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Cosmological Perturbations and the Weyl Tensor

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In a previous work the authors have solved the Einstein equations of General Relativity for a class of metrics with constant spatial curvature where it was found a non vanishing Weyl tensor due to the presence of an anisotropic pressure component associated to a primordial magnetic field. Here, we perform the perturbative analysis of this model in order to study the gravitational stability under linear scalar perturbations. For this purpose, we take the Quasi-Maxwellian formalism of General Relativity as our framework, which offers a naturally covariant and gauge-invariant approach to deal with perturbations that are directly linked to observational quantities. We also consider a generalization of the causal thermodynamics to include the effect of the non-null Weyl tensor by introducing a new "viscosity" term.

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1. Introduction

We consider here the covariant approach to perturbation theory first proposed by Hawking [1] and improved by Olson, Ellis *et al.* and Novello *et al.* [2], based on the Quasi-Maxwellian (QM) formalism of General Relativity [3]. In this framework a set of perturbed quantities is considered as a "good" one if their unperturbed counterparts are null in the background and, therefore, Stewart's lemma ensures that the associated perturbed quantities are gauge-invariant. In Novello *et al.* this method was applied to the Friedmann-Lemaître-Robertson-Walker (FLRW) models.

2. Background model

We briefly review the recent proposal [4] in which it is assumed a Friedmann-like geometry

$$ds^2 = dt^2 - a^2(t)[d\chi^2 + \sigma^2(\chi)d\Omega^2] \tag{1}$$

where χ is for radial coordinate, Ω is used to refer the angular ones.

We consider as source the Maxwell's Lagrangian of electromagnetism $L = -1/4\, F$ where $F \equiv F^{\mu\nu}\, F_{\mu\nu} = 2\,(B^2 - E^2)$. The energy-momentum tensor corresponding to this Lagrangian is

$$T_{\mu\nu} = F_{\mu}{}^{\alpha} F_{\alpha\nu} - L g_{\mu\nu}.$$

Due to the special symmetries of the metric (1), the electromagnetic field can be considered as source of the gravitational field only if an averaging process is performed. In the limit of high conductivity the average electric field vanishes and we are left with a null average magnetic field whose second moment can be written as

$$\overline{B^i B_j} = -\frac{1}{3} B^2 h^i_j - \pi^i{}_j \tag{2}$$

where we introduce an arbitrary traceless matrix $\pi^i{}_j$ that will be identified to an anisotropic pressure term. In the case of constant spatial curvature ${}^{(3)}R$, the time evolution of this cosmological model is driven by the usual Friedmann equations and the anisotropic pressure produces a non-vanishing Weyl tensor. Its components are found to be

$$\pi^2{}_2 = \pi^3{}_3, \qquad \pi^1{}_1 = \frac{2k}{a^2\sigma^3} = -2\pi^2{}_2$$

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where k is a constant. The corresponding QM equations for expansion coefficient θ (see its definition in [6]) and the energy density ρ of this solution read:

$$\dot{\theta} + \frac{\theta^2}{3} = -\frac{1}{2}(\rho + 3p), \tag{3a}$$

$$\dot{\rho} + (\rho + p)\,\theta = 0, \tag{3b}$$

with the constraints $E_{\mu\nu} = -\frac{1}{2}\pi_{\mu\nu}$ and $E^{\alpha}{}_{\mu;\alpha} = 0$ (here and beneath the semicolon and index is for covariant derivative in appropriate coordinate, comma and index – for a partial one).

2.1. Thermodynamical considerations

Let us consider the formulation of the causal thermodynamics proposed by Israel applied particularly to FLRW models, in which the anisotropic pressure represents a shear viscosity related to the shear tensor according to

$$\tau\dot{\pi}_{\mu\nu} + \pi_{\mu\nu} = \xi\sigma_{\mu\nu} \tag{4}$$

where τ is the relaxation time, and ξ is the viscosity parameter (valid only in the linear perturbation regime). In the model we are dealing with, the viscosity is not caused by shear stresses but instead it is due to the curvature tensor, through the electric part of the Weyl tensor. Therefore, we modify the Israel's equation in order to take it into account by proposing the following equation in the presence of what we call "gravitational viscosity",

$$\dot{\pi}_{\mu\nu} + c_o\theta\pi_{\mu\nu} = \xi\theta\sigma_{\mu\nu} + \gamma\theta E_{\mu\nu} \tag{5}$$

where c_o is a constant, and γ is the parameter representing gravitational effects (which would be generated by tidal forces or similar effects due to the magnetic field).

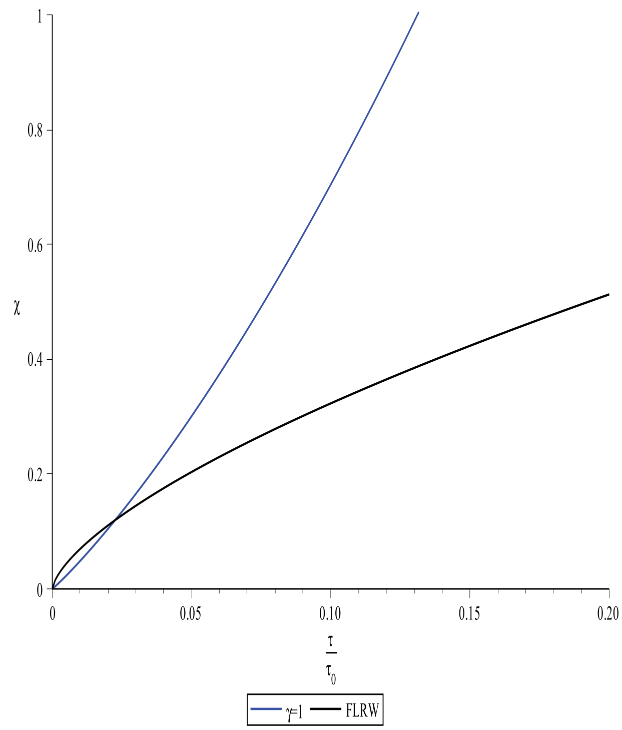

FIG. 1. Growth of the fractional energy density gradient in our model with $\gamma = 1$ compared to the standard FLRW case, both for $\lambda = 0$. (In colour)

3. Covariant perturbations

According to one the constraint equations, we can define

$$X_{\mu\nu} \doteq E_{\mu\nu} + \frac{1}{2}\pi_{\mu\nu}, \tag{6}$$

which is a good variable as it is null in the background, hence a perturbation on it yields a true physical perturbation. To this variable we add the shear $\sigma_{\mu\nu}$ itself, which is also null in the background. Following [5], we also consider as good variables the fractional energy density gradient $\chi_\alpha \doteq h_\alpha{}^\nu \frac{\rho_{,\nu}}{\rho}$ and the gradient of the expansion coefficient $Z_\alpha \doteq h_\alpha{}^\nu\theta_{,\nu}$. We find [a]

$$\ddot{\sigma} + (1-\gamma)\theta\dot{\sigma} + \left(\frac{C}{a^2} - \frac{(1+\lambda)}{2}\rho\right)\sigma = 0, \tag{7}$$

[a] One of the variables was set to zero for simplicity. Details can be found in [6].

$$\chi = \left(2 - \frac{3A}{m^2}\right) \frac{1}{(1+3\lambda)\rho_o\, a_o^2} \left(\frac{t}{t_o}\right)^{\frac{2(1+3\lambda)}{3(1+\lambda)}} \dot{\sigma}$$

where C and A depend only on the wavenumber of the modes. Among all possibilities, the most interesting for structure formation corresponds to $\gamma > 0$ and $C < 0$. In this regime, small values of γ provide a variation in χ larger than the growing mode of the FLRW model without dark matter, see appropriate curves in Fig. 1.

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