PHENOMENOLOGICAL ANALYSES WITH DUAL FIVE-POINT FUNCTIONS.

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I. INTRODUCTION

Several analyses using the dual five-point function[1] $B_5(X,Y,Z,U,V)$ for the description of reactions with three particles in the final state have recently been carried out [2,5] or are in progress[6]. The main advance in these analyses is that the data are now treated in a dual and crossing symmetric way. Duality in this connection implies that all the data are considered at the same time, i.e., one does not have to separate a definite final state like $K^+p \to K^o\pi^+p$ in subprocesses like

$$\begin{aligned} K^+p \to\ & K^*(890)p \\ & K^{**}(1420)p \\ & K^o\Delta^{++} \end{aligned} \qquad (1)$$

as has been done previously[7]. Crossing symmetry, on the other hand, allows one to use the same matrix element for the description of a reaction and all reactions obtained from it by crossing (apart from the non-dual Pomeron part). Thus from $K^+p \to K^o\pi^+p$ one can go to $K^-p \to \overline{K^o}\pi^+p$, $\pi^+p \to K^+\overline{K^o}p$, $\pi^-p \to K^oK^-p$, $\bar{p}p \to K^+\overline{K^o}\pi^-$, etc. In fact, the use of crossing is quite essential for the construction of the matrix element.

It should be remarked at once, that the use of the corresponding dual four-point function

$$\begin{aligned} B_4(X,Y) &= \Gamma(X)\,\Gamma(Y)\,/\,\Gamma(X+Y) \\ &= \sum_{n=0}^{\infty} \binom{n-Y}{n} \frac{1}{X+n} \end{aligned} \qquad (2)$$

in phenomenology has been quite limited[8]. The reason is simply that $B_4(X,Y)$ contains pure Regge poles and exact local duality[9] and its consequences like exchange degeneracy[10]. On the four-point level the experimental information is so accurate that the violations of the degeneracies following from the duality bootstrap are clearly seen and $B_4(X,Y)$ cannot be used to fit the data at least without unreasonably many satellite terms. In addition, there are also the standard problems of unitarity, Regge cuts, Pomeron, unnatural parity trajectories, duality for baryons and the pion, parity doubling, etc. The same difficulties will ultimately have to be faced also on the five-point level but at present the accuracy of the experiments is so low that the use of B_5 is not ruled out.

The detailed properties of B_5 are well known and exposed in many places[11,12] and its numerical evaluation is treated in [13]. For the understanding of the later results the following expansions are useful (fig.1) :

$$B_5(X,Y,Z,U,V) = \sum_{n=0}^{\infty} \frac{R_n(X,Y;\ Z,V)}{n+U} \tag{3a}$$

$$R_n(X,Y;Z,V) = (-)^n \sum_{k=0}^{n} \binom{Z-1}{n-k}\binom{V-Z-Y}{k} B_4(X+k,Y) = B_4(X,Y),\ \left[1 + \frac{XY-YZ-XV}{X+Y}\right] \times B_4(X,\ Y)\ ,\ \ldots \tag{3b}$$

for $n = 0, 1, \ldots$

where $X = -\alpha(S_{ab})$, $Y = -\alpha(t_{ai})$, etc. ; all particles are scalar, $\alpha(S) = \alpha_o + \alpha' S$, $\alpha_o < 0$. Eq. (3a) shows that B_5 can be expanded as a sum of resonances in the channel S_{23} with correct spin structure (fig.2). By cyclic symmetry, any other channel of resonances may be chosen. However, there are no double poles, i.e., when $U = -m$, $Z = -n$, $m,n = 0, 1, 2 \ldots$ there is only a single pole. On the S_{12} vs S_{23} Dalitz plot the predicted structure is thus schematically that shown in fig.3. The situation is similar to the one found by Lovelace[8] on the $S_{\pi^-\pi^-}$ vs $S_{\pi^+\pi^-}$ Dalitz plot in the analysis of $\bar{p}n \to \pi^+\pi^-\pi^-$.

II. APPLICATION TO $\bar{K}N \to \bar{K}\pi N$.

The channel $\bar{K}N \to \bar{K}\pi N$ is chosen since due to the absence (weakness) of KN resonances dual analyses are simplified. Before B_5 can be applied to this realistic case, several problems have to be solved.

a) The resonances are given a finite width by writing :

$$\alpha(S) = \alpha_o + \alpha' S + i \operatorname{Im} \alpha(S) \qquad (4)$$

where in practice (S_o = threshold)

$$\operatorname{Im} \alpha(S) = a(S - S_o)\, \Theta(S - S_o) \qquad (5)$$

For a resonance at spin $\mathcal{J}$ this gives using

$$\frac{1}{\mathcal{J} - \alpha(S)} = \frac{-1/\alpha'}{S - m_R^2 + i m_R \Gamma_R}$$

that

$$m_R \Gamma_R = \operatorname{Im} \alpha(m_R^2) / \alpha' \qquad (6)$$

With this relation, a in (5) can be estimated. Experimentally, especially for baryon resonances, $m_R \Gamma_R$ increases linearly with m_R^2 so that (5) and (6) are a good approximation. Notice also that the integrated width of a Breit-Wigner is proportional to $1/\Gamma$. Since B_5 by (3a) is resonance dominated one has

$$\int dR_n \, (B_5)^2 \sim \frac{1}{a} \qquad (7)$$

$$dR_n = \prod_1^3 \frac{d^3 p_i}{2 E_i} \, \delta^4(p_a + p_b - p_1 - p_2 - p_3)$$

where a is the resonance parameter in (5) for any of the final state resonances. The fits with B_5 are very sensitive to the resonance widths put in.

b) The general problem of including baryon spin has not been solved as yet; if someone had the non-unique general solution it would be meaningless to use it for analyzing the present data. In general, there are four independent amplitudes[9], e.g. ,

$$T = \bar{U}_3 \left[A + B \not{p}_1 + C \not{p}_2 + D \not{e} \gamma_5 \right] \gamma_5 U_b \qquad (8)$$

where $e_\mu = \delta \varepsilon_{\mu\alpha\beta\gamma} p_a^\alpha \, p_1^\beta \, p_2^\gamma$. The problem, then is to determine which quantum numbers contribute to which amplitudes and to write a B_5

representation for each amplitude with realistic trajectories. As such B_5 describes a model with scalar particles with intercepts $\alpha_o < 0$. No general solution to the problem of including realistic trajectories has been given either. A particular approximate solution, involving essentially the neglect of baryon spins, will be mentioned later.

Consider then the reactions $\bar{K}N \to \bar{K}\pi N$. From the point of view of dynamics involved the different charge states can be divided in three different classes. We choose the following representative reactions.

(I) $K^+p \to K^+\pi^o p$ (vector exchange in $p\bar{p}$, Pomeron in $K\bar{K}$)

(II) $K^+p \to K^o\pi^+ p$ (vector exchange in $p\bar{p}$)

(III) $K^+p \to K^+\pi^+ n$ (pion exchange in $p\bar{n}$, Pomeron in $K\bar{K}$)

This classification is based on the following facts :

a) $\sigma(K^*) \sim p_{lab}^{-2}$; thus there is no Pomeron in $p\bar{p}$. If the Pomeron is a 0^+ object this follows also since the reaction $0^-0^+ \to 0^-0^-$ is forbidden .

b) K^* decay distributions and branching ratios tell that ω dominates in $p\bar{p}$ and π in $p\bar{n}$.

Notice that isospin gives the relation

$$- \sqrt{2}\, a_I = a_{II} + a_{III}$$

between the amplitudes of the reactions above.

In order to construct the diagrams telling which permutations of the particles do not contain exotic channels, it is convenient to go to fig. 3 of ref.[12] in which all the 12 possibilities are given explicitly . If there are 1,2,3 exotic channels, the number of nonvanishing diagrams goes down to 6,4,2 respectively. In classes I and II there are two exotic KN channels (4 diagrams), in III also an exotic $K\pi$ channel (2 diagrams). For class I, we then obtain the diagrams shown in fig.4.

Note.

α) To obtain the physical region of the reaction $K^-p \to K^-\pi^o p$ read the diagrams as $a + b \to 1 + 2 + 3$, for $K^+p \to K^+\pi^o p$ as $\bar{a} + 3 \to \bar{1} + \bar{2} + b$, etc.

β) Under charge conjugation $P \leftrightarrow R$, $Q \leftrightarrow Q$, $S \leftrightarrow S$. This gives the trajectories in R in terms of those in P, and requires the trajectories in Q and S to appear symmetrically. Also, the total amplitude has to be of the form

$$A = \alpha(P + R) + \gamma Q + \delta S$$

γ) Q corresponds to a non-planar duality diagram, indicating that $\gamma = 0$. Better still, P and Q differ by a permutation of K^+ and π^o, so that P + Q has K^* poles at 1, 3, etc., P − Q at 2, 4, etc, P alone at 1, 2, etc. Experimentally both K^* and K^{**} are seen so that $\gamma = 0$.

δ) The trajectories are determined by looking at the channels in which they are produced, if possible. Thus Δ is seen in the $\pi^o p$ mass distribution in $K^{\pm}p \to K^{\pm}\pi^o p$, so Δ trajectory is put in P. Similarly, Λ(1520) is seen in $K^-\pi^o p$, so the exchange degenerate $\Lambda_\alpha - \Lambda_\gamma$ trajectory is put in S and R. However, data do not exclude a substantial contribution of $I = \frac{1}{2}$ and $I = 1$ states in $\pi^o p$ and K^-p mass spectra, but at present there is no way of including simultaneously both N and Δ or both Λ and Y (= exchange degenerate $\Sigma_\beta - \Sigma_\delta$ trajectory) trajectories, unless these are degenerate. Also the K^* and A_2 (in the class II reaction $\pi^-p \to K^-K^o p$) are seen to be produced. This direct evidence lacks only for the $p\bar{p}$ channel, but here the indirect evidence mentioned earlier shows the dominance of ω.

ε) The diagrams P and S differ from each other by a permutation of π^o and p. In the data for $K^+p \to K^+\pi^o p$ only the Δ is seenbut not any 5/2 recurrence. Thus R and S have to appear in the combination R + S which kills this recurrence (though not its daughters). Theoretically, however, the Δ should be degenerate with a N_β– trajectory containing a 5/2 particle. This happens if only R appears.

With these trajectories a solution to the problem of including baryon spins (or neglecting them) with realistic trajectories is as follows (T is given in (8)) :

$$\sum_{\text{spins}} |T|^2 \sim |M^i|^2$$

$$M^i = \varepsilon_{\alpha\beta\gamma\delta} \; p_a^\alpha \, p_b^\beta \, p_1^\gamma \, p_2^\delta * B_5^i \quad , \qquad i = P, R, S \tag{9}$$

where

$$B_5^P = B_5 \left(\tfrac{1}{2} - \alpha_\Lambda(S_{ab}),\ 1 - \alpha_{A_2}(t_{a_1}),\ 1 - \alpha_{K^*}(S_{12}),\ \tfrac{3}{2} - \alpha_\Delta(S_{23}), 1 - \alpha_\omega \ (t_{b3})\right)$$

with similar expressions for B_5^S and B_5^R. For the rather sketchy arguments leading to this result as well as for the detailed forms of the trajectories we refer to ref.[4]. Notice just that the square of the ε-factor in (9) is the Gram determinant $\det(p_i \cdot p_j)$, i, j = a, b, 1, 2, of the momentum vectors of the process and that it vanishes at the boundary of this physical region, e.g., when the three-momentum vectors are parallel. This is typical of spin flip amplitudes and (9) in some sense approximates the total amplitude by a spin flip part. Disagreement with data near forward direction is thus to be expected.

Using (9) and one can then calculate the contributions from the different diagrams using Monte Carlo techniques [15] and the Hopkinson program[13] for evaluating B_5. With three B_5's to evaluate the CERN CDC 6600 produces about 3 000 events (CDC 6 400 about 1000) in a minute depending on the number of distributions wanted. Meaningful results require at least about 10 000 events; thus a big computer is clearly needed if one wants to analyze data with B_5's. Numerically, the integrals over the phase space for the different diagrams have the following values for $K^- p \to K^- \pi^o p$ at 4.57 GeV/c :

$$\int dR_3 \; | B_5^i |^2 = 0.120 \quad \text{for P}$$
$$= 0.062 \quad \text{for R}$$
$$= 0.016 \quad \text{for S}$$

P is thus most important and S, containing many baryon exchanges, is very small. In the total amplitude P + R + S the amplitudes interfere destructively for K^-p and constructively for K^+p, the effect of the interference being about 30 %. As functions of energy the cross-sections (integral over phase space | flux factor/P_{lab}) given by P and R behave | × roughly as $P_{lab}^{-1.6}$, S decreasing slightly faster.

For $K^{\pm}p \to K^{\pm}\pi^o p$ one still has to take into account the effect of the Pomeron from the diagram in fig. 5. One can construct a model for this diagram[16] by writing its amplitude in the form:

$$A_{pom} = G_P \; S_{ab} \; e^{2t_{al}} \; V(S_{23}, t_{b3})$$

where V is a dual model amplitude for pion pomeroproduction : $P + p \to \pi^o + p$. The magnitude of G_P can then be estimated from factorization :

$$\sigma_{as}(K^{\pm}p \to K^{\pm}\pi^o p) = \left[\frac{\sigma_T^{as}(Kp)}{\sigma_T^{as}(pp)}\right]^2 \sigma_{as}(pp \to p\pi^o p)$$

$$= \left(\frac{17}{40}\right)^2 \cdot 0.58 \text{ mb} \approx 0.12 \text{ mb}$$

where the amount 0.58 mb is estimated in ref[12]. By isospin one has similarly

$$\sigma_{as}(K^{\pm}p \to K^{\pm}\pi^+ n) \approx 0.24 \text{ mb}$$

At 10 GeV/c :

$$\sigma(K^- p \to K^- \pi^o p) = 0.28 \text{ mb}^{(17)}$$

so that according to this estimate almost 50 % of the total cross-section is due to Pomeron at 10 GeV/c. At lower energies the cross-section from the above Pomeron model decreases slowly with decreasing P_{lab} while $\sigma B_5 \approx P_{lab}^{-1.6}$ for $P_{lab} > 2$ GeV/c, below which the threshold effects win. Thus, the Pomeron contribution is relatively less at lower energies, about 30 % at 5 GeV/c. Notice that the Pomeron and B_5 contributions do not interfere appreciably since one is spin flip and the other non spin flit.

In the class II reactions the situation is simpler since the

Pomeron cannot contribute. Due to different charge states the diagrams are slightly changed from those given above. Figs. 6-8 taken from ref.[4] show some of the fits obtained to the mass distributions of the reactions

$$K^+p \rightarrow K^o\pi^+p$$

$$K^-p \rightarrow \overline{K^o}\pi^-p$$

$$\pi^-p \rightarrow K^oK^-p$$

After the trajectory functions[4] are fixed by using tabulated resonance parameters there is essentially one parameter, the overall normalization, left. As stated earlier, the fits are very sensitive to the resonance parameters. However, it is quite difficult to assess the significance of the results since in a sense one is getting out just what one puts in, namely the resonances. What is unexpected is that everything comes out in right proportions.

A connection between three and two body reactions has been established in ref.[5] by evaluating the B_5 amplitude on a pole, e.g., going in $K^-p \rightarrow \overline{K^o}\pi^-p$ to the neutron pole in the π^-p system, one relates the normalization for the B_5 amplitude, the cross-section for $K^-p \rightarrow \overline{K^o}n$, and the coupling constant $g_{mp\pi^-}$. Reasonable agreement with experiment is obtained.

For class III reactions one has only the diagrams R and S but the situation is greatly complicated by the presence of the pion trajectory in the $p\bar{p}$ channel in R. The pion trajectory is a low-lying unnatural parity trajectory and the dual nature of these is very obscure [14]. Since even the Pomeron can contribute here the analysis of class III reactions is clearly much more complicated than that of classes I and II.

III. FURTHER PROBLEMS.

Combining duality and internal symmetries is known to lead to degeneracies badly in conflict with experiment (as $m_\pi = m_\eta$). This will happen in B_5 phenomenology also immediately if one wants to relate different reactions with isospin. Consider the classes I, II, III of reactions mentioned earlier. The amplitudes will contain the following contributions :

$$a_I = \text{POMERON} + B_5^I(\omega) + \varepsilon B_5^I(\pi) + \ldots$$

$$a_{II} = B_5^{II}(\omega) + \varepsilon' B_5^{II}(\pi) + \ldots$$

$$a_{III} = \text{POMERON} + B_5^{III}(\pi) + \ldots$$

and it is hard to see how the simple isospin relation $\sqrt{2}\, a_I = a_{II} + a_{III}$ can be satisfied. Difficulties arise also since by putting only one trajectory in one channel one restricts the isospin in this channel. In $KN \to K\pi N$, this restriction is strongest in the $K\pi$ - channel since the exotic $I = \frac{3}{2}$ state is in any case supressed by assumption. To see the consequences, consider the going from $K^+p \to K^o\pi^+p$ to $K^+p \to K^+\pi^o p$ by isospin when the $K\pi$ system has isospin $\frac{1}{2}$ and πN $\frac{3}{2}$. Writing :

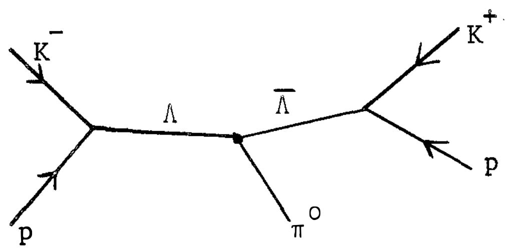
$$A(K^o\pi^+p) = P + R + S$$

$$A(K^+\pi^o p) = \alpha P + \beta(R + S)$$

the combination R + S is imposed by Δ non-exchange degeneracy, one may try to determine α and β by isospin. The fact that $A(K\pi \text{ isospin} = 3/2)=0$ gives $\alpha = -1/\sqrt{2}$ (and no restriction to β since $K^+\pi^o$ isospin is not fixed in R or S) and similarly $A(\pi N \text{ isospin} = 1/2) = 0$ gives $\beta = -\sqrt{2/9}$. Thus

$$A(K^+\pi^o p) = -\frac{1}{\sqrt{2}}\left[P + \frac{2}{3}(R + S)\right]$$

which violates C. A still worse degeneracy is obtained by going in S to the Λ poles :



The central vertex here clearly vanishes by isospin, but it also vanishes if Λ is replaced by the I = 1 Y (since $\langle 10\ 10 \mid 10 \rangle = 0$). Thus isospin forbids the symmetric combinations $\Lambda\Lambda$ and YY and allows only the asymmetric combinations ΛY and YΛ which violate C. The only way to reconcile these two symmetries is to assume that Λ and Y are degenerate. This is strongly in disagreement with experiment since[4] $\alpha_\Lambda(0) \approx -0.68$ and $\alpha_Y(0) \approx -0.22$. Of course, it is also sufficient to assume that Λ is degenerate with the I = 1 Σ_α trajectory, but this couples mainly to Σ_π and not to $\overleftarrow{Kp}$.

The full implications of degeneracies following from the use of B_5 and internal symmetries are not clear as yet but it is obvious that any successful phenomenological analysis has to break them at some stage. The question is just how far one can go.

Concerning the extension of the previous analyses to further processes one may note the following points :

a) In $\pi N \to \pi\pi N$ there are fewer exotic channels, fewer channels with no Pomeron (only $\pi^- p \to \pi^o \pi^o n$) or no (or small) pion exchange contribution. All these difficulties are only partly compensated by better data and the fact that the crossing properties are now particularly simple.

b) In $pN \to N\pi N$ there are many baryons with spin and many baryon trajectories with unclear duality properties. Also the crossed reactions $\pi N \to pN\bar{N}$ or $\bar{p}N \to \pi N\bar{N}$ are more complicated. In any case, it will be difficult to construct a reasonable B_5 amplitude with realistic trajectories.

c) The extension to multiplicity 6 is made difficult by

- large Pomeron and pion contributions
- construction of the model amplitude with realistic properties
- numerical problems

On the other hand, there are very good data for reactions like $Kp \to K\pi\pi p$, etc... .

FIGURE CAPTIONS

Fig.1 The reaction $a + b \to 1 + 2 + 3$ with the variables $X = -\alpha(S_{ab})$, $Y = -\alpha(t_{a1})$, etc.

Fig.2 Approximating $a + b \to 1 + 2 + 3$ by a sum over resonances in the channels 23 or 12.

Fig.3 Predicted structure on the S_{12} vs S_{23} Dalitz plot; the horizontal and vertical lines are the poles at $U = -m$, and $Z = -n$, $n = 0, 1, 2, \ldots$, respectively; the dashed lines are lines of zeroes reducing the double poles to single poles.

Fig.4 The four graphs without exotic channels. All particles are incoming; the trajectories used are indicated between each pair of lines.

Fig.5 A model for Pomeron exchange.

Fig.6 Effective mass distributions for the reaction $\pi^- p \to K^o K^- p$ with predictions from the dual resonance model (solid curves).

Fig.7 As Fig. 6 but for $K^+ p \to K^o \pi^+ p$.

Fig.8 As Fig. 6 but for $K^- p \to \overline{K}^o \pi^- p$.

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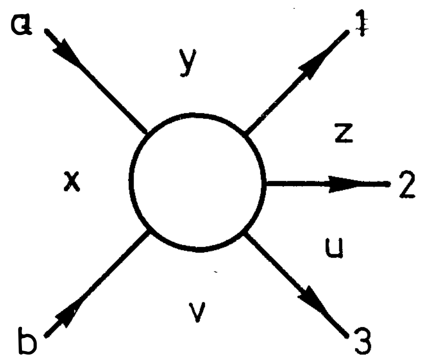


FIG. 1

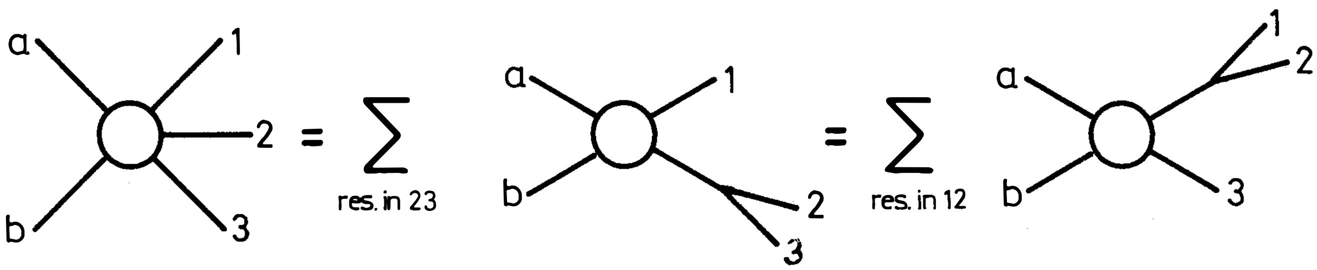


FIG. 2

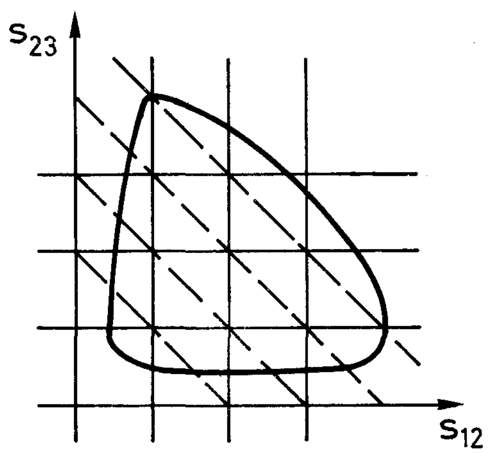


FIG. 3

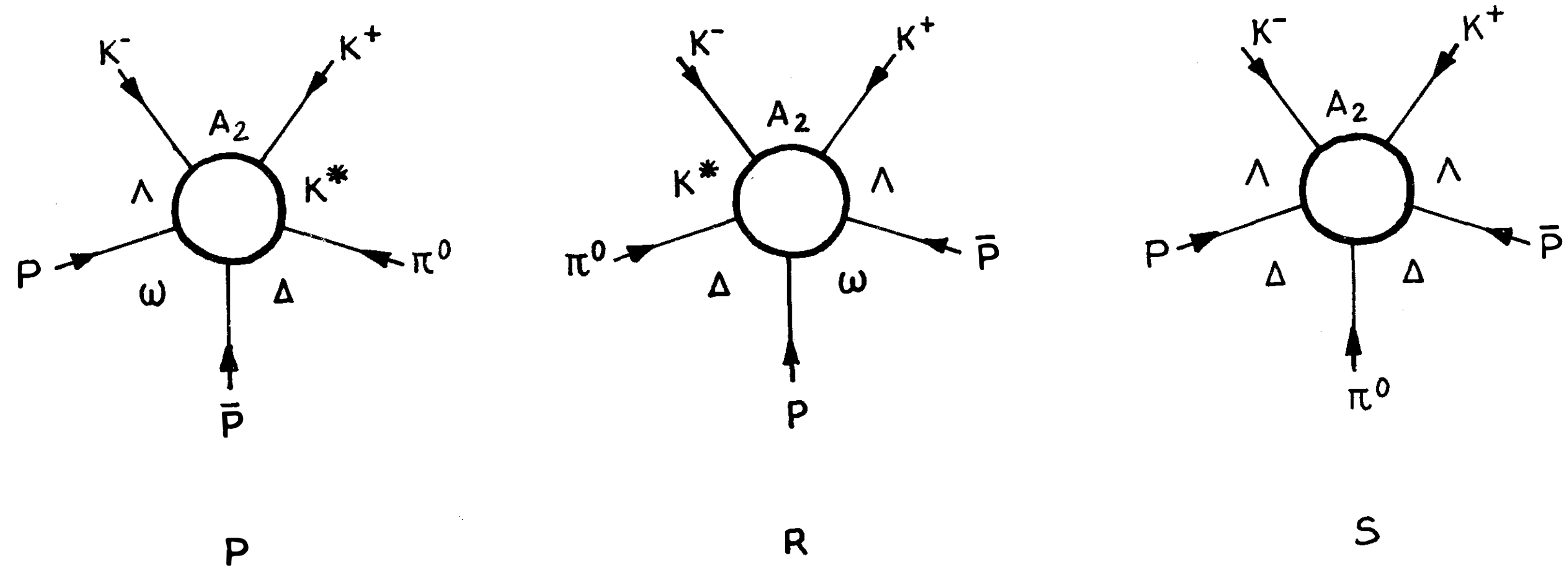


FIG. 4

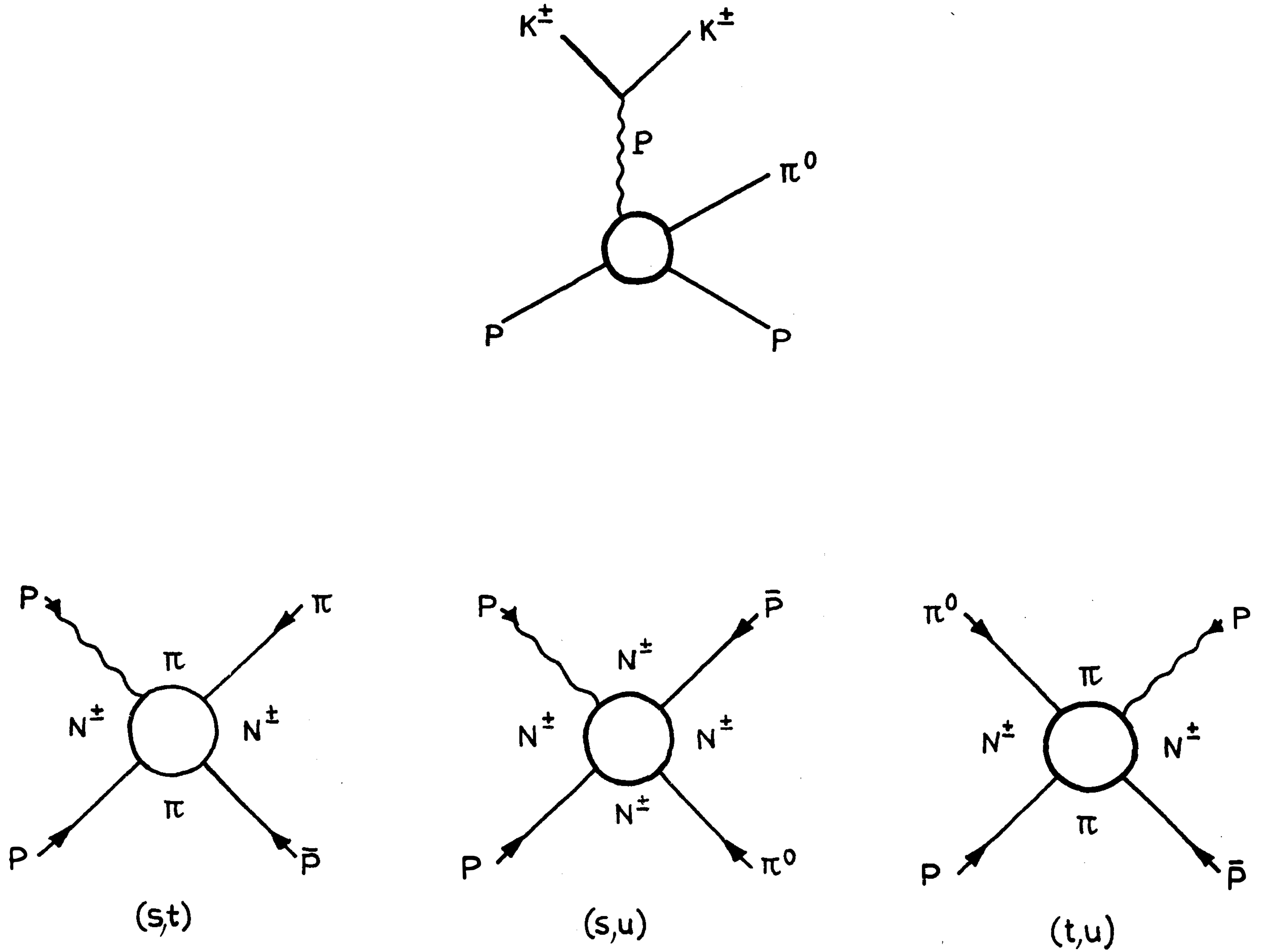


FIG. 5

$\pi^- p \rightarrow K^0 K^- p$

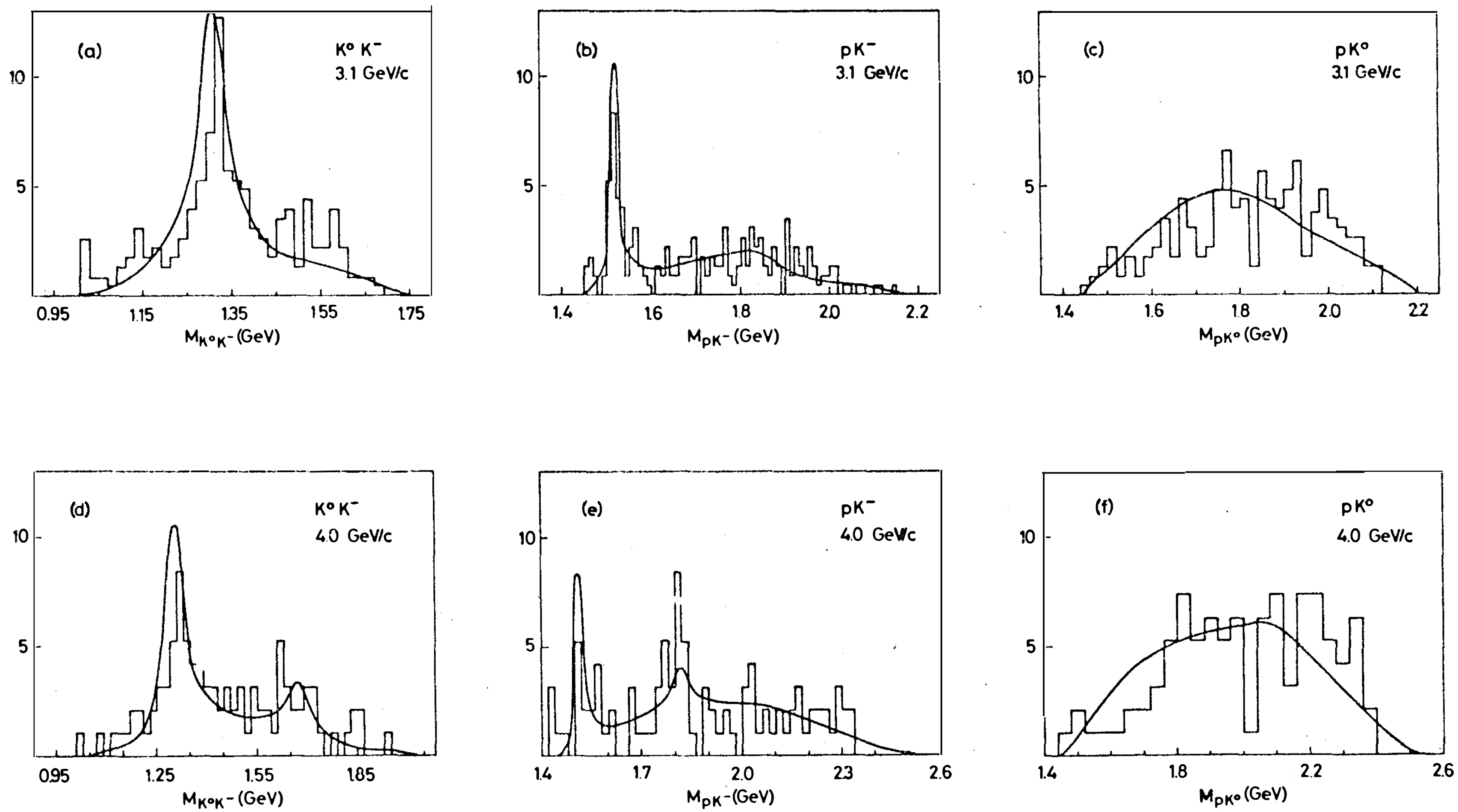
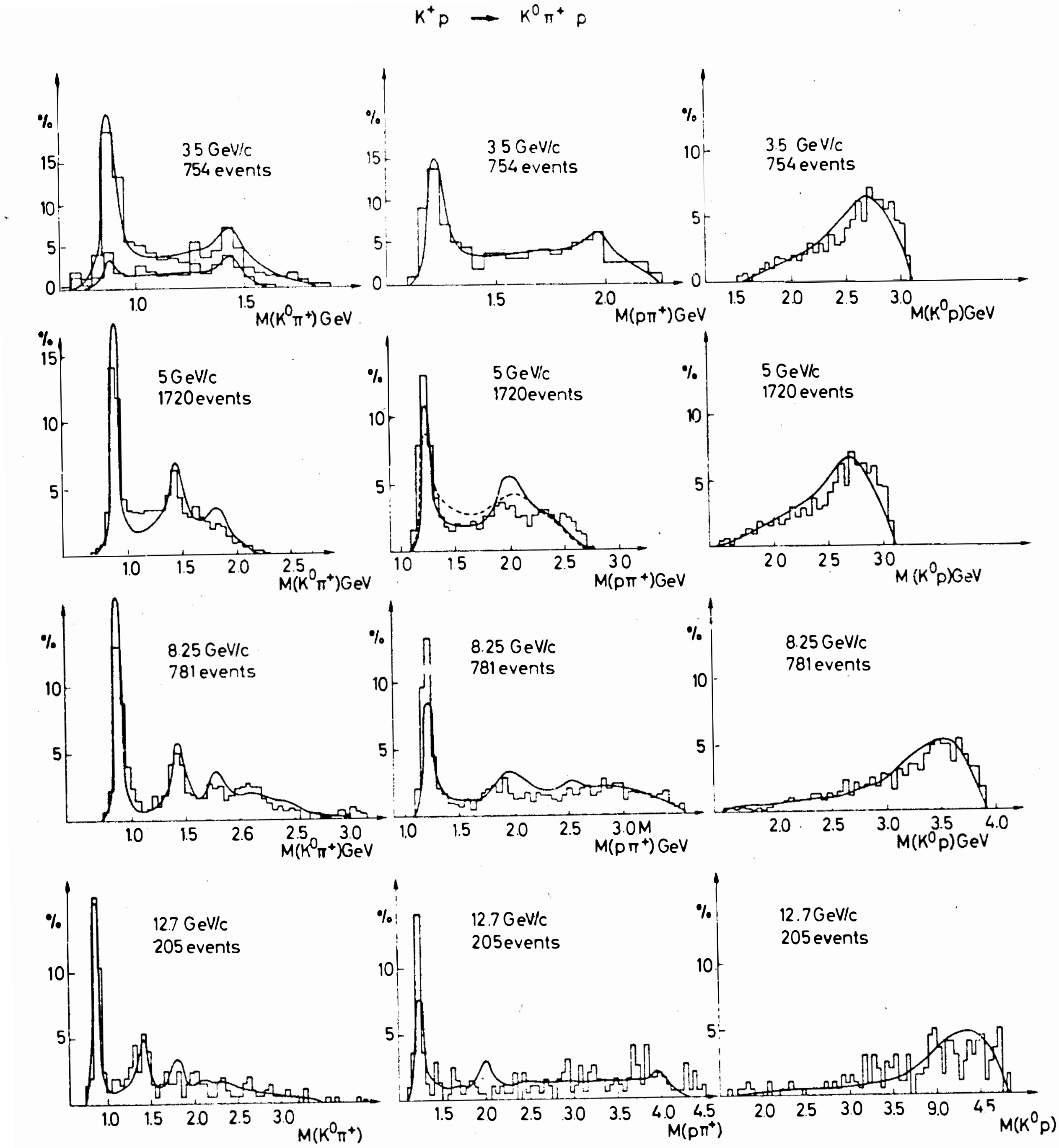
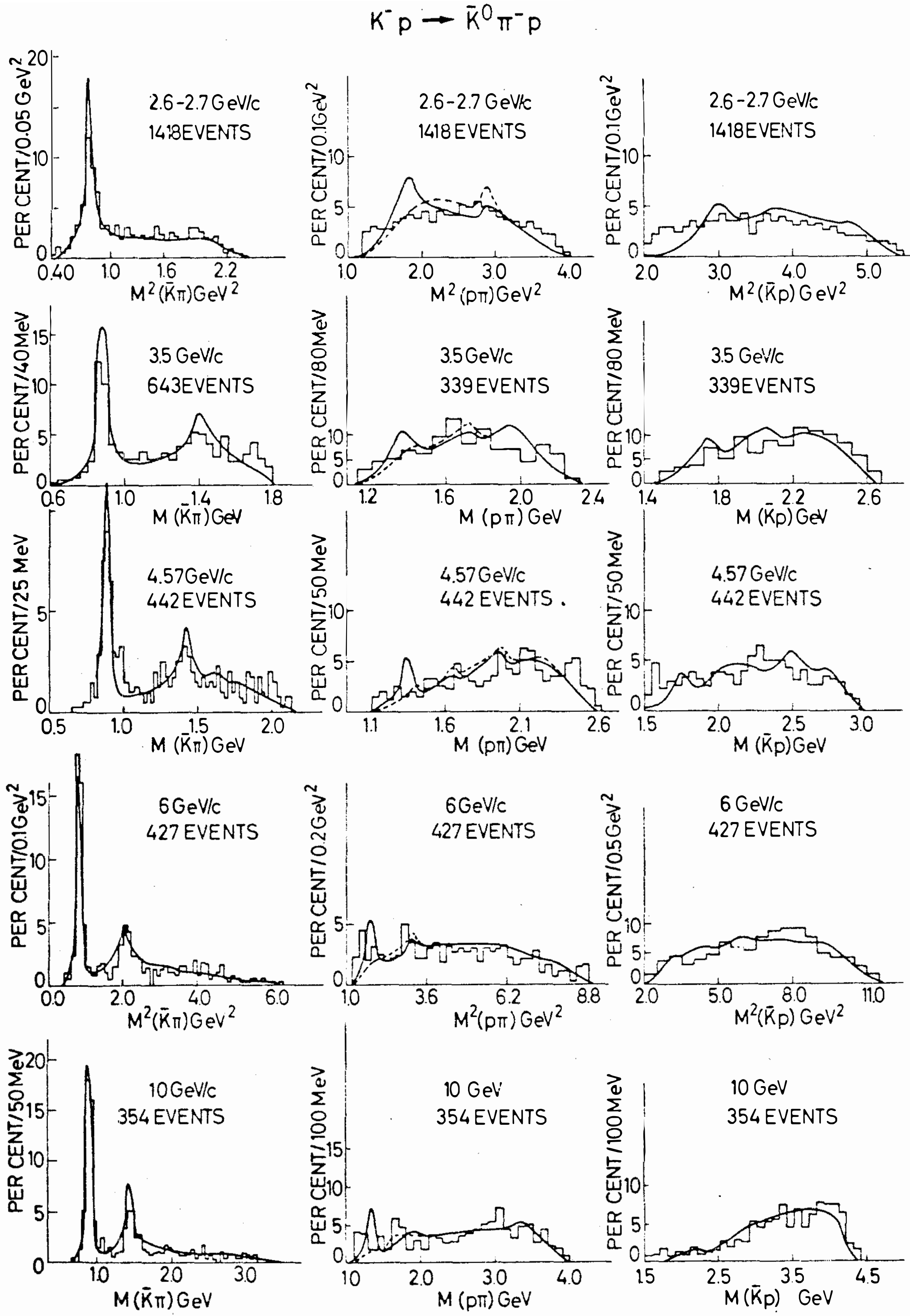


Fig. 6



K⁺p → K⁰π⁺ p
3.5 GeV/c
754 events
M(K⁰π⁺) GeV
M(pπ⁺) GeV
M(K⁰p) GeV
5 GeV/c
1720 events
8.25 GeV/c
781 events
12.7 GeV/c
205 events
M(K⁰π⁺)
M(pπ⁺)
M(K⁰p)

Fig. 7



K⁻p → K̄⁰π⁻p
2.6-2.7 GeV/c
1418 EVENTS
PER CENT/0.05 GeV²
M²(K̄π) GeV²
PER CENT/0.1 GeV²
M²(pπ) GeV²
M²(K̄p) GeV²
3.5 GeV/c
643 EVENTS
339 EVENTS
PER CENT/40 MeV
M (K̄π) GeV
PER CENT/80 MeV
M (pπ) GeV
M (K̄p) GeV
4.57 GeV/c
442 EVENTS
PER CENT/25 MeV
PER CENT/50 MeV
6 GeV/c
427 EVENTS
PER CENT/0.1 GeV²
PER CENT/0.2 GeV²
PER CENT/0.5 GeV²
10 GeV/c
354 EVENTS
10 GeV
PER CENT/50 MeV
PER CENT/100 MeV

Fig. 8