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RESEARCH ARTICLE

Nima Khosravi

Particle creation from vacuum by Lorentz violation

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Abstract It is shown that the vacuum state in the presence of Lorentz violation can be followed by a universe filled with particles at late times similar to the current status of the universe. In this model a modification in dispersion relation (Lorentz violation) appears representing the regime of quantum gravity which has been dominant in the early universe. The existence of the particles can be interpreted as an evidence for quantum effects of gravity at early times. It is concluded that the present observable particles have a geometrical origin due to the well-known correspondence between geometry and gravity.

Keywords Lorentz violation, Particle creation

1 Introduction

The big-bang singularity is a prediction of classical general relativity and it has to be removed in the final theory. This fact[1] points to a regime where general relativity (gravity) is the dominant field with a very high density. From another angle, our knowledge of quantum physics says that in this regime due to enormous density of a the fields, the classical physics is broken and quantum fields become essential. The big-bang and the black-hole, theoretical evidences of quantum gravity, challenge the final theory (at least in its gravity sector). Until now different quantum gravity approaches have grown up e.g. string theory [1; 2] and loop quantum gravity [3; 4]. For a theory, including quantum gravity, it is very crucial to possess some non-trivial accessible predictions, besides the compatibility with their classical counterparts. However, due to the complexity of quantum gravity it is so hard to calculate any physical prediction, verifiable by the present experiments. To make this comparison possible, test theories are needed which fill the gap between

N. Khosravi Department of Physics Shahid Beheshti University G. C., Evin 19839 Tehran, Iran
n-khosravi@sbu.ac.ir

the full theories' predictions and experiments. These test theories should possess the structure of the main theories as much as possible. Also there is another layer between the full quantum gravity theories and test theories which is named falsifiable "quantum gravity theory of not everything" [5]. The latter makes it possible to transit the great gap between test theories and full theories step by step, e.g. noncommutative geometry [6; 7; 8].

As a candidate to model quantum gravity effects is a deviation [5; 9; 10; 11; 12; 13; 14; 15] from the standard dispersion relation among energy and momentum of a particle (Lorentz violation) i.e. $\omega^2 = k^2 + m^2$ where ω, k and m are the energy, momentum's norm and mass of the particle, respectively. Actually, it is well-known that modification to dispersion relation can appear as a consequence of discretization of the space-time on a lattice [5; 16]. Observationally in some cases the modified dispersion relation's imprints can be found in cosmic ray spectrum [17; 18; 19; 20; 21]. This deviation can be represented by a modification to the dispersion relation such that $\omega^2 = k^2 + m^2 - \alpha^2 k^4$ which is a famous example [11]. However, this form of deformation is not a unique choice e.g. introducing a cubic term has been studied in detail [12] and a more general form in [13] which contains an observer independent length besides velocity. The coefficient α^2 is a factor proportional to the minimum length ℓ_P that makes the semi-classical limit to standard form of the dispersion relation trivial due to exiting the quantum gravity regime by taking $\ell_P \to 0$. It is worth mentioning that this deviation is a candidate to go further in the phenomenology of quantum gravity [5; 14; 15]. As mentioned above, the phenomenological aspects of quantum gravity is a controversial issue in theoretical physics. It seems tracing quantum gravity is difficult at the present status of our experiments (due to the large difference between energy levels of theoretical predictions and achievable experiments). But it is believable to accept domination of quantum gravity in the early universe. Therefore, focusing on the physics of the early universe may shed some light on the nature of quantum gravity.

In general, to understand properties of a physical system, understanding of the dynamical rules and the correct initial conditions, are essential. For our universe the common belief is that the initial state is a quantum gravitational state due to domination of gravitational field at early times. This state has reached the present state containing particles without any quantum effects of gravity.[2] So the question is that how the particle-full universe has risen up from an unknown initial quantum gravitational state? Note that the initial state is an unknown state at least at the present. However as it is usual, for different theories with exactly similar predictions, the simpler one is more convenient to choose (Occam's razor). At least, lacking any knowledge on the correct initial state, makes the simpler choice of the theory calculable and hence results in a general understanding of the theory though it would not be complete. The simplest choice for the initial state is the vacuum state, if it does not fall in a trivial prediction. If this simplest choice can predict non-trivial, particle-filled present status of the universe then this assumption is remarkable. Note that the vacuum state is not only the simplest choice but also is a special one. This choice makes the proposal of creation from nothing meaningful [22]. The idea of vacuum creation theory has been considered also in different aspects such that under the action of strong fields which predicts the pos-

[2] To be more precise, in very very tiny effects of quantum gravity.

sibility of creating a particle-anti particle plasma system [23; 24]. As mentioned in [24] the time dependent masses can cause particle creation that makes these kind of models comparable to our model which has a time dependent dispersion relation as it will be shown.

In this work, in a toy model we have shown that the above argument is viable. To construct our toy model we pick up the method of particle creation in the context of the quantum field theory in curved space-times [25]. The deformed dispersion relation is used to model the quantum gravitational regime i.e. $\omega^2 = k^2 + m^2 - \alpha^2 k^4$. We will propose that the quantum gravity parameter α^2 has a time dependence such that for very early times is one and vanishing for late times. This dynamical behavior is appropriate to study the effects of very early quantum gravity at the present time and it is also consistent with what was discussed above.

2 Model

It is generally believed that the notion of particle and as a consequence the notion of vacuum in quantum field theory is not a straightforward manner specially in curved space-times. The crucial point of the definition of states in field theory is the selection of the basis. A field can be expanded due to the appropriate basis $u_k(x)$ as follow

$$\varphi(x) = \sum_k \left(a_k u_k(x) + a_k^* u_k^*(x)\right) \tag{1}$$

where a_k is a complex number, x and k are four-vectors of position and momentum respectively. In the Minkowskian space-time the choice is trivially $e^{\pm i\vec{k}.\vec{x}} e^{\pm i\omega t}$ such that $\omega^2 = \vec{k}^2 + m^2$. The next step is the quantization procedure that transition from complex number coefficients a_k to their corresponding operators $\hat{a}_k$ and consequently a_k^* to $\hat{a}_k^\dagger$, such that the commutation relation $\left[\hat{a}_k, \hat{a}_{k'}^\dagger\right] = i\hbar\delta_{ij}$ is satisfied by $\hat{a}_k$ and $\hat{a}_k^\dagger$. Due to this relation $\hat{a}_k$ and $\hat{a}_k^\dagger$ can be interpreted as annihilation and creation operators respectively. Finally an n_k-particle state with momentum k, $|n_k>$, is defined as $\hat{N}_k = \hat{a}_k^\dagger \hat{a}_k |n_k> = n_k|n_k>$. In the above we reviewed very quickly the structure of definition of states containing particles. As mentioned before the starting point is definition of the appropriate basis that is not trivial for curved space-times [25]. In general cases the symmetries help us to define the appropriate basis. This rapid review was needed to commence our model.

In our model we will suppose a Minkowskian space-time as the background but the dispersion relation is different for two sides of the time interval. For the very early times, i.e. for $t \to -\infty$, it has the form $\omega^2 = \vec{k}^2 + m^2 - \alpha_0^2 \vec{k}^4$ where α_0^2 is the Lorentz violation parameter represents the quantum gravity regime in our model. And for very late times, i.e. $t \to +\infty$, the dispersion relation becomes the standard one, $\omega^2 = \vec{k}^2 + m^2$. For the late times the natural choice of the basis is $e^{\pm i\vec{k}.\vec{x}} e^{\pm i\omega t}$ where $\omega^2 = \vec{k}^2 + m^2$. But for the early times these basis are not the suitable ones since the dispersion relation $\omega^2 = \vec{k}^2 + m^2$ has not any significance in this time region. The natural alternative for this region of time is $e^{\pm i\vec{k}.\vec{x}} e^{\pm i\omega t}$ where $\omega^2 = \vec{k}^2 + m^2 - \alpha_0^2 \vec{k}^4$. Since the definitions of basis for these two different regions are not equivalent then the equivalence of the vacuum's notion for them is not a trivial concept and must re-study again. This feature can cause a deviation between the initial vacuum state and the final vacuum state. This deviation can be interpreted as particle creation during transition from the initial to the final state. Note that this kind of interpretation is a standard one in quantum field theory in curved space-times [25]. The aim of this paper is to study this concept. To do more, supposing the time evolution of the Lorentz violation parameter is

$$\alpha(t)^2 = \frac{\alpha_0^2}{1+e^t}, \tag{2}$$

where α_0^2 is the initial value of the Lorentz violation parameter and the general behavior is such that the parameter for the early times and the late times satisfies our above propositions. Otherwise, the form of the function is picked only to make the equations exactly solvable. It must be noted that the origin of this form of time dependence has not been discussed in this work and it seems that this subject belongs to the quantum gravity and specially that branch discussing on the semi-classical limit of quantum gravity.[3] We consider a mass-less scalar field without any loss of generality. To reach to the Klein-Gordon equation it is sufficient to replace ω and $\vec{k}$ in the dispersion relation by their corresponding operator forms $-i\partial_t$ and $-i\partial_{\vec{x}}$ respectively. So the Klein-Gordon equation[4] becomes

$$\left[\partial_t^2 - \partial_{\vec{x}}^2 - \alpha(t)^2 \partial_{\vec{x}}^4\right] \varphi(x) = 0, \tag{3}$$

where x is position four-vector. Letting $\varphi(x) \propto e^{i\vec{k}.\vec{x}} T_k(t)$ reduces the above equation to

$$\left[\partial_t^2 + k^2 - \alpha(t)^2 k^4\right] T_k(t) = 0, \tag{4}$$

where $k = |\vec{k}|$. The solution of the above second order differential equation is

$$\begin{aligned} T_k(t) = {} & C_1 e^{-i\left(\sqrt{k^2-\alpha_0^2 k^4}\right)t} {}_2F_1\left(a,b;c;-e^t\right) \\ & + C_2 e^{+i\left(\sqrt{k^2-\alpha_0^2 k^4}\right)t} {}_2F_1\left(b^*,a^*;c^*;-e^t\right) \end{aligned} \tag{5}$$

where ${}_2F_1$ is the hypergeometric function and

$$\begin{aligned} a &= -ik - i\sqrt{k^2 - \alpha_0^2 k^4} = -i(\omega_{in} + \omega_{out}) \\ b &= +ik - i\sqrt{k^2 - \alpha_0^2 k^4} = -i(\omega_{in} - \omega_{out}) \\ c &= 1 - 2i\sqrt{k^2 - \alpha_0^2 k^4} = 1 - i\,2\omega_{in} \end{aligned} \tag{6}$$

[3] It is not too bad mentioning this branch of research is an active part without any final conclusion [3; 4; 26; 27].

[4] It is worth mentioning that to reach to Klein-Gordon equation the quantization process is crucial. The details of reaching to above equation is in [28; 29]. It should be notice that in the quantization process the concept of symmetry transformations in presence of Lorentz violation are not trivial. This subject is considered in details in [30].

To go further, we concentrate on the behavior of the both infinite limits. For the very early times, $t \longrightarrow -\infty$, the above solution reduces to

$$T_k(t \longrightarrow -\infty) = C_1 e^{-i\omega_{in} t} + C_2 e^{+i\omega_{in} t}, \tag{7}$$

where the identity ${}_2F_1(a,b;c;0) = 1$ for arbitrary a, b and c is used [31]. The result is fully in agreement with our expectation since for the very early times the energy is $\omega_{in} = \sqrt{k^2 - \alpha_0^2 k^4}$. So the first term in (7) can be interpreted as the temporal part of u_k^{in} that we will need it in the following calculations. For the second infinite limit, the very late times, the solution becomes more complicated such that

$$\begin{aligned} T_k(t \longrightarrow +\infty) = & \left[C_1 \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)} + C_2 \frac{\Gamma(c^*)\Gamma(b^*-a^*)}{\Gamma(b^*)\Gamma(c^*-a^*)} \right] \times e^{-i\omega_{out}\, t} \\ & + \left[C_1 \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)} + C_2 \frac{\Gamma(c^*)\Gamma(a^*-b^*)}{\Gamma(a^*)\Gamma(c^*-b^*)} \right] \times e^{+i\omega_{out}\, t}, \end{aligned} \tag{8}$$

where $\Gamma(x)$ is the gamma function. The important point in the above solutions is that for incoming waves, i.e. $e^{-i\omega_{out}\, t}$, there is a combination of the two terms of the general solutions (5) i.e. C_1 and C_2 both appear in the coefficient of the incoming waves and the same is true for the outgoing waves. This means that the vacuum state of the very early universe, $t \longrightarrow -\infty$, does not coincide to the vacuum state of the very late times, $t \longrightarrow +\infty$. As mentioned before, this interpretation is a standard interpretation in the subject of quantum field theory in curved space-times [25]. To continue we must calculate the Bogolubov coefficients, γ and β, as follows [25]

$$u_k^{in}(x) = \gamma_k u_k^{out}(x) + \beta_k u_{-k}^{out*}(x), \tag{9}$$

where x is the position four-vector. Note that u_k^{in} and u_k^{out} are the first terms in relation (1) corresponding to annihilation operator for *in*-region and *out*-region respectively. The non-vanishing β results to contrast with vacua for *in*-region and *out*-region e.g. in our model, the very early times and the very late times respectively. In mathematical language due to relation (7) in our model $u_k^{in}(x)$ is the first term in relation (5) times $e^{ik.x}$ i.e. it contains only the C_1 factor. But for the *out*-region the result is more complicated such that due to relation (8), $u_k^{out}(x)$ contains a combination of both of terms in (5). The temporal part of the solutions for both of the *in*- and *out*-regions are

$$\begin{aligned} T_k^{in}(t) = & \frac{1}{(4\pi\omega_{in})^{\frac{1}{2}}}\, e^{-i\omega_{in} t}\, {}_2F_1\left(a,b;c;-e^t\right) \\ T_k^{out}(t) = & \frac{(4\pi\omega_{out})^{\frac{1}{2}}}{(4\pi\omega_{in})} \times \left(e^{-i\omega_{in} t} \frac{\Gamma(c^*)\Gamma(a^*-b^*)}{\Gamma(a^*)\Gamma(c^*-b^*)}\, {}_2F_1\left(a,b;c;-e^t\right) \right. \\ & \left. - e^{+i\omega_{in} t} \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)}\, {}_2F_1\left(b^*,a^*;c^*;-e^t\right) \right), \end{aligned}$$

such that by taking the limits, the above solutions reduce to the first terms in their counterpart relations (7) and (8).[5] Note that in the above results the pre-factors

[5] It is worth mentioning again that for the second relation, T_k^{out}, the combination has picked up such that the above relation reduces to its asymptotic counterpart in (8).

Fig. 1 The parameter $\alpha_0^2 = 1$ is positive then the plot is not shown for the forbidden region where the ω_{in} is imaginary (in this case $k > 1$ represents forbidden region)

Fig. 2 This plot is for two negative α_0^2 with no forbidden region. The *solid line* presents the spectrum of created particles for $\alpha_0^2 = -1$ and the *dashed line* for $\alpha_0^2 = -3$

guaranty the normalization of the basis. The second Bogolubov coefficient shows the spectrum of the created particles with respect to the energy i.e. the $|\beta_k^2|$ is the number of particles with energy k, is in the following form

$$N_k = \beta_k^2 = \frac{\sinh^2(\pi(\omega_{out} - \omega_{in}))}{\sinh(2\pi\omega_{in}) \times \sinh(2\pi\omega_{out})}. \tag{10}$$

The above result can be deduced by some algebra (see the Appendix) from definition of Bogolubov coefficients (9) that has been used for relations (10). The figures show the above number density spectrum for different values of α_0^2. Now let us examine the behavior of the result in well known limits. For $\alpha_0^2 = 0$ the expectation is, vanished N_k due to no difference between the early and late times. Obviously since in this case $\omega_{in} = \omega_{out}$ it causes vanishing "$\sinh(\pi(\omega_{in} - \omega_{out}))$" in the numerator of the fraction results in vanished N_k, as expected. Another point is that we have two different choices for α_0^2, a positive one and a negative one. It must be noted that for the positive one, we must restrict the plots to an upper-bound for k's since for the greater values of k, ω_{in} becomes an imaginary number that makes no sense in our conclusions. But for the negative values of α_0^2 the spectrum is valid for all the k's. The Fig. 1 shows the behavior of the N_k with respect to k for a positive α_0^2. In this case as mentioned before, there is a forbidden region due to non-real values of energy ω_{in}. The second figure presents the spectrum of the number of created particles with respect to their energy but for two negative values of α_0^2 (Fig. 2). It is obvious from the plots that in this case, the maximum of the spectrum is changed by different values of Lorentz violation parameters. The energy, that has the maximum number of created particle, decreasing due to increasing of the absolute value of α_0^2. In this case for smaller value of α_0^2 the plot falls and for $\alpha_0^2 \to 0$ it coincides to $N_k = 0$ for all k's, as one expected.

3 Conclusions

We have studied a toy model to describe the effects of Lorentz violation in particle creation [28; 29] in the presence of a time dependent deformed dispersion relation. In this paper, we have shown that the existence of Lorentz violation at very early times' vacuum can result in the existence of particles but in the absence of any Lorentz violation. In the other words, we have combined two legitimate individual ingredients, modification to the dispersion relation [11; 12; 13] and time variation of a fundamental parameter [32; 33], to peruse any interesting features. According to above discussions and the close relation between Lorentz violation and quantum gravity a suggestion can be introduced: quantum gravitational vacuum causes a particle-filled state in classical gravity.[6] This final state is obtainable by making a semi-classical limiting process. It means that particle-filled states are obtained by taking semi-classical limits of the quantum gravitational vacuum. In other words, particles are only some excitations of quantum gravitational vacuum, that is not an obsolete idea in theoretical physics [34; 35]. In this viewpoint, the particles are the evidence of the past existence of Lorentz violation [or, quantum structure of the geometry (gravity)]. Since our model is a toy model and far from complete quantum gravity theory, to see the correctness of our proposal in this paper we should wait until the emergence of a full quantum gravity theory. This can also shed light on the origin of the time-dependence of Lorentz violation parameter that is an ambiguity in our model. Finally, it is again worth mentioning that in our model only the Lorentz violation is picked up, presenting the quantum gravity regime.[7] To present a more accurate discussion we need to study all the quantum gravitational effects in the presence of the curved background and its dynamics.

Finally, it is proper to consider that the origin of the present particles and even the large scale structures can be explained by proposing an inflationary era in the early universe [36]. May be the inflationary scenario would make the results of this paper irrelevant. But it is important to note that the inflationary era is not the first stage after the big-bang where quantum gravitational effects are dominant [37; 38]. Our conclusion is that the created particles by Lorentz violation are just before the inflationary era i.e. in contrast to the standard approach in inflationary models the initial state for this era is not vacuum state. In this viewpoint [37; 38] the natural question will be how one can find any traces of these particles after inflationary era in the last scattering surface or even at present time? Considerably, since the old.[8] particles are created by a geometric effect, they should show themselves by a geometric feature e.g. gravitational waves [39; 40] This way of thinking is still open and in this work we did not consider this problem e.g. the effects on CMB temperature fluctuations etc.

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[6] This suggestion is not provable until reaching a full theory of quantum gravity. But the current results, in the context of a toy model, make the validity of this claim possible.

[7] As mentioned before, this choice makes questionable the validity of the concluding remarks in all the quantum gravity regime. But at least, it sheds some lights on the behavior of this unaccessible regime by analytical results.

[8] Before inflation.

Appendix

In the body of the paper we have used alternatively the following identities among the hypergeometric functions

$$F(a,b;c;z)=F(b,a;c;z) = \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)}(-z)^{-a}F\left(a,1-c+a;1-b+a;\frac{1}{z}\right) + \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)}(-z)^{-b}F\left(b,1-c+b;1-a+b;\frac{1}{z}\right)$$

and the following properties for gamma functions

$$\Gamma(1+iy) = iy\Gamma(iy)$$
$$\Gamma(iy)\Gamma(-iy) = \frac{\pi}{y\sinh(\pi y)}$$

where y is a real number.

References

1. M.B. Green J.H. Schwarz E. Witten (1988) Superstring theory, vol. 1, 2 Cambridge University Press Cambridge
2. J. Polchinski (1998) String theory, vol. 1, 2 Cambridge University Press New York
3. C. Rovelli (2004) Quantum gravity Cambridge University Press New York
4. T. Thiemann (2007) Modern canonical quantum general relativity Cambridge University Press New York
5. Amelino-Camelia, G.: Quantum gravity phenomenology (2008). 0806.0339 [gr-qc]
6. A. Connes (1994) Noncommutative geometry Academic PressNew York
7. Connes, A.: A short survey of noncommutative geometry. J. Math. Phys. **41**, 3832 (2000). hep-th/0003006
8. Majid, S.: Meaning of noncommutative geometry and the planck-scale quantum group. Lect. Notes Phys. **541**, 227 (2000). hep-th/0006166
9. V.A. Kostelecky S. Samuel (1989) Spontaneous breaking of Lorentz symmetry in string theory *Phys. Rev. D* **39** 683
10. Alfaro, J., Morales-Tecotl, H., Urratia, L.: Loop quantum gravity and light propagation. Phys. Rev. **D65**, 103509 (2002). hep-th/0108061
11. Jacobson, T., Mattingly, D.: Generally covariant model of a scalar field with high frequency dispersion and the cosmological horizon problem. Phys. Rev. **D63**, 041502 (2001). hep-th/0009052
12. Amelino-Camelia, G., Ellis, J., Mavromatos, N.E., Nanopoulos, D.V., Sarkar, S.: Potential sensitivity of gamma-ray burster observations to wave dispersion in vacuo. Nature **393**, 763 (1998). astro-ph/9712103

13. Amelino-Camelia, G.: Relativity in space-times with short-distance structure governed by an observer-independent (Planckian) length scale. Int. J. Mod. Phys. **D11**, 35 (2002). gr-qc/0012051
14. Amelino-Camelia, G., Smolin, L.: Prospects for constraining quantum gravity dispersion with near term observations. Phys. Rev. **D80**, 084017 (2009). 0906.3731 [hep-th]
15. Amelino-Camelia, G., Smolin, L., Starodubtsev, A.: Quantum symmetry, the cosmological constant and Planck scale phenomenology. Class. Quant. Grav. **21**, 3095 (2004). hep-th/0306134.
16. 't Hooft, G.: Quantization of point particles in 2+1 dimensional gravity and space-time discreteness. Class. Quant. Grav. **13**, 1023 (1996). gr-qc/9601014
17. Kifune, T.: Invariance violation extends the cosmic ray horizon? Astrophys. J. Lett. **L21**, 518 (1999). astro-ph/9904164
18. Aloisio, R., Blasi, P., Ghia, P.L., Grillo, A.F.: Probing the structure of space-time with cosmic rays. Phys. Rev. **D62**, 053010 (2000). astro-ph/0001258
19. Amelino-Camelia, G., Piran, T.: Planck-scale deformation of Lorentz symmetry as a solution to the UHECR and the TeV-γ paradoxes. Phys. Rev. **D64**, 036005 (2001). astro-ph/0008107
20. Amelino-Camelia, G.: Quantum theory's last challenge. Nature **408**, 661 (2000). gr-qc/0012049
21. Protheroe, R.J., Meyer, H.: An infrared background–TeV gamma-ray crisis? Phys. Lett. **B493**, 1 (2000). astro-ph/0005349
22. Vilenkin, A.: Approaches to quantum cosmology. Phys. Rev. **D50**, 2581 (1994). gr-qc/9403010 and references therein
23. Pervushin, V.N., Skokov, V.V., Reichel, A.V., Smolyansky, S.A., Prozorkevich, A.V.: The kinetic description of vacuum particle creation in the oscillator representation. Int. J. Mod. Phys. **A20**, 5689 (2005). hep-th/0307200
24. Filatov, A.V., Prozorkevich, A.V., Smolyansky, S.A., Toneev, V.D.: Inertial mechanism: dynamical mass as a source of particle creation. Phys. Part. Nucl. **39**, 886 (2008). 0710.0233 [hep-ph]
25. N.D. Birrell P.C.W. Davies (1982) Quantum fields in curved space Cambridge University Press Cambridge
26. Carlip, S.: Quantum gravity: a progress report. Rept. Prog. Phys. **64**, 885 (2001). gr-qc/0108040
27. Rovelli, C.: Graviton propagator from background-independent quantum gravity. Phys. Rev. Lett. **97**, 151301 (2006). gr-qc/0508124
28. Khajeh, E., Khosravi, N., Salehi, H.: Cosmological particle creation in the presence of Lorentz violation. Phys. Lett. **B652**, 217 (2007). 0707.3247 [hep-ph]
29. Rashidi, R., Khosravi, N., Khajeh, E., Salehi, H.: Unruh's detector in the presence of Lorentz symmetry breaking. Astrophys Space Sci. **310**, 333 (2007). 0706.2767 [hep-th]
30. R. Rashidi H. Salehi (2007) On the non-existence of micro-causal sector of Lorentz non-invariant quantum field theory on a continuum *Phys. Lett.*

B **655** 280
31. M. Abramowitz I.A. Stegun eds (1965) Handbook of mathematical functions Dover publications New York
32. Fritzsch, H.: The fundamental constants in physics. Phys. Usp. **52**, 359 (2009). 0902.2989 [hep-ph]
33. C. Brans R.H. Dicke (1961) Mach's principle and a relativistic theory of gravitation *Phys. Rev.* **124** 925
34. Bilson-Thompson, S.O., Markopoulou, F., Smolin, L.: Quantum gravity and the standard model. Class. Quant. Grav. **24**, 3975 (2007). hep-th/0603022
35. Smolin, L., Wan, Y.: Propagation and interaction of chiral states in quantum gravity. Nucl. Phys. **B796**, 331 (2008). 0710.1548 [hep-th]
36. S. Weinberg (2008) Cosmology Oxford University Press New York
37. Easther, R., Greene, B.R., Kinney, W.H., Shiu, G.: Imprints of short distance physics on inflationary cosmology. Phys. Rev. **D67**, 063508 (2003). hep-th/0110226
38. Brandenberger, R.H.: String gas cosmology (2008). 0808.0746 [hep-th]
39. Amelino-Camelia, G.: An interferometric gravitational wave detector as a quantum-gravity apparatus. Nature **398**, 216 (1999). gr-qc/9808029
40. Amelino-Camelia, G.: Gravity-wave interferometers as probes of a low-energy effective quantum gravity. Phys. Rev. **D62**, 024015 (2000). gr-qc/9903080