Triply charged monopole and magnetic quarks

George Lazarides [a],*, Qaisar Shafi [b]

[a] *School of Electrical and Computer Engineering, Faculty of Engineering, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece*
[b] *Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, USA*

A B S T R A C T

We describe the internal composition of a topologically stable monopole carrying a magnetic charge of $6\pi/e$ that arises from the spontaneous breaking of the trinification symmetry $SU(3)_c \times SU(3)_L \times SU(3)_R$ (G). Since this monopole carries no color magnetic charge, a charge of $6\pi/e$ is required by the Dirac quantization condition. The breaking of G to the Standard Model occurs in a number of steps and yields the desired topologically stable monopole ("magnetic baryon"), consisting of three confined monopoles. The confined monopoles ("magnetic quarks") each carry a combination of Coulomb magnetic flux and magnetic flux tubes, and therefore they do not exist as isolated states. We also display a more elaborate configuration ("fang necklace") composed of these magnetic quarks. In contrast to the $SU(5)$ monopole which is superheavy and carries a magnetic charge of $2\pi/e$ as well as color magnetic charge, the trinification monopole may have mass in the TeV range, in which case it may be accessible at the LHC and its planned upgrades.

© 2021 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP³.

The Magnetic Monopole Ninety Years Later

1. Introduction

Grand unification theories (GUTs) based on a single gauge coupling such as $SU(5)$ [1] predict the existence of a topologically stable magnetic monopole which carries one quantum $(2\pi/e)$ of Dirac magnetic charge [2,3]. In contrast to the 't Hooft-Polyakov monopole [4], the $SU(5)$ monopole also carries an appropriate amount of color magnetic flux that is screened because of color electric confinement.

Unification models based on product groups such as $SU(4)_c \times SU(2)_L \times SU(2)_R$ [5] predict the existence of a topologically stable monopole that carries two quanta $(4\pi/e)$ of magnetic charge [6]. One straightforward way to see this is by noting that the underlying group allows the existence of color singlet states that carry electric charges $\pm e/2$ and colored triplets with charges $\pm e/6$. A more explicit realization of this doubly charged monopole was demonstrated in Ref. [7], where it was shown to arise from the merger of two distinct ("confined") monopoles, with each one carrying some Coulomb flux and a magnetic flux tube. This demonstration also reveals the existence of "magnetic dumbbells" in a variety of unified theories.

Very interestingly, following Ref. [7], Volovik has shown [8] how topological structures similar to the doubly charged construction in Ref. [7] may arise in superfluid ³He. Furthermore, the existence of a class of topological structures called "walls bounded by strings" [9] was verified in experiments with superfluid ³He [10]. Motivated by these recent developments and especially the interplay between topological structures in high energy and condensed matter physics, we explore some interesting topological structures that arise in the framework of the trinification gauge symmetry $G = SU(3)_c \times SU(3)_L \times SU(3)_R$ [7,11]. In contrast to $SU(5)$ and $SU(4)_c \times SU(2)_L \times SU(2)_R$, the topologically stable monopole in the trinification model is purely electromagnetic in nature with no color magnetic field accompanying it. It carries three quanta of magnetic charge $(6\pi/e)$ in order to satisfy the Dirac quantization condition, and its mass may be light enough to make it accessible in high energy colliders. To identify the variety of topological substructures potentially associated with this monopole, we assume that the trinification symmetry breaking to the Standard Model (SM) proceeds through a series of steps. This deconstruction procedure allows us to identify the building blocks that make up the triply charged monopole. The latter, it turns out, consists of three distinct constituent monopoles which are bound together by flux tubes. We may thus refer to the triply charged monopole as a "magnetic baryon," and its confined constituent components as "magnetic quarks." It is clear that other bound states such as "magnetic mesons" are also present in this trinification model. We

display an example of a somewhat more elaborate topological configuration referred to as a "fang necklace."

2. Triply charged monopole

The trinification symmetry G is a well known subgroup of E_6 [12], and a variety of topological structures that arise when the latter breaks to the SM have been discussed in Ref. [7]. In this paper we do not insist on this relationship between the two groups, which allows us to contemplate the spontaneous breaking of G at scales lying in the TeV range. Because G implements electric charge conservation, its spontaneous breaking to the SM and subsequently to $SU(3)_c \times U(1)_{em}$ yields a topologically stable magnetic monopole that carries three quanta of Dirac magnetic charge, namely $6\pi/e$ [11].

Recall that in the presence of fractionally charged quarks, say d or s, one naively expects the magnetic monopole to carry this amount $(6\pi/e)$ of magnetic charge from the Dirac quantization condition ($qg/4\pi = n/2$, where q, g denote the electric and magnetic charges respectively and n is an integer) [13]. However, the topologically stable magnetic monopole in $SU(5)$ carries just a single quantum $(2\pi/e)$ of magnetic charge. This is compatible with the Dirac quantization condition because the monopole also carries an appropriate amount of color magnetic charge [3]. In the trinification case this is not the case and so the magnetic charge carried by the monopole is $6\pi/e$. A simple way to see this is to note that G allows, in principle, color singlet states in the representations $(\mathbf{1}, \mathbf{3}, \mathbf{1})$ + h.c., which carry electric charge $\pm 1/3$, and therefore the magnetic monopole must carry a magnetic charge of $6\pi/e$. (Fractionally charged color singlet states accompanied by multiply charged monopoles also appear in string theories [14].) Recall that the observed quarks and leptons reside in bifundamental representations of G such as $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$, etc. The discussion regarding the monopole charge is a bit more subtle if G is embedded in E_6 but the outcome remains intact [7,11]. The monopole is topologically stable because the second homotopy group of the vacuum manifold $\pi_2(G/H) = \mathbb{Z} = \{n = 0, \pm 1, \pm 2, \pm 3, ...\}$, with G being the trinification group and $H = SU(3)_c \times U(1)_{em}$.

We now turn to the breaking of G to $SU(3)_c \times SU(2)_L \times U(1)_{Y_L} \times SU(2)_R \times U(1)_{Y_R}$ at an intermediate scale, which can approach the TeV scale if desired. This is achieved by the vacuum expectation values (VEVs) of the $(\mathbf{1,8,1})$ and $(\mathbf{1,1,8})$ components of a Higgs $\mathbf{78}$-plet under E_6. (It is sometimes convenient to follow the E_6 notation.) Recall that the $SU(3)_{L(R)}$ octet under the $SU(2)_{L(R)} \times U(1)_{Y_L(Y_R)}$ subgroup is decomposed as follows $\mathbf{8} = \mathbf{1}_0 + \mathbf{3}_0 + \mathbf{2}_3 + \mathbf{2}_{-3}$, where the subscripts denote the charges with respect to the generator $T^8_{L(R)} \equiv \mathrm{diag}(1, 1, -2)$ of $U(1)_{Y_L(Y_R)}$. We can further break $SU(2)_R$ to $U(1)_R$ by a VEV along the $\mathbf{3}_0$ component of the $SU(3)_R$ octet. The generator of $U(1)_R$ is $T^3_R = \mathrm{diag}(1, -1)$. The unbroken subgroup is then $SU(3)_c \times SU(2)_L \times U(1)_{Y_L} \times U(1)_R \times U(1)_{Y_R}$.

To be a bit more explicit, the potential for the breaking of $SU(3)_R$ to $SU(2)_R \times U(1)_{Y_R}$, assuming a discrete symmetry $\phi \to -\phi$, with ϕ being the scalar octet, is given by

$$V = -\frac{1}{2}m^2 \mathrm{Tr}\phi^2 + \frac{a}{4}\mathrm{Tr}(\phi^2)^2 + \frac{b}{2}\mathrm{Tr}\phi^4, \tag{1}$$

where m is a mass parameter and a, b dimensionless parameters. The 3×3 matrix ϕ^j_i can be diagonalized by an $SU(3)_R$ rotation, and for suitable choices of a and b, ϕ acquires a VEV

$$\langle \phi \rangle \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \tag{2}$$

which breaks $SU(3)_R$ to $SU(2)_R \times U(1)_{Y_R}$. With a second scalar octet, it is then straightforward to break $SU(2)_R$ to $U(1)_R$. More details will not be provided here.

At this stage, we have the generation of three types of intermediate scale magnetic monopoles. Two of them result from the breaking of $SU(3)_L$ and $SU(3)_R$ to $SU(2)_L \times U(1)_{Y_L}$ and $SU(2)_R \times U(1)_{Y_R}$ and carry one unit of Coulomb magnetic flux along the generators $T^3_L/2 + T^8_L/2$ and $T^3_R/2 + T^8_R/2$ respectively, where $T^3_{L(R)} \equiv \mathrm{diag}(1, -1)$. This is because the $(-1, -1) \in SU(2)_{L(R)} \times U(1)_{Y_L(Y_R)}$ coincides with the identity element as it leaves all the representations of $SU(3)_{L(R)}$ unchanged. Consequently, a rotation by 2π along the generator $T^3_{L(R)}/2 + T^8_{L(R)}/2$, which interpolates between (1,1) and (-1,-1), is a closed loop generating the second homotopy group $\pi_2(SU(3)_{L(R)}/SU(2)_{L(R)} \times U(1)_{Y_L(Y_R)}) = \pi_1(SU(2)_{L(R)} \times U(1)_{Y_L(Y_R)}) = \mathbb{Z}$ of the vacuum manifold. The breaking of $SU(2)_R$ to $U(1)_R$ generates a third monopole which carries one unit of T^3_R magnetic flux corresponding to a 2π rotation along this generator.

We should further break $U(1)_{Y_L} \times U(1)_R \times U(1)_{Y_R}$ to $U(1)_Y$, where $Y = T^3_R/2 + (T^8_L + T^8_R)/6$ is the weak hypercharge. (The electric charge operator is given by $Q = T^3_L/2 + Y$.) First consider the breaking of $U(1)_{Y_L} \times U(1)_{Y_R}$ to $U(1)_{B-L}$, where $B - L = (T^8_L + T^8_R)/3$ is the baryon minus lepton number. This symmetry breaking is achieved by a Higgs field in the fundamental representation of E_6,

$$\mathbf{27} = (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) + (\mathbf{3}, \mathbf{3}, \mathbf{1}) + (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) \equiv \lambda + \mathbb{Q} + \mathbb{Q}^c, \tag{3}$$

where

$$\lambda = \begin{pmatrix} h_u & e^c \\ h_d & \nu^c \\ l & N \end{pmatrix} \tag{4}$$

with the rows being $\bar{\mathbf{3}}$'s of $SU(3)_L$ and the columns $\mathbf{3}$'s of $SU(3)_R$, and

$$\mathbb{Q} = \begin{pmatrix} q \\ g \end{pmatrix} \quad \text{and} \quad \mathbb{Q}^c = (u^c, \quad d^c, \quad g^c) \tag{5}$$

denote an $SU(3)_L$ triplet and an $SU(3)_R$ antitriplet respectively. For simplicity, we use here for the various components of the Higgs $\mathbf{27}$-plet the same symbols as for the corresponding components of the fermion $\mathbf{27}$-plets which contain the ordinary quarks and leptons. The reader should keep this in mind to avoid any confusion. The Higgs $\mathbf{27}$-plet acquires a VEV along its N component which is an $SU(3)_c \times SU(2)_L \times SU(2)_R$ singlet and has $T^8_L = 2$, $T^8_R = -2$. Consequently, the generator $T^8_L + T^8_R = 3(B - L)$ remains unbroken [7]. A rotation by $2\pi/4$ along the orthogonal broken generator

$$\mathcal{B} \equiv T^8_L - T^8_R \tag{6}$$

leaves the VEV of N invariant. Consequently, the cosmic string generated by the breaking of $U(1)_\mathcal{B}$ is a tube with magnetic flux corresponding to this rotation, namely it carries magnetic flux $(T^8_L - T^8_R)/4$.

We next consider the breaking $U(1)_R \times U(1)_{B-L}$ to $U(1)_Y$, where $Y = T^3_R/2 + (B-L)/2$ is the SM weak hypercharge, by a VEV along the ν^c component of the Higgs $\mathbf{27}$-plet which has $T^3_R = -1$ and $B - L = 1$. The normalized generators corresponding to T^3_R and $(B - L)$ are $T^3_R/2$ and $\sqrt{3/8}(B - L)$ and, thus, the orthogonal broken generator is

$$2T_R^3 - 3(B - L). \tag{7}$$

This generator is unbroken by the VEV of N, but breaks by the VEV of ν^c. However, the charges of ν^c imply that a rotation by $2\pi/5$ along this generator remains unbroken and the associated string carries magnetic flux

$$\frac{2}{5}T_R^3 - \frac{3}{5}(B - L). \tag{8}$$

Revisiting the tube with magnetic flux $(T_L^8 - T_R^8)/4$, we see that as we go around it the VEV of ν^c acquires a factor $\exp(2i\pi/4)$ since its relevant charges are $T_L^8 = 2$, $T_R^8 = 1$. To cancel this factor, we should add along the tube an additional magnetic flux $(1/4)\{2T_R^3/5 - 3(B - L)/5\}$ so that ν^c acquires an extra factor $\exp(-2i\pi/4)$. This additional flux does not affect the VEV of N since its relevant charges are $T_R^3 = 0$, $B - L = 0$. In conclusion, we obtain a tube with a combined magnetic flux

$$\frac{1}{4}(T_L^8 - T_R^8) + \frac{1}{4}\{\frac{2}{5}T_R^3 - \frac{3}{5}(B - L)\}. \tag{9}$$

In Ref. [7], it has been shown that the only intermediate scale topological defect which survives in this model, where the symmetry breaking employs the ν^c component of a Higgs **27**-plet rather than the $\nu^c\nu^c$ component of a Higgs $\overline{\textbf{351}'}$, is a triply charged $(6\pi/e)$ magnetic monopole. Therefore, one expects that the three types of intermediate scale monopoles and the two types of magnetic flux tubes mentioned above must combine to generate this monopole. Indeed, when the trinification group is broken to the SM gauge group, the magnetic flux $T_R^3/2 + T_R^8/2$ emerging from the $SU(3)_R$ monopole splits into two parts, one equal to minus the flux in Eq. (9) which forms a tube and one Coulomb flux equal to $6Y/5$. Similarly, the magnetic flux T_R^3 of the $SU(2)_R$ monopole forms a tube with flux given in Eq. (8) and a Coulomb magnetic field with flux $6Y/5$. This tube is absorbed by an $SU(3)_L$ monopole with flux $T_L^3/2 + T_L^8/2$, which also emits the tube with magnetic flux as in Eq. (9) terminating on the $SU(3)_R$ monopole. The remaining magnetic flux $T_L^3/2 + 3Y/5$ forms a Coulomb magnetic field emerging from the $SU(3)_L$ monopole. At this point, it is convenient – for reason to become apparent in the next paragraph – to add to the Coulomb fields of the $SU(3)_R$ and the $SU(2)_R$ monopoles and subtract from the magnetic field of the $SU(3)_L$ monopole a magnetic flux T_L^3. This is legitimate since a rotation by 2π around T_L^3 is homotopically trivial. The sum of the Coulomb magnetic fluxes emerging from the three monopoles is then

$$\frac{3}{2}T_L^3 + 3Y = 3Q, \tag{10}$$

where Q is the electric charge operator. Consequently, the three constituent magnetic monopoles (magnetic quarks) are pulled together by the strings to create a triply charged $(6\pi/e)$ magnetic monopole.

Next we consider the effect of the electroweak symmetry breaking on the two tubes with magnetic fluxes given in Eqs. (8) and (9). The relevant charges of the VEVs $\langle h_u \rangle$ and $\langle h_d \rangle$ of the electroweak doublets h_u and h_d, which couple to the up-type and down-type quarks, are $T_L^3 = -1$, $T_R^3 = 1$, $T_L^8 = -1$, $T_R^8 = 1$, and $T_L^3 = 1$, $T_R^3 = -1$, $T_L^8 = -1$, $T_R^8 = 1$, respectively. Consequently, as we go around the string with magnetic flux as in Eq. (9), the phase of $\langle h_u \rangle$ changes by $(-2/5)2\pi$ and that of $\langle h_d \rangle$ by $(-3/5)2\pi$. The tube must then acquire an extra magnetic flux $-2T_L^3/5$ so that the phase of $\langle h_d \rangle$ changes by -2π and $\langle h_u \rangle$ remains constant around the string. Similarly, as we go around the string with magnetic flux as in Eq. (8), the phases of $\langle h_u \rangle$ and $\langle h_d \rangle$ change by $(2/5)2\pi$ and $(-2/5)2\pi$ respectively. Thus, we must add an extra magnetic flux
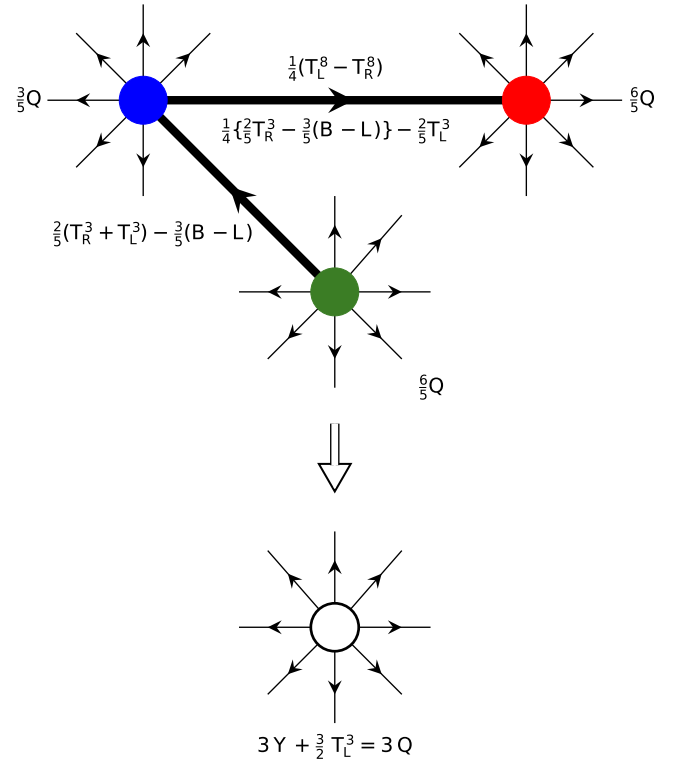

Fig. 1. Emergence of the topologically stable triply charged monopole from the symmetry breaking $G \to SU(3)_c \times SU(2)_L \times U(1)_{Y_L} \times U(1)_{Y_R} \times U(1)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{em}$. An $SU(2)_R$ (green) monopole is connected by a flux tube to an $SU(3)_L$ (blue) monopole which, in turn, is connected to an $SU(3)_R$ (red) monopole by a superconducting flux tube. The constituent monopoles are pulled together to form the triply charged monopole. The fluxes along the tubes and around the monopoles are indicated.

$2T_L^3/5$ along this tube so that both $\langle h_u \rangle$ and $\langle h_d \rangle$ remain constant around the string. This choice is energetically favored since it minimizes the magnetic energy along the strings – see Ref. [7]. The Coulomb magnetic fluxes emerging from the $SU(3)_R$ and $SU(2)_R$ monopoles are $(6/5)(T_L^3/2 + Y) = 6Q/5$ each, and from the $SU(3)_L$ monopole this flux is equal to $(3/5)(T_L^3/2 + Y) = 3Q/5$, in total $3Q$ – see Fig. 1.

The Coulomb magnetic charges accompanying the $SU(3)_R$, $SU(3)_L$, and $SU(2)_R$ constituent magnetic monopoles are, respectively, $(6/5)2\pi/e$, $(3/5)2\pi/e$, and $(6/5)2\pi/e$. These magnetic charges, by construction, are compatible with the Dirac quantization condition because of their accompanying magnetic flux tubes. (Magnetic monopoles carrying a mixture of Coulomb magnetic flux and Z-magnetic flux have been considered in the past [6,15]. For a recent discussion see Refs. [7,16].)

Clearly, each of the three types of constituent magnetic monopoles (magnetic quarks) can alternatively be connected to its own magnetic antiquark by the appropriate flux tube(s) to produce a magnetic meson in the case of the $SU(2)_R$ and $SU(3)_R$ monopoles with a single flux tube connecting it to its antimonopole, or a new type of magnetic mesons in the case of the $SU(3)_L$ magnetic quark with two flux tubes connecting it to its magnetic antiquark. In all three cases, the magnetic quarks and antiquarks eventually annihilate by being pulled together.

Let us briefly discuss the mass of the triply charged magnetic monopole. This mass depends, of course, on the breaking scale M of the trinification symmetry. Since the latter is not a grand unified theory without additional assumptions such as gauge coupling constant unification, there is nothing, in principle, that prevents the scale M to lie in the TeV range, in which case the magnetic monopole mass also is of order M or somewhat larger. This would

make the topologically stable trinification monopole accessible at the LHC [17] and its planned upgrades. For completeness, let us note that the size of the core of each magnetic monopole is determined by gM^{-1}, where g and M denote the relevant gauge coupling constant and symmetry breaking scale. Also, the mass per unit length of the magnetic flux tubes is of order μ^2, with μ being the corresponding symmetry breaking scale. These flux tubes are practically stable with a relatively small hierarchy between M and μ.

Finally, some remarks regarding the observability of this topologically stable triply charged monopole at the LHC are in order here. It has been recognized for quite some time now that the production cross section of a composite coherent quantum state such as this monopole is expected to be exponentially suppressed in Drell-Yan processes involving elementary particles – for a recent review and additional references, see Ref. [18]. This is somewhat analogous to the exponential suppression encountered in tunneling phenomena in quantum mechanics. This suppression of monopole production in Drell-Yan production does not depend on whether the semi-classical monopole solution is spherically symmetric or not. More recently, it has been suggested that this challenge may be overcome at colliders by exploiting the magnetic analogue of the Schwinger mechanism. In the presence of adequately strong magnetic fields the (dual) Schwinger mechanism may lead to an observable cross section for monopole pair production in heavy ion collisions – for a recent discussion and additional references, see Ref. [19]. It is fair to state that the production mechanisms in colliders of more complex topological structures such as necklaces requires additional studies well beyond the scope of this paper.

3. Strings and necklaces

Around the string that connects the $SU(3)_L$ and $SU(3)_R$ monopoles, $\langle h_u \rangle$ remains constant implying that there are no transverse zero modes in the up-type quark sector. However, the phases of $\langle h_d \rangle$ and $\langle N \rangle$ change by -2π and 2π respectively. The masses of the down-type quarks can be written as

$$\mathcal{M}_d = \begin{pmatrix} g^c, & d^c \end{pmatrix} \begin{pmatrix} \langle N \rangle, & 0 \\ \langle v^c \rangle, & \langle h_d \rangle \end{pmatrix} \begin{pmatrix} g \\ d \end{pmatrix}. \tag{11}$$

Three of the four 3×3 blocks in the mass matrix are of the order of $\langle N \rangle$, $\langle v^c \rangle$, and $\langle h_d \rangle$ as indicated with constant unsuppressed coefficients. The fourth block is suppressed by powers of the Planck mass since the relevant direct trilinear Yukawa coupling is forbidden by E_6. Applying the results of Ref. [20], we then see that there exist nine right-moving and nine left-moving zero modes (one for each family and color). A very similar analysis can be done for the charged leptons. We conclude that these strings are superconducting. In contrast, the string that connects the $SU(2)_R$ and $SU(3)_L$ monopoles is not superconducting since $\langle N \rangle$, $\langle h_u \rangle$, and $\langle h_d \rangle$ remain constant as we go around it. It is worth mentioning that the fact that the phase of $\langle v^c \rangle$ changes by -2π around this string does not imply the existence of zero modes in this case. In order to see this, we employ a theorem given in Ref. [20] which states that, if a particular mass matrix element remains constant around the string, we can remove from the mass matrix the row and the column that contain it when calculating the number of transverse zero modes. In our case $\langle N \rangle$ and $\langle h_d \rangle$ remain unaltered around the string, so all rows and columns can be removed and no zero modes appear.

Let us now turn to the alternative case where the symmetry breaking of E_6 employs the $v^c v^c$ component of a Higgs $\overline{\mathbf{351}'}$. In this case, intermediate scale Z_2 topologically stable strings are produced [7,21] in addition to the superheavy Dirac and the intermediate scale triply charged monopoles. A rotation by $2\pi/10$
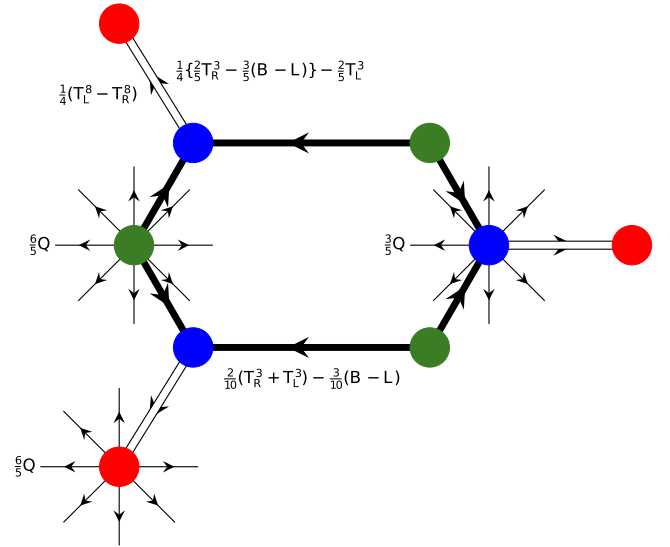

Fig. 2. Necklace configuration with alternating $SU(3)_L$ (blue) and $SU(2)_R$ (green) monopoles from the symmetry breaking $G \rightarrow SU(3)_c \times SU(2)_L \times U(1)_{Y_L} \times U(1)_{Y_R} \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2 \rightarrow SU(3)_c \times U(1)_{em} \times Z_2$. These are connected by half flux tubes along the necklace as indicated. Each $SU(3)_L$ (blue) monopole in the necklace is also connected by a flux tube with an $SU(3)_R$ (red) monopole hanging outside the necklace. We display explicitly only the Coulomb magnetic flux of three of the constituent monopoles and the flux along two of the tubes.

around the generator in Eq. (7) is now left unbroken by the VEV of $v^c v^c$ since its relevant charges are $T_R^3 = -2$, $B - L = 2$. Consequently, the flux tube from the $SU(2)_R$ to $SU(3)_L$ monopole splits into two equivalent tubes with magnetic flux

$$\frac{2}{10} T_R^3 - \frac{3}{10}(B - L). \tag{12}$$

After the electroweak symmetry breaking, this tube acquires an extra magnetic flux $T_L^3/5$ so that $\langle h_u \rangle$, $\langle h_d \rangle$ remain constant around it. One can show that this "half flux tube" is not superconducting. The combined flux tube though is not affected. We can imagine that we break one of the two strings from the $SU(2)_R$ to $SU(3)_L$ monopole which leaves the two monopoles connected by one string and two "loose" strings attached to the two monopoles. One can then connect these latter strings to other similar monopole-string structures in series to form "fang necklaces" – see Fig. 2. More complex fang necklaces can be contemplated where each $SU(3)_L$ monopole (antimonopole) in the necklace is connected by a half tube either to its own antimonopole or an $SU(2)_R$ monopole (antimonopole), and each $SU(2)_R$ monopole (antimonopole) either to its own antimonopole or to an $SU(3)_L$ monopole (antimonopole). Each $SU(3)_R$ monopole (antimonopole) in the necklace is also connected by a flux tube to an $SU(3)_R$ monopole (antimonopole) hanging outside the necklace or to its own antimonopole which participates in a different necklace.

4. Conclusions

The trinification group $SU(3)_c \times SU(3)_L \times SU(3)_L$ implements charge quantization and predicts the existence of a topologically stable monopole of magnetic charge $6\pi/e$. The trinification symmetry breaking to the SM may occur in a number of steps, and we have discussed a scenario in which this monopole may be regarded as a magnetic baryon, in rough analogy with the QCD baryon. It is composed of three confined monopoles (magnetic quarks), where the latter monopoles carry some Coulomb magnetic flux accompanied by a magnetic flux tube. These confined monopoles can yield more elaborate topological configurations and we display one such

example consisting of a fang necklace. In contrast to the super-heavy GUT monopoles the trinification monopole discussed here may be accessible at high energy colliders.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

References

[1] H. Georgi, S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438.
[2] C.P. Dokos, T.N. Tomaras, Phys. Rev. D 21 (1980) 2940.
[3] M. Daniel, G. Lazarides, Q. Shafi, Nucl. Phys. B 170 (1980) 156;
    G. Lazarides, Q. Shafi, W.P. Trower, Phys. Rev. Lett. 49 (1982) 1756.
[4] G. 't Hooft, Nucl. Phys. B 79 (1974) 276;
    A.M. Polyakov, JETP Lett. 20 (1974) 194.
[5] J.C. Pati, A. Salam, Phys. Rev. D 10 (1974) 275;
    J.C. Pati, A. Salam, Phys. Rev. D 11 (1975) 703, Erratum.
[6] G. Lazarides, M. Magg, Q. Shafi, Phys. Lett. B 97 (1980) 87.
[7] G. Lazarides, Q. Shafi, J. High Energy Phys. 10 (2019) 193.
[8] G.E. Volovik, J. Exp. Theor. Phys. 131 (2020) 11;
    G.E. Volovik, Sov. Phys. JETP 131 (2020) 11.
[9] T.W.B. Kibble, G. Lazarides, Q. Shafi, Phys. Rev. D 26 (1982) 435.
[10] J.T. Mäkinen, V.V. Dmitriev, J. Nissinen, J. Rysti, G.E. Volovik, A.N. Yudin, K. Zhang, V.B. Eltsov, Nat. Commun. 10 (2019) 237;
    G.E. Volovik, K. Zhang, Phys. Rev. Res. Int. 2 (2020) 023263;
    K. Zhang, Phys. Rev. Res. Int. 2 (2020) 043356.
[11] Q. Shafi, C. Wetterich, NATO Sci. Ser. B 111 (1984) 47;
    G. Lazarides, C. Panagiotakopoulos, Q. Shafi, Phys. Rev. Lett. 58 (1987) 1707;
    G. Lazarides, Q. Shafi, T.N. Tomaras, Phys. Rev. D 39 (1989) 1239;
    T.W. Kephart, C.-A. Lee, Q. Shafi, J. High Energy Phys. 01 (2007) 088;
    T.W. Kephart, G.K. Leontaris, Q. Shafi, J. High Energy Phys. 10 (2017) 176.
[12] F. Gürsey, P. Ramond, P. Sikivie, Phys. Lett. B 60 (1976) 177;
    Y. Achiman, B. Stech, Phys. Lett. B 77 (1978) 389;
    Q. Shafi, Phys. Lett. B 79 (1978) 301.
[13] P.A.M. Dirac, Proc. R. Soc. Lond. A 133 (821) (1931) 60.
[14] X.-G. Wen, E. Witten, Nucl. Phys. B 261 (1985) 651.
[15] Y. Nambu, Phys. Rev. D 10 (1974) 4262;
    Y. Nambu, Nucl. Phys. B 130 (1977) 505;
    G. Lazarides, Q. Shafi, Phys. Lett. B 94 (1980) 149.
[16] P.Q. Hung, Nucl. Phys. B 962 (2021) 115278.
[17] B. Acharya, et al., MoEDAL Collaboration, J. High Energy Phys. 08 (2016) 067;
    G. Aad, et al., ATLAS Collaboration, Phys. Rev. Lett. 124 (2020) 031802;
    M.M.H. El Sawy, Ph.D. Thesis, preprint CERN-THESIS-2020-191, https://cds.cern.ch/record/2744867.
[18] N.E. Mavromatos, V.A. Mitsou, Int. J. Mod. Phys. A 35 (2020) 203001.
[19] O. Gould, D.L.-J. Ho, A. Rajantie, arXiv:2103.14454.
[20] N. Ganoulis, G. Lazarides, Phys. Rev. D 38 (1988) 547;
    N. Ganoulis, G. Lazarides, Nucl. Phys. B 316 (1989) 443.
[21] T.W.B. Kibble, G. Lazarides, Q. Shafi, Phys. Lett. B 113 (1982) 237.