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VERTICAL QUADRUPOLE ELECTRIC FIELD SYSTEMATICS AND ITS MITIGATION IN THE PROTON-EDM RING

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Abstract

The physics and the ring design of the proton storage ring Electric Dipole Moment (sr-pEDM) experiment have been discussed in detail in reference cited below in the introduction, including the symmetries used to cancel the first-order systematic effects. In this work we discuss the symmetries for the vertical quadrupole electric field systematic effect.

INTRODUCTION

The physics and the ring design of the proton storage ring Electric Dipole Moment (sr-pEDM) experiment have been discussed in detail in ref. [1], including the symmetries used to cancel the first-order systematic effects. In this work we discuss the symmetries for the second-order systematic effects. We discuss the effect of the unwanted vertical electric quadrupole field that account for a false EDM signal and the ring symmetries that would be a cure for this systematic effect. Briefly, the story begins with measurements of the anomalous magnetic moment of the muon in the 1960-70s at CERN. The CERN I [2] experiment was limited by statistics. The breakthrough was to go to a magnetic storage ring. The CERN II Experiment was then limited by the systematics of knowing the magnetic field seen by the muons passing through the quadrupole magnets.

The particle's spin precession rate of the longitudinal component of the spin, i.e., the helicity, due to the magnetic dipole moment $eg\vec{S}/2m$ is:

$$\frac{d(\hat{\beta}\cdot\vec{S})}{dt} = -\frac{e}{m}\vec{S}\cdot\left[\left(\frac{g-2}{2}\right)\hat{\beta}\times\vec{B}+\left(\frac{g\beta}{2}-\frac{1}{\beta}\right)\frac{\vec{E}}{c}\right] \quad (1)$$

The CERN III [2, 3] experiment used a bending magnetic field with electric quadrupoles for focusing at the "magic energy", given by $\beta^2 = 2/g$ – see Eq. (1) electric field term. The CERN III experiment, and BNL E821 [4], were limited by statistics, not systematics. FNAL E989 [5] is anticipated to have equal statistical and systematic errors. The storage ring/magic energy breakthrough gained a factor of 2×10^3 in systematic error.

BNL E821 set a "parasitic" limit on the EDM of the muon: $d_\mu < 1.9 \times 10^{-19} e\cdot cm$ [6]. FNAL E989 is expected to improve this result by an order of magnitude. The statistical and systematic errors will then be roughly equal. The dominant systematic effect is due to radial magnetic fields. For the pEDM experiment, we use a storage ring at the proton magic energy with electric bending and magnetic focusing, which gives a negligible radial magnetic field systematic effect.

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STORAGE RING SPIN DYNAMICS EQUATIONS

The spin $\vec{S}$ precession rate for a particle at rest in the presence of magnetic $\vec{B}$ and electric $\vec{E}$ fields is given as

$$\frac{d\vec{S}}{dt} = \vec{\mu}\times\vec{B} + \vec{d}\times\vec{E} \quad (2)$$

where magnetic and electric dipole moments are defined as $\vec{\mu} = (gq/2m_p)\vec{S}$ and $\vec{d} = (\eta q/2m_p c)\vec{S}$, respectively.

When we boost the proton into the lab frame, if we set $\vec{B} = 0$, choose the particle momentum as the magic momentum (we define $G = (g-2)/2$, as the proton magnetic anomaly, and set $G = 1/(\gamma^2 - 1)$, known as the spin frozen condition) and with horizontal electric field E_x and horizontal $\vec{\beta}$, we would have

$$\frac{dS_y}{dt} = \frac{\eta q}{2m_p}E_x S_s \quad (3)$$

where y is vertical from the ideal orbit, x is radial from the ideal orbit, and longitudinal is along the ideal orbit.

The measurable EDM signal in these setting-ups would be along the vertical direction.

SYMMETRIC RING DESIGN

With the present proton electric dipole moment (pEDM) storage ring design, we would have

- Stored beams going both clockwise (CW) and counter-clockwise (CCW) in the ring.
- Switching the polarity of the quadrupole magnets every fill.
- 24-fold symmetric lattice.

For an ideal ring, the difference of precession rates for CR beams gives us the true EDM signal,

$$\left(\frac{dS_y}{dt}\right)_{EDM} = \frac{1}{2}\left(\frac{dS_y}{dt}\right)_{CW} - \frac{1}{2}\left(\frac{dS_y}{dt}\right)_{CCW} \quad (4)$$

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For an ideal ring, incorporated with switching the polarity of the quadrupole magnets, the EDM signal is given as,

$$\left(\frac{dS_y}{dt}\right)_{EDM} = \frac{1}{2}\left[\frac{1}{2}\left(\frac{dS_y}{dt}\right)_{CW} - \frac{1}{2}\left(\frac{dS_y}{dt}\right)_{CCW}\right]_+ + \frac{1}{2}\left[\frac{1}{2}\left(\frac{dS_y}{dt}\right)_{CW} - \frac{1}{2}\left(\frac{dS_y}{dt}\right)_{CCW}\right]_- \quad (5)$$

Note that we denote the positive helicity beams with + sign, the negative helicity beams with − sign.

ELECTRIC QUADRUPOLE FIELD SYSTEMATICS

For an all-magnetic ring with stray magnetic fields, read the earth's magnetic field, the closed orbit is distorted such that the average radial magnetic field on the closed orbit around the ring is zero: $\langle B_{x,co}\rangle = 0$. When $B_x \neq 0$, it distorts the ideal orbit. The general equation for the closed orbit distortion around the ring is given by:

$$y_{co} = \eta_y(\phi)\sqrt{\beta_y} \quad (6)$$

The β_y is the vertical beta function, which we approximate by

$$\beta_y = \beta_0\,(1 \pm 0.5\cos(24\phi)) \quad (7)$$

showing explicitly the 24-fold symmetry of the ring. $F(\phi)$ is the un-wanted radial magnetic field perturbation. We can expand this un-wanted radial magnetic field perturbation in a Fourier series going around the ring:

$$F(\phi) = \sum_{N=0}^{\infty} F_{M,c,N}\cos(N\phi) + F_{M,s,N}\sin(N\phi) \quad (8)$$

For the closed orbit solution to eq. (6):

$$\begin{aligned} y_{co,c,N}(\phi) &= a_{co,c,N}\cos(N\phi) + b_{co,c,N}\cos(L\phi) \\ &\quad + c_{co,c,N}\cos(M\phi) + \ldots \end{aligned} \quad (9)$$

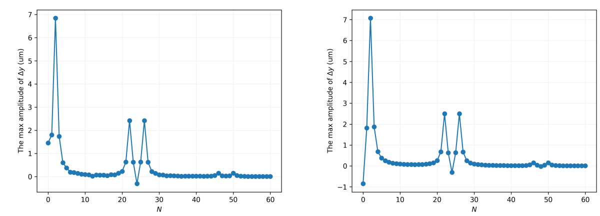
The perturbation $F(\phi)$ does not have L and M Fourier harmonics, but the closed orbit distortion does. This comes from eq.(9) via the trigonometry identity: $2\cos(A)\cos(B) = \cos(A+B) + \cos(A-B)$, etc.

Adding a little bit of unwanted electric quadrupole potential $E_y = k_E y$ gives a non-zero stray radial magnetic field on the closed orbit around the ring, and thus it gives unwanted vertical spin precession:

$$\begin{aligned} \frac{dS_y}{dt} &= \frac{q}{m_p}(G + \frac{1}{\gamma})B_x S_s \\ &= -\frac{q}{m_p}(G + \frac{1}{\gamma})\frac{\langle k_E y_{co}\rangle S_s}{v} \end{aligned} \quad (10)$$

For an ideal ring, we can calculate analytically:

$$\langle B_{x,co}\rangle = -\frac{\langle E_y\rangle}{v} = -\frac{\langle k_E y_{co}\rangle}{v} \quad (11)$$



(a) CW (b) CCW

Figure 1: The amplitude of the closed orbit distortion vs B_x = 1 nT field N harmonic around the ring azimuth, the betatron tune is $\nu_y = 2.245$, CW stands for Clockwise rotating beam, and CCW stands for Counter-Clockwise rotating beam. The three largest COD harmonics of closed orbit distortion happens at $N = 2$, $N = 24 \pm 2$ and $N = 48 \pm 2$ for both CW and CCW.

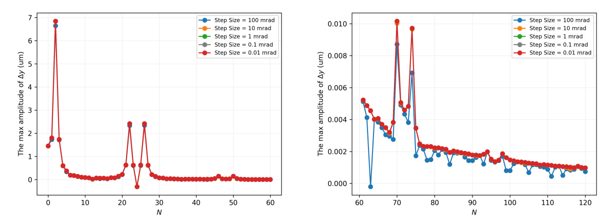


Figure 2: The amplitude of the closed orbit distortion vs B_x = 1 nT field N harmonic around the ring azimuth for various integration step size. The CW rotating beam is shown as an example.

We can expand this unwanted electric quadrupole perturbations in a Fourier series going around the ring

$$k_E(\phi) = \sum_{N=0}^{\infty} k_{E,c,N}\cos(N\phi) + k_{E,s,N}\sin(N\phi) \quad (12)$$

BEAM-SPIN DYNAMICS SYMMETRIES

The $k_{E,c,N=2}$ term, for example, gets coupled with the closed orbit terms with Fourier harmonics 2, 22, and 26:

$$\begin{aligned} &k_{E,c,N=2}\cos(2\phi)[a_{co,c,2}\cos(2\phi) + b_{co,c,22}\cos(2\phi) \\ &+ c_{co,c,26}\cos(2\phi)] + \mathit{sin\ terms} \\ &= k_{E,c,N=2}\cos^2(2\phi)[a_{co,c,2} + b_{co,c,22} + c_{co,c,26}] \\ &+ \mathit{sin\ terms} \end{aligned} \quad (13)$$

The bracketed terms are illustrated in Fig. 3. The EDM-like background is due to the product of the electric quad multipole and the closed orbit distortion. The closed orbit distortion with a $\cos(2\phi)$ term in the top panel has opposite sign for the CW/CCW beams – an EDM background. The $\cos(22\phi)$ and $\cos(26\phi)$ terms in the bottom panel have opposite sign for the CW/CCW beams – an EDM background. The top panel $L = 26$ and $M = 22$ Fourier harmonics, and the bottom panel $L = 22$ and $M = 2$ Fourier harmonic are not backgrounds to the EDM signal.

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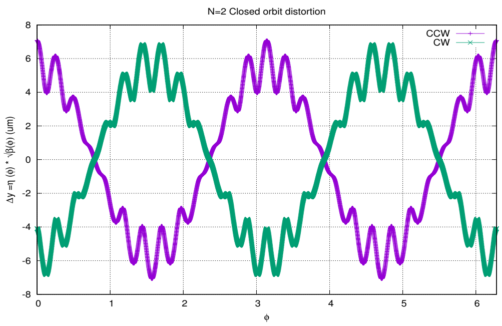
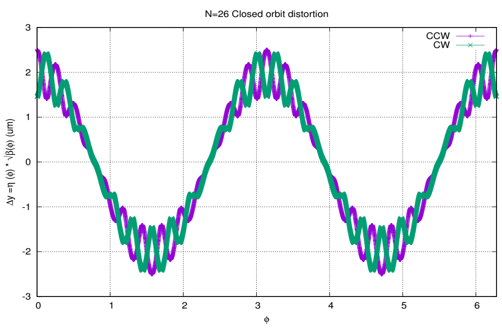


(a) Closed orbit distortion for $N = 2$, $L = 26$, $M = 22$



(b) Closed orbit distortion for $N = 26$, $L = 22$, $M = 2$

Figure 3: CW/CCW beam separation. Both the 26 and 22 harmonics are clearly seen in the four crossings with negative interference at $\phi = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$.

Next we discuss the symmetries associated with flipping the quadrupole magnet currents every fill – see Figs. 4 and 5. The CW/CCW with flipped quadrupole magnet current Fourier terms have opposite symmetry compared to the EDM signal – see the a.) and d.) panels. Likewise for the CCW/CW with flipped quadrupole magnet current Fourier terms – see the b.) and c.) panels.

For the EDM backgrounds we discussed above, we have

$$\left(\frac{dS_y}{dt}\right)_{EDMB1} = \frac{1}{2}\left(\frac{dS_y}{dt}\right)_{CW+} - \frac{1}{2}\left(\frac{dS_y}{dt}\right)_{CCW-} = 0 \quad (14a)$$

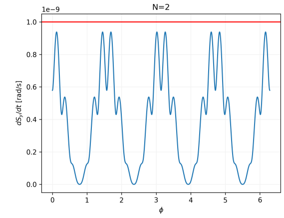
$$\left(\frac{dS_y}{dt}\right)_{EDMB2} = \frac{1}{2}\left(\frac{dS_y}{dt}\right)_{CCW+} - \frac{1}{2}\left(\frac{dS_y}{dt}\right)_{CW-} = 0 \quad (14b)$$

CONCLUSION

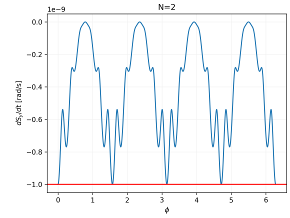
In Figs. 1 and 2, the amplitude of the closed orbit distortion vs B_x = 1 nT field N harmonic around the ring azimuth for various integration step size has been shown, note that for higher N, start from integration step size = 1 mrad, the numerical integration result convergences. In this work, we choose the integration step size as 0.1 mrad.

The first order of electric field E_y systematic effect have been characterized and discussed in ref. [1]. The ref. [1] spin-based alignment procedure for this second order systematic effect is:

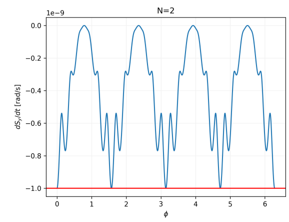
The presence of E^{quad} can be monitored by controlling B_x. The combination of E^{quad} and B_x produces nonzero vertical spin precession rate dS_y/dt. B_x could be made large



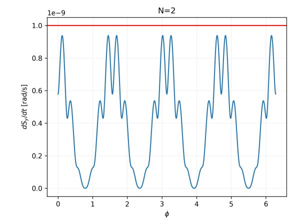
(a) CW+



(b) CCW+

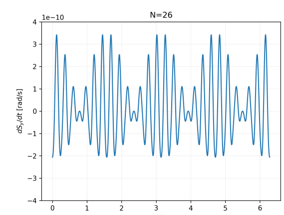


(c) CW-

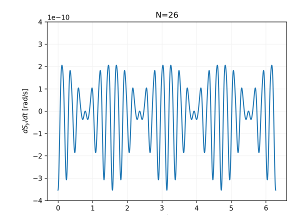


(d) CCW-

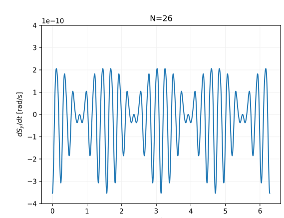
Figure 4: CW+/CW-/CCW+/CCW- vertical spin precession rate around the ring azimuth ϕ for N=2, $\pm$ stands for the positive/negative polarity of the quadrupole magnet current, $k_{2,E} \approx 1.021 \times 10^{-4}\ Vm^{-2}$.



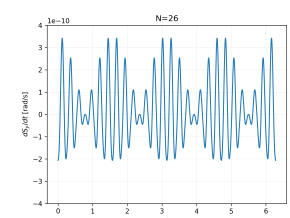
(a) CW+



(b) CCW+



(c) CW-



(d) CCW-

Figure 5: CW+/CW-/CCW+/CCW- vertical spin precession rate around the ring azimuth ϕ for N=26, $\pm$ stands for the positive/negative polarity of the quadrupole magnet current.

on purpose, for example, by controlling dipole correctors. Being able to freely control B_x and all of its harmonics lets us selectively (for each N harmonic) amplify and then reduce the effect of initially unknown E^{quad}.

In principle, the spin-based alignment procedure is enough by itself. However, the new symmetry we discovered for this vertical electric quadrupole systematic effect will provide a valuable check, and will be useful in the future if we are considering a more precise experiment.

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REFERENCES

[1] Z. Omarov *et al.*, "Comprehensive symmetric-hybrid ring design for a proton EDM experiment at below $10^{-29} e \cdot cm$", *Phys. Rev. D*, vol. 105, p. 032001, Feb. 2022. doi:10.1103/PhysRevD.105.032001

[2] F. Combley, F. Farley, and E. Picasso, "The CERN muon (g-2) experiments", *Physics Reports*, vol. 68, pp. 93-119, Jan. 1981. doi:10.1016/0370-1573(81)90028-4

[3] J. Bailey *et al.*, "Final report on the CERN muon storage ring including the anomalous magnetic moment and the electric dipole moment of the muon, and a direct test of relativistic time dilation", *Nucl. Phys. B*, vol. 150, pp. 1-75, Aug. 1979. doi:10.1016/0550-3213(79)90292-X

[4] G. W. Bennett *et al.*, "Final report of the E821 muon anomalous magnetic moment measurement at BNL", *Phys. Rev. D*, vol. 73, p. 072003, Apr. 2006. doi:10.1103/PhysRevD.73.072003

[5] B. Abi *et al.*, "Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm", *Phys. Rev. Lett.*, vol. 126, p. 141801, Apr. 2021. doi:10.1103/PhysRevLett.126.141801

[6] G. W. Bennett *et al.*, "Improved limit on the muon electric dipole moment", *Phys. Rev. D*, vol. 80, p. 052008, Sept. 2009. doi:10.1103/PhysRevD.80.052008