4(\tilde{\sigma}_z - \tilde{\sigma}^\dagger + \tilde{\sigma}^-) + \Omega\tilde{\sigma}_z. \end{aligned} \tag{4}$$

After applying the unitary transformation $U = e^{iH_0t}$ with $H_0 = \Omega\tilde{\sigma}_z$ and performing a unitary transformation e^{-iH_0t} to return to the original interaction picture, we obtain from Eq. (4) (see the supplementary material)

$$\begin{aligned} \tilde{H} = &-(G_1\hat{a}_1^\dagger\hat{a}_1 + G_2\hat{a}_2^\dagger\hat{a}_2 + G_3\hat{a}_3^\dagger\hat{a}_3 + G_4\hat{a}_4^\dagger\hat{a}_4) + \Omega\tilde{\sigma}_z \\ &- G_{13}(\hat{a}_1^\dagger\hat{a}_3 + \hat{a}_3^\dagger\hat{a}_1)\tilde{\sigma}_z - G_{24}(\hat{a}_2^\dagger\hat{a}_4 + \hat{a}_4^\dagger\hat{a}_2)\tilde{\sigma}_z. \end{aligned} \tag{5}$$

After applying the additional unitary transformation $\tilde{U} = e^{i\tilde{H}_0t}$ with $\tilde{H}_0 = -(G_1\hat{a}_1^\dagger\hat{a}_1 + G_2\hat{a}_2^\dagger\hat{a}_2 + G_3\hat{a}_3^\dagger\hat{a}_3 + G_4\hat{a}_4^\dagger\hat{a}_4) + \Omega\tilde{\sigma}_z$, the Hamiltonian in the second new interaction picture will be (see the supplementary material)

$$H_e = -G(\hat{a}_1^\dagger\hat{a}_3 + \hat{a}_3^\dagger\hat{a}_1)\tilde{\sigma}_z + G(\hat{a}_2^\dagger\hat{a}_4 + \hat{a}_4^\dagger\hat{a}_2)\tilde{\sigma}_z, \tag{6}$$

where we have assumed

$$G_1 = G_3, \quad G_2 = G_4, \quad G = G_{13} = -G_{24}. \tag{7}$$

In this work, the two logic states $|0\rangle$ and $|1\rangle$ of a photonic qubit are encoded by two arbitrary orthogonal eigenstates of the photon-number parity operator $\hat{\pi} = e^{i\pi\hat{a}^\dagger\hat{a}}$, denoted as $|\varphi_e\rangle$ with the eigenvalue of 1 and $|\varphi_o\rangle$ with the eigenvalue of -1, i.e.,

$$\begin{aligned} |0\rangle &= |\varphi_e\rangle = \sum_p d_p|p\rangle, \\ |1\rangle &= |\varphi_o\rangle = \sum_q d_q|q\rangle, \end{aligned} \tag{8}$$

where the coefficients d_p and d_q satisfy the normalization conditions $\sum_p |d_p|^2 = \sum_q |d_q|^2 = 1$. The state $|p\rangle$ corresponds to a Fock state within a cavity, representing an arbitrary even count p of photons; while the state $|q\rangle$ represents a Fock state within the cavity, containing an arbitrary odd count q of photons. It is easy to see that the states $|\varphi_e\rangle$ and $|\varphi_o\rangle$ are mutually orthogonal. By applying the operator $\hat{\pi}$ to the states $|\varphi_e\rangle$ and $|\varphi_o\rangle$, one has $\hat{\pi}|\varphi_e\rangle = |\varphi_e\rangle$ and $\hat{\pi}|\varphi_o\rangle = -|\varphi_o\rangle$. This confirms that the states $|\varphi_e\rangle$ and $|\varphi_o\rangle$ are eigenstates of the photon-number parity operator $\hat{\pi} = e^{i\pi\hat{a}^\dagger\hat{a}}$, with corresponding eigenvalues $+1$ and -1, respectively.

As shown in the physical setup depicted in Fig. 1(a), consider a SC qutrit initially in the state $|+\rangle$, which is prepared by applying a microwave pulse resonant with the $|g\rangle \to |e\rangle$ transition of the qutrit in the state $|g\rangle$. The pulse duration is $\frac{\pi}{4\Omega}$, and the initial phase of the pulse is $-\frac{\pi}{2}$. It is noted that the initial state $|+\rangle$ of the SC qutrit remains unaffected by the Hamiltonian (6). Thus, the part of the Hamiltonian (6) related to the SC qutrit can be disregarded. Hence, from Eq. (6), we obtain

$$H_e = -G(\hat{a}_1^\dagger\hat{a}_3 + \hat{a}_3^\dagger\hat{a}_1) + G(\hat{a}_2^\dagger\hat{a}_4 + \hat{a}_4^\dagger\hat{a}_2). \tag{9}$$

Cavities 1 and 2 are initially in the entangled state $a|\varphi_e\rangle_1|\varphi_e\rangle_2 + b|\varphi_o\rangle_1|\varphi_o\rangle_2$, whereas cavities 3 and 4 are initially in the vacuum state $|0\rangle_3|0\rangle_4$. Here, the subscripts 1, 2, 3, and 4 represent cavities 1, 2, 3, and 4, respectively. The initial state of the cavity system is thus given by

$$|\psi(0)\rangle = (a|\varphi_e\rangle_1|\varphi_e\rangle_2 + b|\varphi_o\rangle_1|\varphi_o\rangle_2)|0\rangle_3|0\rangle_4. \tag{10}$$

Under the Hamiltonian H_e of Eq. (9), the state of the cavity system will evolve to the following state (see the supplementary material):

$$\begin{aligned} |\psi\rangle = &\, a|0\rangle_1|0\rangle_2 \sum_p d_p e^{i\frac{p}{2}\pi}|p\rangle_3 \sum_p d_p e^{-i\frac{p}{2}\pi}|p\rangle_4 \\ &+ b|0\rangle_1|0\rangle_2 \sum_q d_q e^{i\frac{q}{2}\pi}|q\rangle_3 \sum_q d_q e^{-i\frac{q}{2}\pi}|q\rangle_4. \end{aligned} \tag{11}$$

Note that the Hamiltonian (6) and (9) are expressed in the second new interaction picture. Thus, in order to complete the entangled states transfer in the original interaction picture, one needs to apply a unitary transformation $e^{-i\tilde{H}_0t}$ to return to the original interaction picture, the state (11) becomes (see the supplementary material)

$$|\psi\rangle_f = e^{-i\phi_0}|0\rangle_1|0\rangle_2(a|\varphi_e\rangle_3|\varphi_e\rangle_4 + b|\varphi_o\rangle_3|\varphi_o\rangle_4), \tag{12}$$

where $\phi_0 = \Omega\pi/(2G)$ is a global phase that can be ignored.

Based on the above-mentioned description, the following can be seen:

(i) Each photonic qubit is encoded by two arbitrary orthogonal eigenstates $|\varphi_e\rangle$ (with eigenvalue 1) and $|\varphi_o\rangle$ (with eigenvalue -1) of the photon-number parity operator $\hat{\pi} = e^{i\pi\hat{a}^\dagger\hat{a}}$ of a cavity, which allows for various encodings of the photonic qubits. For instance, they can have the following encodings: (i) $|\varphi_e\rangle$ is the vacuum state $|0\rangle$ and $|\varphi_o\rangle$ is the single-photon state $|1\rangle$; (ii) $|\varphi_e\rangle$ is an even cat state $|cat\rangle$ with $|cat\rangle = \mathcal{N}_+(|\alpha\rangle + |-\alpha\rangle)$, and $|\varphi_o\rangle$ is an odd cat state $|\overline{cat}\rangle$ with $|\overline{cat}\rangle = \mathcal{N}_-(|\alpha\rangle - |-\alpha\rangle)$, where $\mathcal{N}_+$ ($\mathcal{N}_-$) is a normalization factor; (iii) $|\varphi_e\rangle$ is an even Fock state $|2m\rangle$, and $|\varphi_o\rangle$ is an odd Fock state $|2n+1\rangle$; and (iv) $|\varphi_e\rangle$ is a squeezed vacuum state $|\xi\rangle$ with $|\xi\rangle$ being a superposition state of Fock states with even-number photons, and $|\varphi_o\rangle$ is an odd cat state $|\overline{cat}\rangle$, and so on.

(ii) The entanglement transfer is completed in just one step.

(iii) During the state transfer process, the third energy level $|f\rangle$ of the coupler qutrit is not occupied, and thus decoherence from this level is diminished.

(iv) The utilization of only a single SC qutrit as the coupler significantly reduces the circuit resources.

(v) The operation time is (see the supplementary material)

$$t = \pi/(2G), \tag{13}$$

which should be much shorter than both the decoherence time of the qutrit and the cavity decay time, ensuring that the system dissipation is negligibly small during the state transfer.

(vi) The above-mentioned condition (7) turns out into

$$g_1 = g_3, \quad g_2 = g_4, \quad g_1 g_3/\delta = -g_2 g_4/\delta', \tag{14}$$

which can be satisfied by adjusting the cavity frequency, the qutrit's energy-level spacings, and the coupling strengthens (e.g., through varying the capacitance between the SC qutrit and the cavities).

In the following, we present a discussion on the experimental feasibility of transferring quantum entangled states of two photonic qubits from two cavities to the other two cavities by employing four 3D microwave cavities coupled to a SC flux qutrit (Fig. 2).[15,89,90] Here, each photonic qubit is encoded via a squeezed vacuum state $|\xi\rangle$ and a cat state $|\overline{cat}\rangle$. As mentioned previously, the encoding with a squeezed vacuum state $|\xi\rangle$ and a cat state $|\overline{cat}\rangle$ is an example for the encoding states $|\varphi_e\rangle$ and $|\varphi_o\rangle$.

When considering the unwanted cavities-qutrit interactions, the unwanted pulse-induced interactions, and the inter-cavity crosstalk, the Hamiltonian (2) is modified as $H' = H_{\rm I} + \delta H + \epsilon_1 + \epsilon_2$. Here, δH represents the unwanted inter-cavity crosstalk, which can be expressed as

$$\begin{aligned}\delta H = {} & g_{12} e^{-i\delta_{12} t} \hat{a}_1 \hat{a}_2^\dagger + g_{13} e^{-i\delta_{13} t} \hat{a}_1 \hat{a}_3^\dagger \\ & + g_{14} e^{-i\delta_{14} t} \hat{a}_1 \hat{a}_4^\dagger + g_{23} e^{-i\delta_{23} t} \hat{a}_2 \hat{a}_3^\dagger \\ & + g_{24} e^{-i\delta_{24} t} \hat{a}_2 \hat{a}_4^\dagger + g_{34} e^{-i\delta_{34} t} \hat{a}_3 \hat{a}_4^\dagger + \text{H.c.}.\end{aligned} \tag{15}$$

Here, g_{kl} and $\delta_{kl} = \omega_k - \omega_l$ represent the coupling strength and frequency detuning between cavities k and l, respectively, ($kl = 12, 13, 14, 23, 24, 34$); and ϵ_1 represents the unwanted pulse-induced $|e\rangle \leftrightarrow |f\rangle$ transition of the SC qutrit, which can be expressed as

$$\epsilon_1 = \Omega_{fe} e^{i\delta_p t} |f\rangle\langle e| + \text{H.c.}, \tag{16}$$

where $\delta_p = \omega_{fe} - \omega_{eg}$, and Ω_{fe} represents the Rabi frequency of the pulse (Fig. 3). In addition, ϵ_2 represents the unwanted cavity-induced $|g\rangle \leftrightarrow |e\rangle$ transition of the SC qutrit, which can be expressed as

$$\epsilon_2 = \tilde{g}_3 e^{i\tilde{\delta} t} \hat{a}_3 |e\rangle\langle g| + \tilde{g}_4 e^{i\tilde{\delta}' t} \hat{a}_4 |e\rangle\langle g| + \text{H.c.}, \tag{17}$$

where the first term (the second term) describes the unwanted coupling between cavity 3 (cavity 4) and the $|g\rangle \leftrightarrow |e\rangle$ transition of the qutrit with coupling constant $\tilde{g}_3$ ($\tilde{g}_4$) and detuning $\tilde{\delta} = \omega_{eg} - \omega_3$ ($\tilde{\delta}' = \omega_{eg} - \omega_4$) (Fig. 3). Due to $\omega_{fg} \gg \omega_{fg}, \omega_{eg}$, the $|g\rangle \leftrightarrow |f\rangle$ transition induced by the cavities 3 and 4 or the pulse can be neglected, and thus is not considered in our numerical simulations.

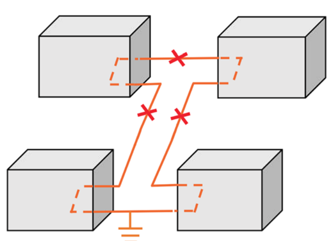

FIG. 2. Schematic circuit of four 3D microwave cavities coupled to a SC flux qutrit. Each box represents a 3D microwave cavity, and the loop with three Josephson junctions represents a SC flux qutrit.

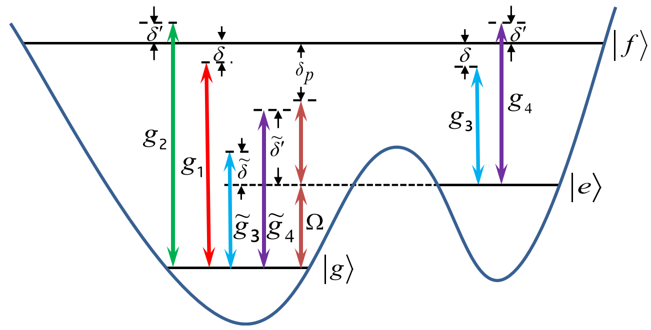

FIG. 3. Illustration of the unwanted coupling between cavity 3 (cavity 4) and the $|g\rangle \leftrightarrow |e\rangle$ transition of the qutrit with coupling constant $\tilde{g}_3$ ($\tilde{g}_4$) and detuning $\tilde{\delta} = \omega_{eg} - \omega_3$ ($\tilde{\delta}' = \omega_{eg} - \omega_4$) as well as the unwanted pulse-induced $|e\rangle \leftrightarrow |f\rangle$ transition of the SC qutrit with the Rabi frequency Ω_{fe} and detuning $\delta_p = \omega_{fe} - \omega_{eg}$.

When considering the dissipation of the cavities and the decoherence of the SC qutrit, the master equation for the dissipative system is determined by

$$\begin{aligned}\frac{d\rho}{dt} = {} & -i[H', \rho] + \sum_{j=1}^{4} \kappa_j \mathcal{L}[\hat{a}_j] + \gamma_{eg} \mathcal{L}[\sigma_{eg}^-] \\ & + \gamma_{fe} \mathcal{L}[\sigma_{fe}^-] + \gamma_{fg} \mathcal{L}[\sigma_{fg}^-] \\ & + \gamma_{\varphi,e} (\sigma_{ee} \rho \sigma_{ee} - \sigma_{ee} \rho/2 - \rho \sigma_{ee}/2) \\ & + \gamma_{\varphi,f} (\sigma_{ff} \rho \sigma_{ff} - \sigma_{ff} \rho/2 - \rho \sigma_{ff}/2),\end{aligned} \tag{18}$$

where H' is the modified Hamiltonian given earlier, $\sigma_{eg}^- = |g\rangle\langle e|$, $\sigma_{fe}^- = |e\rangle\langle f|$, $\sigma_{fg}^- = |g\rangle\langle f|$, $\sigma_{ee} = |e\rangle\langle e|$, $\sigma_{ff} = |f\rangle\langle f|$, and $\mathcal{L}[\Lambda] = \Lambda \rho \Lambda^\dagger - \Lambda^\dagger \Lambda \rho/2 - \rho \Lambda^\dagger \Lambda/2$, with ($\Lambda = \hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4, \sigma_{eg}^-, \sigma_{fe}^-, \sigma_{fg}^-$). Additionally, κ_j is the decay rate of cavity j ($j = 1, 2, 3, 4$); γ_{eg} is the energy relaxation rate of the level $|e\rangle$ for the decay path $|e\rangle \to |g\rangle$; γ_{fe} (γ_{fg}) is the energy relaxation rate of the level $|f\rangle$ for the decay path $|f\rangle \to |e\rangle$ ($|f\rangle \to |g\rangle$), and $\gamma_{\varphi,e}$ ($\gamma_{\varphi,f}$) is the dephasing rate of the level $|e\rangle$ ($|f\rangle$) of the qutrit.

The fidelity of the quantum entangled state transfer can be expressed as

$$F = \sqrt{\langle \psi_{\rm id} | \rho | \psi_{\rm id} \rangle}. \tag{19}$$

Here, $|\psi_{\rm id}\rangle$ represents the ideal output state, which can be expressed as $|\psi_{\rm id}\rangle = |0\rangle_1 |0\rangle_2 \frac{1}{\sqrt{2}} (|\xi\rangle_3 |\xi\rangle_4 + |\overline{cat}\rangle_3 |\overline{cat}\rangle_4) |+\rangle$ according to Eq. (12) with $a = b = \frac{1}{\sqrt{2}}$ and because the initial state $|+\rangle$ of the SC qutrit remains unchanged during the state transfer. Meanwhile, ρ is the density operator that is calculated by solving the master equation (18), which takes into account the system's dissipation, the unwanted interactions, and the inter-cavity crosstalk.

Additional parameters employed in our numerical simulations are as follows: (i) $\gamma_{eg}^{-1} = T$ μs, $\gamma_{fg}^{-1} = T$ μs, $\gamma_{fe}^{-1} = T$ μs, $\gamma_{\varphi e}^{-1} = \gamma_{\varphi f}^{-1} = T/2$ μs; (ii) $\kappa_1 = \kappa_2 = \kappa_3 = \kappa_4 = \kappa$; (iii) $g_{12} = g_{13} = g_{14} = g_{23} = g_{24} = g_{34} = g_c$; (iv) $\alpha = 0.5$; (v) $\tilde{g}_3 = g_3$, $\tilde{g}_4 = g_4$; (vi) $\Omega/2\pi = \Omega_{fe}/2\pi = 50$ MHz; and (vii) $\xi = 0.5$. Note that the coupling

TABLE I. Parameters used in the numerical simulation.

$\omega_{eg}/(2\pi) = 5.5\,\text{GHz}$	$\omega_{fe}/(2\pi) = 10.5\,\text{GHz}$	$\omega_{fg}/(2\pi) = 16\,\text{GHz}$
$g_1/(2\pi) = 55\,\text{MHz}$	$g_2/(2\pi) = 70\,\text{MHz}$	$g_3/(2\pi) = 55\,\text{MHz}$
$g_4/(2\pi) = 70\,\text{MHz}$	$\delta/2\pi = 0.8\,\text{GHz}$	$\delta'/2\pi = -1.3\,\text{GHz}$
$\omega_1/2\pi = 15.2\,\text{GHz}$	$\omega_2/2\pi = 17.3\,\text{GHz}$	$\omega_3/2\pi = 9.7\,\text{GHz}$
$\omega_4/2\pi = 11.8\,\text{GHz}$	$\delta_{12}/2\pi = -2.1\,\text{GHz}$	$\delta_{13}/2\pi = 5.5\,\text{GHz}$
$\delta_{14}/2\pi = 3.4\,\text{GHz}$	$\delta_{23}/2\pi = -7.6\,\text{GHz}$	$\delta_{24}/2\pi = 5.5\,\text{GHz}$
$\delta_{34}/2\pi = -2.1\,\text{GHz}$	$\delta_p/2\pi = 5\,\text{GHz}$	$\tilde{\delta}/2\pi = -4.2\,\text{GHz}$
$\tilde{\delta}'/2\pi = -6.3\,\text{GHz}$		

strength between a SC flux qubit and a microwave cavity can reach $2\pi \times 636$ MHz.[91,92]

By solving the master equation (18), we calculate the fidelity of entangled state transfer with the parameters chosen above and in Table I. By setting $T = 40\ \mu s$ in numerical simulations, we plot Fig. 4, which shows how fidelity varies with κ^{-1} for $g_c = 0,\ 0.01 g_{\max}$, and $0.1 g_{\max}$, where $g_{\max} = \max\{g_1, g_2, g_3, g_4, \tilde{g}_3, \tilde{g}_4\}$. According to Fig. 4, we can observe that the fidelity exceeds 99.01% for $\kappa^{-1} \geq 40\ \mu s$ and $g_c = 0.01 g_{\max}$. It is worth noting that the inter-cavity crosstalk strength can be made $0.01 g_{\max}$ by a prior design of the sample in experiments.[69] It is worth noting that the inter-cavity crosstalk strength can be made $0.01 g_{\max}$ by a prior design of the sample in experiments,[69] and the decoherence times of the flux qutrit are achievable due to experimental reports showing decoherence times ranging from 70 μs to 1 ms.[15,91,93]

Based on the parameters in Table I, the operational time is estimated as $0.14\,\mu s$, which is much shorter than both the decoherence time (20–40 μs) of the qutrit and the dissipation time of the cavity (10–150 μs) used in numerical simulations. For the cavity dissipation time $\kappa^{-1} = 40\ \mu s$, the quality factors of the four cavities are $Q_1 \sim 3.84 \times 10^6$, $Q_2 \sim 4.32 \times 10^6$, $Q_3 \sim 2.44 \times 10^6$, and $Q_4 \sim 2.96 \times 10^6$, which are achievable as a high $Q \sim 3.5 \times 10^7$ for a 3D microwave cavity has been demonstrated in experiments.[36–38]

The above-mentioned analysis implies that high-fidelity transfer of quantum entangled states of two photonic qubits encoded by a

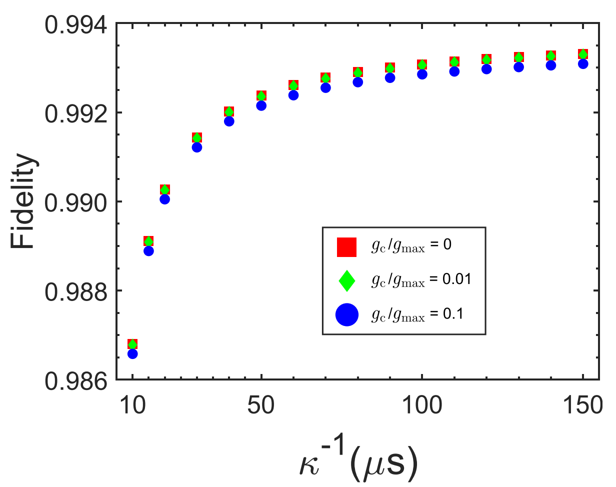

FIG. 4. Fidelity vs κ^{-1} for $g_c = 0,\ 0.01 g_{\max},\ 0.1 g_{\max}$, and $T = 40\ \mu s$.

squeezed vacuum state $|\xi\rangle$ and a cat state $|\overline{cat}\rangle$ from two microwave cavities to the other two microwave cavities can be achieved within current circuit QED techniques. Our proposal is universal and can be applied to accomplish the same task in a variety of physical systems, which consists of a three-level artificial atom (e.g., a quantum dot, a NV center, a magnon, a superconducting qutrit with different types) coupled to four microwave or optical cavities.

See the of supplementary material, subsection I, for the derivation of Eqs. (5) and (6); subsection II for the derivation of Eqs. (11)–(13); and subsection III for the analysis of fidelity under various experimental imperfections.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Liang Bin: Conceptualization (lead); Writing – original draft (lead). **Dong-Xuan Zhang:** Software (equal); Writing – review & editing (equal). **Lei Chen:** Formal analysis (equal); Writing – review & editing (equal). **Yi-Hao Kang:** Software (equal); Writing – review & editing (equal). **Zhi-Rong Zhong:** Supervision (lead); Writing – review & editing (equal). **Qi-Ping Su:** Software (equal); Writing – review & editing (equal). **Chui-Ping Yang:** Project administration (lead); Supervision (lead); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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