Non-linear evolution of matter power spectrum in a closure theory

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Abstract

We study the non-linear evolution of cosmological power spectra in a closure theory. Governing equations for matter power spectra have been previously derived by a non-perturbative technique with closure approximation. Solutions of the resultant closure equations just correspond to the resummation of an infinite class of perturbation corrections, and they consistently reproduce the one-loop results of standard perturbation theory. We develop a numerical algorithm to solve closure evolutions in both perturbative and non-perturbative regimes. The present numerical scheme is particularly suited for examining non-linear matter power spectrum in general cosmological models, including modified theory of gravity. As a demonstration, we apply our numerical scheme to the Dvali-Gabadadze-Porrati braneworld model.

1 Introduction

Probing the nature of dark energy, accelerating the late-time universe, is one of the most tough issues in cosmology and astrophysics. So far it remains to be clarified what the dark energy really is. A simple solution is that it is the cosmological constant, or described by dynamics of unknown scalar fields. Another solution may lie in the sector of gravity. The attention has been focused on the test of general relativity (GR) in both solar system experiments and cosmological contexts. Several modified theories of gravity beyond GR have passed these tests and been viable. A key to distinguish these possibilities is to precisely predict the matter power spectrum considering non-linear dynamics of matter perturbations. This situation motivates to develop a framework to compute the non-linear matter power spectrum in a variety of cosmological models.

The naive perturbative approach (see [1] for a review) has been frequently used for predictions of the power spectra, as well as fully numerical approach such as N-body simulations. Recently, alternative to perturbation theory, several authors have recently proposed the renormalisation/resummation technique for the infinite series of the loop calculation appeared in the naive expansion of the perturbative quantities. In those treatments, the fundamental quantities are not the density/velocity perturbations but propagators, power spectra and vertex functions of density and velocity divergence [2]. In our previous paper [3], we have derived evolution equations of the non-perturbative quantities, power spectra and propagators of matter fluctuations on the basis of the closure approximation used in the statistical theory of turbulence [4]. This approach is suited for numerical purpose where we set the initial conditions and track the time evolution of those quantities.

In this paper, we demonstrate the closure equations in the case of Dvali-Gabadadze-Poratti (DGP) braneworld model. For the numerical scheme, please see [5] and, for the numerical analysis, [6].

2 Evolution equations for perturbations

We consider the cold dark matter plus baryon system as a pressureless perfect (irrotational) fluid neglecting the contribution from massive neutrinos. The density contrast, $\delta \equiv \delta\rho/\rho$, and the velocity divergence,

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$\theta \equiv \nabla \cdot \mathbf{u}$, are governed by the Euler and the continuity equations. The Newton potential, ϕ, satisfies the Poisson equation with non-linear source terms, if exists,

$$-\frac{k^2}{a^2}\phi(\mathbf{k},\tau) = 4\pi G_{\rm eff}(\mathbf{k},\tau)\rho_m\widetilde{\delta}(\mathbf{k},\tau) + (\text{nonlinear terms}), \quad G_{\rm eff} = G\left[1+\frac{1}{3}\frac{(k/a)^2}{\Pi(k)}\right], \tag{1}$$

where we take the time coordinate as $\tau = \log(a/a_0)$, and $G_{\rm eff}$ represents the effective gravity constant generalising the Newton constant G in GR with a model-dependent function, $\Pi(k)$.

For the purpose of quantitative estimation of the density/velocity perturbations, we consider their statistical quantities, namely, power spectra. The perturbations are recasted in a vector form, $\Phi_a(\mathbf{k},\tau) = (\widetilde{\delta}(\mathbf{k},\tau), -\widetilde{\theta}(\mathbf{k},\tau))^{\rm T}$. The standard approach of perturbative estimation of power spectra is then to expand $\Phi_a = \Phi_a^{(0)} + \Phi_a^{(1)} + \cdots$, where $\Phi_a^{(i\geq 1)}$ is iteratively obtained from the Euler and the continuity equations, and to substitute it into the definition of the power spectra,

$$\langle \Phi_a(\mathbf{k},\tau)\Phi_b(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta_D(\mathbf{k}+\mathbf{k}')P_{ab}(|\mathbf{k}|;\tau). \tag{2}$$

In addition to this quantity, we here introduce the propagator, $G_{ab}(\mathbf{k}|\tau,\tau')$, and the power spectra between different times, $R_{ab}(\mathbf{k};\tau,\tau')$, defined as [2, 3]

$$\left\langle \frac{\delta\Phi_a(\mathbf{k},\tau)}{\delta\Phi_b(\mathbf{k}',\tau')}\right\rangle = G_{ab}(\mathbf{k}|\tau,\tau')\delta_D(\mathbf{k}-\mathbf{k}'), \quad \langle\Phi_a(\mathbf{k},\tau)\Phi_b(\mathbf{k}',\tau')\rangle = (2\pi)^3\delta_D(\mathbf{k}+\mathbf{k}')R_{ab}(|\mathbf{k}|;\tau,\tau'), \quad (\tau>\tau'). \tag{3}$$

Then using those quantities, the above naïve expansion is re-organised in a non-perturbative way.

In the closure theory, we truncate the non-perturbative expansions of the power spectra and the propagator up to the one-loop level, and make them close. The resultant equations are [3, 5, 6]

$$\hat{\Lambda}_{ab}G_{bc}(k|\tau,\tau') = \int_{\tau'}^{\tau} d\tau''\, M_{as}(k;\tau,\tau'')G_{sc}(k|\tau'',\tau') + S_{ar}(k;\tau)G_{rc}(k|\tau,\tau'), \tag{4}$$

$$\hat{\Lambda}_{ab}R_{bc}(k;\tau,\tau') = \int_{\tau_0}^{\tau} d\tau''\, M_{as}(k;\tau,\tau'')R_{\overline{sc}}(k;\tau'',\tau') + \int_{\tau_0}^{\tau'} d\tau''\, N_{a\ell}(k;\tau,\tau'')G_{c\ell}(k|\tau',\tau'') + S_{ar}(k;\tau)R_{rc}(k;\tau,\tau'), \tag{5}$$

where we defined $k = |\mathbf{k}|$ and $k' = |\mathbf{k}'|$, and

$$M_{as}(k;\tau,\tau'') = 4\int\frac{d^3\mathbf{k}'}{(2\pi)^3}\gamma_{apq}(\mathbf{k}-\mathbf{k}',\mathbf{k}')\gamma_{\ell rs}(\mathbf{k}'-\mathbf{k},\mathbf{k})G_{q\ell}(k'|\tau,\tau'')R_{pr}(|\mathbf{k}-\mathbf{k}'|;\tau,\tau''), \tag{6}$$

$$N_{a\ell}(k;\tau,\tau'') = 2\int\frac{d^3\mathbf{k}'}{(2\pi)^3}\gamma_{apq}(\mathbf{k}-\mathbf{k}',\mathbf{k}')\gamma_{\ell rs}(\mathbf{k}-\mathbf{k}',\mathbf{k}')R_{qs}(k';\tau,\tau'')R_{pr}(|\mathbf{k}-\mathbf{k}'|;\tau,\tau''), \tag{7}$$

$$S_{ar}(k;\tau) = 3\int\frac{d^3\mathbf{k}'}{(2\pi)^3}\sigma_{apqr}(\mathbf{k}',-\mathbf{k}',\mathbf{k};\tau)P_{pq}(k';\tau). \tag{8}$$

Here the functions γ_{acd} and σ_{abcd} are called as the vertex functions described as a simple function of wavenumber. The non-linear terms in the Euler and continuity equations give rise to three non-vanishing components, $\gamma_{112}, \gamma_{121}$ and γ_{222}. In the case where the Poisson equation has non-linear source terms, the components, γ_{211} and σ_{2111}, become non-zero, while the other components remain to be zero. The operator $\hat{\Lambda}_{ab}$ is defined by

$$\hat{\Lambda}_{ab} = \delta_{ab}\frac{\partial}{\partial\tau} + \Omega_{ab}(\tau), \quad \Omega_{ab}(\tau) = \begin{pmatrix} 0 & -1 \\ -4\pi G_{\rm eff}\frac{\rho_{\rm m}}{H^2} & 2+\frac{\dot{H}}{H^2}\end{pmatrix}. \tag{9}$$

Eqs. (4)(5) contain the non-linear quantities in those source terms. Since we truncated at the one-loop level when the equations were derived, it is easy to see the recovery of the one-loop power spectrum in the SPT by replacing all power spectra, R_{ab}, and propagators, G_{ab}, in the source terms with those calculated in the linear theory, $R^{\rm L}_{ab}$, and $G^{\rm L}_{ab}$ [3]. Here the linear quantities are given by Eqs. (4)(5) with neglecting all of right-hand sides. Furthermore, it needs to be emphasised that, the formal solution of the closure equations has been confirmed to coincide with the renormalised one-loop results presented by Crocce and Scoccimarro apart from the vertex renormalisation [2]. Hence, the closure equation is basically equivalent to the RPT truncated at the one-loop level.

3 Results

3.1 Non-linear solution : ΛCDM model

We first present the results of closure equations (4)(5) in the flat ΛCDM model with the cosmological parameters : $\Delta_R^2(k_0) = 2.457 \times 10^{-9}$, $n_s = 0.960$, $\Omega_{M0} = 0.279$, $h = 0.701$. In this case, $\gamma_{211} = \sigma_{2111} = 0$, and $\Pi(\mathbf{k}) = (k/a)^2/3$, resulting in $G_{\rm eff} = G$.

In the left panel of Fig. 1, the ratio of the non-linear power spectrum to the linear one at $z = 3$ is plotted. We found that the non-linear effects of the closure equations suppress the amplitude on small scales in comparison with the SPT (dashed line, see [1]). The dotted and the dotted-dashed lines are the power spectra analytically obtained with the $1^{\rm st}$ and $2^{\rm nd}$ Born approximation, respectively, [3]. We confirmed that the higher order approximation tends to agree with the result of the closure equations.

The behaviour of the propagators in the closure equations is shown in the middle panel where we define $\widetilde{G}_1 \equiv G_{11} + G_{12}$. The amplitude is normalised to unity on large scales by multiplying the linear growth rate. While the propagator computed in SPT goes negative infinity shown as the dotted line, the numerical solution in the closure equations tends to converge to zero on small scales. This damping feature is caused by the non-linear effects as a result of taking into account the higher-order corrections neglected in the SPT, leading to the suppression of the power on small scales.

As we mentioned in [3], in the small-scale limit, the propagator behaves as the Bessel function of the first kind. On the other hand, in the large-scale limit, the propagator should coincide with the one-loop result in the SPT. The dashed line in the right panel is the approximate solution matching the both limits obtained in [3]. We found that the approximate solution agrees well with the numerical solution of the closure equations even on intermediate scales between the limits.

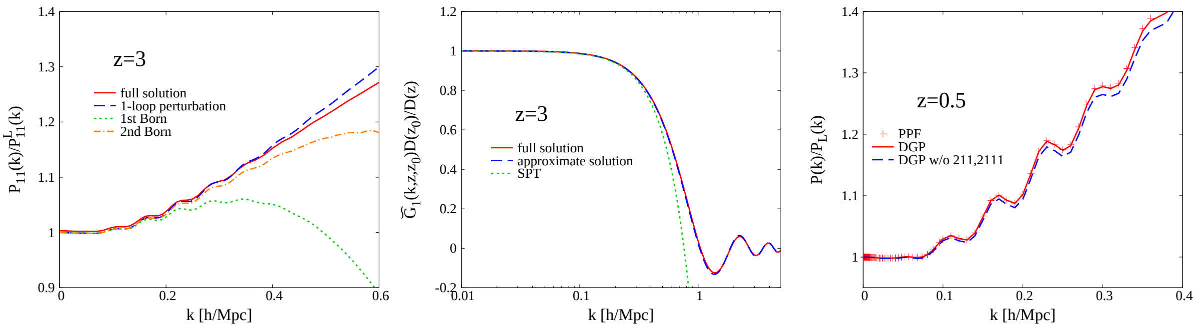

Figure 1: [*Left*] The resultant power spectra divided by the linear power spectra at $z = 3$, [*Middle*] the propagators $\widetilde{G}_1(k, z, z_0) \equiv G_{11} + G_{12}$ evaluated at $z = 3$, and [*Right*] the perturbative solution normalised by the linear power spectrum in the DGP model at $z = 0.5$ (next subsection). 'PPF' is obtained from the fitting formula proposed by [8] with $c_{nl} \approx 0.36$.

3.2 Perturbative solution : DGP model

As an application of our formalism, we apply it to the DGP braneworld model to compute the power spectrum in the SPT. The perturbative solution, namely, the calculation in the SPT, can be done by a simple replacement of G_{ab} and R_{ab} in the right-hand sides of the closure equations (4)(5) with the linear propagator and power spectra, $G_{ab}^{\rm L}$ and $R_{ab}^{\rm L}$, as discussed in the end of Sec.2.

The modified Friedman equation in the self-accelerating branch is given by

$$\frac{H}{r_c} = H^2 - \frac{\kappa^2}{3}\rho, \tag{10}$$

where r_c is the parameter in this model which is a ratio between the 5D Newton constant and the 4D Newton constant.

In this model, gravity becomes 5D on large scales larger than r_c. On small scales, gravity becomes 4D but it is not described by GR. According to the quasi-static perturbations [7], the brane-bending mode couples to the Newton potential on the brane. Therefore we have

$$\gamma_{211}(\mathbf{k}_1,\mathbf{k}_2;\tau) = -\frac{1}{12H^2}\left(\frac{8\pi G\rho_{\rm m}}{3}\right)^2\left(\frac{k_{12}^2}{a^2}\right)\frac{M(\mathbf{k}_1,\mathbf{k}_2)}{\Pi(\mathbf{k}_{12})\Pi(\mathbf{k}_1)\Pi(\mathbf{k}_2)} \tag{11}$$

$$\sigma_{2111}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3;\tau) = \frac{8\pi G\rho_{\rm m}}{324H^2}\left(\frac{k_{123}^2/a^2}{\Pi(\mathbf{k}_{123})\Pi(\mathbf{k}_1)\Pi(\mathbf{k}_2)\Pi(\mathbf{k}_3)}\right)\left(\frac{M(\mathbf{k}_1,\mathbf{k}_2+\mathbf{k}_3)M(\mathbf{k}_2,\mathbf{k}_3)}{\Pi(\mathbf{k}_{23})}+\text{perm.}\right) \tag{12}$$

where

$$M(\mathbf{k}_1,\mathbf{k}_2) = 2\frac{r_c^2}{a^4}[k_1^2k_2^2-(\mathbf{k}_1\cdot\mathbf{k}_2)^2],\quad \beta(\tau) = 1-2Hr_c\left(1+\frac{\dot{H}}{3H^2}\right) \tag{13}$$

For the numerical calculation, we use the best fit parameters for the flat self-accelerating universe $\Omega_m = 0.257, \Omega_r = 0.138, h = 0.66, n_s = 0.998$. The resultant power spectrum normalised by the linear one is presented as the solid line in the right panel of Fig. (1). The dashed line is the same calculation neglecting the extra functions, γ_{211} and σ_{2111}. From this figure, we found that γ_{211} gives a positive contribution to the mode coupling, while σ_{2111} gives a negative and smaller contribution. Therefore the resultant power is enhanced on small scales.

We also plotted the prediction in the Parameterised Post-Friedmann (PPF) framework proposed in [8] as 'PPF' in the right panel. It is possible to recover the numerical solution for the non-linear power spectrum very well with the non-linear parameter $c_{nl} \approx 0.36$ introduced in [8].

4 Summary

In this paper, we showed the non-linear matter power spectra as a demonstration of our numerical scheme for the closure equations derived in [3]. From the calculation in the ΛCDM model, we observed the extra suppression of the power on small scales due to the non-linear effects which are neglected in the SPT, as shown in the left panel of Fig (1). Moreover, the middle panel shows that the resultant propagator converges to zero on small scales, and coincides with the approximate solution obtained in an analytic way in [3]. These facts confirm that the closure equations contain non-linear contributions more than the SPT, and that our numerical scheme works well from the qualitative viewpoint.

As an application, we applied the closure equations to compute the perturbative prediction of the matter power spectrum in the DGP model. The perturbative calculation, namely, the calculation in the SPT can be performed by the replacement of the right-hand sides of the closure equations (4)(5) with the linear propagator and power spectra, $G_{ab}^{\rm L}$ and $R_{ab}^{\rm L}$. As as result, the extra vertex function, γ_{211}, gives a positive, and large, contribution to the mode coupling, enhancing the power on small scales.

More detailed analysis can be seen in [6] together with the application to a $f(R)$ gravity model.

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