A new relation to estimate Nuclear Radius

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It has long been known that the electric quadrupole transition strength $B(E2; 2_1^+ \rightarrow 0_1^+)$ for the lowest 2^+state of a nonmagic even-even nucleus is strongly correlated with the corresponding excitation energy $E2_1^+$[1]. Furthermore, the vibrational or rotational models ~VRM of Bohr and Mottelson [2] predict a dependence of $BE2$ on the excitation energy $E2_1^+$, the nuclear charge Z, and the nuclear mass A, which reproduces reasonably well that found in practice, apart from an overall normalization constant [3].

The deformation parameter β and γ of the collective model of Bohr and Mottelson are basic descriptors of the nuclear equilibrium shape and structure. In recent past the sets of deformation parameters (β, γ) have been extracted from both level energies and E2 transition rates in even Xe, Ba and Ce nuclei [4] and Hf, W, Os, Pt and Hg nuclei [5] using rigid triaxial rotor model of Davydov – Filippov [6] . The researchers found that the values of γ obtained from energies (= γ_e) are nearly equal to the value of γ derived from transition rate (= γ_b) in both the mass regions. It is well established fact that asymmetry exist in nuclei and for calculating asymmetry parameter (γ) the relation of rigid rotor model is competent.

The uncertainty found in Grodzins semi empirical relation [5] therefore may be due to the non – consideration of asymmetry in the relation. In the present work we propose a new relation connecting $B(E2; 2_1^+ \rightarrow 0_1^+)$ and $E2_1^+$ with asymmetric parameter γ. The Grodzins Semi-empirical relation for symmetric nucleus –

$$B(E2; 2_1^+ \rightarrow 0_1^+).E2_1^+ = (2.5 \pm 1) \times 10^{-3} Z^2 A^{-1} \quad \text{(MeV.e}^2\text{b}^2\text{)} \qquad (1)$$

For asymmetric nucleus, we propose that the numerical constant in equation (1) i.e. $(2.5 \pm 1) \times 10^{-3}$ is proportional to the $\frac{9-[81-72\sin^2 3\gamma]^{1/2}}{4\sin^2 3\gamma}$ which is a γ- dependent function. The above γ- dependent function is the energy of first excited state of ground band in the unit of $6\hbar^2/2I_0$. Therefore equation (1) may be rewritten as

$$\frac{A.B(E2;2_1^+\rightarrow 0_1^+).E2_1^+}{Z^2} = K\epsilon_{21} \qquad (2)$$

Where, $\epsilon_{21} = \frac{9-[81-72\sin^2 3\gamma]^{1/2}}{4\sin^2 3\gamma}$ and K is proportionality constant.

In Present work, we have used equation (2) to evaluate asymmetric parameter γ for ^{150}Sm, ^{152}Gd, ^{156}Er, $^{188\text{-}192}$Os and ^{188}Pt nuclei along with the other known method of evaluating γ [3]. The calculated values of γ are listed in Table I and are found to be nearly equal.

In order to relate the two basic deformation β and γ, we put values of $B(E2)$ in terms of β

$$B(E2; 2_1^+ \rightarrow 0_1^+) = \frac{e^2 Q_0^2}{16\pi} \qquad (3)$$

$$\text{And } Q_0 = \frac{3ZR^2\beta}{\sqrt{5\pi}} \qquad (4)$$

Where, $R = R_0 A^{1/3}$ and $R_0 = 1.21\ fm$.

The relation proposed above among the physical quantities atomic mass (A),

quadrupole deformation (Q_0), asymmetry parameter (γ) and nuclear radius (R) have been used to evaluate the R by feeding the other known values. By evaluating R, we can find the value of R_0 for the considered nuclei and are listed in Table – 1. It is noted here that the values of R_0 is not same for all the considered nuclei. Our estimate of R_0 lies between 1.056 and 1.374 i.e. $R_0 = 1.21 \pm 0.16$fm. The average value is same however; the deviation in present work from 1.21 is ± 0.16. Recently, the new relation were presented connecting B (E2) values and excitation energies in which the values of R_0 were estimated using cluster model that yields $R_0 = 1.21 \pm 0.04$fm in heavy mass nuclei [7].

It is concluded that in heavy mass nuclei the nuclear radius remains uncontroversial as the present value, cluster model value and the earlier value i.e. (1.21fm) are almost same. It is interesting that the procedure presented here is simple and capable of handling the nuclear radius.

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Table 1

Nucl.	γ_e	$\gamma_{present}$	r_0	γ_b
^{150}Sm	20.5^0	20^0	1.20	18^0
^{152}Gd	21.5^0	22^0	1.32	-
^{156}Er	23.5^0	22.5^0	1.21	-
^{188}Os	19.2^0	20^0	1.056	16.7^0
^{190}Os	22.3^0	22^0	1.056	20.2^0
^{192}Os	25.2^0	21^0	1.056	21.7^0
^{188}Pt	25.9^0	24^0	1.374	25.70

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