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3 Quark and Lepton Masses and Mixing From a Gauged $SU(3)_F$ Family Symmetry With a Light $\mathcal{O}(eV)$ Sterile Dirac Neutrino

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Abstract. In the framework of a complete vector-like and universal gauged $SU(3)_F$ family symmetry, we report a global region in the parameter space where this approach can account for a realistic spectrum of quark masses and mixing in a 4×4 non-unitary V_{CKM}, as well as for the known charged lepton masses and the squared neutrino mass differences reported from neutrino oscillation experiments. The $SU(3)_F$ family symmetry is broken spontaneously in two stages by heavy SM singlet scalars, whose hierarchy of scales yield and approximate $SU(2)_F$ global symmetry associated to the almost degenerate boson masses of the order of the lower scale of the $SU(3)_F$ SSB. The gauge symmetry, the fermion content, and the transformation of the scalar fields, all together, avoid Yukawa couplings between SM fermions. Therefore, in this scenario ordinary heavy fermions, top and bottom quarks and tau lepton, become massive at tree level from Dirac See-saw mechanisms, while light fermions, including light neutrinos, obtain masses from radiative corrections mediated by the massive gauge bosons of the $SU(3)_F$ family symmetry. The displayed fit parameter space region solution for fermion masses and mixing yield the vector-like fermion masses: $M_D \approx 3.2\,\text{TeV}$, $M_U \approx 6.9\,\text{TeV}$, $M_E \approx 21.6\,\text{TeV}$, $SU(2)_F$ family gauge boson masses of $\mathcal{O}(2\,\text{TeV})$, and the squared neutrino mass differences: $m_2^2 - m_1^2 \approx 7.5 \times 10^{-5}\ eV^2$, $m_3^2 - m_2^2 \approx 2.2 \times 10^{-3}\ eV^2$, $m_4^2 - m_1^2 \approx 0.81\ eV^2$.

Povzetek. Avtor ponudi razširitev *standardnega modela*, ki k poznanim grupam doda še družinsko (umeritveno) grupo $SU(3)_F$. Poišče območje v prostoru parametrov, ki ponudi eksperimentalno sprejemljive lastnosti kvarkov in leptonov. Družinsko simetrijo zlomi v dveh korakih. Težki fermioni - kvarka t in b ter lepton tau - postanejo masivni na drevesnem nivoju (z mehanizmom gugalnice, see-saw), ostali pa s popravki višjih redov. Mase fermionov, pri katerih so kvantna števila levoročnih in desnoročnih fermionov enaka, so nekaj TeV ali več: $M_D \approx 3.2\,\text{TeV}$, $M_U \approx 6.9\,\text{TeV}$, $M_E \approx 21.6\,\text{TeV}$. Mase bozonov z družinskimi kvantnimi števili so $\mathcal{O}(2\,\text{TeV})$, razliki kvadratov nevtrinskih mass pa so:$m_2^2 - m_1^2 \approx 7.5 \times 10^{-5}\ eV^2$, $m_3^2 - m_2^2 \approx 2.2 \times 10^{-3}\ eV^2$, $m_4^2 - m_1^2 \approx 0.81\ eV^2$.

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3.1 Introduction

The origen of the hierarchy of fermion masses and mixing is one of the most important open problems in particle physics. Any attempt to account for this hierarchy introduce a mass generation mechanism which distinguish among the different Standard Model (SM) quarks and leptons.

After the discovery of the scalar Higgs boson on 2012, LHC has not found a conclusive evidence of new physics. However, there are theoretical motivations to look for new particles in order to answer some open questions like; neutrino ossccillations, dark matter, stability of the Higgs mass against radiative corrections,,etc.

In this article, we address the problem of charged fermion masses and quark mixing within the framework of an extension of the SM introduced by the author in [1]. This Beyond Standard Model (BSM) proposal include a vector gauged $SU(3)_F$ family symmetry[1] commuting with the SM group and introduce a hierarchical massgeneration mechanism in which the light fermions obtain masses through loop radiative corrections, mediated by the massive bosons associated to the $SU(3)_F$ family symmetry that is spontaneously broken, while the masses of the top and bottom quarks as well as for the tau lepton, are generated at tree level from "Dirac See-saw"[3] mechanisms implemented by the introduction of a new set of $SU(2)_L$ weak singlets U, D, E and N vector-like fermions. Due to the fact that these vector-like quarks do not couple to the W boson, the mixing of U and D vector-like quarks with the SM quarks gives rise to and extended 4×4 non-unitary CKM quark mixing matrix [4].

3.2 Model with $SU(3)_F$ flavor symmetry

3.2.1 Fermion content

We define the gauge symmetry group

$$G \equiv SU(3)_F \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \tag{3.1}$$

where $SU(3)_F$ is a completely vector-like and universal gauged family symmetry. That is, the corresponding gauge bosons couple equally to Left and Right Handed ordinary Quarks and Leptons, including right handed neutrinos. $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the "Standard Model" (SM) gauge group, with g_H, g_s, g and g' the coupling constants. The content of fermions assumes the ordinary quarks and leptons assigned under G as:

Ordinary Fermions: $q^o_{iL} = \begin{pmatrix} u^o_{iL} \\ d^o_{iL} \end{pmatrix}$, $l^o_{iL} = \begin{pmatrix} \nu^o_{iL} \\ e^o_{iL} \end{pmatrix}$, $Q = T_{3L} + \frac{1}{2} Y$

$$\Psi^o_q = (3, 3, 2, \frac{1}{3})_L = \begin{pmatrix} q^o_{1L} \\ q^o_{2L} \\ q^o_{3L} \end{pmatrix} \quad , \quad \Psi^o_l = (3, 1, 2, -1)_L = \begin{pmatrix} l^o_{1L} \\ l^o_{2L} \\ l^o_{3L} \end{pmatrix}$$

[1] See [1,2] and references therein for some other $SU(3)_F$ family symmetry model proposals.

$$\Psi_u^o = (3,3,1,\frac{4}{3})_R = \begin{pmatrix} u_R^o \\ c_R^o \\ t_R^o \end{pmatrix} \quad , \quad \Psi_d^o = (3,3,1,-\frac{2}{3})_R = \begin{pmatrix} d_R^o \\ s_R^o \\ b_R^o \end{pmatrix}$$

$$\Psi_e^o = (3,1,1,-2)_R = \begin{pmatrix} e_R^o \\ \mu_R^o \\ \tau_R^o \end{pmatrix}$$

where the last entry corresponds to the hypercharge Y, and the electric charge is defined by $Q = T_{3L} + \frac{1}{2}Y$. The model also includes two types of extra fermions:

Right Handed Neutrinos: $\Psi_{\nu_R}^o = (3,1,1,0)_R = \begin{pmatrix} \nu_{e_R} \\ \nu_{\mu_R} \\ \nu_{\tau_R} \end{pmatrix}$,

and the $SU(2)_L$ weak singlet vector-like fermions

Sterile Neutrinos: $N_L^o, N_R^o = (1,1,1,0)$,

The Vector Like quarks:

$$U_L^o, U_R^o = (1,3,1,\frac{4}{3}) \quad , \quad D_L^o, D_R^o = (1,3,1,-\frac{2}{3}) \tag{3.2}$$

and

The Vector Like electrons: $E_L^o, E_R^o = (1,1,1,-2)$

The transformation of these vector-like fermions allows the mass invariant mass terms

$$M_U\, \bar{U}_L^o\, U_R^o \,+\, M_D\, \bar{D}_L^o\, D_R^o \,+\, M_E\, \bar{E}_L^o\, E_R^o + h.c.\,, \tag{3.3}$$

and

$$m_D\, \bar{N}_L^o\, N_R^o \,+\, m_L\, \bar{N}_L^o\, (N_L^o)^c \,+\, m_R\, \bar{N}_R^o\, (N_R^o)^c \,+\, h.c \tag{3.4}$$

The above fermion content make the model anomaly free. After the definition of the gauge symmetry group and the assignment of the ordinary fermions in the usual form under the standard model group and in the fundamental 3-representation under the $SU(3)_F$ family symmetry, the introduction of the right-handed neutrinos is required to cancel anomalies[5]. The $SU(2)_L$ weak singlets vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions through Dirac See-saw mechanisms. These vector like fermions play a crucial role to implement a hierarchical spectrum for quarks and charged lepton masses, together with the radiative corrections.

3.3 $SU(3)_F$ family symmetry breaking

We need to be consistent with low energy Standard Model (SM) and simultaneously we would like to generate and account for the hierarchy of fermion masses and mixing after spontaneously symmetry breaking (SSB) down to $SU(3)_C \times U(1)_Q$. Previous basic assumptions of this BSM define the required scalar fields and V.E.V's to achieve the desired symmetry breaking.

To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve the SSB of $SU(3)_F$, we introduce the flavon scalar fields: η_i, $i = 2, 3$,

$$\eta_i = (3, 1, 1, 0) = \begin{pmatrix} \eta^o_{i1} \\ \eta^o_{i2} \\ \eta^o_{i3} \end{pmatrix} \quad , \quad i = 2, 3$$

with the "Vacuum ExpectationValues" (VEV's):

$$\langle \eta_2 \rangle^T = (0, \Lambda_2, 0) \quad , \quad \langle \eta_3 \rangle^T = (0, 0, \Lambda_3) \, . \tag{3.5}$$

The above scalar fields and VEV's break completely the $SU(3)_F$ flavor symmetry. The corresponding $SU(3)_F$ gauge bosons are defined in Eq.(3.17) through their couplings to fermions. Thus, the contribution to the horizontal gauge boson masses from Eq.(3.5) read

- $\langle \eta_2 \rangle$: $\frac{g_{H_2}^2 \Lambda_2^2}{2} (Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2}^2 \Lambda_2^2}{4} (Z_1^2 + \frac{Z_2^2}{3} - 2 Z_1 \frac{Z_2}{\sqrt{3}})$
- $\langle \eta_3 \rangle$: $\frac{g_{H_3}^2 \Lambda_3^2}{2} (Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda_3^2 \frac{Z_2^2}{3}$

These two scalars in the fundamental representation is the minimal set of scalars to break down completely the $SU(3)_F$ *family symmetry.*

$$SU(3)_F \times G_{SM} \xrightarrow{\langle \eta_3 \rangle, \langle \eta_2 \rangle} SU(2)_F \, ? \times G_{SM} \xrightarrow{\langle \eta_2 \rangle, \langle \eta_3 \rangle} G_{SM}$$

FCNC ?

$\Lambda_3 (\Lambda_2)$: 5 very heavy boson masses ($\geq 100\,\text{TeV's}$)

$\Lambda_2 (\Lambda_3)$: 3 heavy boson masses (a few TeV's).

Notice that the hierarchy of scales $\Lambda_3 \gg \Lambda_2$ define an "approximate $SU(2)_F$ global symmetry" in the spectrum of $SU(3)_F$ gauge boson masses. To suppress properly the FCNC like, for instance PDG 2016 [9] : $\mu \to e \gamma$ ($Br < 5.7 \times 10^{-13}$), $\mu \to e\, e\, e$ ($Br < 1 \times 10^{-12}$), $K^o - \bar{K}^o$, it is relevant which gauge bosons become massive at the lower scale of the $SU(3)_F$ symmetry breaking.

Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain the gauge boson mass terms.

$$M_2^2 \, Y_1^+ Y_1^- + M_3^2 \, Y_2^+ Y_2^- + (M_2^2 + M_3^2) \, Y_3^+ Y_3^- + \frac{1}{2} M_2^2 \, Z_1^2 + \frac{1}{2} \frac{M_2^2 + 4 M_3^2}{3} \, Z_2^2 - \frac{1}{2} (M_2^2) \frac{2}{\sqrt{3}} \, Z_1 \, Z_2 \tag{3.6}$$

$$M_2^2 = \frac{g_H^2 \Lambda_2^2}{2} \quad , \quad M_3^2 = \frac{g_H^2 \Lambda_3^2}{2} \quad , \quad y \equiv \frac{M_3}{M_2} = \frac{\Lambda_3}{\Lambda_2} \tag{3.7}$$

	Z_1	Z_2
Z_1	M_2^2	$-\frac{M_2^2}{\sqrt{3}}$
Z_2	$-\frac{M_2^2}{\sqrt{3}}$	$\frac{M_2^2+4M_3^2}{3}$

Table 3.1. $Z_1 - Z_2$ mixing mass matrix

Diagonalization of the $Z_1 - Z_2$ squared mass matrix yield the eigenvalues

$$M_-^2 = \frac{2}{3}\left(M_2^2 + M_3^2 - \sqrt{(M_3^2 - M_2^2)^2 + M_2^2 M_3^2}\right)_- \tag{3.8}$$

$$M_+^2 = \frac{2}{3}\left(M_2^2 + M_3^2 + \sqrt{(M_3^2 - M_2^2)^2 + M_2^2 M_3^2}\right)_+ \tag{3.9}$$

$$M_2^2\, Y_1^+ Y_1^- + M_3^2\, Y_2^+ Y_2^- + (M_2^2 + M_3^2)\, Y_3^+ Y_3^- + M_-^2\, \frac{Z_-^2}{2} + M_+^2\, \frac{Z_+^2}{2} \tag{3.10}$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \tag{3.11}$$

$$\cos\phi\ \sin\phi = \frac{\sqrt{3}}{4}\, \frac{M_2^2}{\sqrt{M_2^4 + M_3^2(M_3^2 - M_2^2)}}$$

Due to the $Z_1 - Z_2$ mixing, we diagonalize the propagators involving Z_1 and Z_2 gauge bosons according to Eq.(3.11)

$$Z_1 = \cos\phi\ Z_- - \sin\phi\ Z_+ \quad , \quad Z_2 = \sin\phi\ Z_- + \cos\phi\ Z_+$$

3.4 Electroweak symmetry breaking

Recently ATLAS[6] and CMS[7] at the Large Hadron Collider announced the discovery of a Higgs-like particle, whose properties, couplings to fermions and gauge bosons will determine whether it is the SM Higgs or a member of an extended Higgs sector associated to a BSM theory. For electroweak symmetry breaking we introduction two triplets of $SU(2)_L$ Higgs doublets, namely;

$$\Phi^u = (3,1,2,-1) = \begin{pmatrix} \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_1^u \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_2^u \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_3^u \end{pmatrix} \quad , \qquad \Phi^d = (3,1,2,+1) = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_1^d \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_2^d \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_3^d \end{pmatrix} ,$$

and the VEV?s

$$\Phi^u\rangle = \begin{pmatrix} \langle\Phi_1^u\rangle \\ \langle\Phi_2^u\rangle \\ \langle\Phi_3^u\rangle \end{pmatrix} \quad , \quad \langle\Phi^d\rangle = \begin{pmatrix} \langle\Phi_1^d\rangle \\ \langle\Phi_2^d\rangle \\ \langle\Phi_3^d\rangle \end{pmatrix} ,$$

where

$$\Phi_i^u\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{ui} \\ 0 \end{pmatrix} \quad , \quad \langle\Phi_i^d\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{di} \end{pmatrix} .$$

The contributions from $\langle\Phi^u\rangle$ and $\langle\Phi^d\rangle$ yield the W and Z gauge boson masses and mixing with the $SU(3)_F$ gauge bosons

$$\frac{g^2}{4}\,(v_u^2 + v_d^2)\,W^+W^- + \frac{(g^2 + g'^2)}{8}\,(v_u^2 + v_d^2)\,Z_o^2$$

$$+\tfrac{1}{4}\sqrt{g^2 + g'^2}\,g_H\,Z_o\,\Big[\,(v_{1u}^2 - v_{2u}^2 - v_{1d}^2 + v_{2d}^2)\,Z_1$$

$$+(v_{1u}^2 + v_{2u}^2 - 2v_{3u}^2 - v_{1d}^2 - v_{2d}^2 + 2v_{3d}^2)\,\tfrac{Z_2}{\sqrt{3}}$$

$$+2\,(v_{1u}v_{2u} - v_{1d}v_{2d})\,\tfrac{Y_1^+ + Y_1^-}{\sqrt{2}} + 2\,(v_{1u}v_{3u} - v_{1d}v_{3d})\,\tfrac{Y_2^+ + Y_2^-}{\sqrt{2}}$$

$$+2\,(v_{2u}v_{3u} - v_{2d}v_{3d})\,\tfrac{Y_3^+ + Y_3^-}{\sqrt{2}}\Big]$$

$$+ \text{ tiny contributions to the } SU(3) \text{ gauge boson masses } ,$$

$v_u^2 = v_{1u}^2 + v_{2u}^2 + v_{3u}^2$, $v_d^2 = v_{1d}^2 + v_{2d}^2 + v_{3d}^2$. Hence, if we define as usual $M_W = \frac{1}{2} g v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

$$Y_j^1 = \frac{Y_j^+ + Y_j^-}{\sqrt{2}} \quad , \quad Y_j^{\pm} = \frac{Y_j^1 \mp i Y_j^2}{\sqrt{2}} \tag{3.12}$$

The mixing of Z_o neutral gauge boson with the $SU(3)_F$ gauge bosons modify the couplings of the standard model Z boson with the ordinary quarks and leptons

3.5 Fermion masses

3.5.1 Dirac See-saw mechanisms

Now we describe briefly the procedure to get the masses for fermions. The analysis is presented explicitly for the charged lepton sector, with a completely analogous procedure for the u and d quarks and Dirac neutrinos. With the fields of particles introduced in the model, we may write the gauge invariant Yukawa couplings, as

$$h\,\bar{\psi}_l^o\,\Phi^d\,E_R^o\;+\;h_2\,\bar{\psi}_e^o\,\eta_2\,E_L^o\;+\;h_3\,\bar{\psi}_e^o\,\eta_3\,E_L^o\;+\;M\,\bar{E}_L^o\,E_R^o\;+h.c \tag{3.13}$$

where M is a free mass parameter (because its mass term is gauge invariant) and h, h_2 and h_3 are Yukawa coupling constants. When the involved scalar fields acquire VEV's we get, in the gauge basis ${\psi_{L,R}^o}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms $\bar{\psi}_L^o \mathcal{M}^o \psi_R^o + h.c$, where

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & h\,v_1 \\ 0 & 0 & 0 & h\,v_2 \\ 0 & 0 & 0 & h\,v_3 \\ 0 & h_2\Lambda_2 & h_3\Lambda_3 & M \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & M \end{pmatrix} . \tag{3.14}$$

Notice that $\mathcal{M}^o$ has the same structure of a See-saw mass matrix, here for Dirac fermion masses. So, we call $\mathcal{M}^o$ a **"Dirac See-saw"** mass matrix. $\mathcal{M}^o$ is diagonalized by applying a biunitary transformation $\psi_{L,R}^o = V_{L,R}^o\,\chi_{L,R}$. The orthogonal matrices V_L^o and V_R^o are obtained explicitly in the Appendix A. From V_L^o and V_R^o, and using the relationships defined in this Appendix, one computes

$${V_L^o}^T \mathcal{M}^o\;V_R^o = Diag(0, 0, -\lambda_3, \lambda_4) \tag{3.15}$$

$${V_L^o}^T \mathcal{M}^o {\mathcal{M}^o}^T\;V_L^o = {V_R^o}^T {\mathcal{M}^o}^T \mathcal{M}^o\;V_R^o = Diag(0, 0, \lambda_3^2, \lambda_4^2)\,. \tag{3.16}$$

where λ_3^2 and λ_4^2 are the nonzero eigenvalues defined in Eqs.(3.50-3.51), λ_4 being the fourth heavy fermion mass, and λ_3 of the order of the top, bottom and tau mass for u, d and e fermions, respectively. We see from Eqs.(3.15,3.16) that at tree level the See-saw mechanism yields two massless eigenvalues associated to the light fermions:

3.6 One loop contribution to fermion masses

Subsequently, the masses for the light fermions arise through one loop radiative corrections. After the breakdown of the electroweak symmetry we can construct the generic one loop mass diagram of Fig. 1. Internal fermion line in this diagram represent the Dirac see-saw mechanism implemented by the couplings in Eq.(3.13). The vertices read from the $SU(3)_F$ flavor symmetry interaction Lagrangian

$$i\mathcal{L}_{int} = \frac{g_H}{\sqrt{2}} \left(\bar{e^o}\gamma_\mu \mu^o Y_1^+ + \bar{e^o}\gamma_\mu \tau^o Y_2^+ + \bar{\mu^o}\gamma_\mu \tau^o Y_3^+ + h.c. \right)$$
$$+ \frac{g_H}{2} \left(\bar{e^o}\gamma_\mu e^o - \bar{\mu^o}\gamma_\mu \mu^o \right) Z_1^\mu + \frac{g_H}{2\sqrt{3}} \left(\bar{e^o}\gamma_\mu e^o + \bar{\mu^o}\gamma_\mu \mu^o - 2\bar{\tau^o}\gamma_\mu \tau^o \right) Z_2^\mu \quad (3.17)$$

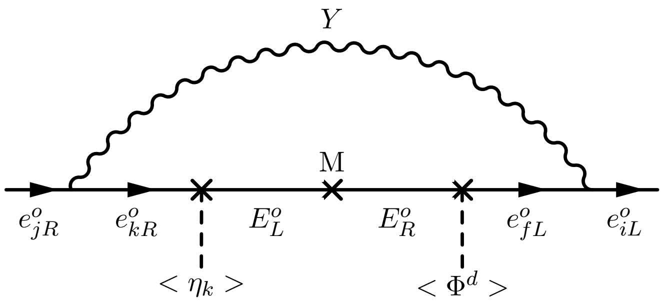

Fig. 3.1. Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

where g_H is the SU(3)$_F$ coupling constant, Z_1, Z_2 and Y_i^j , $i = 1,2,3$, $j = 1,2$ are the eight gauge bosons. The crosses in the internal fermion line mean tree level mixing, and the mass M generated by the Yukawa couplings in Eq.(3.13) after the scalar fields get VEV's. The one loop diagram of Fig. 1 gives the generic contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o\ (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi} \quad (3.18)$$

where M_Y is the gauge boson mass, c_Y is a factor coupling constant, Eq.(3.17), $m_3^o = -\sqrt{\lambda_3^2}$ and $m_4^o = \lambda_4$ are the See-saw mass eigenvalues, Eq.(3.15), and $f(x,y) = \frac{x^2}{x^2-y^2} \ln \frac{x^2}{y^2}$. Using the results of Appendix A, we compute

$$\sum_{k=3,4} m_k^o\ (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i\ b_j\ M}{\lambda_4^2 - \lambda_3^2} F(M_Y)\,, \quad (3.19)$$

$i = 1,2,3$, $j = 2,3$, and $F(M_Y) \equiv \frac{M_Y^2}{M_Y^2-\lambda_4^2} \ln \frac{M_Y^2}{\lambda_4^2} - \frac{M_Y^2}{M_Y^2-\lambda_3^2} \ln \frac{M_Y^2}{\lambda_3^2}$. Adding up all the one loop SU(3)$_F$ gauge boson contributions, we get the mass terms $\bar{\psi}^o_L \mathcal{M}_1^o\ \psi^o_R +$ h.c.,

$$\mathcal{M}_1^o = \begin{pmatrix} D_{11} & D_{12} & D_{13} & 0 \\ 0 & D_{22} & D_{23} & 0 \\ 0 & D_{32} & D_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{\alpha_H}{\pi}\,, \quad (3.20)$$

$$
\begin{aligned}
D_{11} &= \mu_{11}\left(\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12} + F_m\right) + \frac{1}{2}(\mu_{22}F_1 + \mu_{33}F_2) \\
D_{12} &= \mu_{12}\left(-\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12}\right) \\
D_{13} &= -\mu_{13}\left(\frac{F_{Z_2}}{6} + F_m\right) \\
D_{22} &= \mu_{22}\left(\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12} - F_m\right) + \frac{1}{2}(\mu_{11}F_1 + \mu_{33}F_3) \\
D_{23} &= -\mu_{23}\left(\frac{F_{Z_2}}{6} - F_m\right) \\
D_{32} &= -\mu_{32}\left(\frac{F_{Z_2}}{6} - F_m\right) \\
D_{33} &= \mu_{33}\frac{F_{Z_2}}{3} + \frac{1}{2}(\mu_{11}F_2 + \mu_{22}F_3)\,,
\end{aligned}
$$

Here,

$$
F_1 \equiv F(M_{Y_1}) \quad , \quad F_2 \equiv F(M_{Y_2}) \quad , \quad F_3 \equiv F(M_{Y_3})
$$

$$
F_{Z_1} = \cos^2\phi\, F(M_-) + \sin^2\phi\, F(M_+)\ ,\ F_{Z_2} = \sin^2\phi\, F(M_-) + \cos^2\phi\, F(M_+)
$$

$$
M_{Y_1}^2 = M_2^2 \quad , \quad M_{Y_2}^2 = M_3^2 \quad , \quad M_{Y_3}^2 = M_2^2 + M_3^2
$$

$$
F_m = \frac{\cos\phi\sin\phi}{2\sqrt{3}}\,[\,F(M_-) - F(M_+)\,]
$$

with M_2, M_3, M_-, M_+ the horizontal boson masses, Eqs.(3.7-3.9),

$$
\mu_{ij} = \frac{a_i\, b_j\, M}{\lambda_4^2 - \lambda_3^2} = \frac{a_i\, b_j}{a\, b}\,\lambda_3\, c_\alpha\, c_\beta\ , \tag{3.21}
$$

and $c_\alpha \equiv \cos\alpha\ ,\ c_\beta \equiv \cos\beta\ ,\ s_\alpha \equiv \sin\alpha\ ,\ s_\beta \equiv \sin\beta$, as defined in the Appendix, Eq.(3.52). Therefore, up to one loop corrections we obtain the fermion masses

$$
\bar{\psi}_L^o \mathcal{M}^o\, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o\, \psi_R^o = \bar{\chi}_L\, \mathcal{M}\, \chi_R\ , \tag{3.22}
$$

with $\mathcal{M} \equiv \left[Diag(0,0,-\lambda_3,\lambda_4) + V_L^{o\,T} \mathcal{M}_1^o\, V_R^o\right]$.

Using V_L^o, V_R^o from Eqs.(3.45-3.46) we get the mass matrix up to one loop radiative corrections:

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta\, m_{13} & s_\beta\, m_{13} \\ m_{21} & m_{22} & c_\beta\, m_{23} & s_\beta\, m_{23} \\ c_\alpha\, m_{31} & c_\alpha\, m_{32} & (-\lambda_3 + c_\alpha c_\beta\, m_{33}) & c_\alpha s_\beta\, m_{33} \\ s_\alpha\, m_{31} & s_\alpha\, m_{32} & s_\alpha c_\beta\, m_{33} & (\lambda_4 + s_\alpha s_\beta\, m_{33}) \end{pmatrix} , \qquad (3.23)$$

where

$$m_{11} = \delta\, c_1\, \pi_1 \quad , \quad m_{21} = -\delta\, s_1\, s_2\, \pi_1 \quad , \quad m_{31} = \delta\, c_2\, s_1\, \pi_1$$

$$m_{12} = \delta\, s_1\, s_r\, (c_1\, c_2\, c_r\, \Delta + \pi_3)$$

$$m_{13} = -\delta\, s_1\, (c_1\, c_2\, Fm - c_1\, c_2\, s_r^2\, \Delta + c_r\, \pi_3)$$

$$m_{22} = \delta(-3\, c_2\, c_r\, s_2\, s_r\, F_m + c_1^2\, c_2\, c_r\, s_2\, s_r\, \Delta + c_2\, c_r\, \pi_2 + c_1\, s_2\, s_r\, \pi_3)$$

$$m_{23} = \delta(c_2\, s_2\, (1 + s_1^2 - 3s_r^2)\, F_m + c_1^2\, c_2\, s_2\, s_r^2\, \Delta + c_2\, s_r\, \pi_2 - c_1\, c_r\, s_2\, \pi_3)$$

$$m_{32} = \delta(-c_r\, s_r\, (-1 + 3s_2^2)\, F_m + c_r\, (c_2^2 s_1^2 + s_2^2)\, s_r\, \Delta + c_r\, s_2\, \pi_2 \\ -c_1\, c_2\, s_r\, \pi_3)$$

$$m_{33} = \delta(-\tfrac{F_{Z_2}}{6} - (c_2^2\, s_1^2 - s_2^2 - s_r^2 + 3s_2^2\, s_r^2)\, F_m + (c_2^2\, s_1^2 + s_2^2)\, s_r^2\, \Delta \\ +s_2\, s_r\, \pi_2 + c_1\, c_2\, c_r\, \pi_3).$$

$s_1, s_2, s_r, s_\alpha, s_\beta, \lambda_3, \lambda_4$ come from the diagonalization of the tree level mass matrix $\mathcal{M}^o$, Eq. (3.14), are defined in Appendix 3.9.

$$\delta = \frac{\alpha_H}{\pi}\, c_\alpha\, c_\beta\, \lambda_3 \ , \ \ \Delta = \tfrac{1}{4}(F_{Z_2} - F_{Z_1})\, , \, \pi_1 = \frac{1}{2}(c_1\, c_2\, c_r\, F_2 + F_1\, s_2\, s_r)$$

$$\pi_2 = \frac{1}{2}(c_1\, c_2\, c_r\, F_3 + F_{Z_1}\, s_2\, s_r)\ , \ \ \pi_3 = \tfrac{1}{2}(c_1\, c_2\, c_r\, F_{Z_2} + F_3\, s_2\, s_r)$$

The diagonalization of $\mathcal{M}$, Eq.(3.23) gives the physical masses for u and d quarks, e charged leptons and ν Dirac neutrino masses.

Using a new biunitary transformation $\chi_{L,R} = V^{(1)}_{L,R}\ \Psi_{L,R}$; $\bar{\chi}_L\ \mathcal{M}\ \chi_R = \bar{\Psi}_L\ {V^{(1)}_L}^T \mathcal{M}\ V^{(1)}_R\ \Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$V_L^{(1)^T} \mathcal{M}\, \mathcal{M}^T\, V_L^{(1)} = V_R^{(1)^T} \mathcal{M}^T\, \mathcal{M}\, V_R^{(1)} = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\,, \qquad (3.24)$$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. Therefore, the transformation from massless to mass fermions eigenfields in this scenario reads

$$\psi_L^o = V_L^o\, V_L^{(1)}\, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\, V_R^{(1)}\, \Psi_R \qquad (3.25)$$

It is worth to comment here that neutrinos may also obtain left-handed and right-handed Majorana masses both from tree level and radiative corrections.

3.6.1 Quark $(V_{CKM})_{4\times 4}$ and Lepton $(U_{PMNS})_{4\times 8}$ mixing matrices

Within this $SU(3)_F$ family symmetry model, the transformation from massless to physical mass fermion eigenfields for quarks and charged leptons is

$$\psi_L^o = V_L^o\, V_L^{(1)}\, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\, V_R^{(1)}\, \Psi_R\,,$$

Recall now that vector like quarks, Eq.(3.2), are $SU(2)_L$ weak singlets, and hence, they do not couple to *W* boson in the interaction basis. In this way, the interaction of L-handed up and down quarks; $f_{uL}^{o\ T} = (u^o, c^o, t^o)_L$ and $f_{dL}^{o\ T} = (d^o, s^o, b^o)_L$, to the *W* charged gauge boson is

$$\frac{g}{\sqrt{2}}\, \bar{f^o}_{uL} \gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}}\, \bar{\Psi}_{uL}\; [(V_{uL}^o\, V_{uL}^{(1)})_{3\times 4}]^T\; (V_{dL}^o\, V_{dL}^{(1)})_{3\times 4}\; \gamma_\mu \Psi_{dL}\; W^{+\mu}\,, \qquad (3.26)$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times 4} = [(V_{uL}^o\, V_{uL}^{(1)})_{3\times 4}]^T\; (V_{dL}^o\, V_{dL}^{(1)})_{3\times 4} \qquad (3.27)$$

3.7 Numerical results

To illustrate the spectrum of masses and mixing from this scenario, let us consider the following fit of space parameters at the M_Z scale [8]

Using the input values for the $SU(3)_F$ family symmetry:

$$M_2 = 2\,\text{TeV} \quad , \quad M_3 = 2000\,\text{TeV} \quad , \quad \frac{\alpha_H}{\pi} = 0.2 \qquad (3.28)$$

with M_2, M_3 horizontal boson masses, Eq.(3.7), and the coupling constant, respectively, and the tree level mixing angles

$$s_{1d} = s_{1e} = 0.6 \qquad s_{2d} = s_{2e} = 0.1047$$
$$s_{1u} = s_{1\nu} = 0.575341 \qquad s_{2u} = s_{2\nu} = 0.0925127$$

we obtain the following tree level $\mathcal{M}_f^o$, one loop $\mathcal{M}_f$, $f = u, d, e, \nu$ mass matrices, mixing and mass eigenvalues:

3.7.1 Quark masses and $(V_{CKM})_{4\times4}$ mixing

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 108921. \\ 0 & 0 & 0 & 17589.5 \\ 0 & 0 & 0 & 154844. \\ 0 & -6.42288\times10^6 & 462459. & 2.5111\times10^6 \end{pmatrix} \text{MeV}\,, \tag{3.29}$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} 7.19764 & -626.533 & -1479.88 & -3792.15 \\ -0.468392 & -81.7707 & -197.807 & -506.875 \\ 5.04103 & 1502.25 & -172425. & 12057.2 \\ 0.0504129 & 15.0233 & 47.0554 & 6.91226\times10^6 \end{pmatrix} \text{MeV} \tag{3.30}$$

and the u-quark masses

$$(\, m_u\,,\, m_c\,,\, m_t\,,\, M_U\,) = (\, 1.396\,,\, 644.835\,,\, 172438\,,\, 6.912\times10^6\,)\ \text{MeV} \tag{3.31}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 2860.87 \\ 0 & 0 & 0 & 501.98 \\ 0 & 0 & 0 & 3814.49 \\ 0 & -2.3645\times10^6 & 323661. & 2.17117\times10^6 \end{pmatrix} \text{MeV} \tag{3.32}$$

$$\mathcal{M}_d = \begin{pmatrix} -4.22954 & 3.26664 & 26.4239 & 29.045 \\ 19.9418 & -41.6 & -57.7027 & -63.4265 \\ -2.27726 & -31.0285 & -2859.26 & 755.343 \\ -0.002277 & -0.031028 & 0.687179 & 3.2264\times10^6 \end{pmatrix} \text{MeV} \tag{3.33}$$

$$(\, m_d\,,\, m_s\,,\, m_b\,,\, M_D\,) = (\, 2.501\,,\, 45.803\,,\, 2860.14\,,\, 3.226\times10^6\,)\ \text{MeV} \tag{3.34}$$

and the quark mixing

$$(V_{CKM})_{4\times 4} = \begin{pmatrix} -0.97445 & 0.224576 & -0.003514 & -0.000021 \\ -0.224523 & -0.973562 & 0.042015 & -0.000010 \\ 0.006011 & 0.041720 & 0.999041 & -0.001233 \\ -0.000219 & -0.0011268 & -0.011702 & 0.000014 \end{pmatrix} \tag{3.35}$$

3.7.2 Lepton masses and $(U_{PMNS})_{4\times 4}$ mixing:

Charged leptons:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 129165. \\ 0 & 0 & 0 & 22663.9 \\ 0 & 0 & 0 & 172220. \\ 0 & -337398. & 32029.6 & 2.16401 \times 10^7 \end{pmatrix} \text{MeV} \tag{3.36}$$

$$\mathcal{M}_e = \begin{pmatrix} 2.2376 & -66.4545 & -394.792 & -6.18239 \\ -0.175708 & -9.01417 & -57.8818 & -0.906422 \\ 1.66889 & 164.714 & -1693.76 & 26.556 \\ 0.016689 & 1.64723 & 16.9589 & 2.16438 \times 10^7 \end{pmatrix} \text{MeV} \tag{3.37}$$

fit the charged lepton masses:

$$(m_e\,,\, m_\mu\,,\, m_\tau\,,\, M_E) = (0.486\,,\, 102.702\,,\, 1746.17\,,\, 2.164 \times 10^7\,)\ \text{MeV} \tag{3.38}$$

Dirac neutrino masses:

$$\mathcal{M}_\nu^o = \begin{pmatrix} 0 & 0 & 0 & 0.076760 \\ 0 & 0 & 0 & 0.012395 \\ 0 & 0 & 0 & 0.109124 \\ 0 & -0.108392 & -0.264395 & 0.854133 \end{pmatrix} \text{eV} \tag{3.39}$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0.015703 & -0.004190 & 0.009713 & 0.003179 \\ -0.001021 & -0.01824 & -0.005614 & -0.001837 \\ 0.010890 & 0.004245 & -0.048705 & -0.002164 \\ 0.001539 & 0.000600 & -0.000934 & 0.909297 \end{pmatrix} \text{eV} \tag{3.40}$$

fit the light neutrino masses:

$$(m_1\,,\, m_2\,,\, m_3\,,\, m_4) = (\,0.017127\,,\, 0.0192\,,\, 0.050703\,,\, 0.909309\,)\ \text{eV} \tag{3.41}$$

the squared mass differences:

$$m_2^2 - m_1^2 = 0.000075 \text{ eV}^2 \quad , \quad m_3^2 - m_2^2 = 0.00220 \text{ eV}^2 \tag{3.42}$$

and the lepton mixing

$$(U_{PMNS})_{4\times4} = \begin{pmatrix} 0.610887 & -0.786302 & -0.092369 & 0.003816 \\ -0.709911 & -0.595411 & 0.374805 & 0.032066 \\ -0.349473 & -0.164968 & -0.912482 & -0.133926 \\ 0.001821 & 0.000257 & 0.009733 & 0.001377 \end{pmatrix} \tag{3.43}$$

3.8 Conclusions

Within the frame work of a gauged SU(3)$_F$ family symmetry model, we have reported in section 7 a global fit region of the parameter space where this scenario can accommodate a realistic spectrum for the ordinary quark masses and mixing in a non-unitary $(V_{CKM})_{4\times4}$, for the charged lepton masses and the squared neutrino mass differences, within allowed values reported in PDG 2016 [9].

Simultaneously, some of extra particles introduced in this scenario; horizontal gauge bosons and vector-like fermions are predicted to lie within a few TeV's region, and hence, within current LHC energies.

It is worth to comment that the gauge symmetry $G \equiv SU(3)_F \times G_{SM}$, the fermion content, and the transformation of the scalar fields, all together, avoid Yukawa couplings between SM fermions. So, the scalar fields introduced to break the symmetries in the model: η_2, η_3, Φ^u and Φ^d couple ordinary fermions with their corresponding vector-like fermion U, D, E and N, through the tree level Yukawa couplings. Therefore, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.

3.9 Appendix: Diagonalization of the generic Dirac See-saw mass matrix

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & c \end{pmatrix} \tag{3.44}$$

Using a biunitary transformation $\psi_L^o = V_L^o \chi_L$ and $\psi_R^o = V_R^o \chi_R$ to diagonalize $\mathcal{M}^o$, the orthogonal matrices V_L^o and V_R^o may be written explicitly as

$$V_L^o = \begin{pmatrix} c_1 & -s_1 s_2 & s_1 c_2 c_\alpha & s_1 c_2 s_\alpha \\ 0 & c_2 & s_2 c_\alpha & s_2 s_\alpha \\ -s_1 & -c_1 s_2 & c_1 c_2 c_\alpha & c_1 c_2 s_\alpha \\ 0 & 0 & -s_\alpha & c_\alpha \end{pmatrix} \tag{3.45}$$

$$V_R^o = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_r & s_r\, c_\beta & s_r\, s_\beta \\ 0 & -s_r & c_r\, c_\beta & c_r\, s_\beta \\ 0 & 0 & -s_\beta & c_\beta \end{pmatrix} \tag{3.46}$$

$$s_1 = \frac{a_1}{a_n} \quad , \quad c_1 = \frac{a_3}{a_n} \quad , \quad s_2 = \frac{a_2}{a} \quad , \quad c_2 = \frac{a_n}{a} \quad , \quad s_r = \frac{b_2}{b} \quad , \quad c_r = \frac{b_3}{b} \tag{3.47}$$

$$a_n = \sqrt{a_1^2 + a_3^2} \quad , \quad a = \sqrt{a_1^2 + a_2^2 + a_3^2} \quad , \quad b = \sqrt{b_2^2 + b_3^2} \tag{3.48}$$

$$a_1 = a\, s_1\, c_2 \quad , \quad a_2 = a\, s_2 \quad , \quad a_3 = a\, c_1\, c_2 \quad , \quad b_3 = b\, c_r \quad , \quad b_2 = b\, s_r \tag{3.49}$$

$$\lambda_3^2 = \frac{1}{2}\left(B - \sqrt{B^2 - 4D}\right) \quad , \quad \lambda_4^2 = \frac{1}{2}\left(B + \sqrt{B^2 - 4D}\right) \tag{3.50}$$

are the nonzero eigenvalues of $\mathcal{M}^o \mathcal{M}^{oT}$ ($\mathcal{M}^{oT} \mathcal{M}^o$), and

$$B = a^2 + b^2 + c^2 = \lambda_3^2 + \lambda_4^2 \quad , \quad D = a^2 b^2 = \lambda_3^2 \lambda_4^2 \, , \tag{3.51}$$

$$\begin{aligned} \cos\alpha &= \sqrt{\frac{\lambda_4^2 - a^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\alpha = \sqrt{\frac{a^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \, , \\ \cos\beta &= \sqrt{\frac{\lambda_4^2 - b^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\beta = \sqrt{\frac{b^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \, . \end{aligned} \tag{3.52}$$

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